Taylor-Couette Flow

Honours Progress Presentation

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Introduction

Source: Siddharth Krishnamoorthy, *YouTube*, 2007

Basics of fluid dynamics

- Understanding fluids
- Eulerian and Lagrangian frames:

$$\frac{DF}{Dt} = \frac{\partial F}{\partial t} + \boldsymbol{u} \cdot \nabla F$$

- Continuity equations:

$$\frac{1}{\rho}\frac{\partial \rho}{\partial t} + \nabla \cdot \boldsymbol{u} = 0$$

$$\rho \frac{\partial u}{\partial t} + \rho(\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla p + \rho \boldsymbol{g}$$

- Navier-Stokes equations:

$$\rho \frac{\partial \boldsymbol{u}}{\partial t} + \rho(\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla p + \mu \nabla^2 \boldsymbol{u} + \rho \boldsymbol{g}$$

Small experiments

- Double pendulum
- Dripping faucet
- Burette experiments

Lattice Boltzmann method

- Flow past a cylinder
- Based on collision models
- Problem with boundary conditions

Studying Partial Differential Equations

- Understanding what a PDE is, what it contains
- PDE's encountered in the past:
  - Laplace's equation: $\nabla^2 u = 0$
  - Poisson's equation: $\nabla^2 u = f(x, y, z)$
  - Diffusion equation: $\nabla^2 u = \frac{1}{\alpha^2}\frac{\partial u}{\partial t}$
  - Wave equation: $\nabla^2 u = \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2}$
  - Linear and nonlinear convection: $\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0, \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0$
  - Burgers' equation: x term: $\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$

    y term: $\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$

Analytical methods of solving PDEs

- Laplace's equation for steady-state temperature in a rectangular plate: $\nabla^2 T = 0, \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$
- Diffusion equation for heat flow: $\nabla^2 u = \frac{1}{\alpha^2} \frac{\partial u}{\partial t}$
- Wave equation: $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$
- Methods used:
  - Trial solution, separation of variables, partial separation, Boundary value problems: Dirichlet and Neumann conditions

Numerical Methods of solving PDEs:

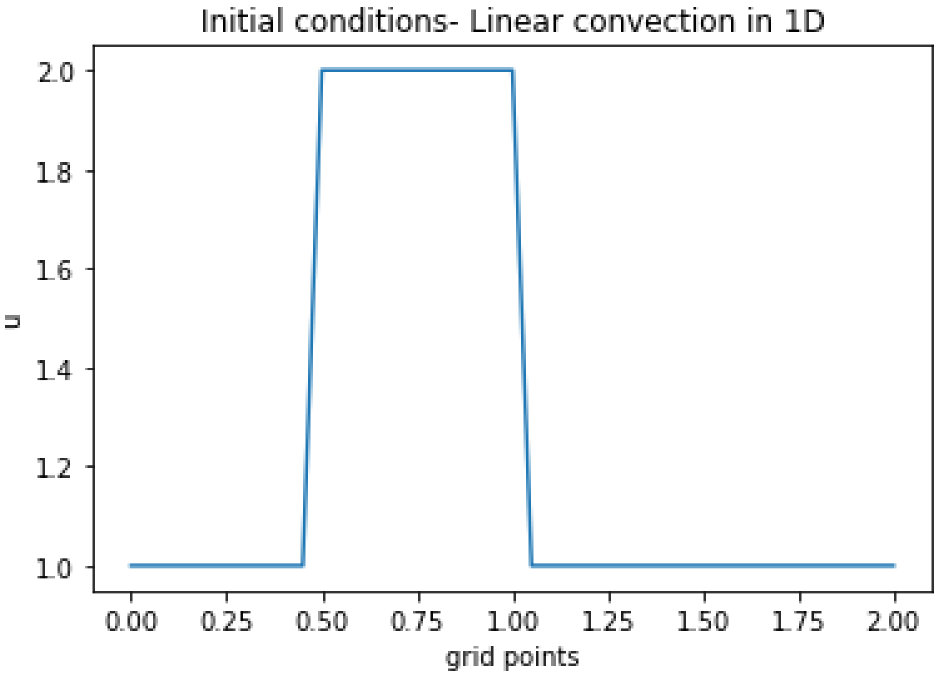
- Nonlinearity
- Finite Difference Method:
  - Applying forward, backward and central differences
  - Discretization of terms
- Finite Volume method

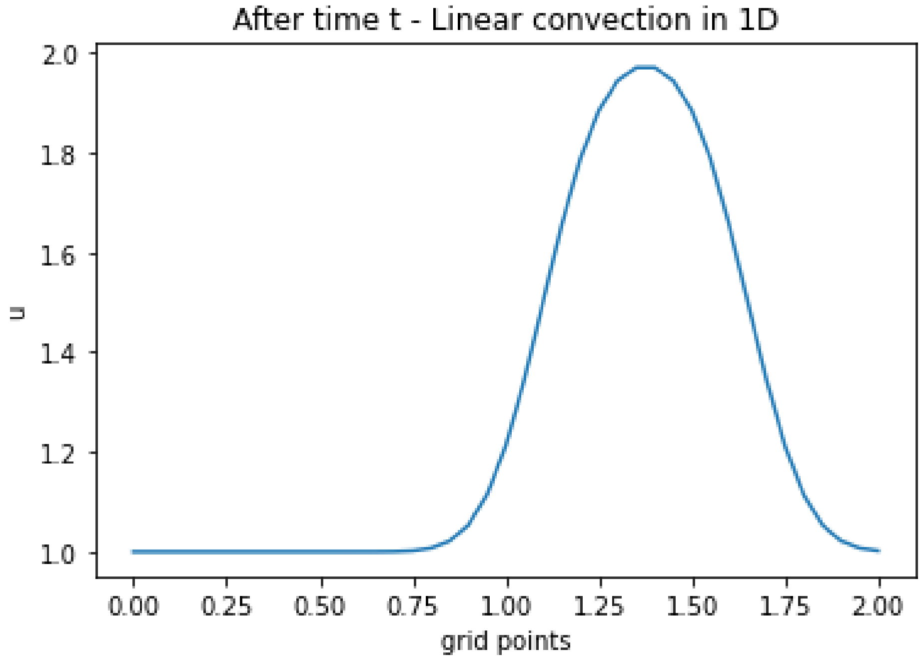
Computation of numerical methods:

- Applying FDM to the previous PDEs in 1D and 2D in python
- Plots of velocity profiles in fluids

Linear Convection in 1D

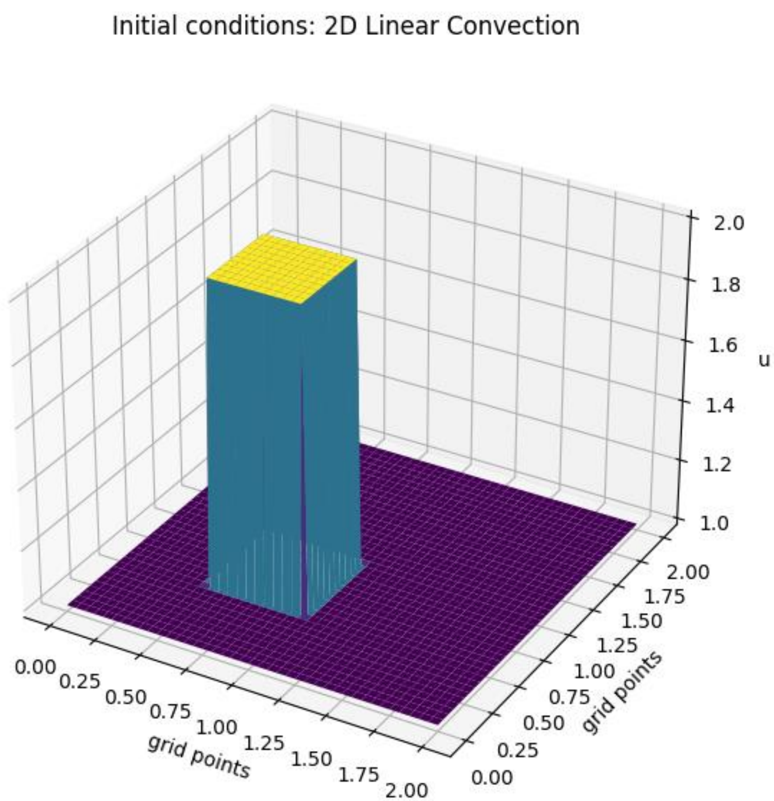
$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$$

