

Lecture 5

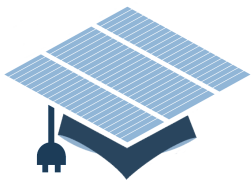
Junctions!

R. Treharne

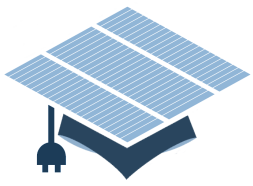
Nov 6th 2014



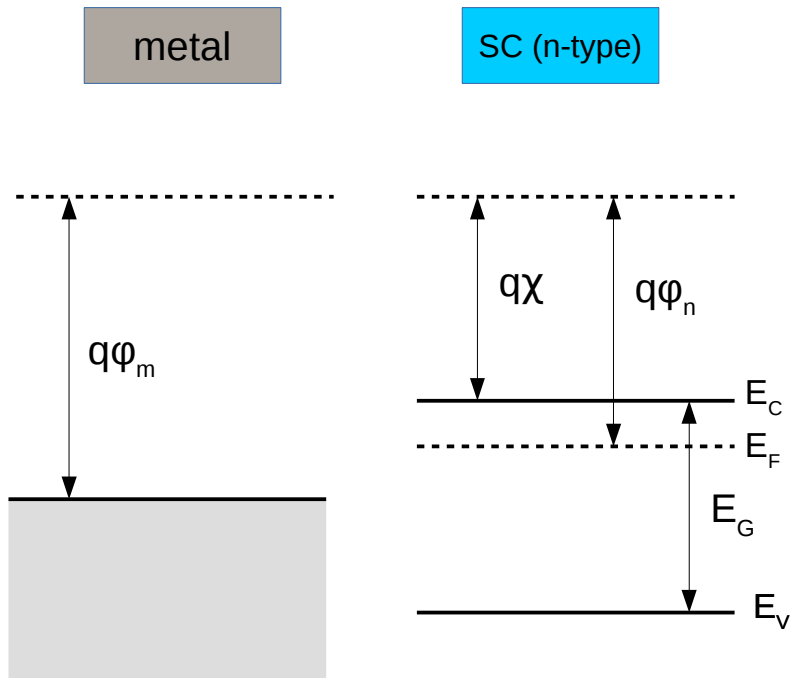
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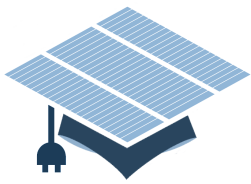


- Drawing Junctions
- Junction Formation – A Physical Picture
- Abrupt Junction
- Deriving Important Junction parameters
- Deriving diode equation
- Ideality Factor - Recombination

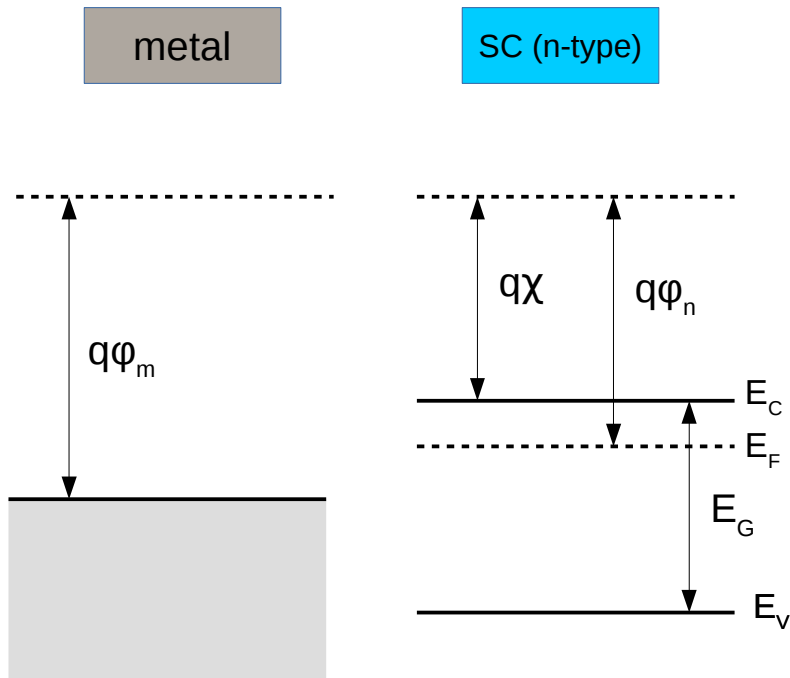


Schottky Junction



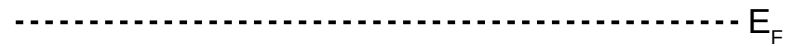


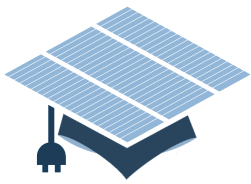
Metal-Semiconductor



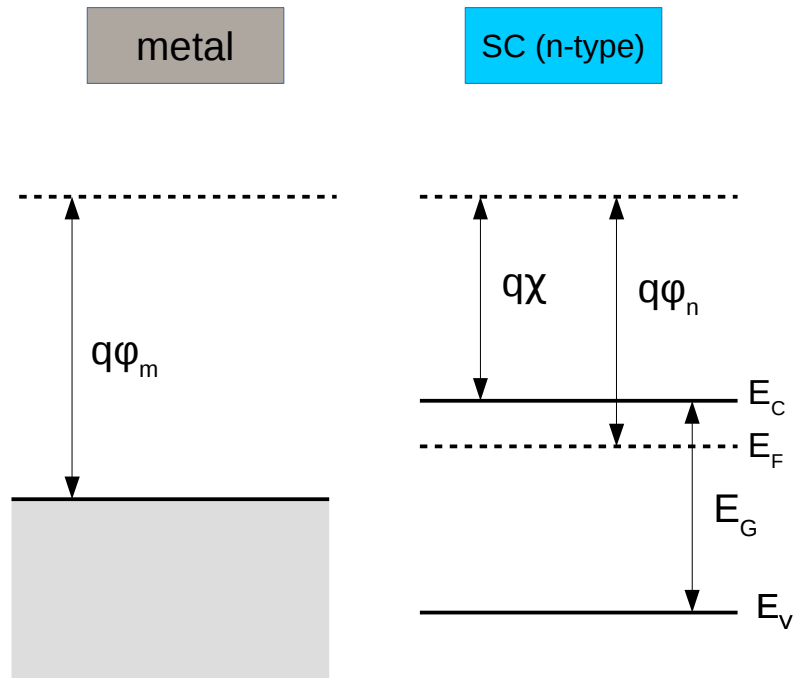
One Rule!

- Fermi level must be in equilibrium

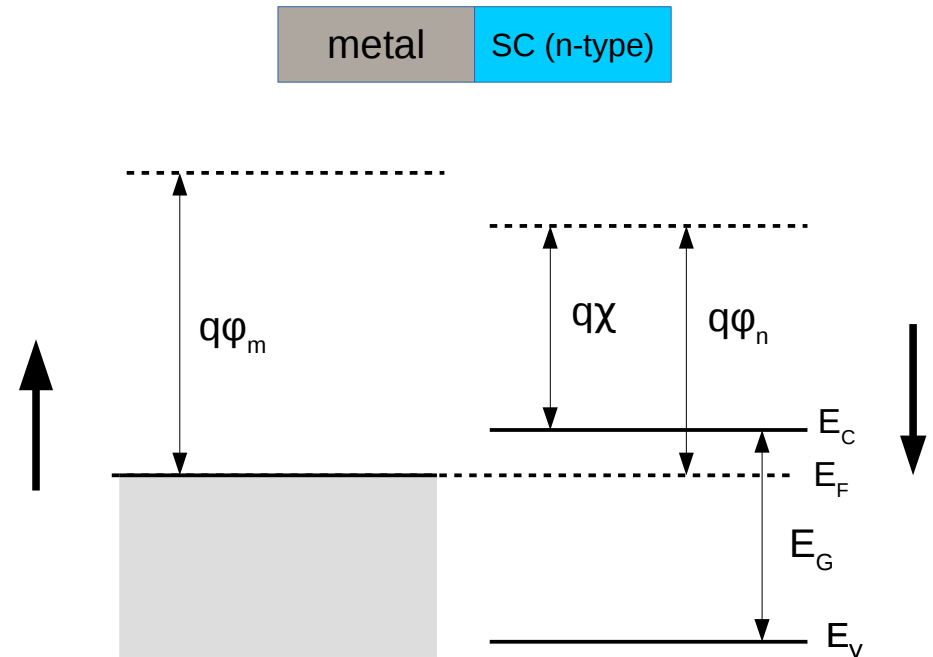


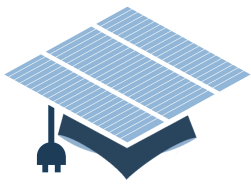


Metal-Semiconductor

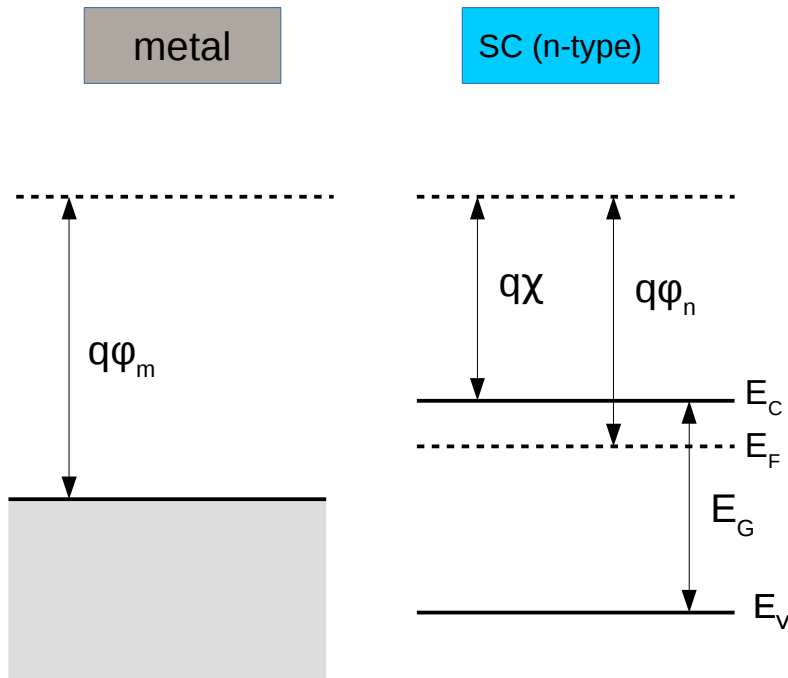


“Intimate Contact”

