NENG 685 Fall 2017
HW 5 Due Dec. 4, 2017

Name: Torzilli

On homework:

- Well organized and documented work scores better. If I cannot figure out what is going on, then I am less likely to "intuit" what you intended, and the score will be reflective of this fact.
- If you work with anyone else, document what you worked on together.
- Show your work.
- Always clearly label plots (axis labels, a title, and a legend if applicable).
- Homework should be done "by hand" (i.e. not with a numerical program such as MATLAB, Python, or Wolfram Alpha) unless otherwise specified. You may use a numerical program to check your work.
- If you use a numerical program to solve a problem, submit the associated code, input, and output.
- ***I should not have to run your code to see your answers.*** The attached code is an additional form of feedback for me and a method to give partial credit. If you want full credit, then include the outputs (plots, tables, answers, etc.) in your write-up.

Do not write in the table to the right.

Problem	Points	Score
1	5	3
2	25	17
3	20	16
4	10	7
5	20	14
6	15	13
Total:	95	75

+5

Problem 1 (5 points) *discuss the significance of the spectral radius for the iterative solution of*

A~x =~b, including how it is used to determine convergence and how it is related to the rate of convergence.

The spectral radius determines the speed of convergence and if it is smaller than 1 it converges very slowly. So the larger the spectral radius the faster the convergence.

-2pts: <1 required for convergence; smaller is faster

Problem 2 Print the solution vector from each method converged to an absolute tolerance of 10^{-6}.

-2pts: No run instructions found

Jacobi:

Jacobi iterations: 25

Jacobi Solution Vector

[[61.11105884]

[83.33323925]

[88.88878435]

[83.33323925]

[61.11105884]]

These are relative, not absolute errors

Tolerance met at: 7.92128941434e-07

Gauss Seidel:

GSI iterations: 23

GSI Solution Vector

[[59.99996436]

[79.99994233]

[79.99994233]

[79.99994233]

[59.99996436]]

Tolerance met at: 5.88096677098e-07

-3pts: Looks like indexing error; Converged solutions are not correct (compare to analytic or Jacobi)

SOR:

SOR iterations: 19

SOR Solution Vector

[[59.99996898]

[79.99994956]

[79.99994956]

[79.99994956]

[59.99996898]]

Tolerance met at: 6.76783457974e-07

-3pts: Looks like indexing error; Converged solutions are not correct (compare to analytic or Jacobi)

Problem 3

Part A (10 points)

Iterations:

************ 1e-06 ***********

Jacobi iterations: 25

Jacobi Solution Vector

[[61.11105884]

[83.33323925]

[88.88878435]

[83.33323925]

[61.11105884]]

Tolerance met at: [9.40839318e-07]

GSI iterations: 23

GSI Solution Vector

[[59.99996436]

[79.99994233]

[79.99994233]

[79.99994233]

[59.99996436]]

Tolerance met at: [6.15658431e-07]

SOR iterations: 19

SOR Solution Vector

[[59.99996898]

[79.99994956]

[79.99994956]

[79.99994956]

[59.99996898]]

Tolerance met at: [7.06217714e-07]

************ 1e-08 ***********

GS and SOR errors follow from above

Jacobi iterations: 34	GSI iterations: 30	SOR iterations: 25
Jacobi Solution Vector	GSI Solution Vector	SOR Solution Vector
[[61.11111072]	[[59.99999953]	[[59.99999966]
[83.33333269]	[79.99999923]	[79.99999946]
[88.88888811]	[79.99999923]	[79.99999946]
[83.33333269]	[79.99999923]	[79.99999946]
[61.11111072]]	[59.99999953]]	[59.99999966]]
Tolerance met at: [6.19481960e-09]	Tolerance met at: [8.17342752e-09]	Tolerance met at: [7.65949318e-09]

-2pts: No absolute error results

Compare: Iterations for 10E-6 for convergence versus relative error

-1pts: You've compared relative to relative

Jacobi: Same iteration different breakout convergence

Gauss Seidel: Same iteration different breakout convergence

SOR: Same iteration different breakout convergence

Which method required the least amount of iterations? By comparing the convergence values we can infer that the SOR using the relative error would require the least amount of iterations.