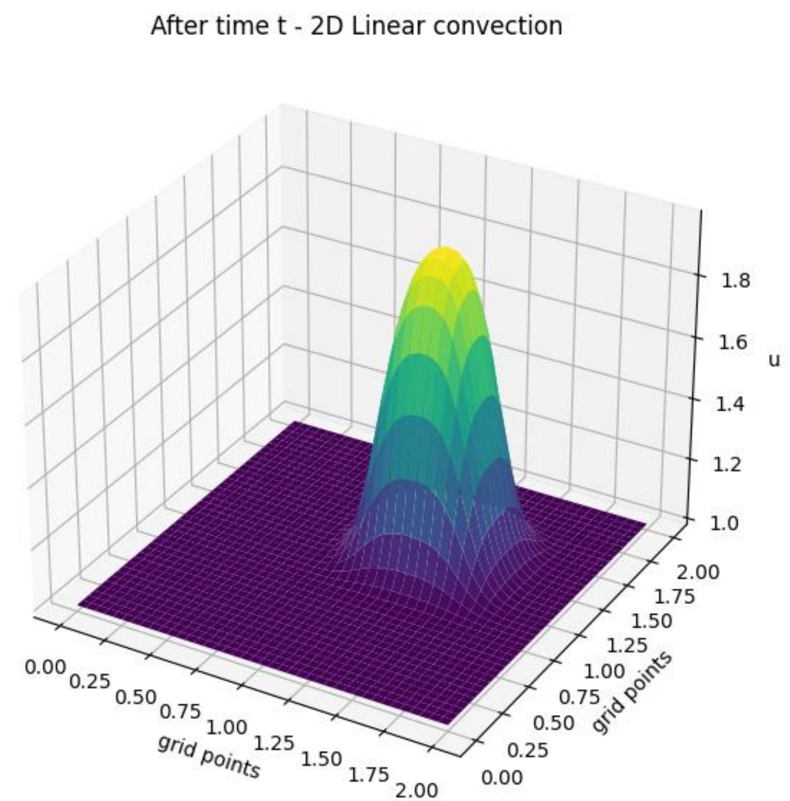




Linear Convection in 2D

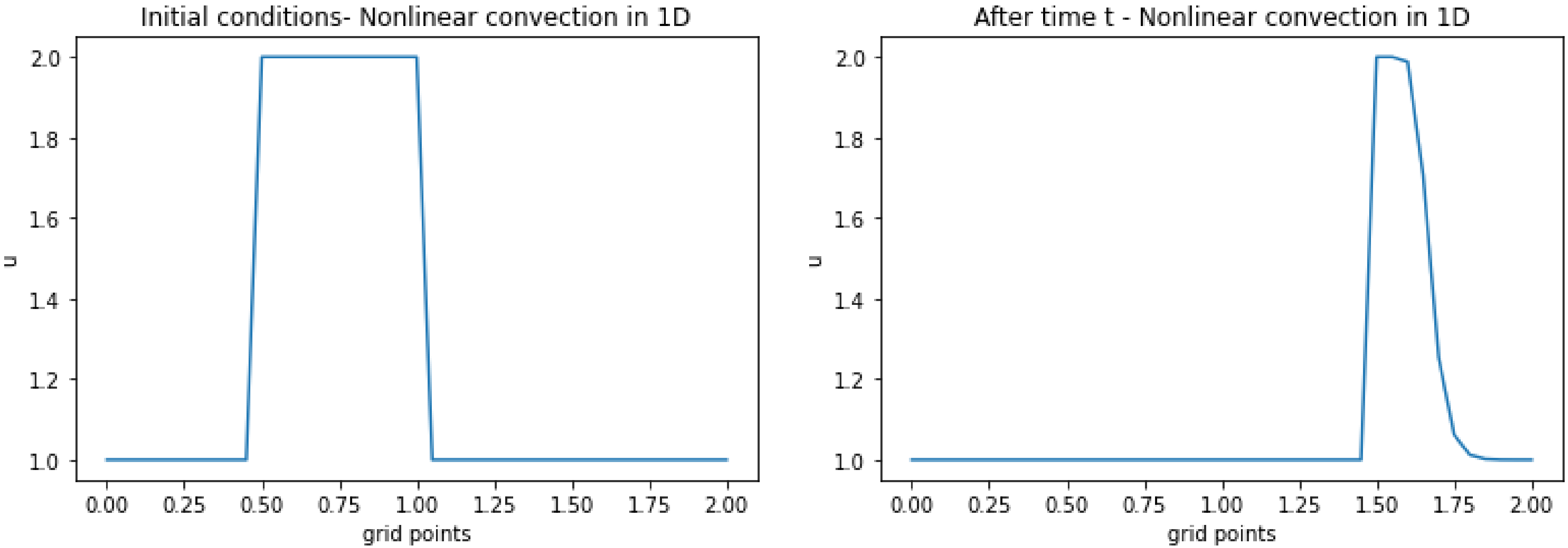
$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} + c\frac{\partial u}{\partial y} = 0$$





