Hands-On Seminar: Experimental Optics and Atomic Physics Report
Radio frequencies, RF filters, RF mixers and FM Radio

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This report discusses various RF devices and components and their working and operations are studied. First, coaxial cables and signal transmission through them is studied, with different cases of impedance matching. RF components such as filters, amplifiers and mixers are studied and their frequency responses are obtained. Finally, these devices are used to build an FM radio.

COAXIAL CABLES

A coaxial cable or coax cable is a commonly used type of electric cable used for the transmission of signals, typically radio frequency signals. They are called coaxial cables because the transmission line of such cables is made of two conductors - an inner conductor and an outer conductor in a coaxial manner. A dielectric shield is used as an insulator between the two conductors. The typical structure is seen in figure (1). Coax cables are of great importance as they carry high-frequency electrical signals with low losses and they have a broad range of uses and applications [2].

Equivalent circuit

An equivalent circuit can be drawn for a coaxial cable with components as seen in figure (2). All the quantities corresponding to the different components are per unit length and in order to get them in total quantities they need to be integrated along the transmission line. The different components and their quantities represent the following:

- Series resistance R': this represents the resistance of the wire or the finite resistance of the conductor.
- Capacitance C': the two coaxial conductors are placed parallelly and behave like a capacitor.

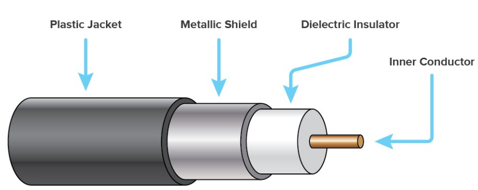

FIG. 1. Schematic of a coaxial cable, taken from [1]

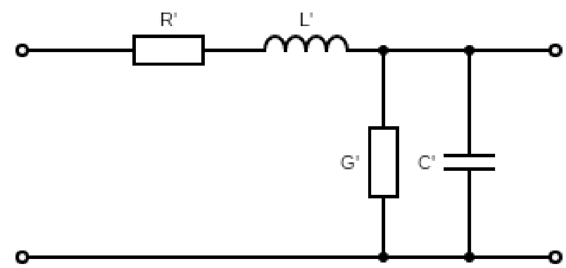

FIG. 2. Equivalent circuit for a coaxial cable

- Parallel resistance G': this is the resistance between the two conductors because of the dielectric, which makes the two not perfectly isolated.
- Inductance L': due to transmission through the conductor, a field is produced and this field leaks and induces a field in the other conductor. This also happens because the two conductors are not perfectly isolated.

Impedance

The impedance of a coaxial cable is given by:

$$z_0 = \sqrt{\frac{R' + i\omega L'}{G' + i\omega C'}} \tag{1}$$

The impedance is a complicated expression and is a complex number with separable real and imaginary parts. But the cases where it is a real number are considered and there are two:

- For the lossless case: $R' = G' = 0$
- For high frequencies like radio frequencies (RF) $z_0 = \sqrt{\frac{L'}{C'}}$

Typical values for the characteristic impedance are around 50 Ω.

Reflection in a coaxial cable

When an RF field is passed through a coaxial cable, it gets transmitted to the end of the cable where it undergoes some reflection, depending on what happens at the end of the cable. The parameter that conveys this is the reflection coefficient:

$$r = \frac{Z_L - Z_0}{Z_L + Z_0} \tag{2}$$

where Z_L is the impedance of the load connected to the cable and Z_0 is the impedance of the cable.
Based on this reflection coefficient, the different cases that could occur can be distinguished:

1. A load with impedance same as that of the cable is attached: matched impedance case
   $Z_L = Z_0 \Rightarrow r = 0$
2. A load impedance of zero: short circuit case
   $Z_L = 0 \Rightarrow r = -1$
3. An infinite load impedance: open-ended case
   $Z_L = \infty \Rightarrow r = 1$

To physically look at the three cases and determine the speed of transmission in the cable, the following are required: First, a short pulse that can be sent through the cable and the reflection of this pulse is observed. A function generator is used to create this short pulse of minimum width, setting certain low and high-level parameters and the correct mode. The pulse is required to be as short as possible to avoid overlap of the reflected pulse, which would make it hard to distinguish. The pulse is split using a T-junction, with one part passing through the delay line and the other to visualise on the oscilloscope. The three cases of load and cable impedance can be studied with the following:

