NENG 685 Fall 2017
HW 3 Due Nov. 1, 2017

Name: Robert Torzilli

On homework:

- If you work with anyone else, document what you worked on together.
- Show your work.
- Always clearly label plots (axis labels, a title, and a legend if applicable).
- Homework should be done "by hand" (i.e. not with a numerical program such as MATLAB, Python, or Wolfram Alpha) unless otherwise specified. You may use a numerical program to check your work.
- If you use a numerical program to solve a problem, submit the associated code, input, and output (email submission is fine).

Do not write in the table to the right.

Problem	Points	Score
1	10	
2	10	
3	10	
4	5	
5	5	
6	10	
Total:	50	

1. (10 points) Determine the macroscopic scattering cross section of UO_2 as a function of a generic enrichment factor $\gamma = N_{U-235}/N_{U-238}$ where N is the atom density. Find its value assuming a density of $10\,\mathrm{g/cm}^3$, $\sigma_s^U \simeq 8.9\,\mathrm{b}$ and $\sigma_s^O \simeq 3.75\,\mathrm{b}$, and 5% weight enrichment.

$$\Sigma_{UO_2} = \sigma_{UO_2} N_{UO_2} \qquad N_{UO_2} = \frac{\rho N_A}{M_{UO_2}} \qquad \rho = 10\ \mathrm{g/cm^3}$$

$$N_A = 6.022 \times 10^{23}\ \mathrm{atoms/mol}$$

$$M(U) = \frac{M(U^{235})\, M(U^{238})}{\varepsilon M(U^{235}) + [1-\varepsilon] M(U^{238})}$$

$\uparrow$ enrichment %

$$M(UO_2) = M(U) + M(O_2) \qquad \gamma = \frac{N_{235}}{N_{238}} = \frac{\frac{\rho N_A}{M U_{235}}}{\frac{\rho N_A}{M U_{238}}} = \frac{M(U_{238})}{M(U_{235})}$$

$$\frac{1}{M_{UO_2}} = \frac{1}{M(U) + M(O_2)}$$

$$\frac{1}{M(U)} = \frac{\varepsilon M(U^{235}) + [1-\varepsilon] M(U^{238})}{M(U^{235})\, M(U^{238})}$$

$$= \frac{\varepsilon + (1-\varepsilon)\gamma}{M(U^{238})}$$

$$M(U) = \frac{M(U^{238})}{(1-\varepsilon)\gamma + \varepsilon} \quad \leftarrow \text{to incorporate given } \gamma$$

$$\sigma_{UO_2} = \sigma_U + 2\sigma_O \quad \leftarrow \text{Assuming Homogeneous}$$

So

$$\boxed{\Sigma_{UO_2} = \sigma_{UO_2} \left\{ \frac{\rho N_A}{2M(O) + \frac{M(U^{235})}{(1-\varepsilon)\gamma + \varepsilon}} \right\}}$$

Given

$$\sigma_U = 8.9\ \mathrm{b} \qquad \sigma_{O_2} = 3.75\ \mathrm{b} \qquad \rho = 10\ \mathrm{g/cm^3}$$

$$N_A = 6.022 \times 10^{23} \qquad M(O) = 15.999 \qquad M(U^{235}) = 235.043924$$

$\varepsilon = 5\%$ $\qquad\qquad$ $M(^{238}U) = 238.050941$

$\sigma_{UO_2} = 8.9 + 2(3.75)$

$$\Sigma_{UO_2} = 16.4 \left\{ \frac{10\ (6.022 \times 10^{23})}{2(15.999) + \dfrac{235.043924}{1.01279(.95) + .05}} \right\}$$

$$\Sigma_{UO_2} = 3.73782 \times 10^{23} \left\{ b \ \frac{g}{cm^3} \ \frac{atoms}{mol} \ \frac{1}{amu} \right\}$$

$$\times 10^{-24} \ \cancel{cm^2} \ \frac{\cancel{g}}{cm^{\cancel{3}}} \ \frac{atoms}{\cancel{mol}} \ \frac{\cancel{mol}}{\cancel{g}}$$

$\Sigma_{UO_2} = 2.25093 \times 10^{-1}\ [cm^{-1}]$

$$\boxed{\Sigma_{UO_2} = .225093\ [cm^{-1}]}$$

2. (10 points) Briefly describe what each term in the Transport Equation [Eqn. (2)] physically represents.

$$\underbrace{[\hat{\Omega}\cdot\nabla\psi(\vec{r},\hat{\Omega},E)}_{A} + \underbrace{\Sigma(\vec{r},E)\psi(\vec{r},\hat{\Omega},E)}_{B} = \underbrace{\int_0^\infty dE' \int_{4\pi} d\hat{\Omega}'\,\Sigma_s(\vec{r},E'\to E,\hat{\Omega}'\cdot\hat{\Omega})\psi(\vec{r},\hat{\Omega}',E')}_{C}$$
$$+\underbrace{\frac{\chi(E)}{k}\int_0^\infty dE'\,\nu\Sigma_f(\vec{r},E')\int_{4\pi} d\hat{\Omega}'\,\psi(\vec{r},\hat{\Omega}',E')}_{D} \qquad (2)$$

Assuming Time independent

A = streaming loss rate

B = Total interaction loss rate

C = Inscattering source rate

D = Fission Source rate

3. (10 points) List three assumptions *needed* to get from the Transport Equation to the Diffusion Equation.

Assume

(1) Angular Flux depends only weakly on direction $\hat{\Omega}$

(2) $\Sigma_s(\hat{\Omega}' \to \hat{\Omega})$ is azimuthally symmetric

(3) Scattering is at most linearly anisotropic

angular flux is at most linearly anisotropic

Fission neutrons are included in source term S

4. (5 points) List four locations where the Diffusion Equation is not valid because the underlying assumptions do not hold; for each explain why the assumptions do not hold

Void (Vacuum)

Boundary

Source

strong Absorber

The basic premise for why these invalidate the diffusion equation is the same for all of them

At large differences the DE is no longer linear or in the case of a vacuum non continuous so the problems boundaries are extrapolated.

and the boundary condition placed on the flux essentially goes to ∞ breaking one of the requirements

the

5. (a) (2 points) Write the **steady state**, 2D diffusion equation and explain how we typically characterize it from the viewpoint of labeling second order linear PDEs. Assume D is not a function of x or y for this part.

$$\frac{1}{v}\frac{\partial}{\partial t}\phi(\vec{r},t) - \nabla \cdot D \nabla \phi(\vec{r},t) = v\Sigma_f \phi(\vec{r},t) + S(\vec{r},t)$$

We classify this according to $B^2 - 4AC$

(b) (3 points) Can you think of any physical cases in which this characterization would change?

Boundaries
Points of extreme changes

6. (10 points) At what energy is the lowest isolated resonance of ^{235}U, ^{238}U, ^{239}Pu, and ^{240}Pu? Why do we care about that?

We care since it tells us where we should aim to "collect" energy from the reactor. It's a design parameter since higher resonances have a better chance of capturing the fast neutrons we try to thermalize for energy production

$^{235}U = 2.8351 \times 10^{-1}$ eV

$^{238}U = 6.6791$ eV

$^{239}Pu = 2.9139 \times 10^{-1}$ eV

$^{240}Pu = 1.0451$ eV