Math 394 HW1

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Problem 1

1) Of the seven symbols, there are 3 spots that need to be letters, and the other four are digits. We must choose which of the seven must be letters, each of which have 26 choices (assuming all capital letters). By choosing the initial three spots to be letters, we are implicitly choosing the remaining four spots to be digits, for which there are 10 options per spot. By the counting principle (?) there are a total of:

$$\binom{7}{3} * (26 * 26 * 26) * (10 * 10 * 10 * 10) = \binom{7}{3} * 26^3 * 10^4$$

possible license plates.

2) If there are now no restrictions on how many spots are reserved for letters/digits, we essentially can choose either a digit or a letter at any spot, for a total of 36 choices at each spot. Thus there are a total of:

$$36^7$$

possible license plates.

Problem 2

The system is functioning if either A and/or B occur with C:

$$functioning = \{(A \cap C) \cup (B \cap C)\}$$

Problem 3

The sample space of rolling two fair six sided die is:

$$\begin{aligned} SS = \{&(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),\\ &(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),\\ &(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),\\ &(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),\\ &(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),\\ &(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\} \end{aligned}$$

where the first element of each tuple in the sample space represents the value of the first roll and the second element of each tuple represents the value of the second roll.

The event space of A is:

$$A = \{(1,2),(1,3),(1,4),(1,5),(1,6),(2,3),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6),(4,5),(4,6),(5,6)\}$$

Of the 36 possible outcomes from rolling two fair six sided die, 15 outcomes belong in A, so

$$P(A) = \frac{15}{36}$$

Problem 4

$$\binom{10}{4} = \frac{10!}{4!6!} = 210$$
$$\binom{12}{9} = \frac{12!}{9!3!} = 220$$
$$\binom{8}{2} = \frac{8!}{2!6!} = 28$$

Problem 5

$$\begin{aligned}(1+x)^5 &= \sum_{k=0}^{5} \binom{5}{k} 1^k * x^{5-k} \\ &= x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1\end{aligned}$$

Problem 6

Suppose there are a group of n males and m females for a total of $n+m$ people. We want to count the number of ways to select p people from this group, which is:

$$\binom{n+m}{p}$$

Another way to count the number of ways to select p people (given that there are only males and females in the group) are to first pick a males and then b females where $a+b=p$. For given values of a and b, there are $\binom{n}{a}\binom{m}{b}$ ways to choose. Furthermore, since values of a can range from 0 to p for which values of b would range from $p-a$ to 0, we can count the number of ways to select groups of a males and b females for all values of a and b as:

$$\sum_{a=0}^{p} \binom{n}{a}\binom{m}{p-a}$$

Problem 7

Let A={first computer works}, B={second computer works}, C={printer works}. The event in which at least two of the devices work can be expressed as:

$$functioning = \{(A \cap B) \cup (B \cap C) \cup (A \cap C)\}$$