1. Case I: Matched impedance
   A 50 Ω resistor is connected to the end of the cable. The output on the oscilloscope is as seen in figure (3a). As expected, there is no second pulse seen because when the impedance is matched, there is no reflection.
2. Case II: Short circuit
   Attaching a thin short wire, so that there is a short circuit between the inner conductor and outer conductor, so a propagating field in the opposite direction is expected. The output obtained on the oscilloscope is as seen in figure (3b). As expected, the back reflected field returns with opposite polarity.
3. Case III: Infinite load
   The cable is left open-ended. The output obtained from the oscilloscope is as seen in figure (3c). As expected, a copy of the pulse with some time delay is obtained as it is reflected through the cable. Here, the effect of some losses are also seen as the amplitude of the second pulse is smaller than the first.

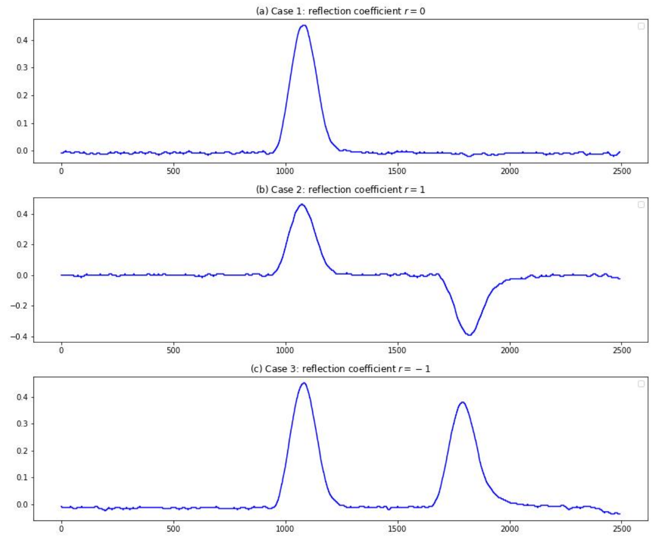

FIG. 3. Oscilloscope data for the three cases of cable load and impedance

To measure the transmission speed the length of the cable is measured and the data recorded from the oscilloscope is used to obtain the time delay between two consecutive pulses. The length of the cable was about 6.4 m. The channels between consecutive peaks correspond to 708 channels, the horizontal scale is 25 ns per division. The time delay thus obtained is (708/25 ns) $\times$ 2 = 56.64 ns. The speed thus obtained is $1.1299435 \times 10^8 m/s$. This is very fast as expected and is roughly 1/3 rd the speed of light.

RF FILTERS

RF filters are devices used to filter or select only desired frequencies (high frequencies in the radio range in the cases discussed). They can, for instance, perform the following functions:

- Low-pass filter: allowing only frequencies below a threshold to pass through
- High-pass filter: allowing only frequencies above a threshold to pass through
- Band-pass filter: allowing a selected band of frequencies to pass through

To understand the working of such filters, we look at the frequency response of the filter. This involves studying the transmission spectrum through the filter and this requires a lot of different frequencies and this can be done using a spectrum analyzer. An analog spectrum analyzer is used. The setup of such a system is seen in figure(4).

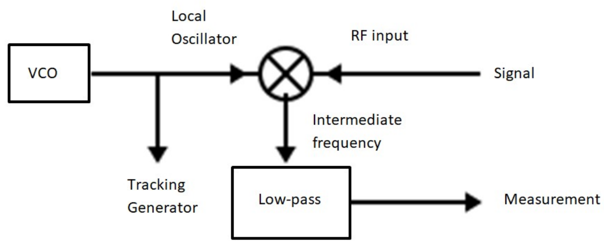

FIG. 4. Schematic of the setup to study RF filters, adapted from [5]

It has an RF mixer (with the symbol X), an input which is an RF signal that goes into the input of the mixer. A local oscillator (LO) goes into the other input port of the mixer, and the LO is connected to a voltage-controlled oscillator (VCO). The output is an intermediate frequency, which is a 'mixed' signal of the two, a multiplied signal which is passed through a low-pass filter and is measured or read out.