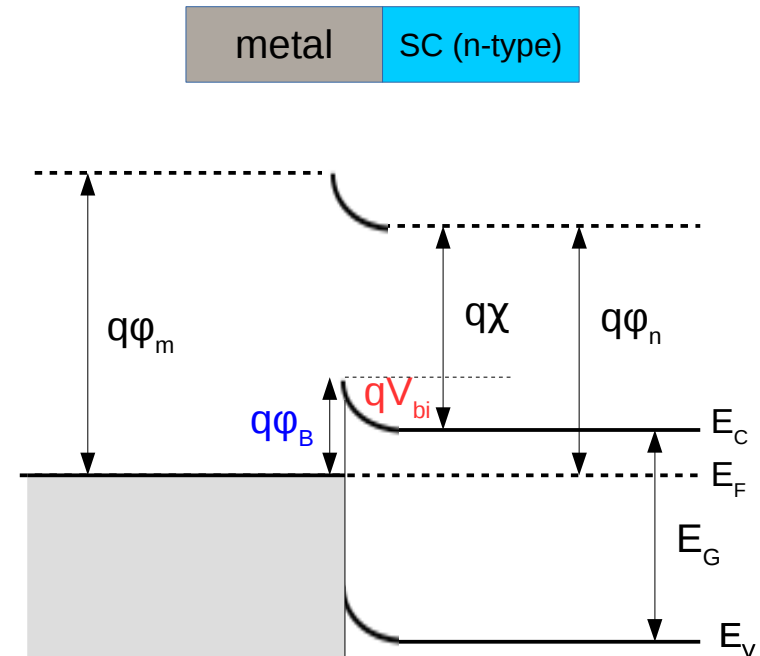




Metal-Semiconductor



BAND BENDING!

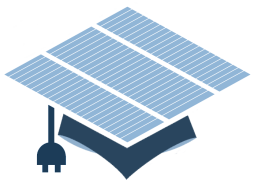


“Built-in Voltage”

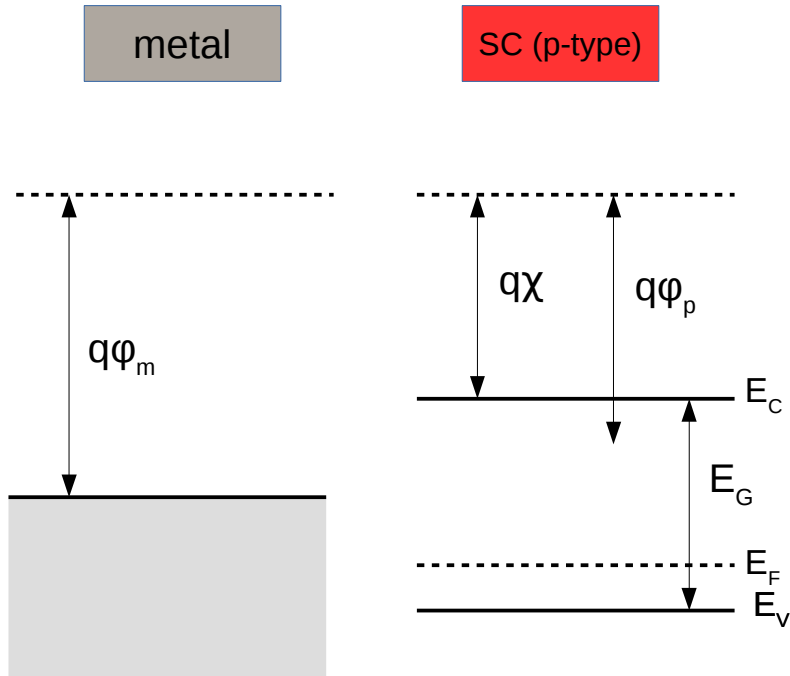
$$V_{bi} = \phi_m - \phi_s$$

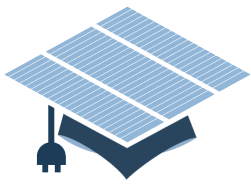
$$\phi_B = \phi_m - \chi$$

Barrier Height

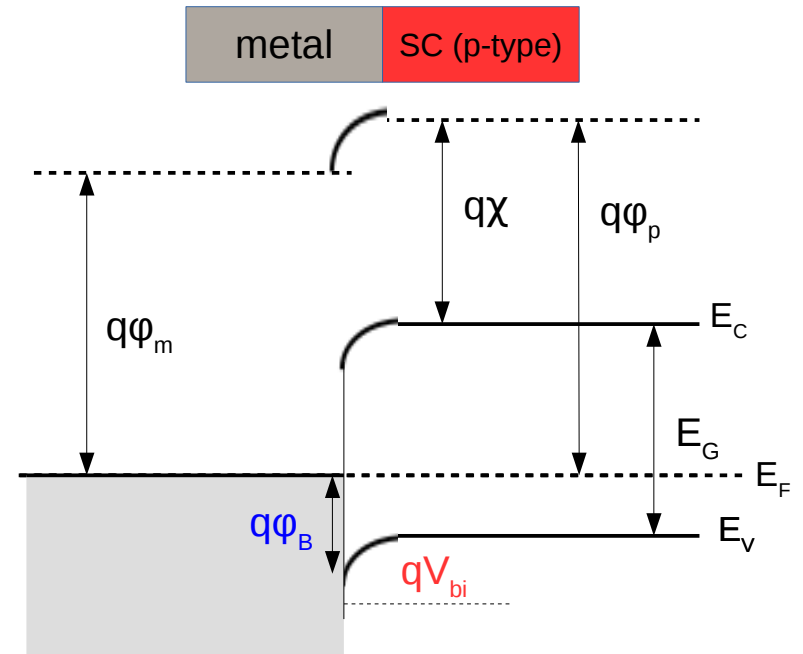
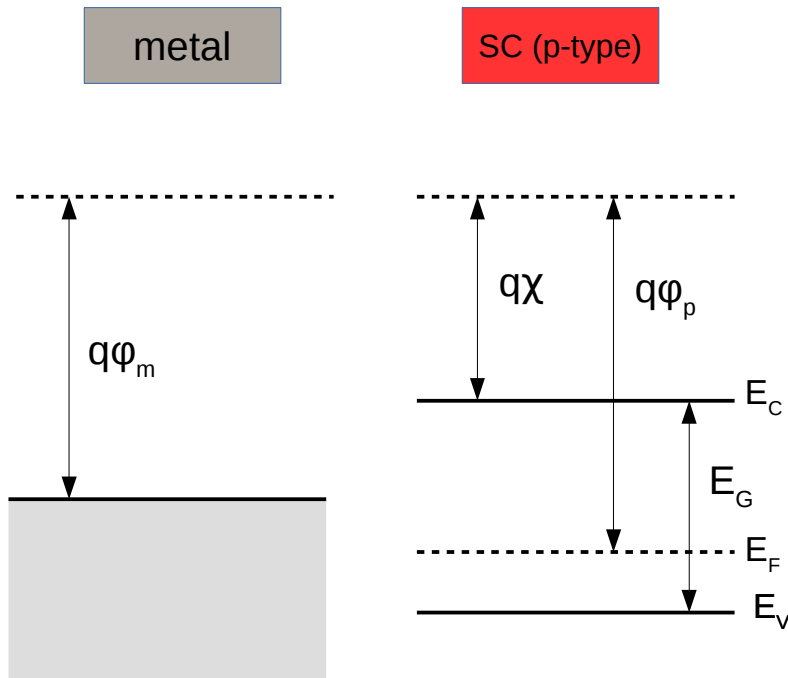