What do you observe about reaching a tighter convergence tolerance? An increased amount of iterations is observed when the break out tolerance is lowered.

Part B (10 points) Perform an experiment to determine !opt for SOR. Explain your procedure and include the results.

I choose to use the brute force method by continuously looping until the difference in my perceived optimal values for *w* is less than the tolerance I set, in this case 1E-6.

I define my optimal *w* by creating two lists that append values as I move throughout all the possible values of *w* (0 to 2). The first list is comprised by the amount of iterations while the second is a list of the current *w;* these list of course end up being the same size. By finding the min of the iteration list and using that index for the *w* list I can determine the perceived optimal value. However to keep the loop going for better precision I reset my bounds of possible *w* by using the two indexed values around my “best” *w* as the new range. To break out of the loop I compare the relative difference of the “best” *w* and the previous “best” *w*

Wopt = 1.17331963004

-1pts: Correct Method, incorrect results. Likely follows from previously noted SOR error.

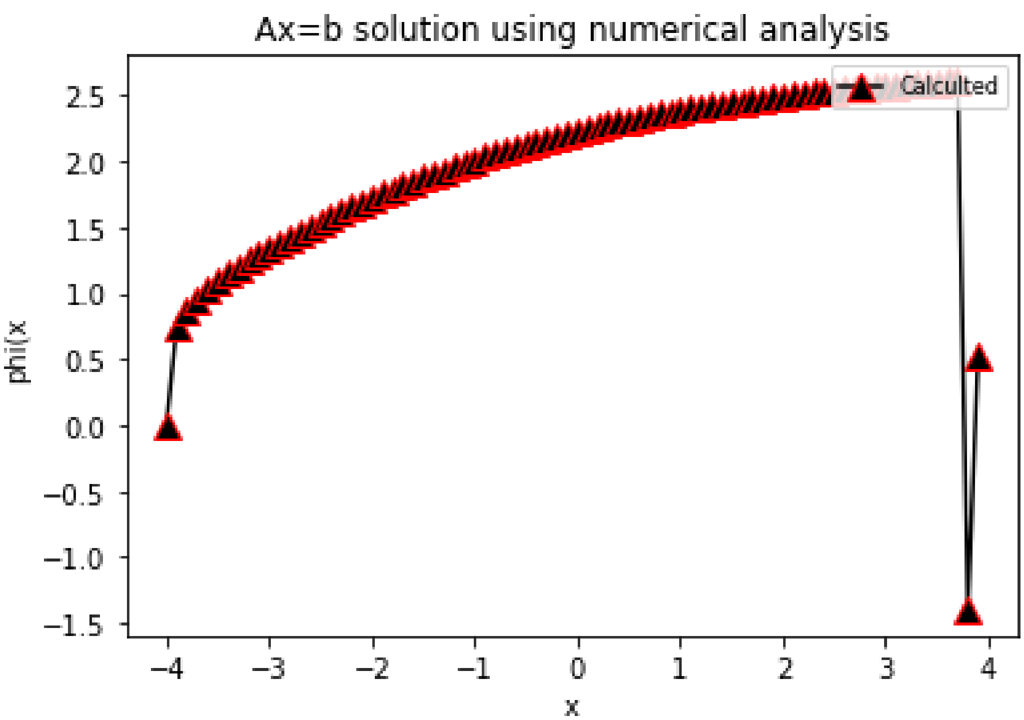
Problem 4 Attached as an analytic solution

Definite integral:

$$\int_{-4}^{4} \frac{1}{8} \times 0.2 \left(\frac{e^{-\frac{x}{\sqrt{0.2}}} + e^{\frac{x}{\sqrt{0.2}}}}{e^{-\frac{4}{\sqrt{0.2}}} + e^{\frac{4}{\sqrt{0.2}}}} + \frac{8}{0.2} \right) dx = 8.02236$$

-3pts: Incorrect result ; incorrect C1

Problem 5



Plot:

Max Error for h = 0.1 is: 14.1509454215

-6pts: Incorrect answer; Incorrect problem setup

Compare answer to solution for Analytic solution in prb 4 using max error

Problem 6

Repeat 5 for h= 1, .5, .1, .05, .01

Max Error for h = 1 is: 333.996611559 Note, these errors are really large!