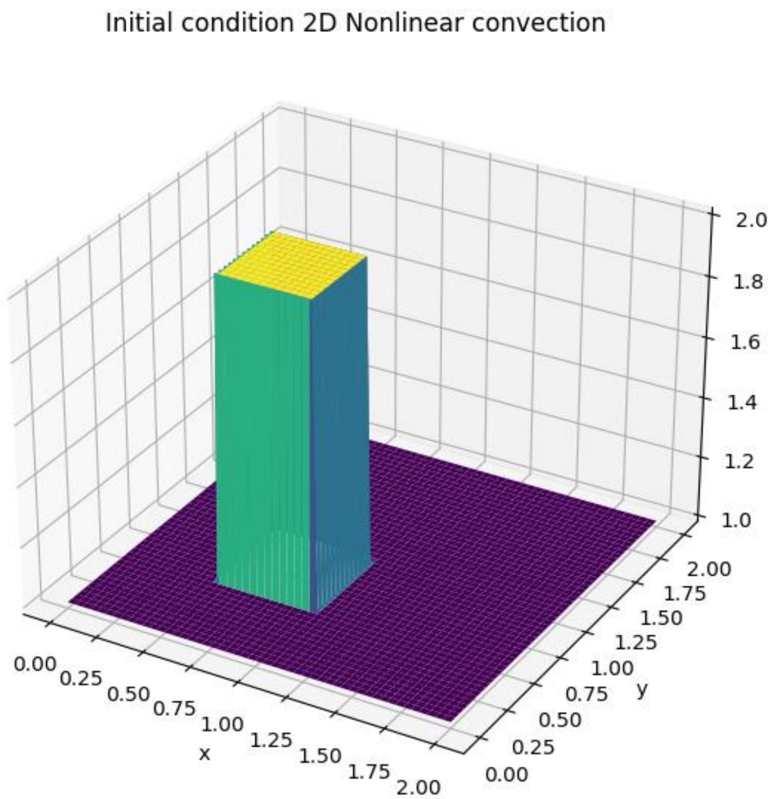
Nonlinear Convection in 1D

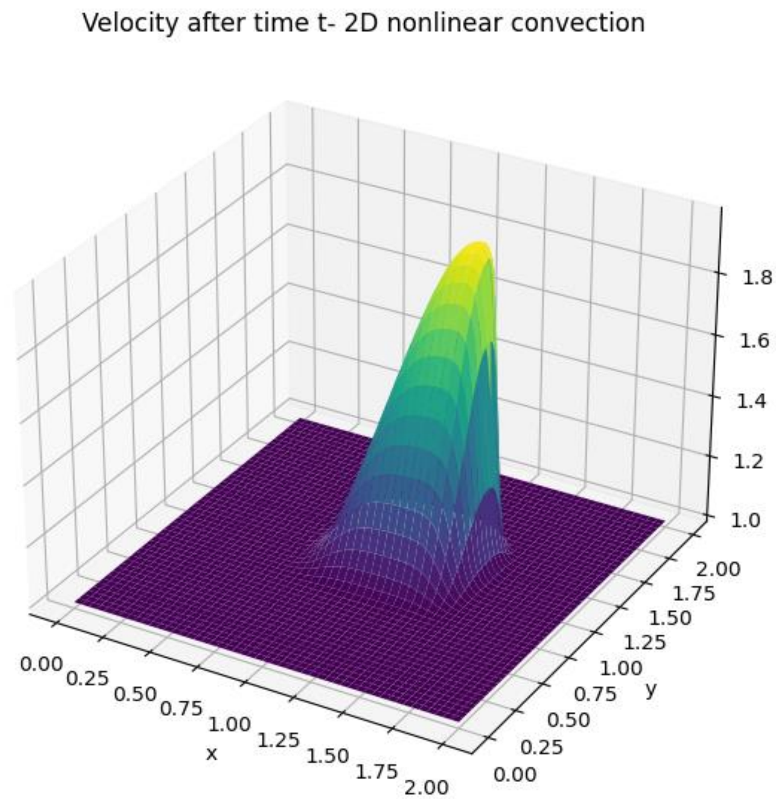
$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0$$



Nonlinear Convection in 2D

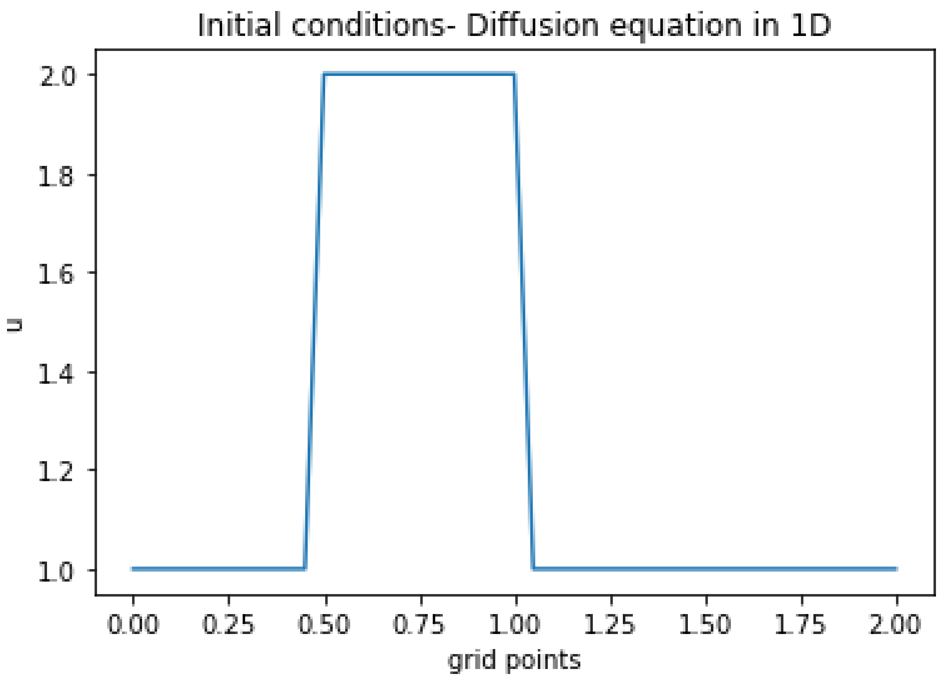
$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = 0$$