Metal-Semiconductor



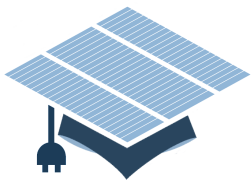


Metal-Semiconductor



$$V_{bi} = \phi_m - \phi_p$$

$$\phi_B = E_G/q - (\phi_m - \chi)$$

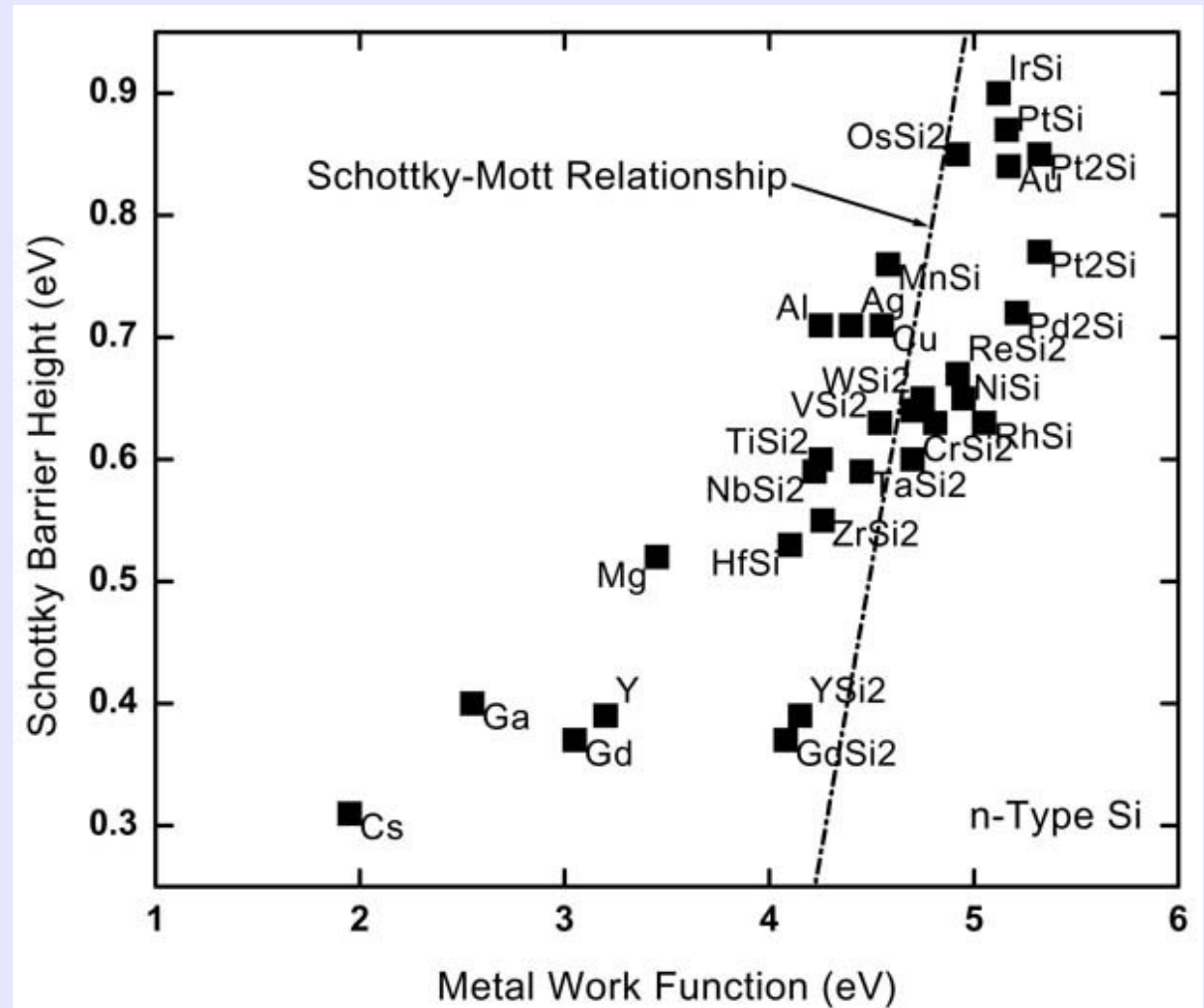


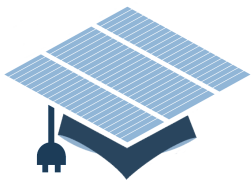
Metal-Semiconductor

$$\phi_B = \phi_m - \chi \longrightarrow \text{Why doesn't this rule hold?}$$

Metal – SC interface effects

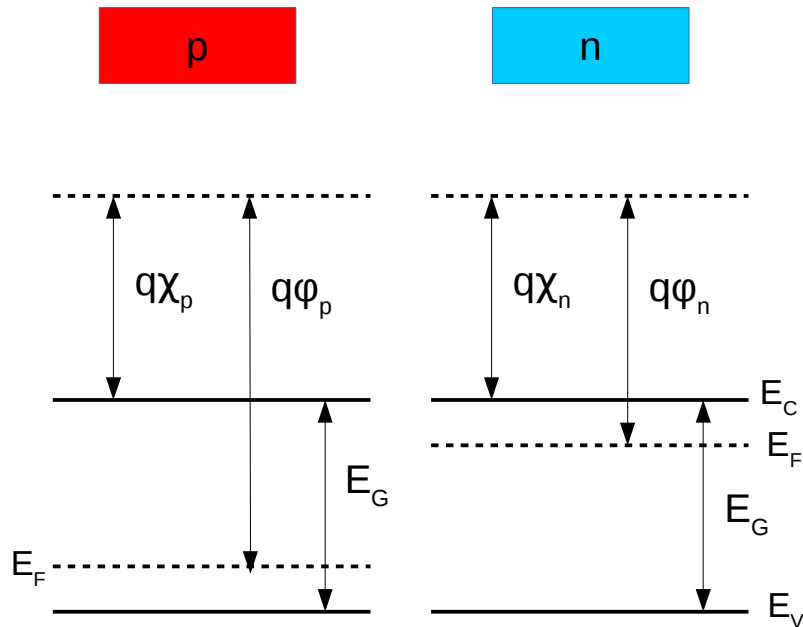
- Lattice Mis-match
- High density of defects
- Interface states in gap

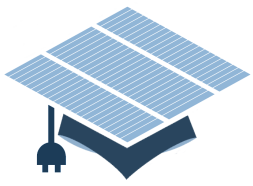




Semiconductor-Semiconductor

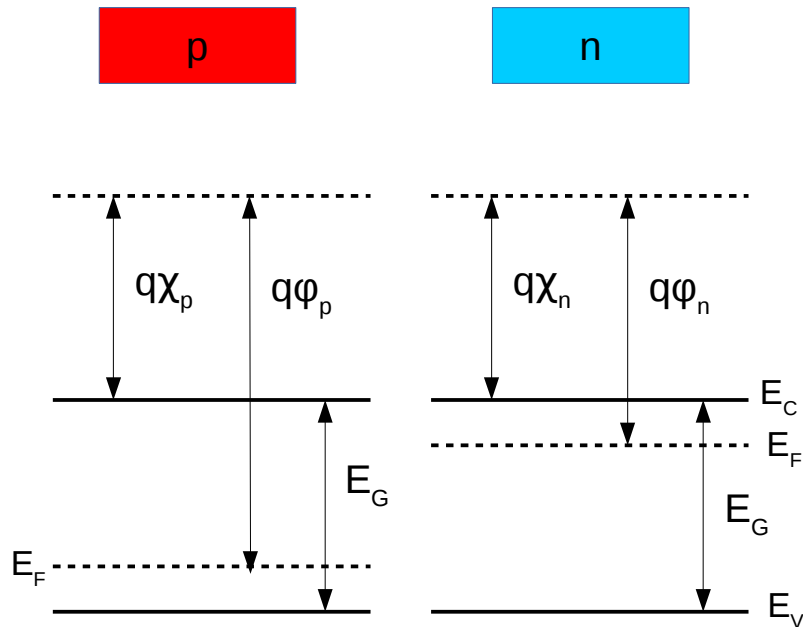
p-n homojunction



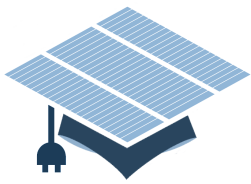


Semiconductor-Semiconductor

p-n homojunction

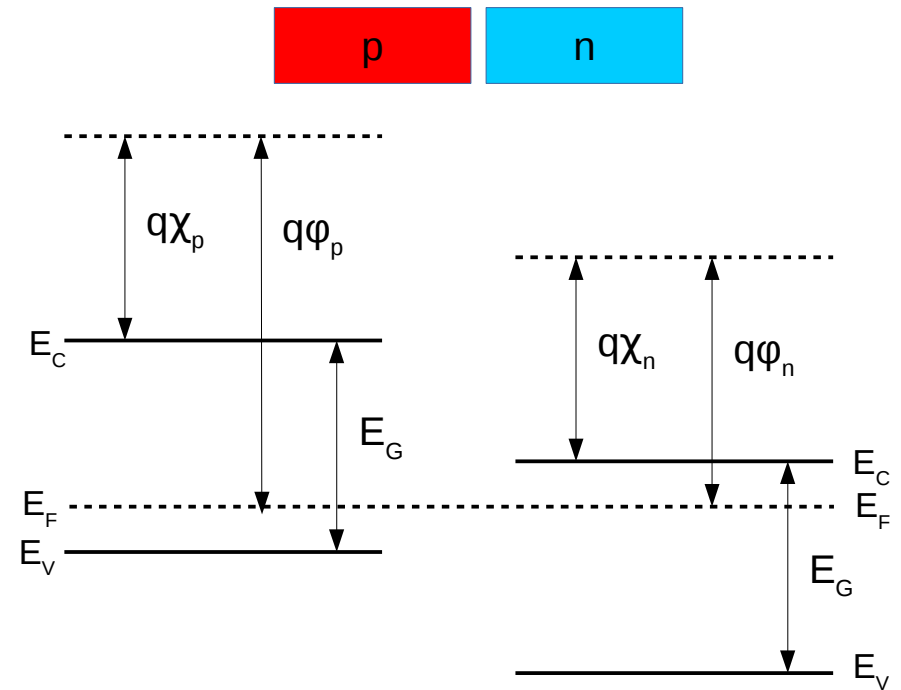
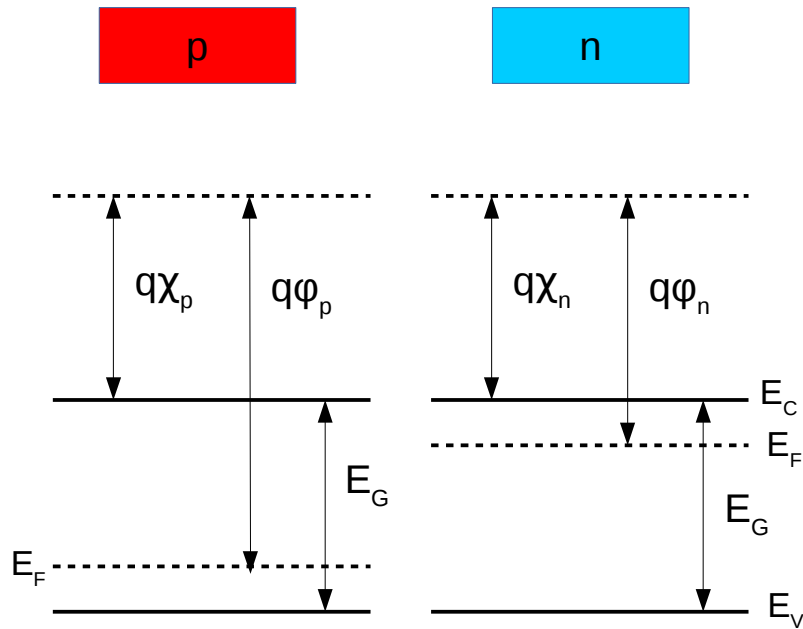


