AAMO Final Exam

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Polykarp Kusch was awarded the Nobel Prize in the year 1955 for the precision determination of the magnetic moment of the electron. In this essay, the research and impact of the experiments that led to the measurement are discussed.

The most significant early experiment to study the magnetic moments of atoms was the Stern-Gerlach experiment in 1922. They aimed to measure the quantizations and projections of angular momentum in space, and expected to observe splitting into discrete components along the quantization axis with an amount of splitting according to the Bohr magneton μ_B, which they had independently determined. They expected to find three lines but discovered only two and the success of the experiment was the demonstration of the quantization of the projections of angular momentum along an axis. But the splitting of levels into two in the fine structure of the atomic spectra, the anomalous Zeeman Effect, remained unexplained. In response to this, Uhlenbeck and Goudsmit in 1925 postulated that the electron has an additional internal degree of freedom called the spin, which has a quantum number $\frac{1}{2}$ and a magnetic moment approximately equal to one Bohr magneton. The magnetic moment of the electron is by virtue of its angular momentum about a spin axis. The observation of this magnetic moment (μ_s) was twice as large as expected for an orbital angular momentum of $\hbar/2$. This was quantified by the g-factor of the electron g_s given by $\mu_s = -g_s\mu_B/\hbar S$ [4].

The Stern-Gerlach experiment showed the g_s had a value approximately equal to 2. Following this, a theoretical treatment was performed on electric spin, by Dirac's relativistic quantum theory, which postulated the spin and magnetic moment to be consequences of the relativistic invariance of the Dirac equation. The theory predicts the value of $g_s = 2$. This is considered to be one of the biggest achievements of the theory [4]. But later two limitations of the Dirac theory were found: this value of g_S which was not accurate and the Lamb shift. g_S is related to the magnetic moment and if one knows the angular momentum of the system, the magnetic moment can be obtained from the g-value [6]. Explanations for the Lamb shift was discussed extensively in the lecture [4], but the value of g_S was studied further and serves as the factor of interest in most of the research discussed here that led to the Nobel Prize for Kusch. The explanations for these anomalies go beyond Dirac theory. Kusch points out that the Dirac electron theory did not consider the interaction of the quantized electromagnetic field with the electron, and this idea was integral to his research.

Kusch's contributions begin with his participation in the development of experimental techniques that allowed the precise determination of the magnetic moment. This involved research with atomic and molecular beams used as spectroscopic devices for the observation of spectral lines, which was relevant at the time and pioneered by Isador Rabi. Rabi won the Nobel Prize in 1944 (cite something) for magnetic resonance techniques, which were extensively used in the research done by Kusch, which he contributed to [8]. The measurements by molecular beams magnetic resonance by Rabi's group involved the determination of nuclear g values by the observation in a molecule of the nuclear resonance frequency in a classically determined magnetic field [6]. At the time, ratios of different g values could be found without any limits caused by the magnetic field. Still, the determination of the nuclear moment in terms of the Bohr magneton had limited accuracy due to uncertainties in the measurement of the field and a prior measurement of the Bohr magneton. The need for this thus became apparent to Kusch.

Initially, the molecular beam magnetic resonance method was used to determine the nuclear g values for molecules without net electronic orbital or spin angular momentum. But the method could also be used to investigate HFS of atoms as well by observing the transitions between different levels and you also get the quantities $g_J\mu_0 H/h$ and g_N/g_J, which is found with limited precision. g_J is the g-value associated with total electronic angular momentum and g_N is the g-value for the nuclear magnetic moment. This extension of the molecular beam magnetic resonance to atoms was performed by Kusch, Millman and Rabi in 1940 on isotopes of alkali atoms (Lithium and Potassium) to determine their hyperfine structure constants. This was achieved with very high precision since it involved only measurements of frequency and were in agreement with measurements by spectroscopic and atomic beams previously measured and the radiofrequency method thus had greater precision than other known methods at the time [5]. This work later led to further research on atomic HFS to study higher-order moments in nuclei and radioactive nuclei using these methods.