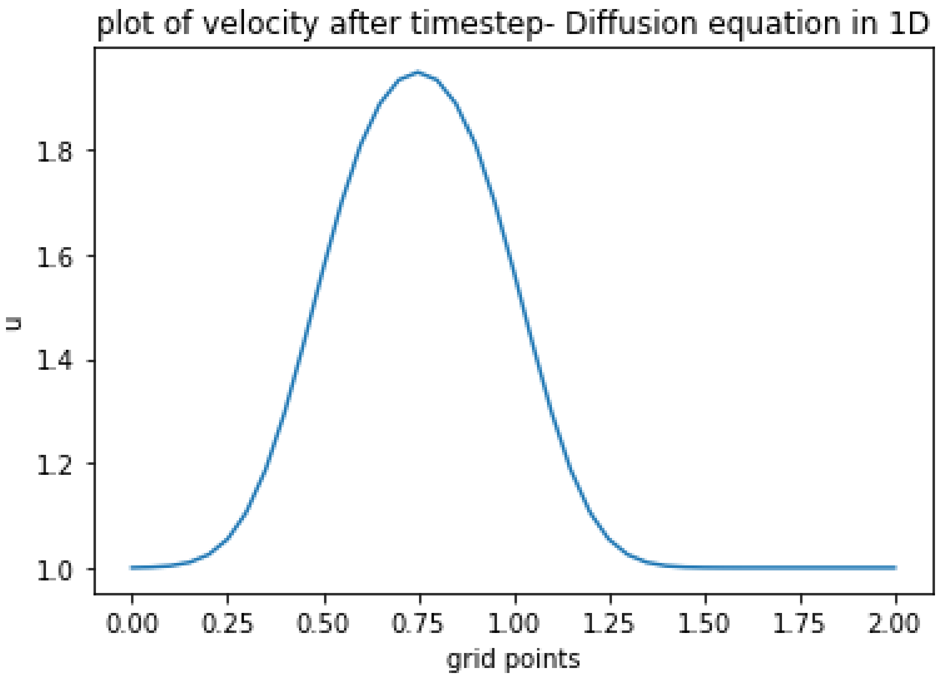




Diffusion Equation in 1D

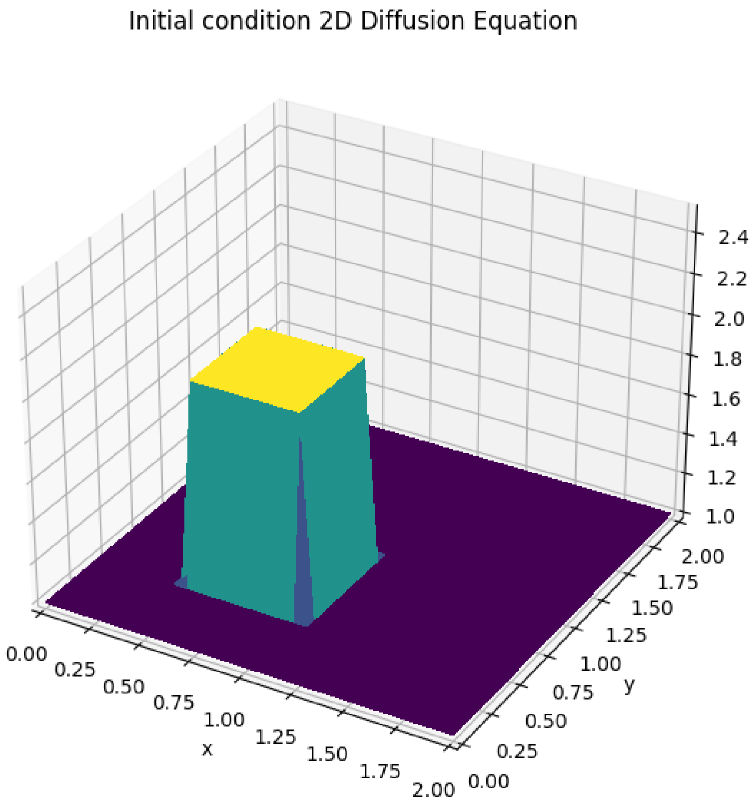
$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial x^2}$$

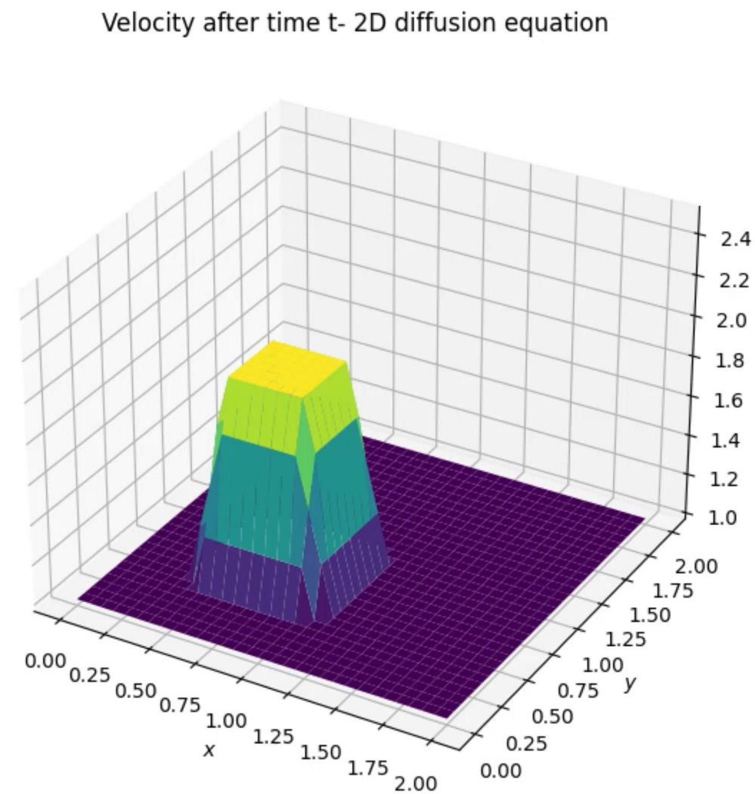




Diffusion Equation in 2D

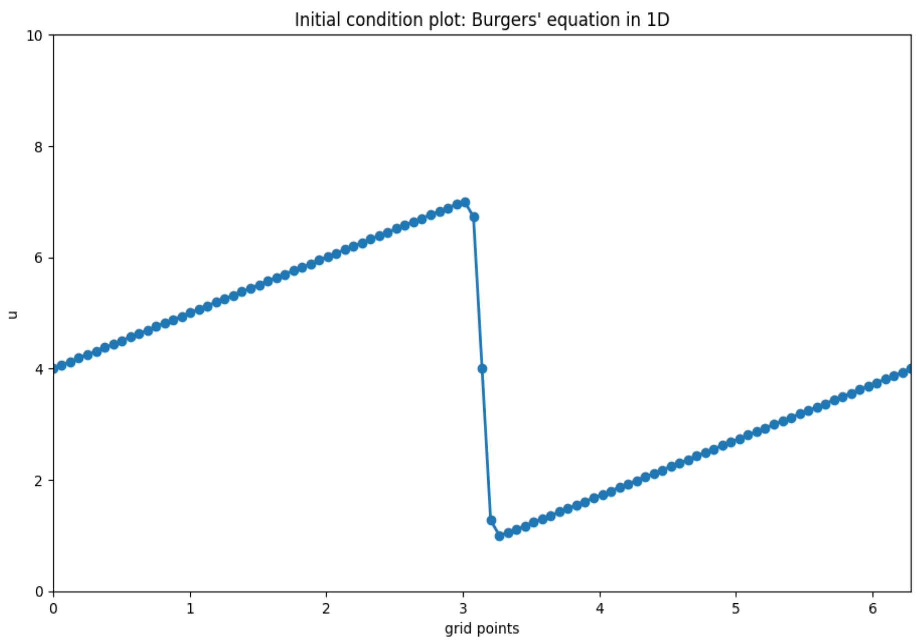
$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial x^2} + \nu \frac{\partial^2 u}{\partial y^2}$$