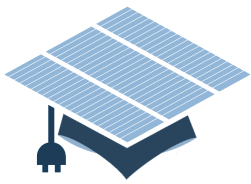
E_F ----- E_F



Semiconductor-Semiconductor

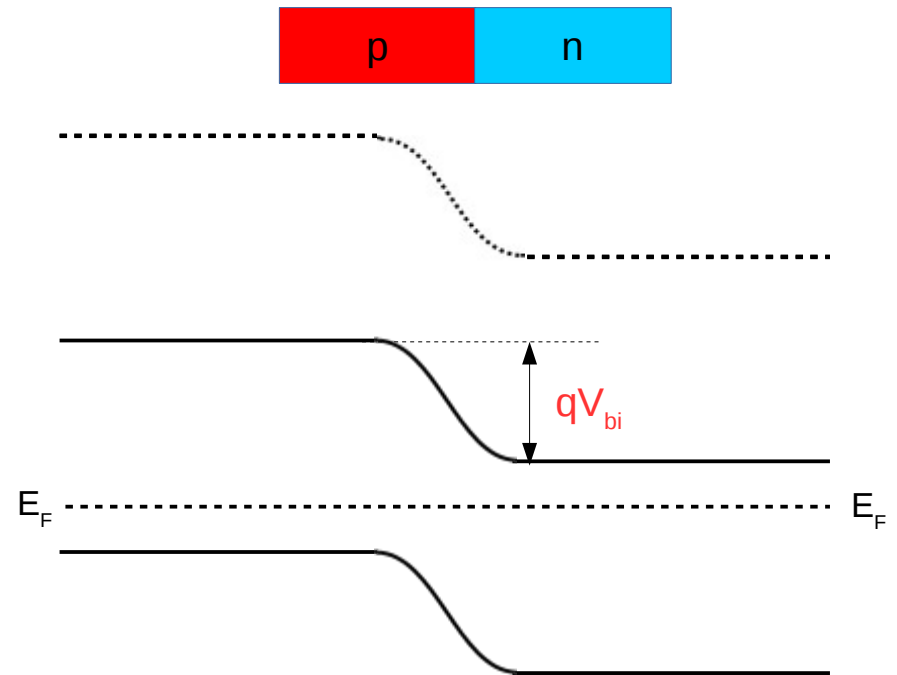
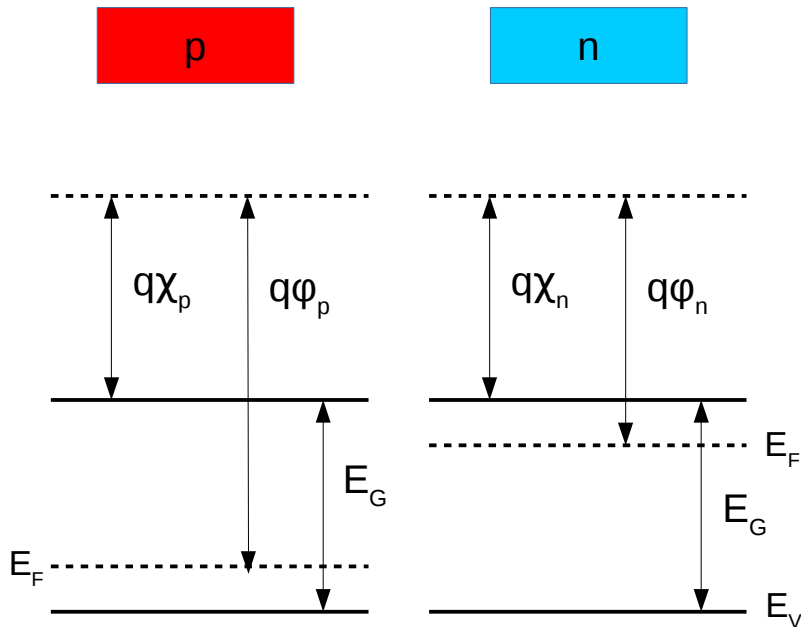
p-n homojunction

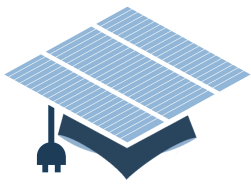




Semiconductor-Semiconductor

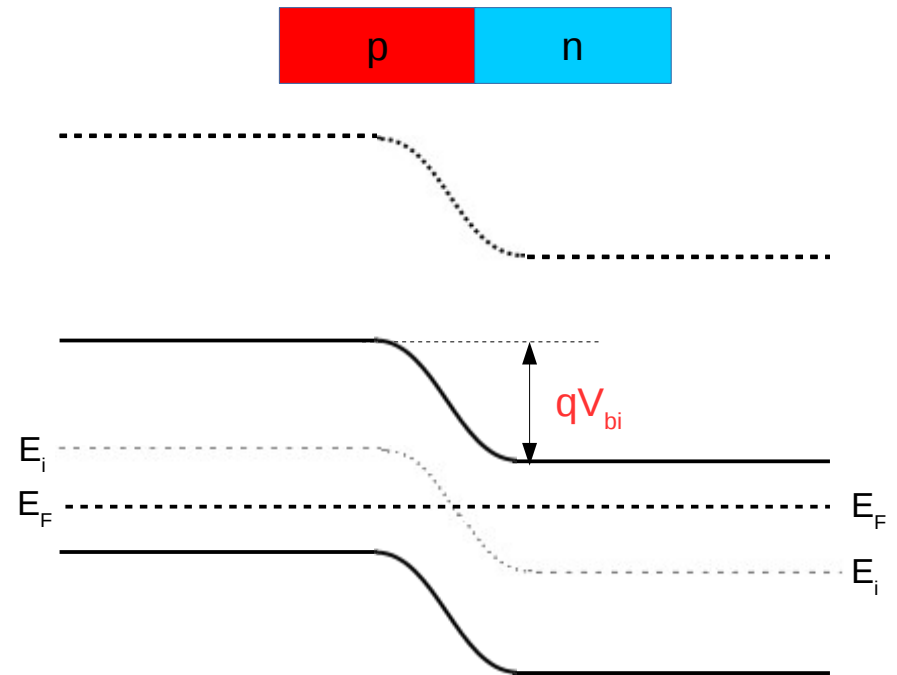
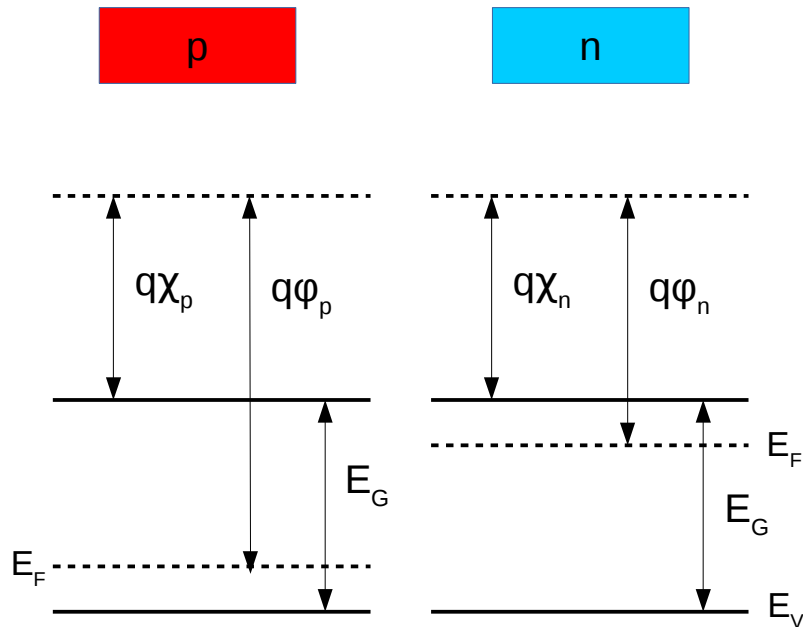
p-n homojunction