Errors appear to follow from above

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Monday, December 4, 2017 8:19 PM

$$-D\frac{d^2\phi(x)}{dx^2} + \Sigma_a \phi(x) = S(x)$$

B.C. $= 0$

$S(x) = S_o$ for $x \in [-a, a]$

$$m^2 + (0)m - \frac{\Sigma_a}{D} = \frac{-S_o}{D} = 0$$

recall

$$\frac{\Sigma_a}{D} = L^2$$

$$(m+L)(m-L) = 0$$

so

$$\phi_c(x) = A e^{-x/L} + B e^{x/L}$$

now

$$\frac{d^2\phi}{dx^2} + (0)\frac{d\phi}{dx} - L^2\phi = -\frac{S_o}{D}$$

guess $\phi_p = \frac{A S_o}{\Sigma_a}$

skipping $\frac{d^2}{dx^2}\phi$

$$-L^2\left(\frac{A S_o}{\Sigma_a}\right) = \frac{-S_o}{D}$$

$$A \cancel{L^2} = \cancel{\frac{\Sigma_a}{D}}$$

$$A = 1$$

Thus

$$\phi_p = \frac{S_o}{\Sigma_a}$$

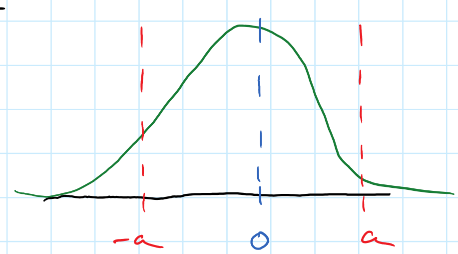
Finally

$$\phi(x) = \phi_c(x) + \phi_p(x)$$

$$\phi(x) = A e^{-x/L} + B e^{x/L} + \frac{S_o}{\Sigma_a}$$

Solve A and B

note



B.C. $\phi(\pm a) = 0$

thus $S(x) = S_o$

see symmetry

note max

$$\frac{d\phi}{dx} = 0 \text{ at } x = 0$$

Solve at $X = 0$

$$\frac{d\phi(x)}{dx} = \frac{-A}{L} e^{-x/L} + \frac{B}{L} e^{x/L}$$

$$0 = \frac{-A}{L} + \frac{B}{L}$$

so

$$A = B$$

Solve at $X = a$

$$\phi(a) = 0 = A e^{-a/L} + A e^{a/L} + \frac{S_0}{\Sigma_a}$$

$$A = \frac{-\Sigma_a}{S_0} \left(e^{-a/L} + e^{a/L} \right)^{-1}$$

Inverted constant

Going Back

$$\phi(x) = A\left(e^{-x/L} + e^{x/L}\right) + S_0/\Sigma_a$$

$$\boxed{\phi(x) = \frac{-\Sigma_a}{S_0} \left(\frac{e^{-x/L} + e^{x/L}}{e^{-a/L} + e^{a/L}} \right) + \frac{S_0}{\Sigma_a}}$$