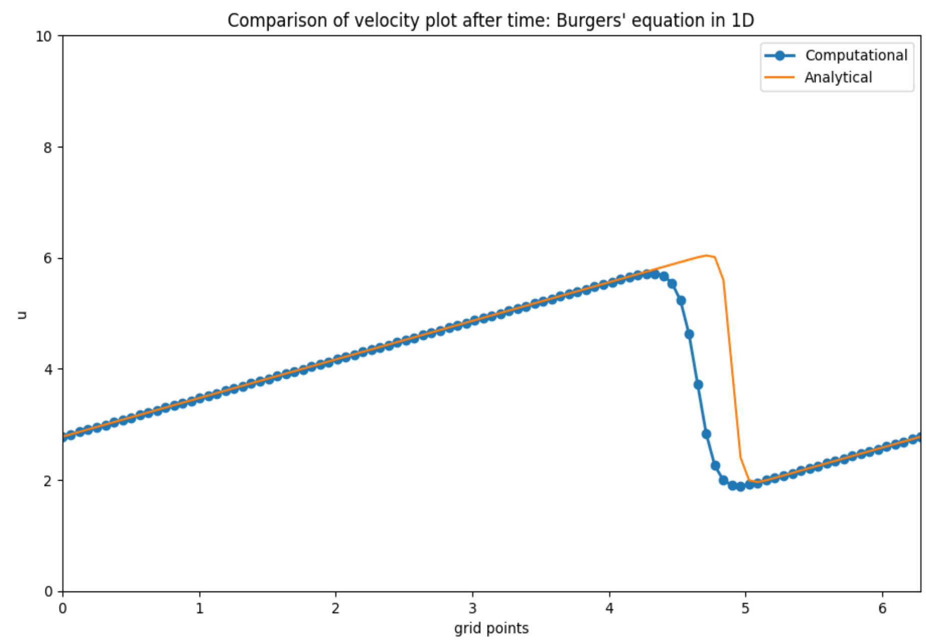




Burgers’ equation in 1D

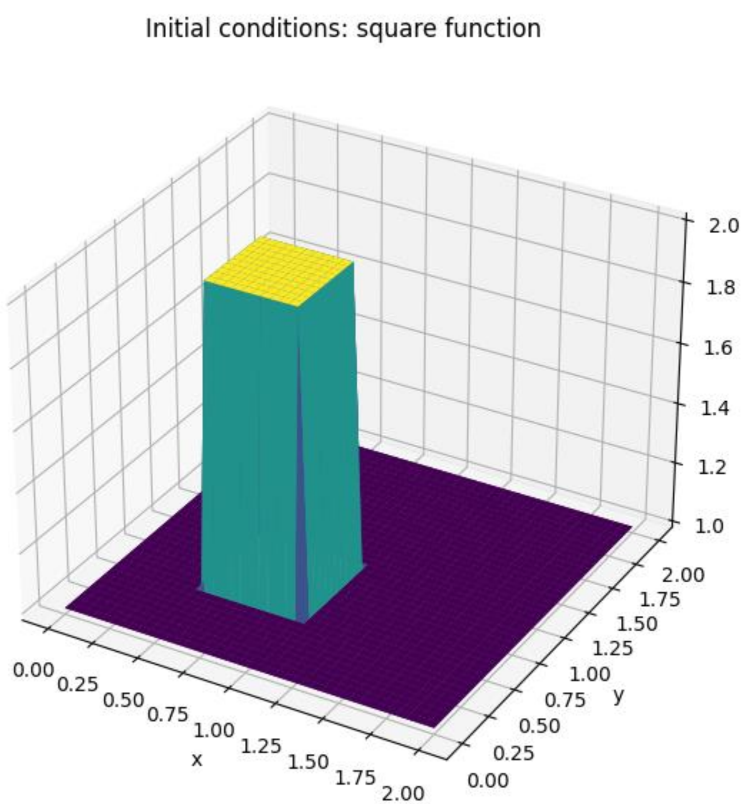
$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu \frac{\partial^2 u}{\partial x^2}$$

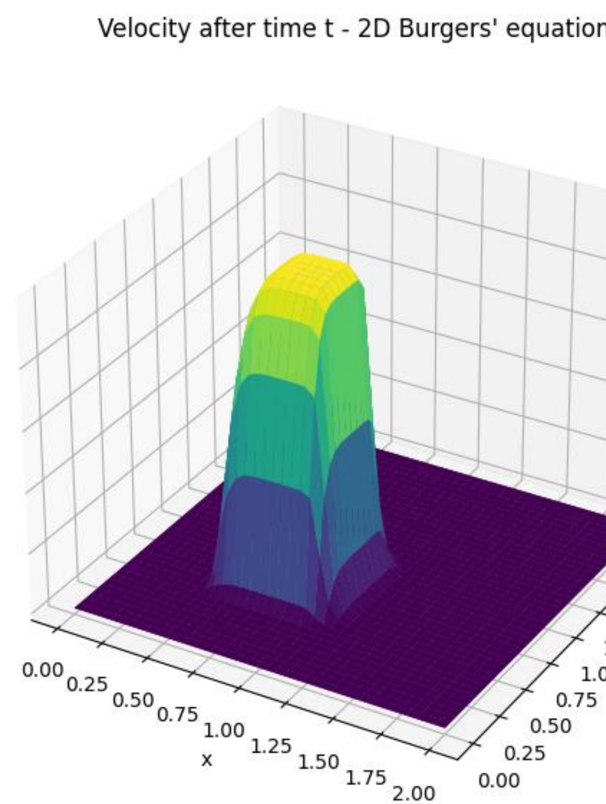




Burgers' equation 2D

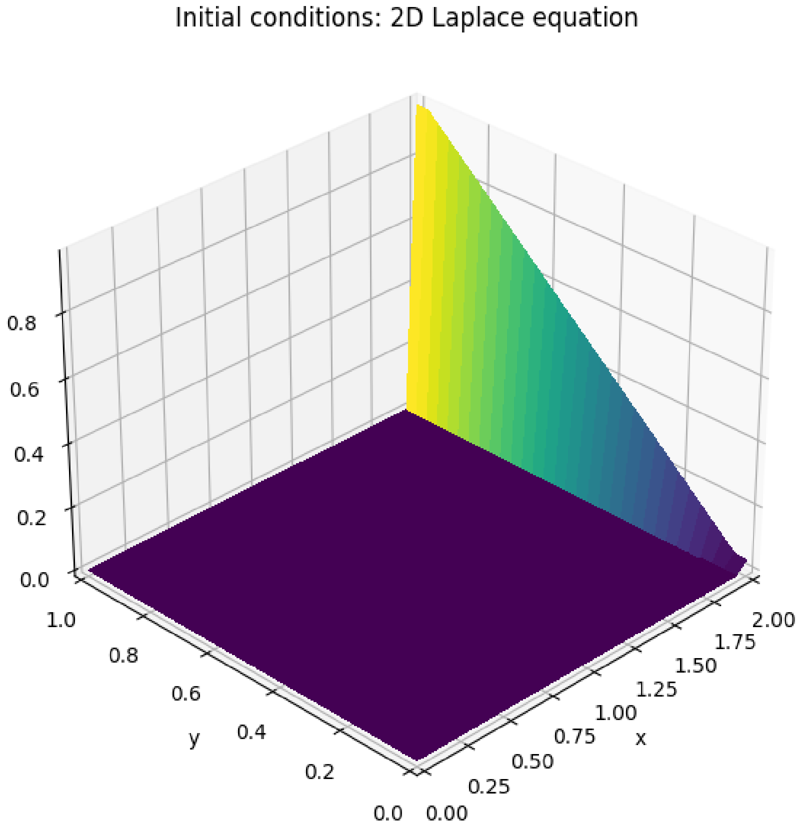
$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$
$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