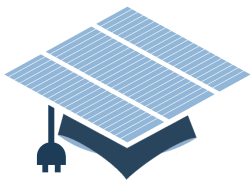




Semiconductor-Semiconductor

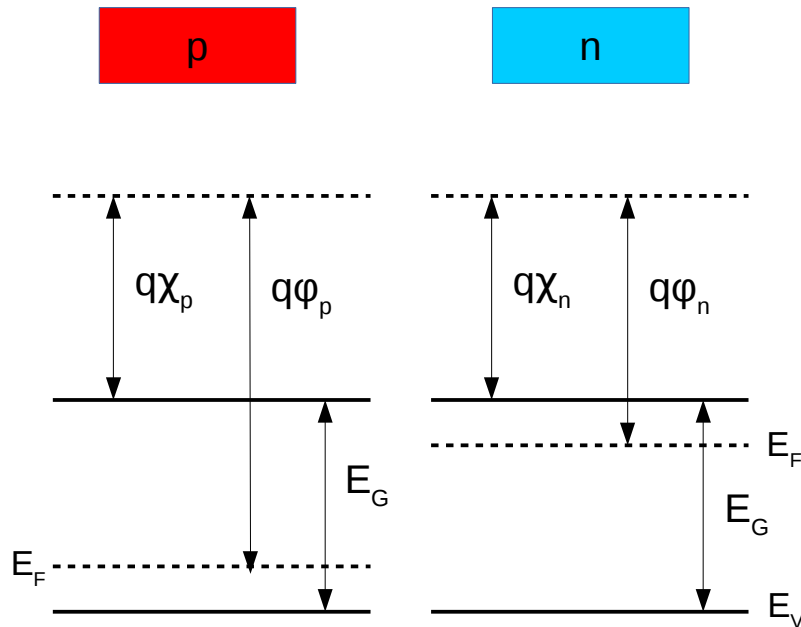
p-n homojunction





Semiconductor-Semiconductor

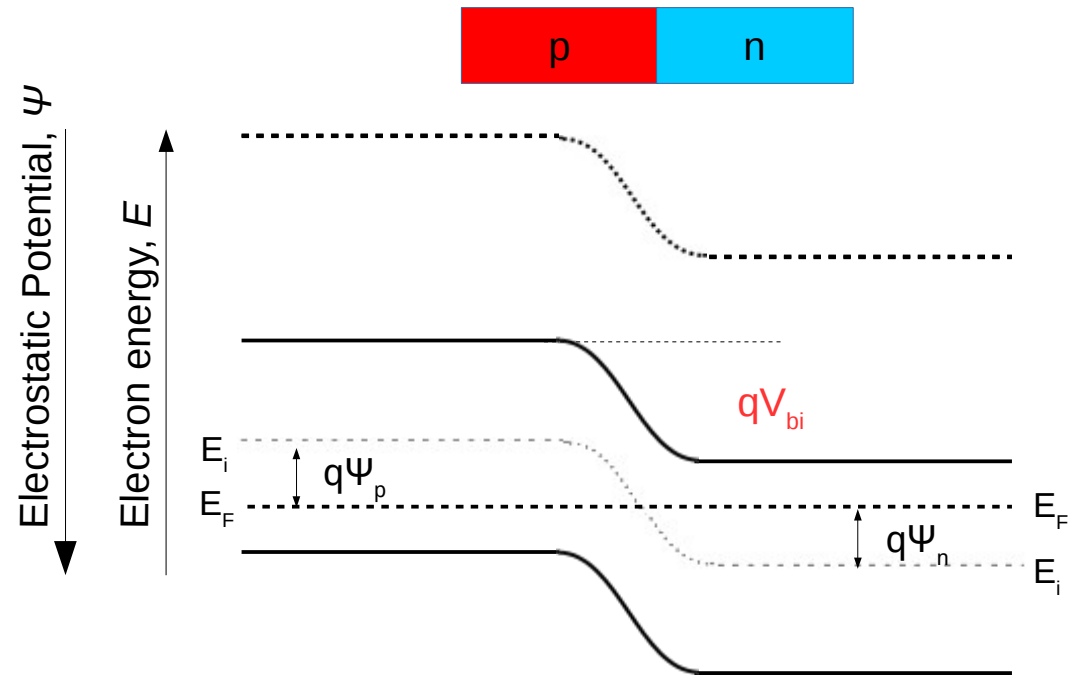
p-n homojunction



KNOW HOW TO DRAW THIS!

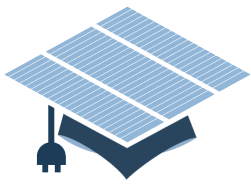
Defining Electrostatic potential, Ψ , for a SC

- Difference between E_i and E_f



$$V_{bi} = \Psi_n - \Psi_p$$

Coming back here shortly to
get a more physical picture



Semiconductor-Semiconductor

Everybody's favourite homojunction - **Silicon**

Every bit of electronics you own is jam packed with silicon homojunctions



iPhone 5:
1 billion transistors!
~ 45 nm wide



Mono-C solar cells

~750 μm thick
cell size = 1 cm^2

Extrinsically doped

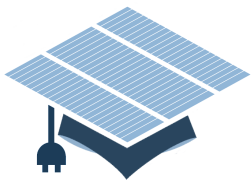
p-type

n-type

5 B Boron 2.34	6 C Carbon 2.62	7 N Nitrogen 1.251
13 Al Aluminum 2.70	14 Si Silicon 2.33	15 P Phosphorus 1.82
31 Ga Gallium 5.91	32 Ge Germanium 5.32	33 As Arsenic 5.72

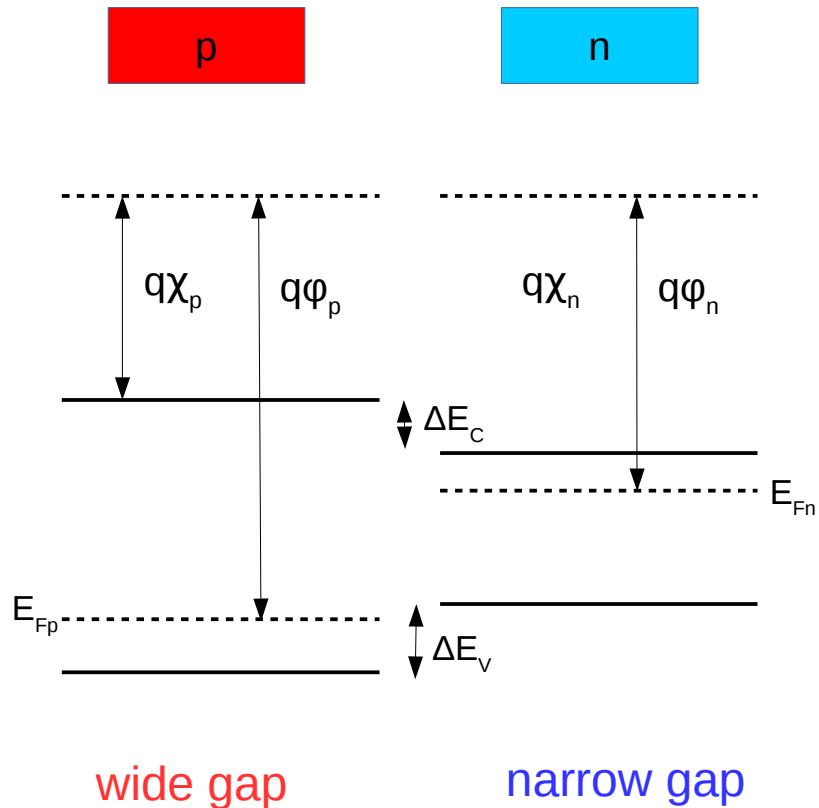
©2001 HowStuffWorks

<http://electronics.howstuffworks.com/diode1.htm>



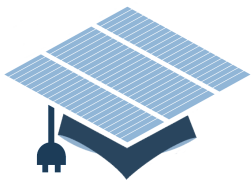
Semiconductor-Semiconductor

p-n heterojunction: **Different band gaps**



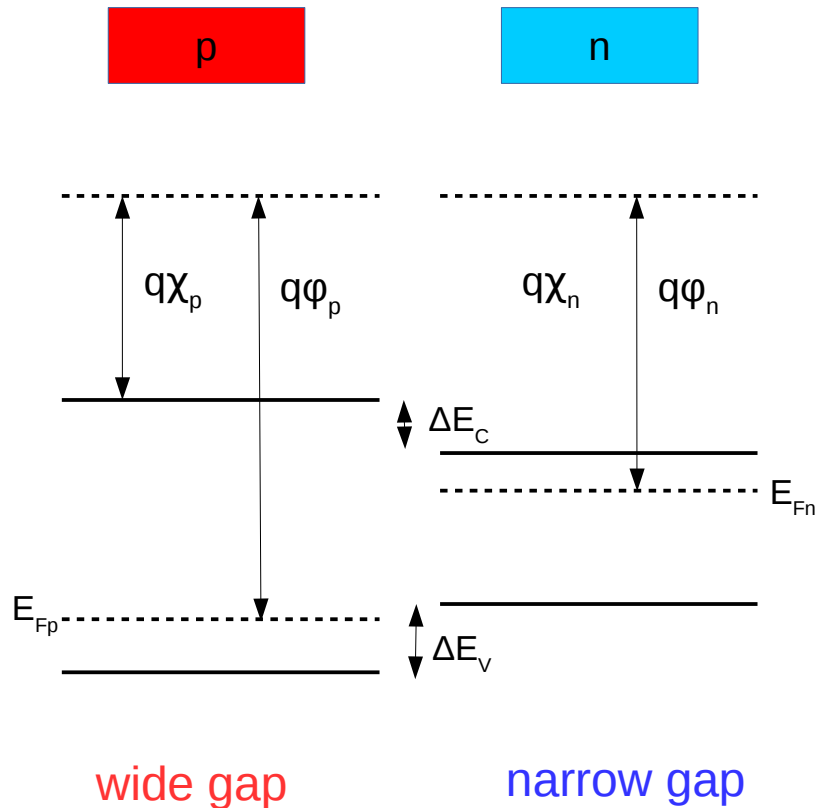
1. Align the Fermi level. Leave some space for transition region.



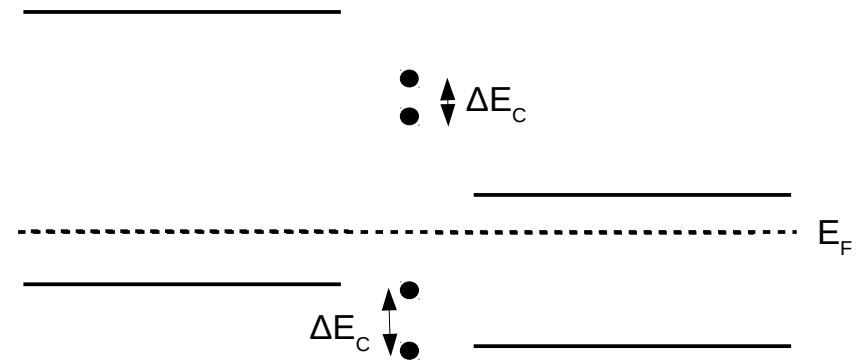


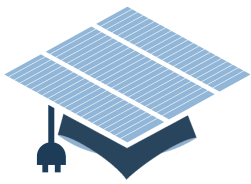
Semiconductor-Semiconductor

p-n heterojunction: **Different band gaps**



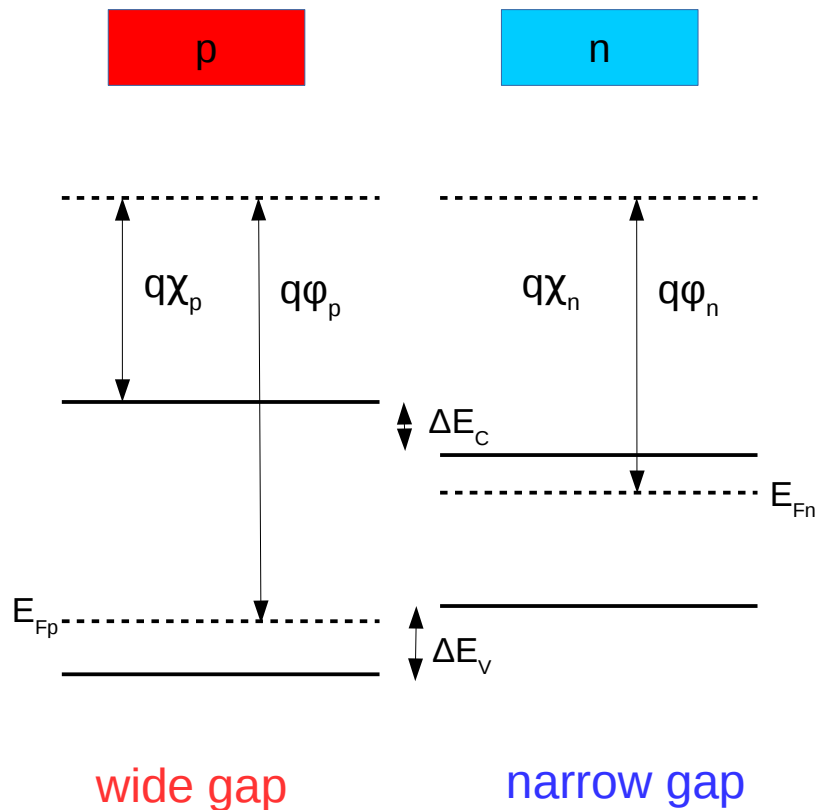
2. Mark out ΔE_c and ΔE_c at mid-way points