The spectrum analyzer scans over many frequencies with time and displays the amplitude against the frequency of the signal. The VCO generates an output signal (a saw-tooth wave in this case), it sweeps the spectrum of the input signal and the different components are displayed on the spectrum analyzer. The signal from the LO is also displayed, this is called the tracking generator (TG) that follows the frequency sweep of the spectrum analyzer. By connecting the TG output to the input, and setting the center frequency to 500 MHz, spanning 1000 MHz. The output of the TG appears as an almost flat line, possibly due to changes in the amplitude during processing or because the VCO does not consistently put out the same amplitude. Moving the marker on the spectrum analyzer within the 1 dB range makes it flat, allowing it to be used as a useful reference for the filters.

Next, with the filter, there is a cutoff of the signal, changing the span a kink in the spectrum can be seen. Since it is in the log scale, it is a straight line, and the small frequency components are damped exponentially. Low pass circuits as a function of frequency are exponential. Looking for the point where the filter cuts off, the -3 dBm point is at 80 MHz, which means that the 100 MHz labelled is still completely transmitted. Only 20 MHz below this starts to act like a filter. This was a high-pass filter. Next using a low-pass filter centered around 150 MHz. Compared to the case of the high-pass filter, the frequency response was not as steep and the kink, looking at the -3 dBm point, is at 160 MHz, which is closer to the centre frequency. At 150 MHz the attenuation or damping starts.

Combining the low pass and high pass filters gives the intermediate part between 80-150 MHz, this is the bandpass filter where the complete transmission profile of both filters is obtained.

AMPLIFIER

The next component studied is the amplifier. Voltage is supplied to an amplifier and it has an input and output port.

They have a typical gain of 30 dB. Unlike before, the TG cannot be connected directly to the input and then the output to the SA, as nothing will be seen without control. Equivalent attenuators need to be used before the amplifier to reduce the input by 30 dB before amplifying. An attenuation of 30 dB is used (10+20 dB) and the frequency response of the amplifier is obtained. Initially, a flat line within the given frequency range, the same level as before is seen. Looking at a broader range of frequencies with 500 MHz central frequency and 1 GHz span, the spectrum starts dropping off after 500 MHz, which is expected. The amplifier and the filters combined produce this effect. Looking at them individually the same drop off is obtained so it adds up with the combination.

RF MIXERS

A mixer is a device that has three ports: the RF port, the LO port and the intermediate frequency port.

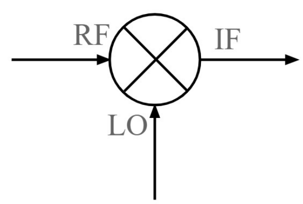

FIG. 5. Schematic of a mixer, taken from [6]

The mathematical operation performed by the mixer is multiplication. Two fields are mainly considered, the RF signal field and the LO field:

$$U_{RF} = A_{RF}\cos(\omega_{RF}t + \phi_{RF}(t)) \tag{3}$$

$$U_{LO} = A_{LO}\cos(\omega_{LO}t + \phi_{LO}(t)) \tag{4}$$

The mixer takes the two and multiplies them to give the intermediate frequency (IF) field:

$$\begin{aligned} U_{IF} = A_{RF}A_{LO}[&\cos(\omega_{LO} + \omega_{RF})t + \phi_{LO}(t) + \phi_{RF}(t)] \\ &\cos(\omega_{LO} - \omega_{RF})t + \phi_{LO}(t) - \phi_{RF}(t) \end{aligned} \tag{5}$$

The two main components in the IF field are the sum frequency and the difference frequency components. To visualize this on the SA, with a low-pass filter, the difference frequency part is transmitted while the sum part is neglected. On the SA the video bandwidth and radio bandwidth are set, which are essentially the bandwidths of the low pass filter. This can be set to reduce the amplitudes of the spectrum while making narrower lines of the spectrum.

In literature, there is a distinction between the types of a down-converter and an up-converter but in practice, it does not make a difference as it depends on the mixer. The inputs for the two types change: the LO is always the input but for a down converter you supply the RF and get out IF and for an up-converter, you reverse the RF and IF. The only convention is that the intermediate frequency is lesser than the radio frequency.