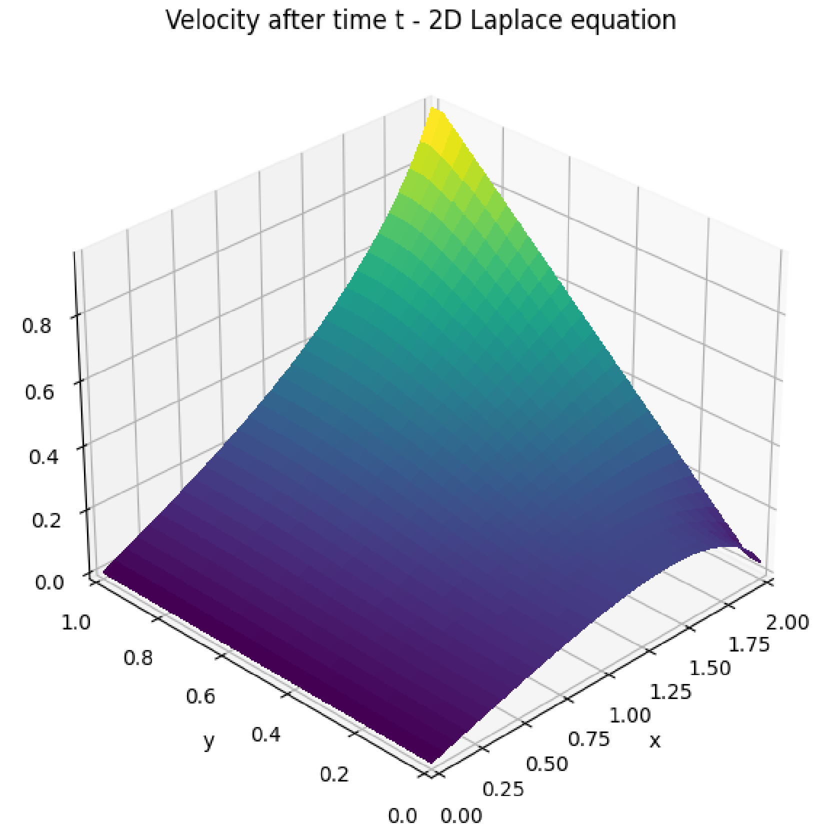




Laplace equation in 2D

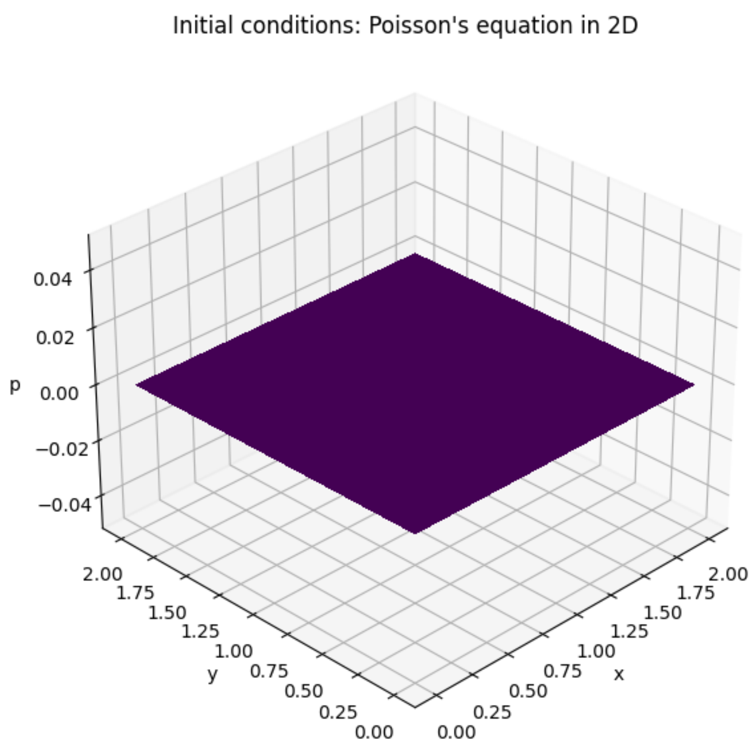
$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 0$$