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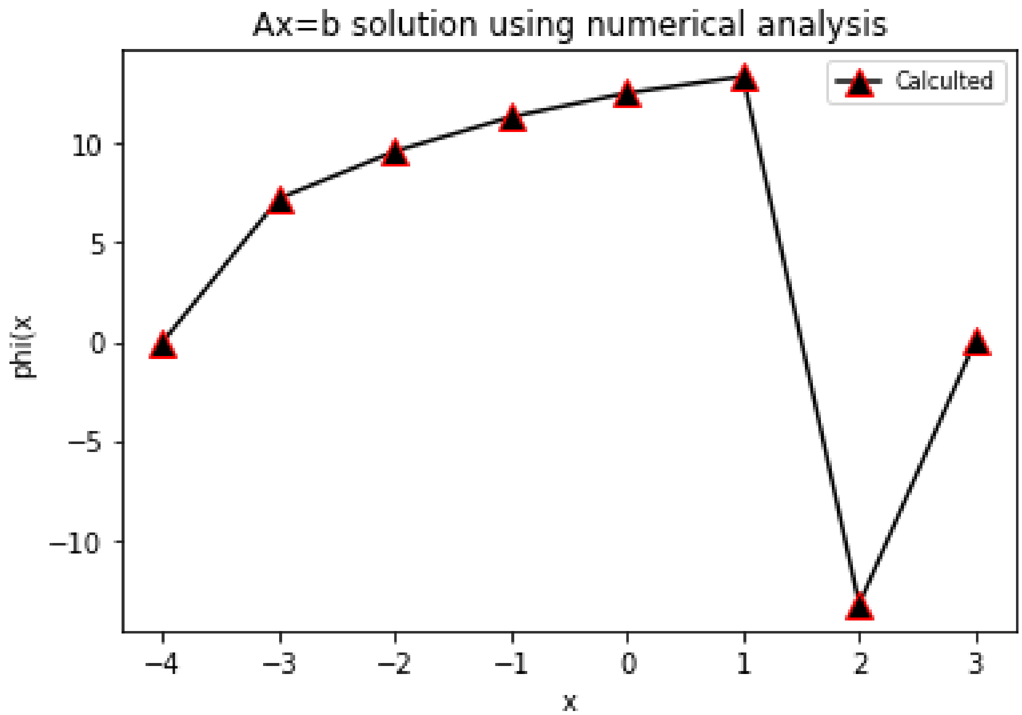
5) Numerically Solve

$$\phi(x) = \frac{-\Sigma_a}{S_0} \left(\frac{e^{-x/L} + e^{x/L}}{e^{-a/L} + e^{a/L}} \right) + \frac{S_0}{\Sigma_a}$$

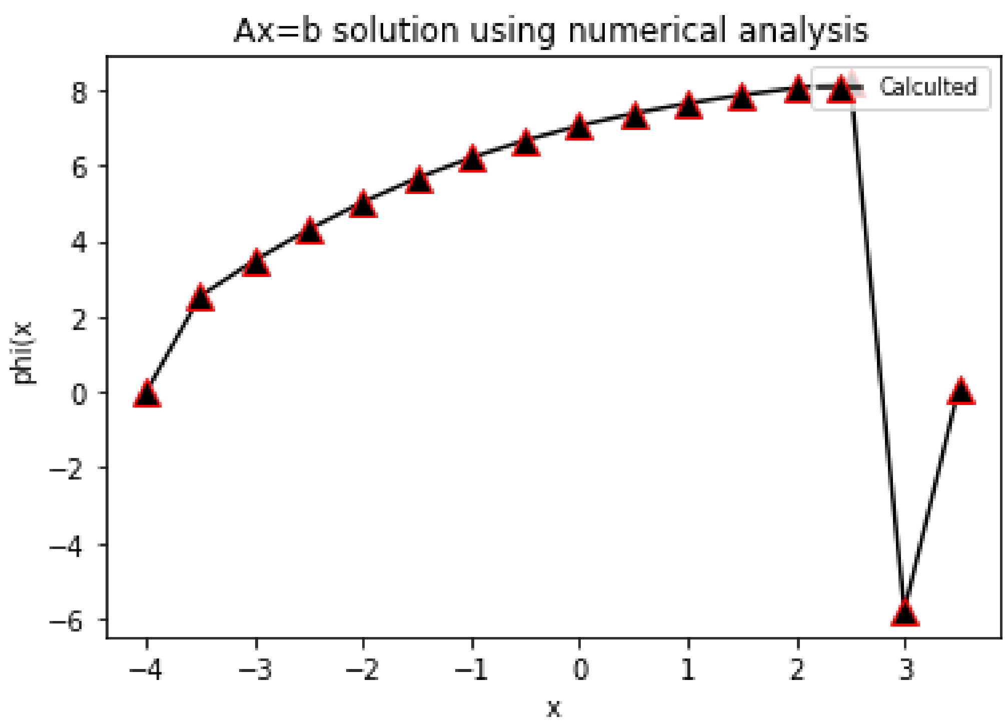
$a = 4\text{ cm}$ $\quad$ $\Sigma_a = .2\text{ cm}^{-1}$