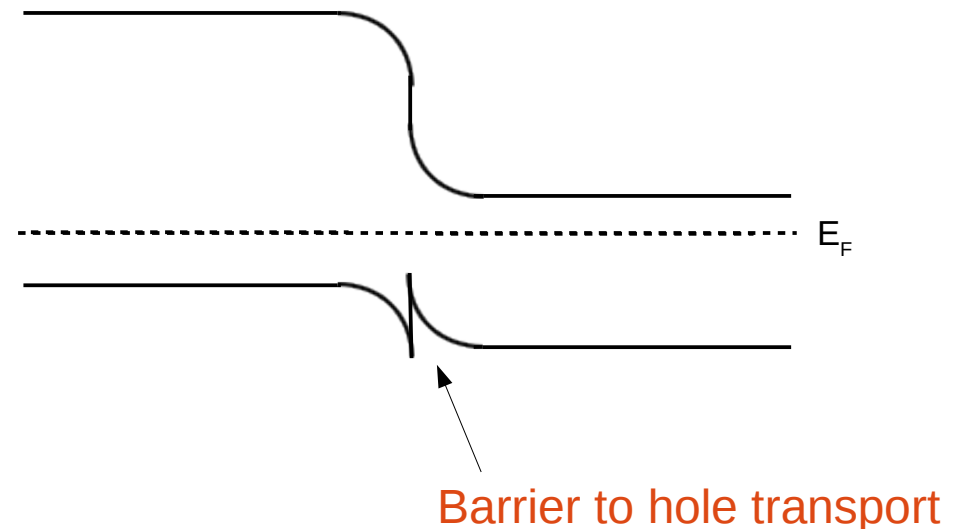
Semiconductor-Semiconductor

p-n heterojunction: **Different band gaps**



3. Connect the C.B. and V.B.
keeping the band gap constant in
each material

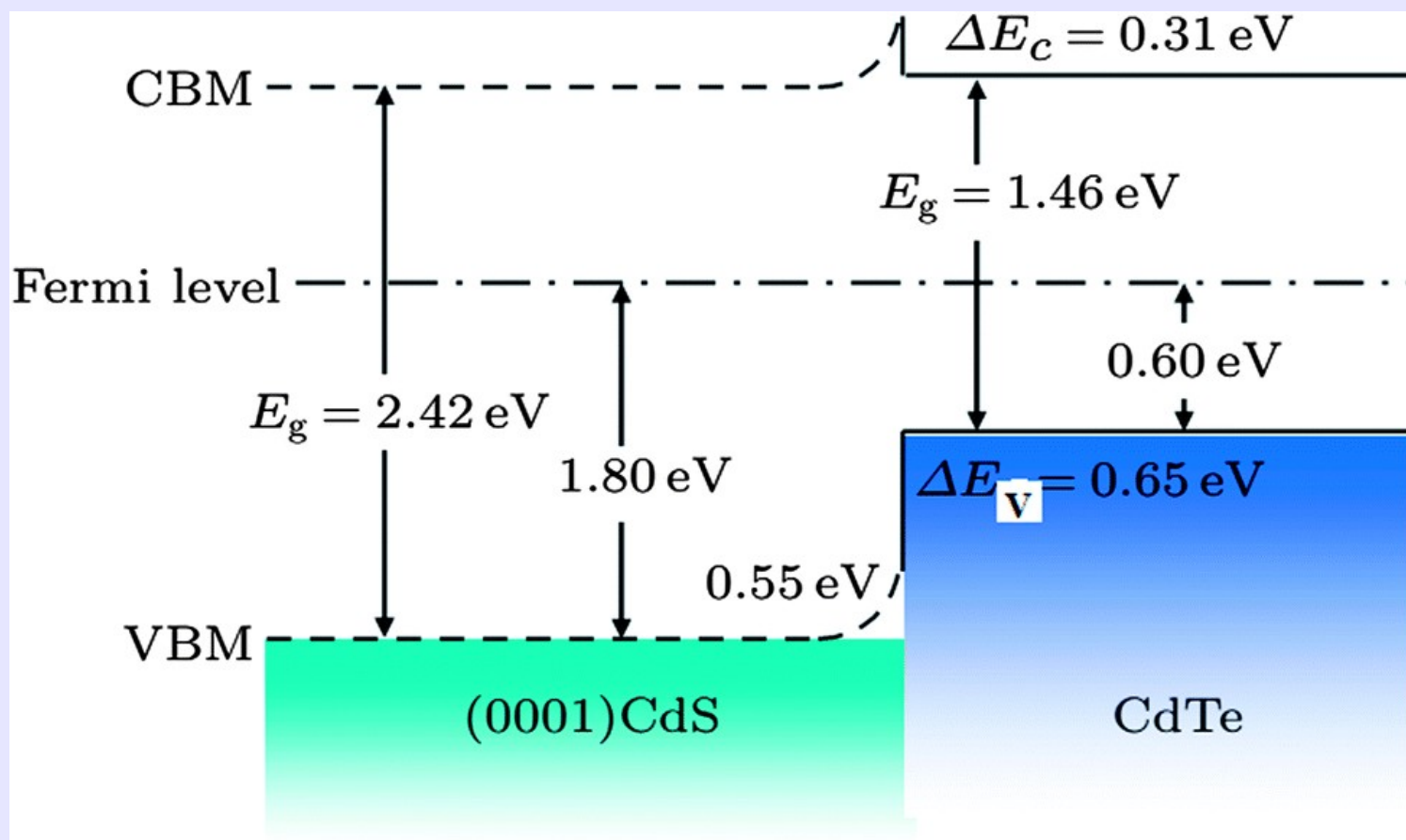
Difference in band gaps give rise to
discontinuities in band diagrams. This
limits the carrier transport by introducing
potential barriers at the junction