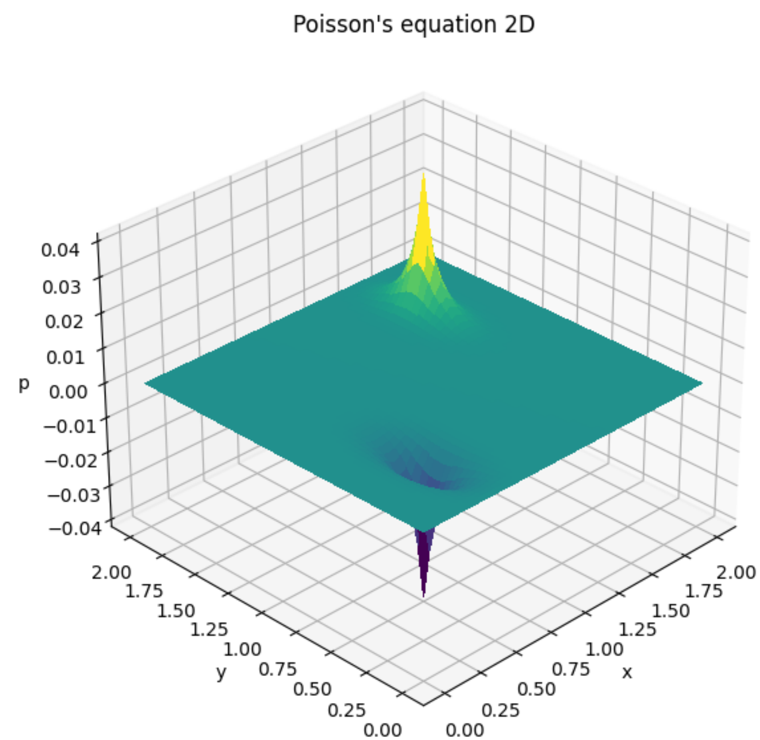




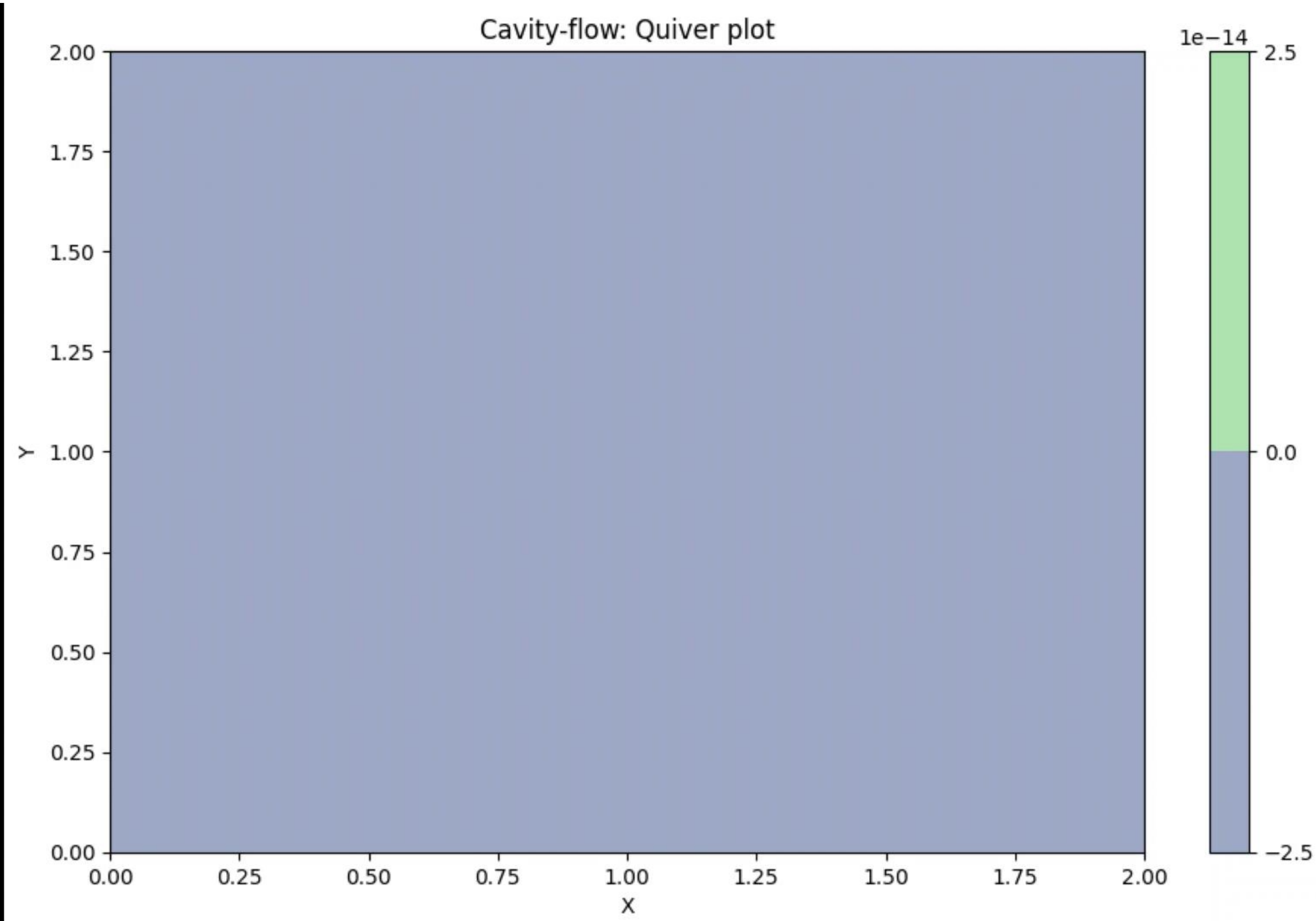
Poisson Equation in 2D

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = b$$

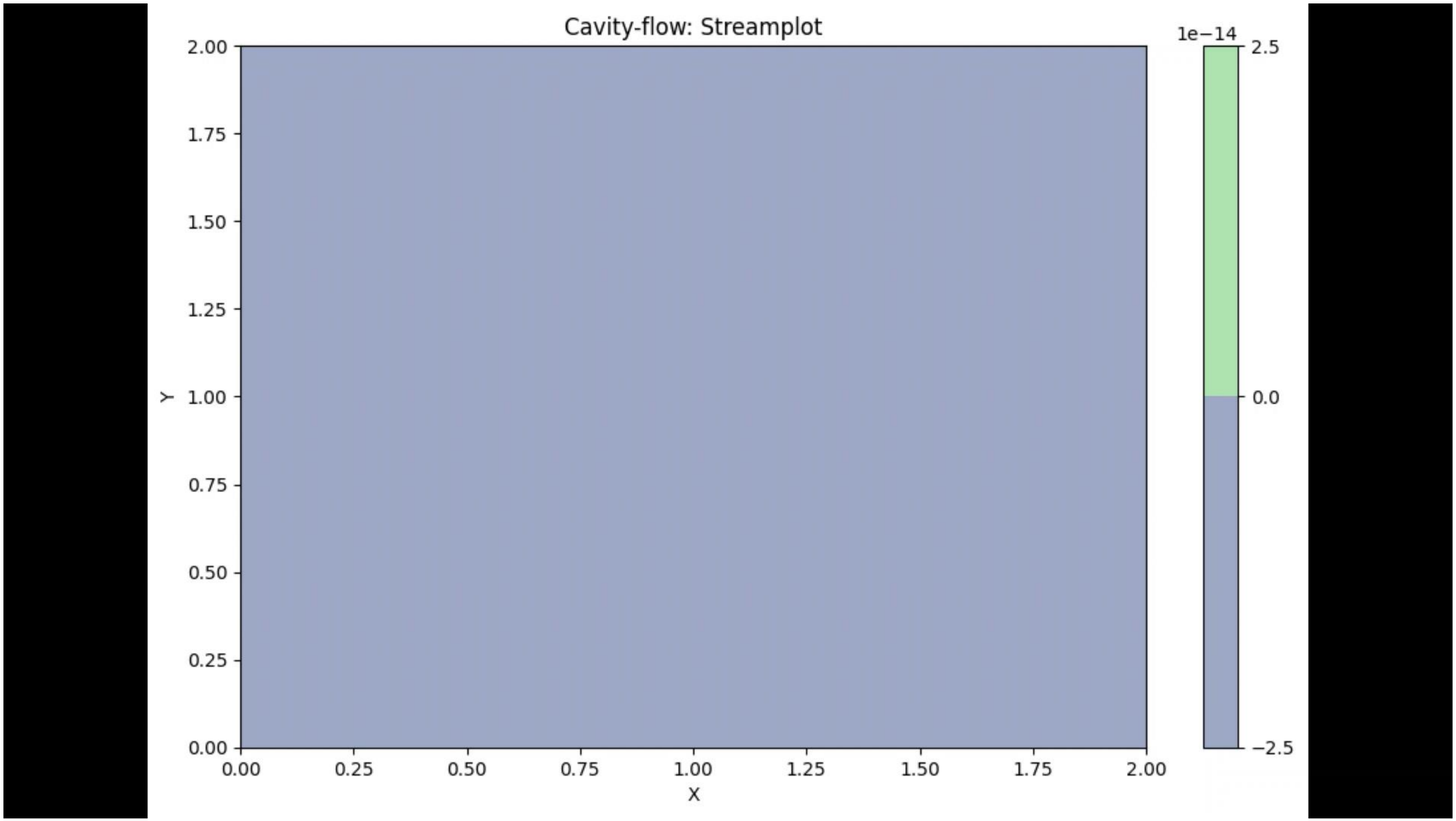




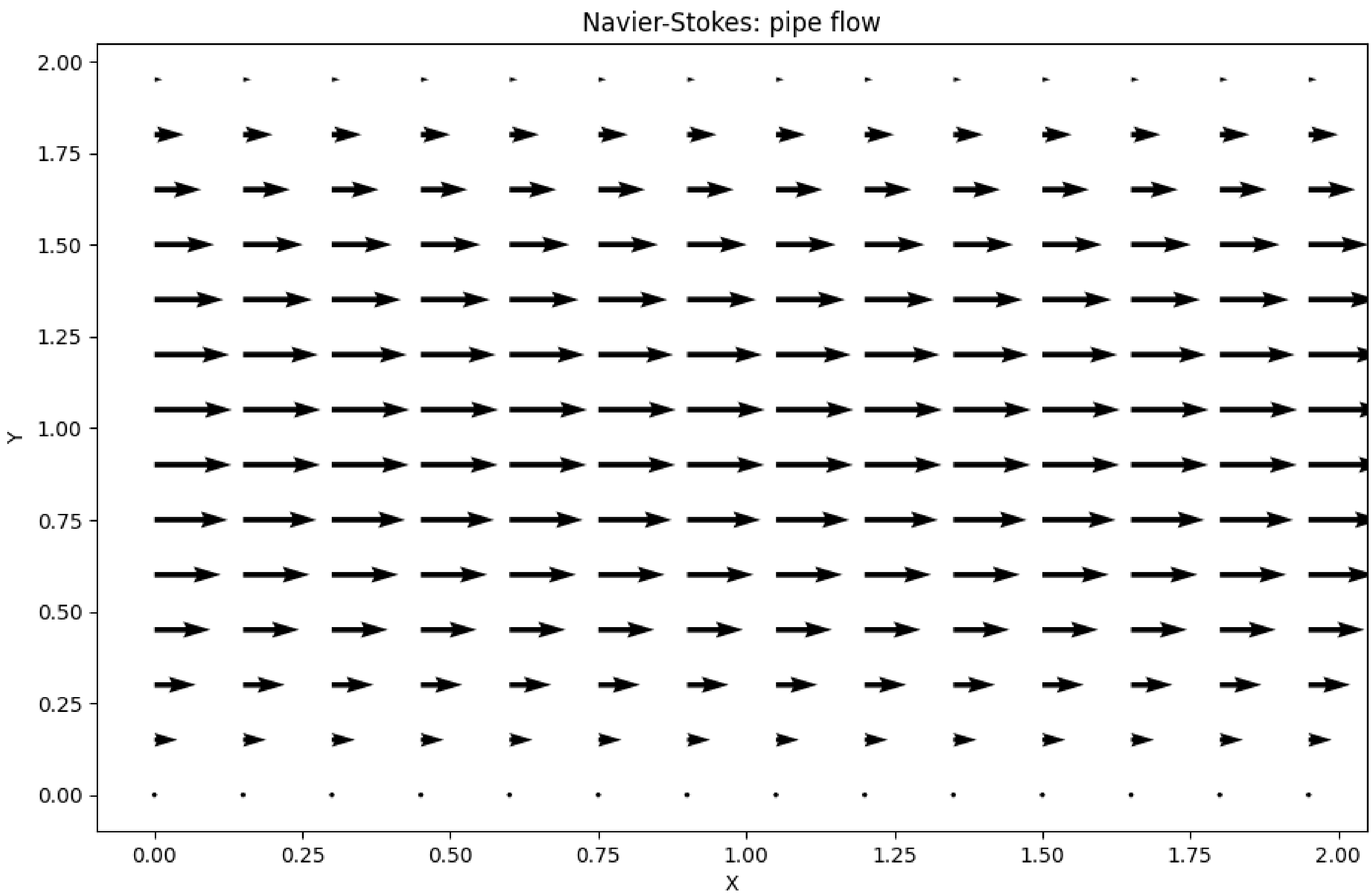
Simulation of Navier-Stokes equation: Cavity Flow Quiverplot