$D = 1\text{ cm}$ $\quad$ $S = 8 \frac{n}{cm^3 \cdot s}$

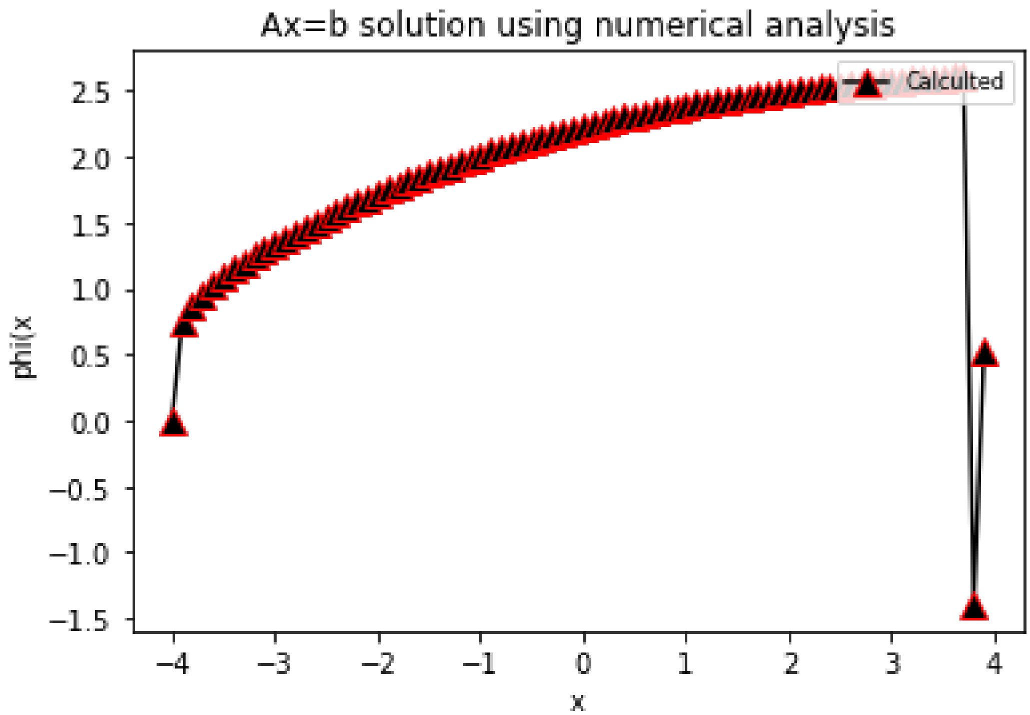
$h = .1\text{ cm}$



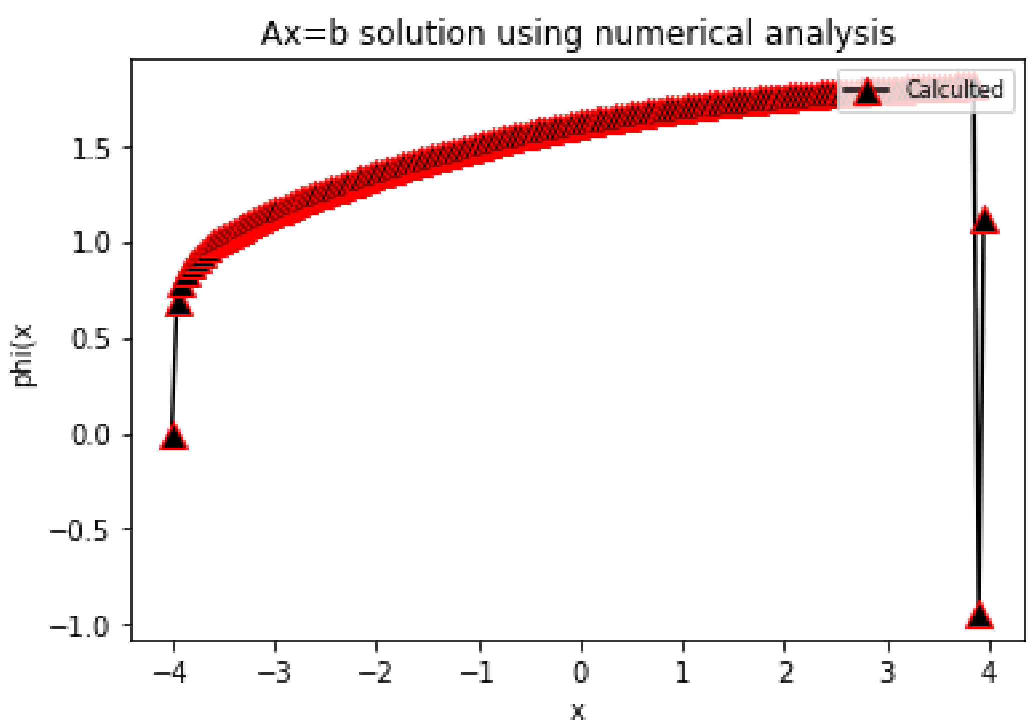
Max Error for h = 0.5 is: 110.827037012



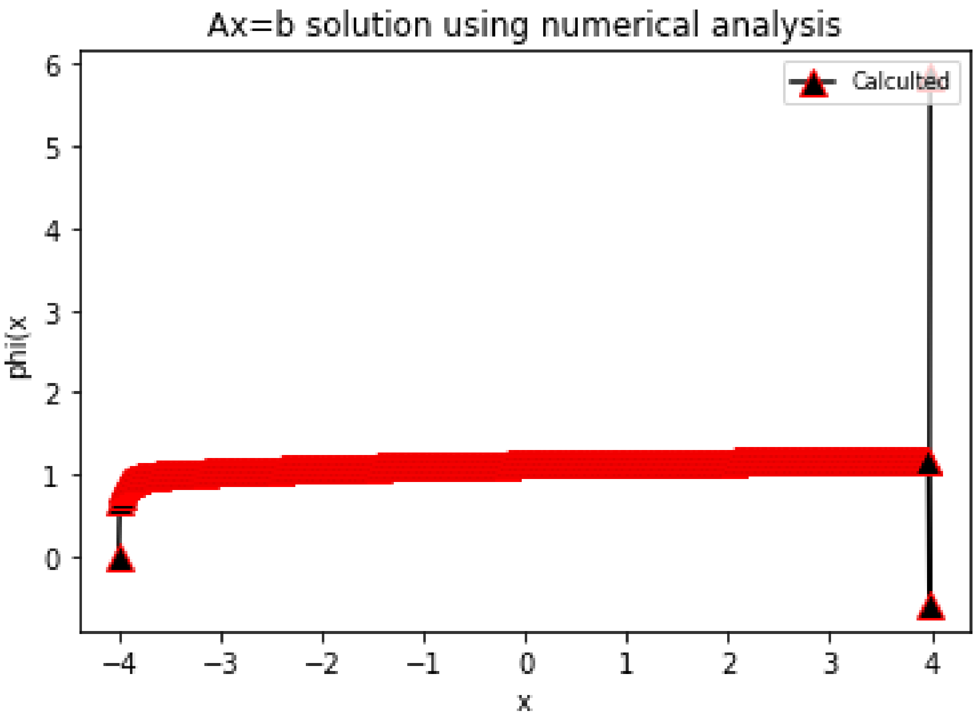
Max Error for h = 0.1 is: 14.1509454215



Max Error for h = 0.05 is: 10.687944599



Max Error for h = 0.01 is: 14.6884176906



What can you conclude about the relationship between the maximum error and the total number of meshes?

Max error goes down as the mesh number goes up.

What is the order convergence?

-2pts