To see this experimentally, two RF input fields are used the and function generator is used to produce a sine wave for both channels, equal in amplitudes. The low level is set at 0 and the high level is 500 mV for both channels. This gives two sine waves with the same amp but different frequencies. The IF is displayed on the spectrum analyzer with a 50 MHz center frequency, and attenuation is adjusted appropriately with a span of 100 MHz. The output on the spectrum analyzer is a collection of peaks with the component at 25 MHz being the tallest, and intermediate peaks at 10, 15, 20, 30, 35, 40 MHz (in multiples of around 5). Peaks at 40 and 10 MHz are expected from the sum and difference of the frequencies, the other peaks come from combinations of other input frequencies, as there are more frequencies than that from the formula. This is from the mixer and depends on how the mixer is built. The mixer consists of a bunch of diodes. Diodes are nonlinear devices and any circuit that uses diodes to perform operations will have nonlinear effects, and this with the process of frequency mixing creates higher harmonics or other combinations of frequencies.

To operate it as a down-converter, the IF and RF ports are interchanged and it is observed that the frequency components don't change, but amplitudes are very different.

SETTING UP AN FM RADIO RECEIVER

The devices described and studied until now form the components used to build an FM radio receiver. FM stands for frequency modulation, and it is the technique used for the working of such radio devices. An FM signal consists of a high frequency carrier signal and an audio signal that is in the KHz range is modulated on top of this accordingly (hence, it is called frequency modulation). This modulated signal is then transmitted over long distances, made possible with the high radio frequencies, and the task of the radio receiver is to find the carrier frequency and unmodulate the signal to retrieve the audio signal.

The field for the radio signal can be described as:

$$U_{RF} = A_{RF} \cos(\omega_{RF} t + \phi_{RF}(t)) \tag{6}$$

Based on this expression, the frequency ω_{RF} is fixed, but the phase $\phi_{RF}(t)$ is time-dependent. For a time-dependent signal, the frequency as a function of time is:

$$\omega(t) = \frac{d\phi}{dt} = \frac{d}{dt}(\omega_{RF} t + \phi_{RF}(t)) = \underbrace{\omega_{RF}}_{\text{carrier}} + \underbrace{\dot{\phi}_{RF}(t)}_{\text{FM signal}} \tag{7}$$

The receiver's task is to extract this function from the radio signal to find $\phi_{RF}(t)$. To obtain the phase, a mixer and a low-pass filter are used. simplify the expression above: use the low pass filter, we have the RF field and the LO, and we can use the LO by setting it to the carrier frequency ω_{RF}. Assume we have

$$\omega_{LO} = \omega_{RF} + LP$$

where LP is the low-pass filter. The signal we get out of the mixer has the form:

$$U_{IF} = \frac{A_{RF} A_{LO}}{2} (\cos\left(\underbrace{\phi_{LO}(t) - \phi_{RF}(t)}_{\Delta\phi(t)}\right) \tag{8}$$

From the equation for the intermediate frequency field above, we see that there are no frequencies any more, only time-dependent phase terms remain.

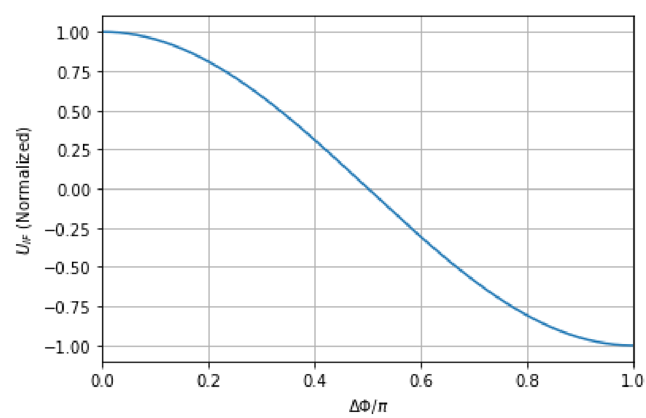

FIG. 6. Intermediate frequency field as a function of phase difference as described in equation(8)