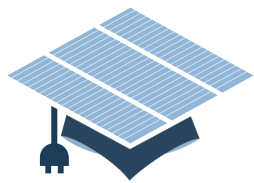




Examples of p-n heterojunctions

Most thin-film PV technologies

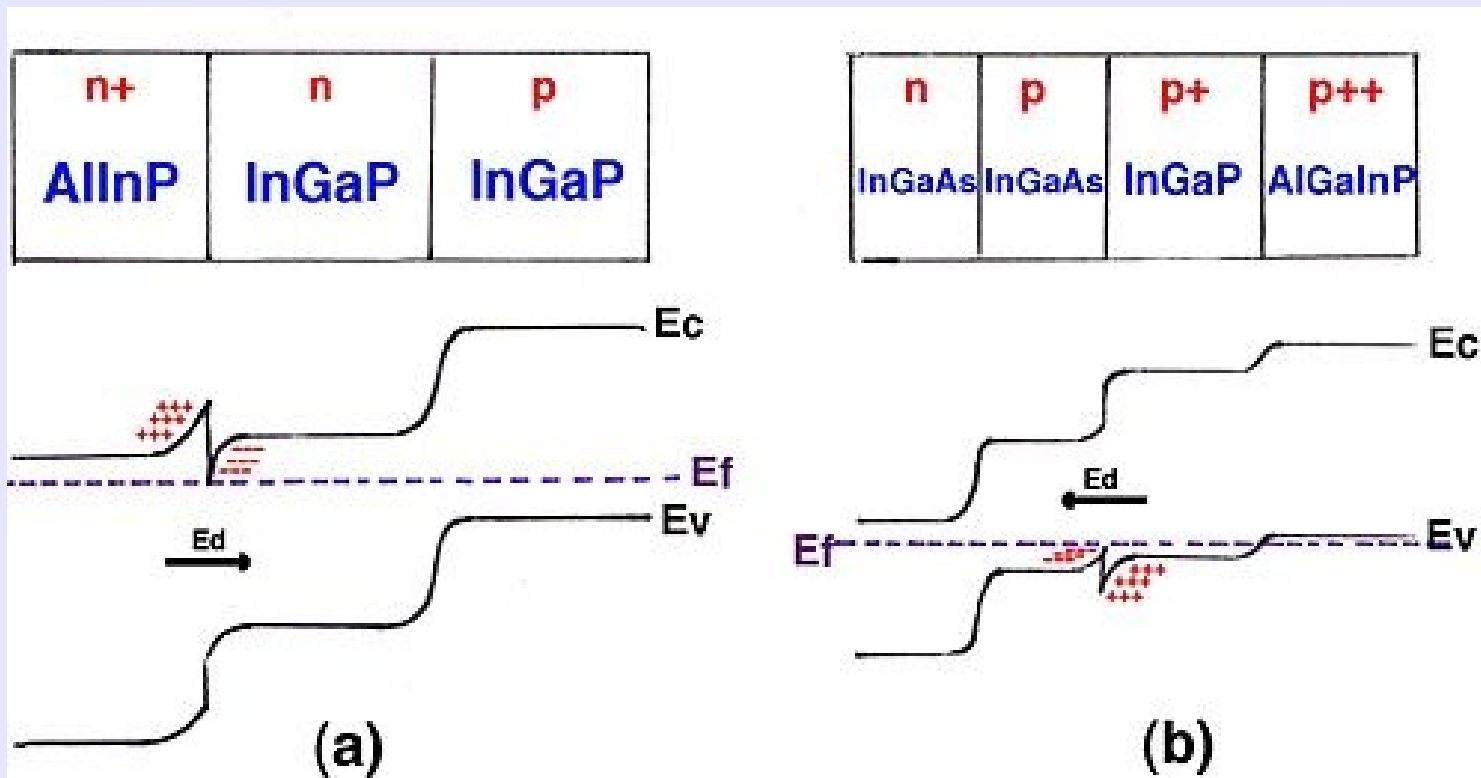




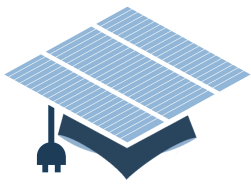
Semiconductor-Semiconductor

Examples of p-n heterojunctions

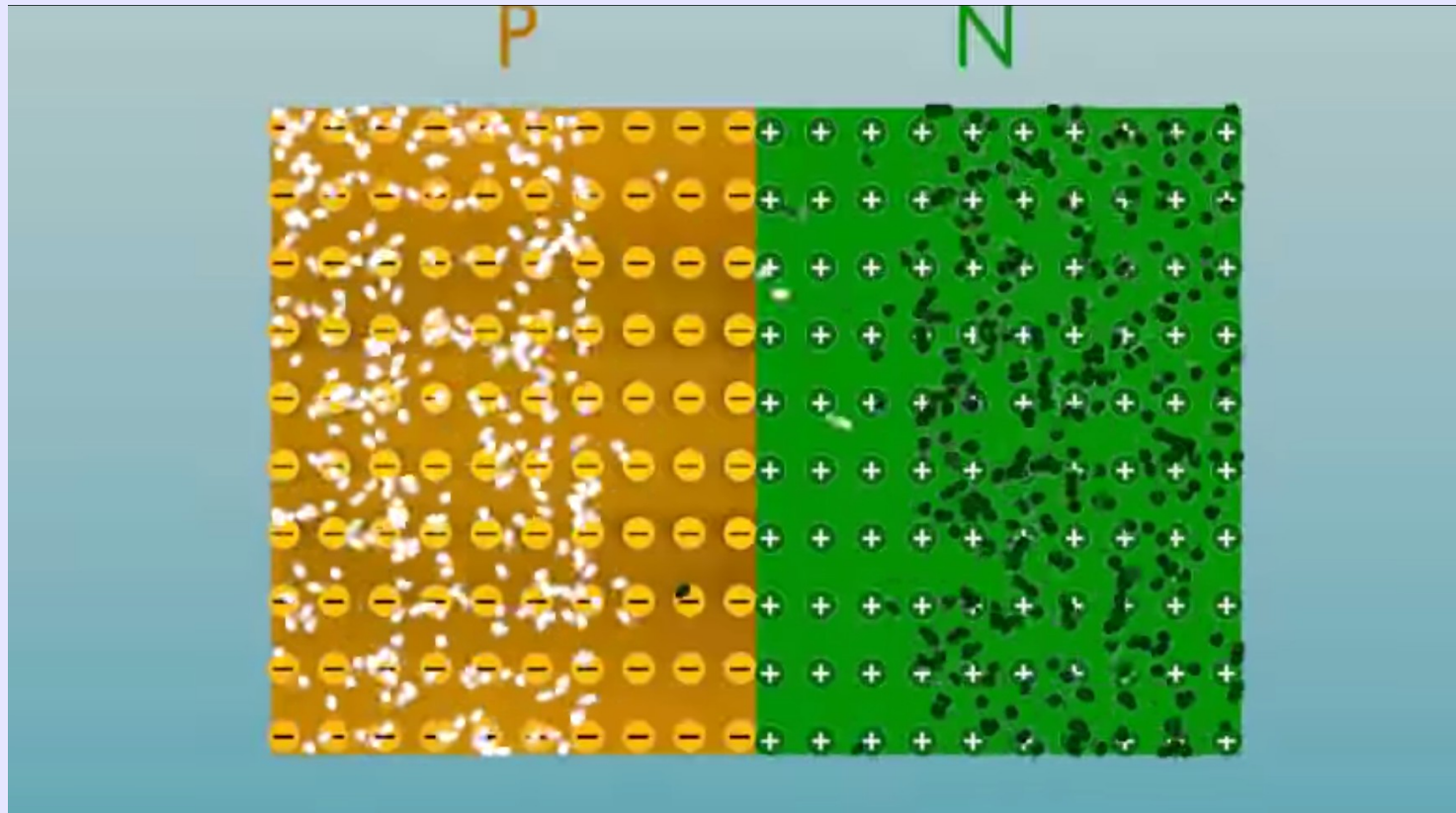
III-V multi-junction devices



Being able to control the band offsets of a material can help eliminate barriers

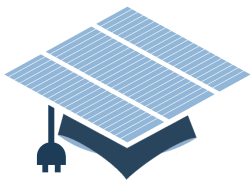


Drawing band schematics for junctions all day is fun, but what is actually going on here?



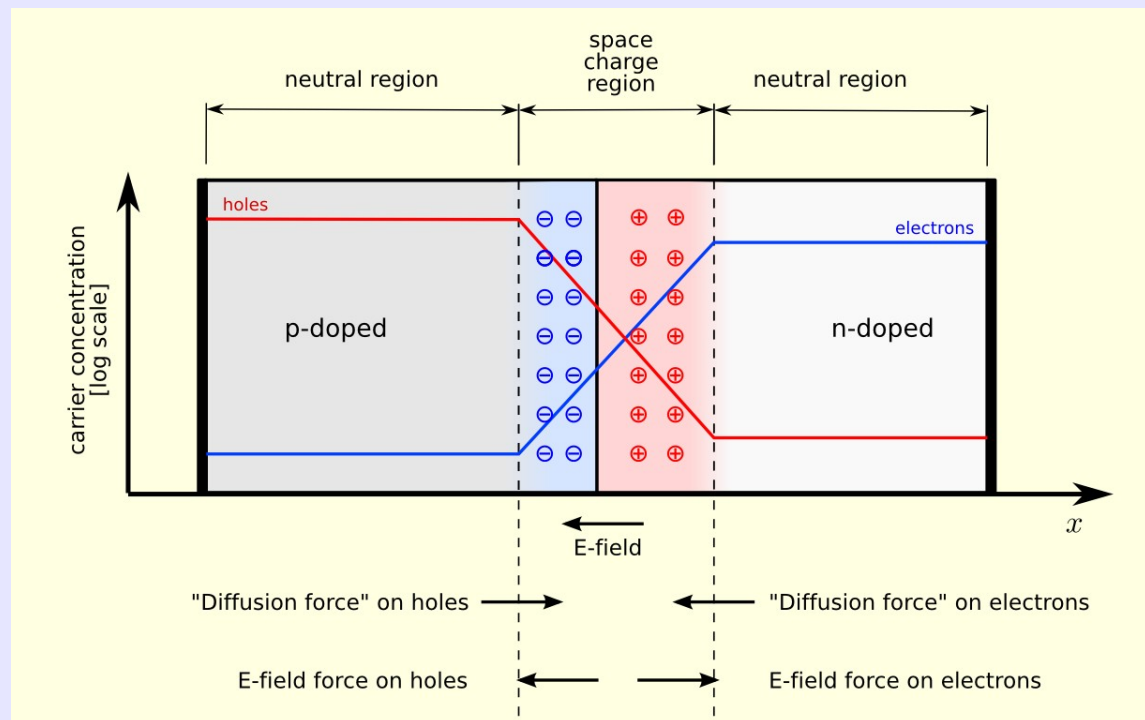
<http://www.youtube.com/watch?v=JBtEckh3L9Q>

Excellent qualitative description
6:27 – Didactic Model of Junction formation



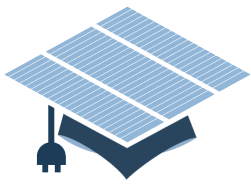
Equilibrium Fermi Levels – Explained – our one rule for drawing band schematics!

In thermal equilibrium, i.e. steady state: **not net current flows**



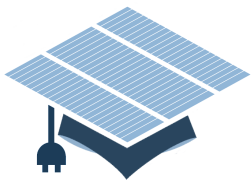
notice position
of “metallurgical”
junction. Usually
call this $x=0$

$$J = J(\text{drift}) + J(\text{diffusion}) = 0$$



Lets look at hole current first:

$$\begin{aligned} J_p &= J_p(\text{drift}) + J_p(\text{diffusion}) \\ &= q \mu_p p \xi - q D_p \frac{dp}{dx} \end{aligned}$$



Lets look at hole current first:

$$J_p = J_p(\text{drift}) + J_p(\text{diffusion})$$
$$= q\mu_p p \xi - qD_p \frac{dp}{dx}$$

mobility:

$$\mu_p = \frac{e\tau}{m_h}$$

hole distribution:

$$p = n_i e^{(E_i - E_F)/k_B T}$$

OK to use Boltzmann as
approx to Fermi-Dirac

$$\frac{dp}{dx} = \frac{p}{k_B T} \left(\frac{dE_i}{dx} - \frac{dE_F}{dx} \right) \longrightarrow \text{will use this shortly}$$

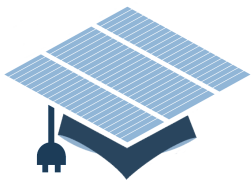
e-field:

$$\xi = \frac{1}{e} \frac{dE_i}{dx}$$

diffusion coefficient

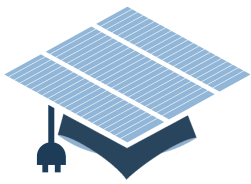
$$D_p = (k_B T / e) \mu_p$$

Einstein relation



$$\begin{aligned}J_p &= J_p(\text{drift}) + J_p(\text{diffusion}) \\&= q\mu_p p \xi - qD_p \frac{dp}{dx} \\&= \mu_p p \frac{dE_i}{dx} - \mu_p p \left(\frac{dE_i}{dx} - \frac{dE_F}{dx} \right) \\0 &= \mu_p p \frac{dE_F}{dx}\end{aligned}$$