Simulation of Navier-Stokes equation: Cavity Flow Streamplot



Simulation of Navier-Stokes equation: Channel Flow



Learnings from CFD

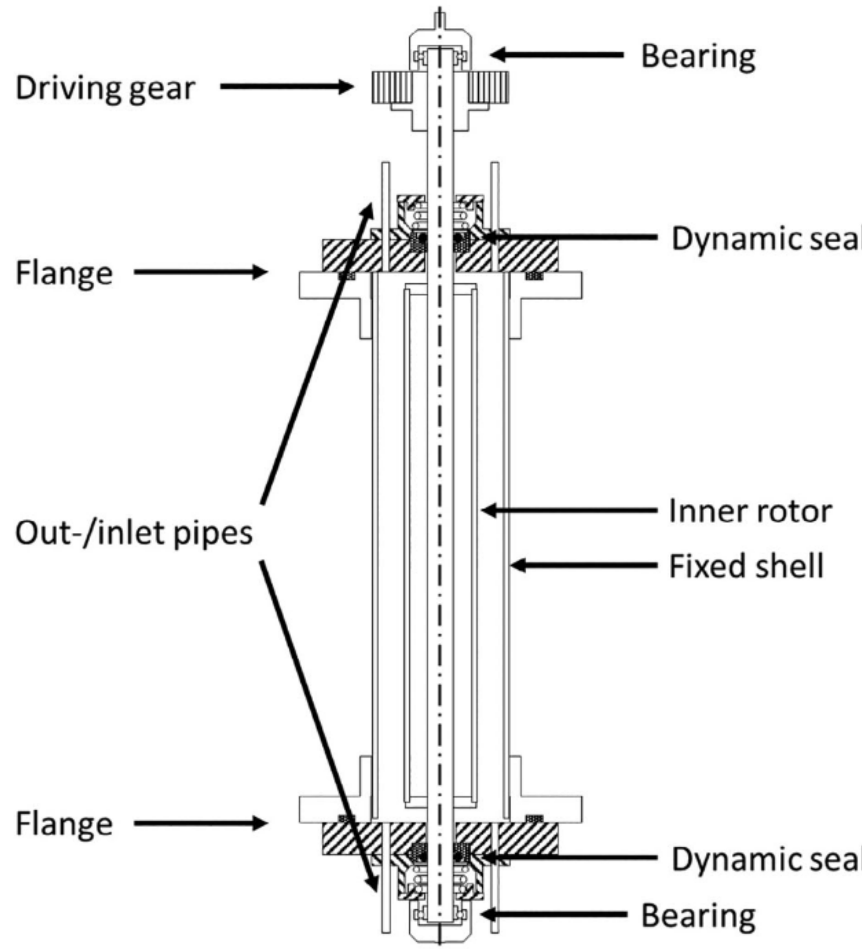
- Solving PDEs by FDM
- Understanding initial and boundary conditions (Dirichlet and Neumann problems)
- Creating 3D plots on Python

Literature review for the set up:

- Stability of a Viscous Liquid contained between Two Rotating Cylinders, G I Taylor, 1922.
- An Experimental Study of the Motion of a Viscous Liquid contained between Two Coaxial Cylinders, J W Lewis, 1928.
- Taylor-Couette Flow: The Early Days, R J Donnelly, *American Institute of Physics*, 1991.
- Taylor-Couette Reactor: Principles, Designs, Applications, Schrimpf et al, *AIChE Journal*, 2021.
- Simulation of Taylor-Couette flow: Part 1: Numerical Methods and Comparison with Experiment, Part 2: Numerical Results for wavy Vortex flow, Philip S Marcus, *Journal of Fluid Mechanics,* 1984.

Taylor-Couette Apparatus setup

- Mechanical arrangement: Coaxial plexiglass cylinders (different diameters), attached to a stepper motor, inner cylinder rotating.
- Fixed ends with stainless steel flanges. L shaped outlet at the bottom.
- Working fluid: water-glycerol mixture at different compositions.
- Visualisation: Aluminum pigment powder/artists silver powder as opposed to rheoscopic fluids. Also create contrast by painting or using black working fluid.



Schrimpf et al.,*AIChE journal*, 2021.

Additional Variations (long-term)

- Rotating second cylinder (changing directions)
- Adding axial/azimuthal flow
- Different orientations of the apparatus, changing aspect ratios
- Quantitative aspect: signal processing

Plan for the semester:

- Set up the apparatus
- Work on computation
- Finish writing thesis

References:

- Stability of a Viscous Liquid contained between Two Rotating Cylinders, G I Taylor, 1923. https://royalsocietypublishing.org/doi/pdf/10.1098/rsta.1923.0008
- An Experimental Study of the Motion of a Viscous Liquid contained between Two Coaxial Cylinders, J W Lewis, 1928. https://royalsocietypublishing.org/doi/pdf/10.1098/rspa.1928.0006
- Taylor-Couette Flow: The Early Days, R J Donnelly, *American Institute of Physics*, 1991. Taylor-Couette flow: The early days. (ntu.edu.tw)
- Taylor-Couette Reactor: Principles, Designs, Applications, Schrimpf et al, *AIChE Journal*, 2021. https://aiche.onlinelibrary.wiley.com/doi/10.1002/aic.17228?af=R
- Simulation of Taylor-Couette flow: Part 1: Numerical Methods and Comparison with Experiment, Part 2: Numerical Results for wavy Vortex flow, Philip S Marcus, *Journal of Fluid Mechanics,* 1984.
- CFD: 12 Steps to Navier-Stokes equations, Lorena A Barba, *Lorena Barba Group*, 2013. https://lorenabarba.com/blog/cfd-python-12-steps-to-navier-stokes/
- A Lattice Boltzmann Study of 2D Steady and Unsteady flows around a confined cylinder, Filip Strnisa et al, *Journal of the Brazillian Society of Mechanical Sciences and Engineering*, 2020. https://link.springer.com/article/10.1007/s40430-020-2176-y#Abs1
- Velocity field for Taylor–Couette flow with an axial flow, Werely and Lupetow, *AIP Physics of Fluids*, 1999. https://aip.scitation.org/doi/10.1063/1.870228
- Numerical study of Taylor–Couette flow with an axial flow, Hwang and Yang, *Computers and fluids, Science Direct*, 2004. https://www.sciencedirect.com/science/article/abs/pii/S0045793003000331
- Finite Difference Method, *Berkely Python Numerical Methods*, https://pythonnumericalmethods.berkeley.edu/notebooks/chapter23.03-Finite-Difference-Method.html.
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- Instability of Taylor-Couette Flow between Concentric Rotating Cylinders, Dou et al, *International Journal of Thermal Science*, 2008. https://arxiv.org/ftp/physics/papers/0502/0502069.pdf
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- Burgers' equation, *Wikipedia*. https://en.wikipedia.org/wiki/Burgers%27_equation
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- The Visual Room: Notes on Project Work, CFD, Programming and Computing. https://www.thevisualroom.com/index.html
- Lid-driven Cavity, Guido Dhondt, MIT, 2014. https://web.mit.edu/calculix_v2.7/CalculiX/ccx_2.7/doc/ccx/node14.html