From the plot of the function, it is apparent that the regime of interest is where the cosine function can be simplified. This happens when the phase difference is $\pi/2$, where $U_{IF} = 0$ and the function can be linearized. Here,

$$U_{IF} \propto -\Delta\phi(t) \tag{9}$$

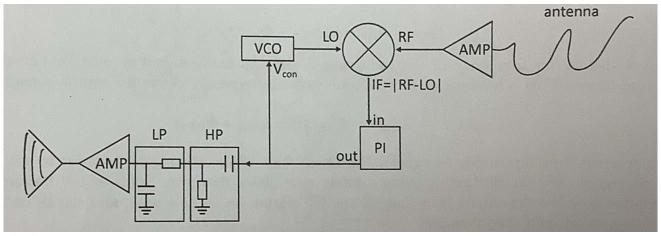

FIG. 7. Schematic of the setup for the FM radio receiver, taken from [5]

The intermediate frequency is proportional to the phase difference. This information is important as the phase of the LO which is known can be set, and the only unknown is ϕ_{RF}. If we make the IF remain at zero, we can obtain the RF signal based on the relation. To do so, a circuit that stabilizes the LO is required, with a feedback to the LO to ensure that this value remains at zero. For this, an additional device, a Proportional and Integral (PI) controller or a lock-box, is used which forms the integral feedback circuit. This linear variation of U_{IF} around 0 can be used to follow the modulation of the RF signal with the help of the PI controller.

The figure(7) shows the setup used to implement the phase lock. The radio signal is picked up by the antenna, and since the signal is weak it is first amplified. It is then passed on as the RF input of the mixer. The LO input is obtained from the VCO which can be controlled and the output signal from the mixer is the IF, which is the input for the PI controller. The PI controller already has a small bandwidth so it acts like a low-pass by default. The output of the PI controller is split and one part is used as the control input for the VCO, which is used as a feedback to the oscillator to ensure that the intermediate frequency output from the mixer does not change with time, and stays constant at zero. The remaining part is sent through a low-pass and high-pass filter to get rid of residual noise, and it is amplified and connected to speakers to obtain the audio signal output.

The oscilloscope is used to visualise the frequency of the channel, and a sine curve is seen. The monitor signal is noisy as it is the signal coming from the intermediate part of the mixer and it has some time-dependence. After locking the PI controller, the DC offset added to the input is adjusted, which controls the frequency of the LO through the feedback, and this is made to match a known radio frequency of a local radio station. The output on the speaker with some fine-tuning of the frequency and adjusting the gain of the amplifier gives a clean audio output. This way, an FM radio receiver is set up and the FM radio signal is down-converted to obtain the audio signal output.

APPLICATIONS

The devices discussed here have a multitude of applications across different fields. Coax cables are used in applications such as telephone trunk lines, broadband internet cables, computer networks and cable television [4]. BNC cables used in labs are also a type of coaxial cable.

The RF devices: filters, amplifiers and mixers are also important. One application as discussed is for the setup of a FM radio receiver. Most RF devices operated in this frequency range would require these devices, particularly filters. This includes cell phones, broadcast radio and television, and wireless connections such as wi-fi. Mixers are also used for processes that involve converting signals from one frequency range to another, for example, superheterodyne receivers [3]. These are used for various purposes, for instance, in radio astronomy.

CONCLUSION

RF devices and their components were successfully studied. Understood how signals travel through a coaxial cable and looked at the different cases of impedance matching. Different RF components such as filters, amplifiers and mixers and their operation were studied to see how they work, and these were all used with a PI controller to set up an FM radio receiver and build an FM radio. The different applications were also discussed.

[1] L. Bronston, *What is Coax Cable Internet?*, Telnet Worldwide, May 14, 2020.

[2] Wikipedia contributors, *Coaxial Cable*, `https://en.wikipedia.org/wiki/Coaxial_cable`.

[3] Wikipedia contributors, *Frequency mixer*, `https://en.wikipedia.org/wiki/Frequency_mixer`.

[4] Wikipedia contributors, *RF and microwave filter*, `https://en.wikipedia.org/wiki/RF_and_microwave_filter`.

[5] M. Weitz, Andreas Redman *Hands-on Seminar Guideline*, 2023.

[6] Electronics Notes Contributors, *How to buy the best RF mixer for your application*, `https://www.electronics-notes.com/articles/radio/rf-mixer/how-to-buy-select-best-rf-mixer.php`.