4

Monday, December 4, 2017 8:19 PM

$$-D\frac{d^2\phi(x)}{dx^2} + \Sigma_a \phi(x) = S(x)$$

B.C. = 0

$S(x) = S_0$ for $x \in [-a, a]$

$$m^2 + (0)m - \frac{\Sigma_a}{D} = \frac{-S_0}{D} = 0$$

recall $\frac{\Sigma_a}{D} = L^2$

$$(m + L)(m - L) = 0$$

so

$$\phi_c(x) = A e^{-x/L} + B e^{x/L}$$

now

$$\frac{d^2\phi}{dx^2} + (0)\frac{d\phi}{dx} - L^2\phi = -\frac{S_0}{D}$$

guess $\phi_p = \frac{A S_0}{\Sigma_a}$

skipping $\frac{d^2}{dx^2}\phi$

$$-L^2\left(\frac{A S_0}{\Sigma_a}\right) = \frac{-S_0}{D}$$

$$A \cancel{L^2} = \cancel{\frac{\Sigma_a}{D}}$$

$$A = 1$$

Thus

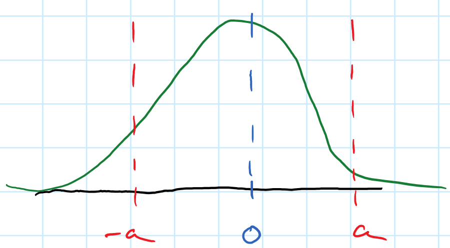
$$\phi_p = \frac{S_0}{\Sigma_a}$$

Finally

$$\phi(x) = \phi_c(x) + \phi_p(x)$$

$$\phi(x) = A e^{-x/L} + B e^{x/L} + \frac{S_0}{\Sigma_a}$$

Solve A and B note



B.C. $\phi(\pm a) = 0$

thus $S(x) = S_0$

see symmetry

note max

$$\frac{d\phi}{dx} = 0 \text{ at } x = 0$$

Solve at $X = 0$

$$\frac{d\phi(x)}{dx} = \frac{-A}{L} e^{-x/L} + \frac{B}{L} e^{x/L}$$

$$0 = \frac{-A}{L} + \frac{B}{L}$$

so

$$A = B$$

Solve at $X = a$

$$\phi(a) = 0 = A e^{-a/L} + A e^{a/L} + \frac{S_o}{\Sigma_a}$$

$$A = \frac{-\Sigma_a}{S_o} \left(e^{-a/L} + e^{a/L} \right)^{-1}$$

Going Back

$$\phi(x) = A\left(e^{-x/L} + e^{x/L}\right) + S_o/\Sigma_a$$

$$\boxed{\phi(x) = \frac{-\Sigma_a}{S_o} \left(\frac{e^{-x/L} + e^{x/L}}{e^{-a/L} + e^{a/L}} \right) + \frac{S_o}{\Sigma_a}}$$

---

5) Numerically Solve

$$\phi(x) = \frac{-\Sigma_a}{S_o} \left(\frac{e^{-x/L} + e^{x/L}}{e^{-a/L} + e^{a/L}} \right) + \frac{S_o}{\Sigma_a}$$

$a = 4\,cm$ $\quad \Sigma_a = .2\,cm^{-1}$

$D = 1\,cm$ $\quad S = 8\,\frac{n}{cm^3 \cdot s}$

$h = .1\,cm$