Mechanisms of symmetry breaking in optimal networks

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Finding the optimal flow through a given network is relevant in a wide range of systems, including urban transportation, power grids, river basins, plant and animal venation. Gaining insight into the mechanisms behind the structure of optimal networks is of crucial importance, especially in the context of designing transport systems. Many such systems involve multiple modes or layers of transport, motivating the study of optimal flow in multiplex networks [1].

In this article we introduce an analytically solvable model for symmetry breaking in optimal networks based on [2,3]. We provide a comprehensive solution for elementary model networks and demonstrate two different mechanisms of symmetry breaking. First, the optimal state can undergo a bifurcation, such that the symmetric state vanishes entirely. Second, the symmetric and the symmetry-broken state co-exist and exchange roles as global and local minima of the optimisation problem. Examples of these two different mechanisms are shown in Fig. 1.

We extend our fundamental results to (i) study the impact of uncertain supply and demand and (ii) the optimal structure of multilayer networks. Combining analytic and numeric tools, we show how noise impacts the emergence to symmetry breaking and construct the phase diagram. For multi-layer networks, we demonstrate symmetry breaking between layers, where one layer carries the entire flow despite a symmetry in supply and demand between the layers. Our results can hold importance for future energy systems, where electricity, gas and heat networks are integrated and generation becomes increasingly uncertain [4].

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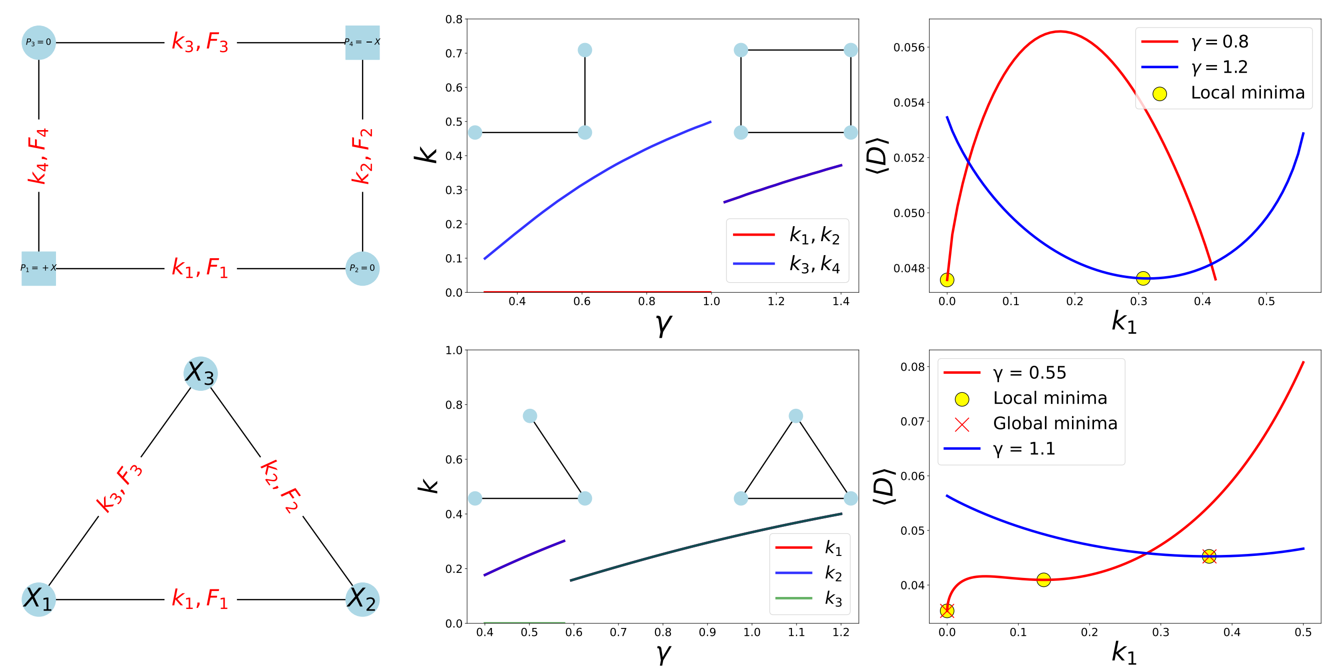

Figure 1: Two mechanisms of symmetry breaking under the change of a cost exponent γ (cf. [2,3]): a bifurcation (top row) and the exchange of local and global minima (bottom row).