An important consequence of these experiments was that for the measurement of the HFS, one can observe nuclear resonances of frequency $g_N\mu_0 H/h$ and transitions among the magnetic components of the HFS levels had a frequency dependence of the order $g_J\mu_0 H/h$. This could be used to calculate the magnetic moment of the proton in terms of the Bohr magneton, which was undertaken by Kusch and Millman in 1941. The experiment conducted by them, as observed by Kusch, had many problems and this led them to determine the ratio of the proton moment and electron spin magnetic moment as the Bohr magneton. One way to calculate the HFS of hydrogen is done in terms of the magnetic moment of the proton, the spin magnetic moment of the electron and the electronic wave function at the nucleus. And experiment done by Nafe, Nelson and Rabi led to the first measurement of the HFS of hydrogen in the ground state. But when the magnetic moment of the proton measured by Millman and Kusch was used to calculate the HFS there was a discrepancy of about one quarter percent. The calculation uses the spin moment of the electron twice and assumes that it is equal to the Bohr magneton. This led to a theoretical prediction by Breit suggesting in a paper in 1947 that the electron may possess an intrinsic magnetic moment and evidence did not disprove it having a value of the order $\alpha\mu_0$ [1].

This ultimately led to the investigation of the existence of such an anomalous magnetic moment of the electron by Kusch and Foley in 1947-48.

An important conceptual procedure that allowed this was that g_J is a linear combination of the electronic orbital and spin g-values: $g_J = g_L + g_S$, the orbital motion and the spin of the electron contribute to its total magnetic moment. The Russell-Saunders coupling, which is an LS coupling scheme was used since only atoms with single valence electrons were considered. So, comparing the g_J values of different atoms in different spectroscopic states would give the ratio of g_S and g_L with an accuracy that is limited by the precision of observation and the precision with which the coefficients relating the different g-values are known. (cite lecture).

To compare the g-values to obtain g_S with the method described, atoms in different spectroscopic states needed to be available for observation. This was made possible by studies after Kusch's experiment mentioned previously on alkali atoms which were in the ${}^2S_{1/2}$ state, which included work on the HFS of indium in the ${}^2P_{1/2}$ state by Hardy and Millman, work by Kusch and Becker on two isotopes of Gallium in ${}^2P_{3/2}$ states for which they had interaction constants.

The principle used to determine the ratio of two g_J values was by observing transitions for which F is constant and m_F is subject to selection rule $\Delta m_F = 1$. These are dipole magnetic transitions. Here, F refers to the total angular momentum quantum number. In the low-field limit which is, as discussed in the lecture, the Zeeman regime, all lines for a given value of F have the same frequency and the ratio of frequencies for these lines is the ratio of the g_F values. From this ratio and some properties of the nuclei of each atom, the ratio g_S/g_L for the electron could be obtained. But the HFS splitting of atomic states is generally small and the energies of the levels are not linearly dependent on magnetic field at usefully high fields. But it was possible to obtain expressions for the energies of all levels in the HFS in terms of the zero-field HFS, g_N/g_J and $g_J\mu_0 H/h$. From the observed frequencies of appropriate lines and prior knowledge of interaction constants for an atom in a given state, $g_J\mu_0 H/h$ could be determined and this measurement at the same field for two different atoms gave right away the ratio g_S/g_L. The importance of this was that it was independent of knowledge of the magnetic field of fundamental constants, which was the source of the uncertainty that could not be overcome before this.

Experimental methods that facilitated this measurement with high precision involved the choice of field that ensured all observed lines had a frequency of the order of 1 megacycle per gauss and excessive distortion of the lines from the inhomogeneity of the field was avoided by adjusting the field meticulously. Arrangements were also made to allow the rapid interchange of ovens for the atomic beams so that measurements for different atoms could be made in rapid succession. Lots of oscillators were used so that several frequencies which differed by large factors could be applied to the RF circuits that induced the transitions. The field used was not fixed but varied monotonically throughout the series of measurements and this had the drawback of requiring a lot of data to establish the frequencies of two or more lines at fixed fields but it prevented errors that arise from repetitions of reading a fixed quantity. From this experiment, they were able to measure and compare the g_J values for different alkali metals and this showed that they deviated from the nominal values of the Bohr magneton. Subsequent independent experiments performed by Kusch and Mann and Taub for Indium and Sodium at much larger fields provided further evidence. The ratio of g_S/g_J obtained was 2(1+0.00119) which was in agreement with measurements for different fields. Based on all this evidence, they were able to conclude that the electron does possess an intrinsic or anomalous magnetic moment greater than that predicted by Dirac theory and about 0.119% of the Bohr magneton [6].

The impact of these results is seen in the immediate formulation of new procedures in quantum electrodynamics theory, with which Schwinger in 1948 came up with a theoretical result for the value and obtained a value of $g_S/g_L = 2(1.00116)$, which was in excellent agreement with their experimental measurement. The increased electron moment comes from the quantization of the electromagnetic field which always has a residual zero-point amplitude. The existence of this field had not been recognized and hence it was not possible to accurately deal with the interaction before this theoretical formulation [6]. This is perhaps the most significant impact the result of the experiment had, which is the formulation of contemporary quantum electrodynamics.

These results also demanded further studies on the magnetic moment of the electron, to also be able to address theoretical difficulties in interpreting g-values for complex atoms with higher precision. The experimental methods of spectroscopy by atomic beams could be used to obtain results that would be sufficiently precise to test QED calculations of higher orders than those by Schwinger. For further investigation, Kusch with Koenig and Prodell determined the g value for the hydrogen atom. The hydrogen atom was chosen since it is the closest to a free electron for which the spin moment could be measured for and in the ground state, there is no contribution to the electronic magnetic moment from the orbital motion, it comes entirely from the spin g'_S. g_J was found to be equal to g'_S for very high orders of accuracy. This varies slightly from the free electron g_S by some small relativistic effect but corrections for this were well known and could be made, and by combining their results with Gardner and Purcell for the cyclotron frequency of the electron, they obtained the desired value. They also determined the HFS of hydrogen with high precision [6].

Until then magnetic gaps contributed to the inhomogeneity when internal to the vacuum system but they had the magnet external to the vacuum envelope which had much better field homogeneity and the magnets were current-carrying conductors which reduced distortions as opposed to conventional iron magnets ([6]). Another component used was a device that inserted a cylindrical sample of water or mineral oil as closely as possible to a region where an atomic line was observed to provide diamagnetic shielding and assess homogeneity. All these corrections were accounted for and the nuclear g-value in a spherical sample of mineral oil was found. Gardner and Purcell also did their measurement for a spherical sample of mineral oil and on applying corrections and combining their results they obtained: $g_S = 2\mu_S = 2(1.001146 \pm 0.000012)$. Subsequent studies by Beringer and Heald used different experimental methods to obtain results in good agreement [6].

The result of their experiment gave evidence for the anomalous magnetic moment of the electron and its deviation from the nominal value was about $\alpha\mu_0/2\pi$. Higher order theoretical calculations were made by Karpus and Kroll and agreed with the experiment, with a value of $g_S = 2(1.0011454)$, giving validity to calculations of the order α^2, providing validity for the newly formulated procedures of QED at the time.

As of today, there has been extensive research since Kusch's experiment to determine the value of the electron magnetic moment and these have reached higher orders of magnitude, with far more advanced experimental techniques that have been able to isolate single electrons and have offered more precise measurements. The current method of measurements involves using a Penning trap or the geonium, which was developed by H G Dehmelt and R S Van Dyck [10], and they used this for precise measurement of the magnetic moment of the free electron in 1987 [11] and Dehmelt was awarded the Nobel prize for the same in 1989. This remained the most precise measurement until 2006 [7] and then in 2008 by the same group [3]. The current forerunner 14 years past then is research done in 2022 by remnants of the same group led by G Gabrielse [2].

But the significance of Kusch's efforts has to do with providing evidence of the existence of the anomalous magnetic moment and its behaviour in hydrogen, having been the first to do so and this led to the validity of the newly formulated theoretical calculations of QED at the time, which is what has led to its advancement today. The magnetic moment also plays a significant role in particle physics, being the most precise standard model prediction yet. Further, the experimental methods used back then in molecular beam magnetic resonance became crucial to the development of NMR spectroscopy techniques in analytical chemistry and MRI in medicine. The improvements in NMR also contributed to advanced electronics [9].

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