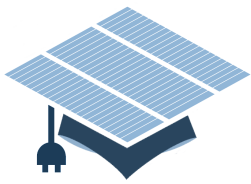
Condition for steady state



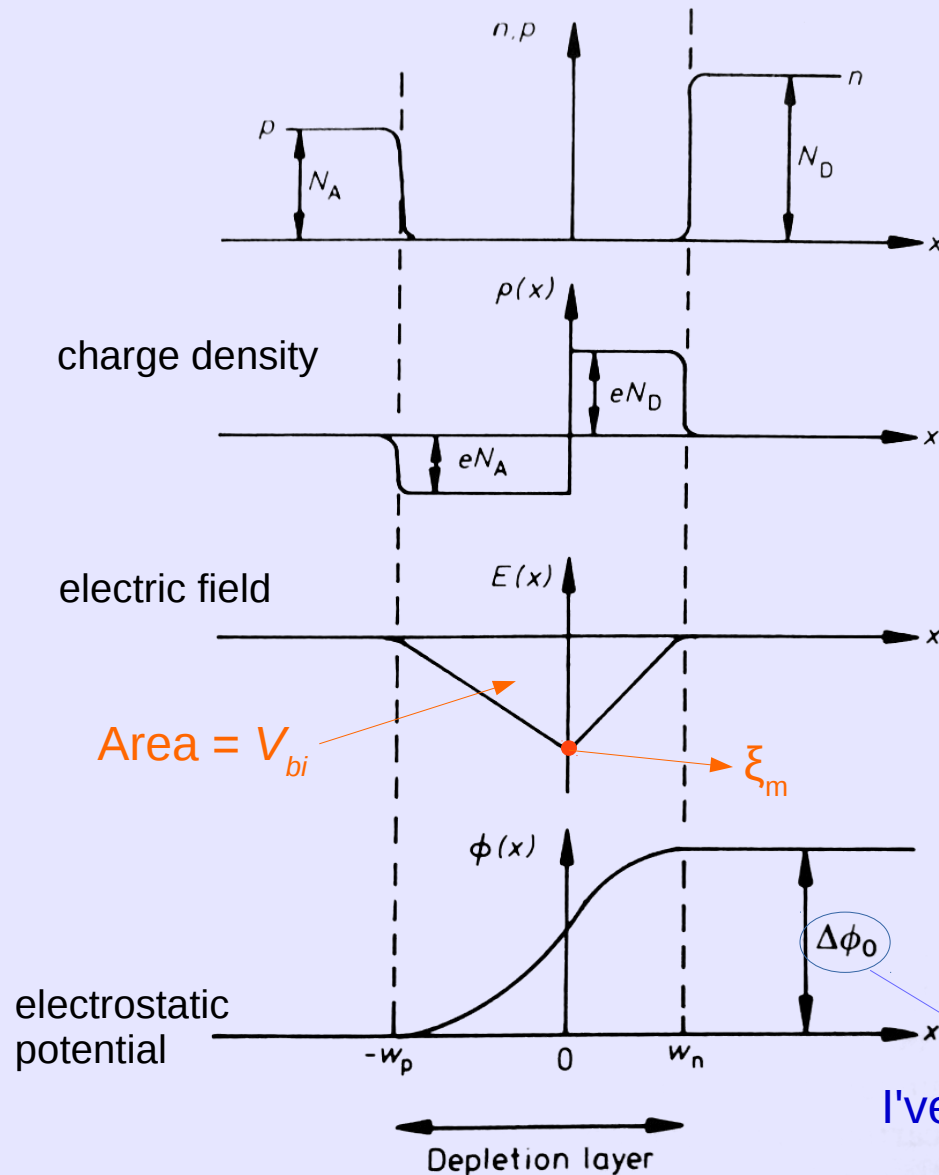
TA DA!

$$\frac{dE_F}{dx} = 0$$

The same is true from consideration of net electron current, J_n



Abrupt Junction



outside depletion region:

$$p = N_A \text{ and } n = N_D$$

Space-charge condition

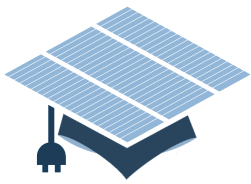
$$N_A w_p = N_D w_n$$

i.e. areas of rectangles must be the same

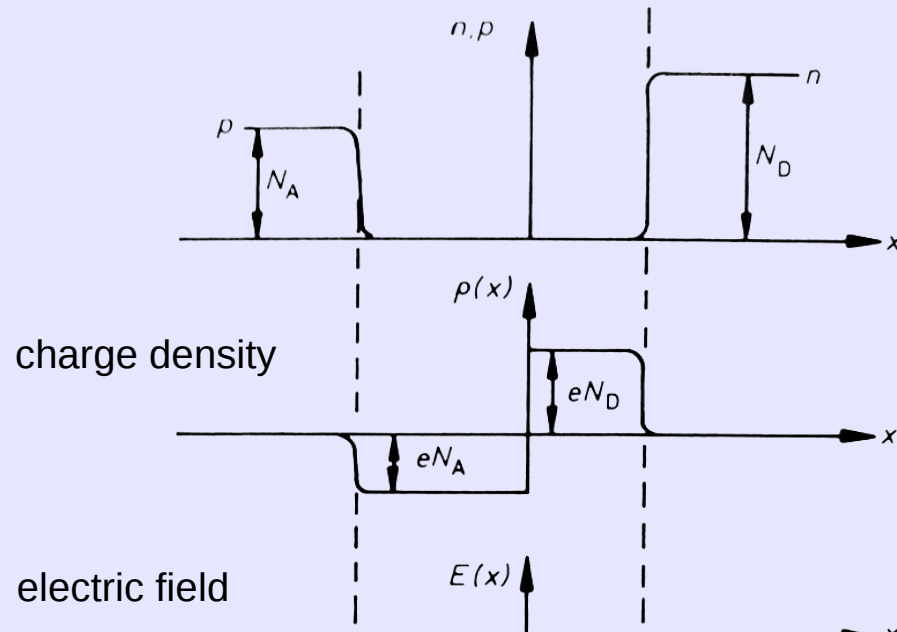
Max field (at $x=0$):

$$\xi_m = qN_D x_n / \epsilon_s = qN_A x_p / \epsilon_s$$

$$V_{bi} = \varphi_n - \varphi_p = \frac{k_B T}{q} \ln \left(\frac{N_A N_D}{n_i^2} \right)$$



Abrupt Junction



outside depletion region:

$$p = N_A \text{ and } n = N_D$$

Space-charge condition

$$N_A w_p = N_D w_n$$

i.e. areas of rectangles must be the same

Max field (at $x=0$):

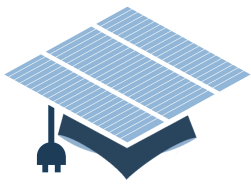
$$\xi_m = qN_D x_n / \epsilon_s = qN_A x_p / \epsilon_s$$

Lets derive this!
(but skip if I'm being slow)

$$V_{bi} = \varphi_n - \varphi_p = \frac{k_B T}{q} \ln \left(\frac{N_A N_D}{n_i^2} \right)$$

already called this V_{bi}

Depletion layer



From Earlier:

$$\varphi = -\frac{1}{e}(E_i - E_F)$$

What is φ in p and n regions away from junction?

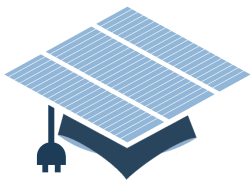
Here

$$p = n_i e^{(E_i - E_F)/k_B T} \quad \text{and} \quad n = n_i e^{(E_i - E_F)/k_B T}$$

$$\varphi_p \equiv -\frac{1}{e}(E_i - E_F) \Big|_{x \leq -x_p} = -\frac{k_B T}{e} \left(\frac{N_A}{n_i} \right)$$

$$\varphi_n \equiv -\frac{1}{e}(E_i - E_F) \Big|_{x \geq x_n} = -\frac{k_B T}{e} \left(\frac{N_D}{n_i} \right)$$

S. M. Sze, "Semiconductor
Devices: Physics and
Technology", Chap 4, p.90



Hence:

$$V_{bi} = \varphi_n - \varphi_p = \frac{k_B T}{q} \ln \left(\frac{N_A N_D}{n_i^2} \right)$$

want to know what this is for Si?

Go to: <http://pveducation.org/pvcdrom/pn-junction/intrinsic-carrier-concentration>

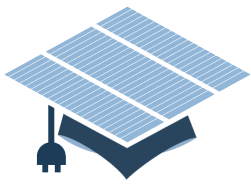
Next Question:

How do I calculate φ in the depletion region? (And why would I want to?)

Must solve **Poisson's Equation**

$$\nabla^2 \varphi = -\frac{\rho}{\epsilon}$$

Go read some electrostatics if you're interested



Abrupt Junction

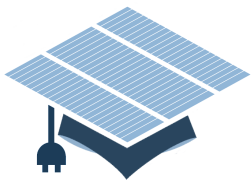
$$\nabla^2 \varphi = -\frac{\rho}{\epsilon} \quad \rho(x) = \begin{cases} -N_A e & -w_p < x < 0 \\ +N_D e & 0 < x < w_n \\ 0 & \text{elsewhere} \end{cases}$$

INTEGRATE ONCE

$$\xi = -\frac{d\varphi}{dx} = \begin{cases} -\frac{N_A e}{\epsilon} (x + w_p) & -w_p < x < 0 \\ \frac{N_D e}{\epsilon} (x + w_n) & 0 < x < w_n \end{cases}$$

INTEGRATE TWICE

$$\varphi(x) = \begin{cases} -\frac{N_A e}{2\epsilon} (x + w_p)^2 & -w_p < x < 0 \\ \frac{N_D e}{2\epsilon} (x + w_n)^2 & 0 < x < w_n \end{cases}$$



Abrupt Junction

$$V_{bi} = \varphi(x = w_n) - \varphi(x = -w_p)$$

$$V_{bi} = \frac{e}{2\epsilon} (N_A w_p^2 + N_D w_n^2)$$

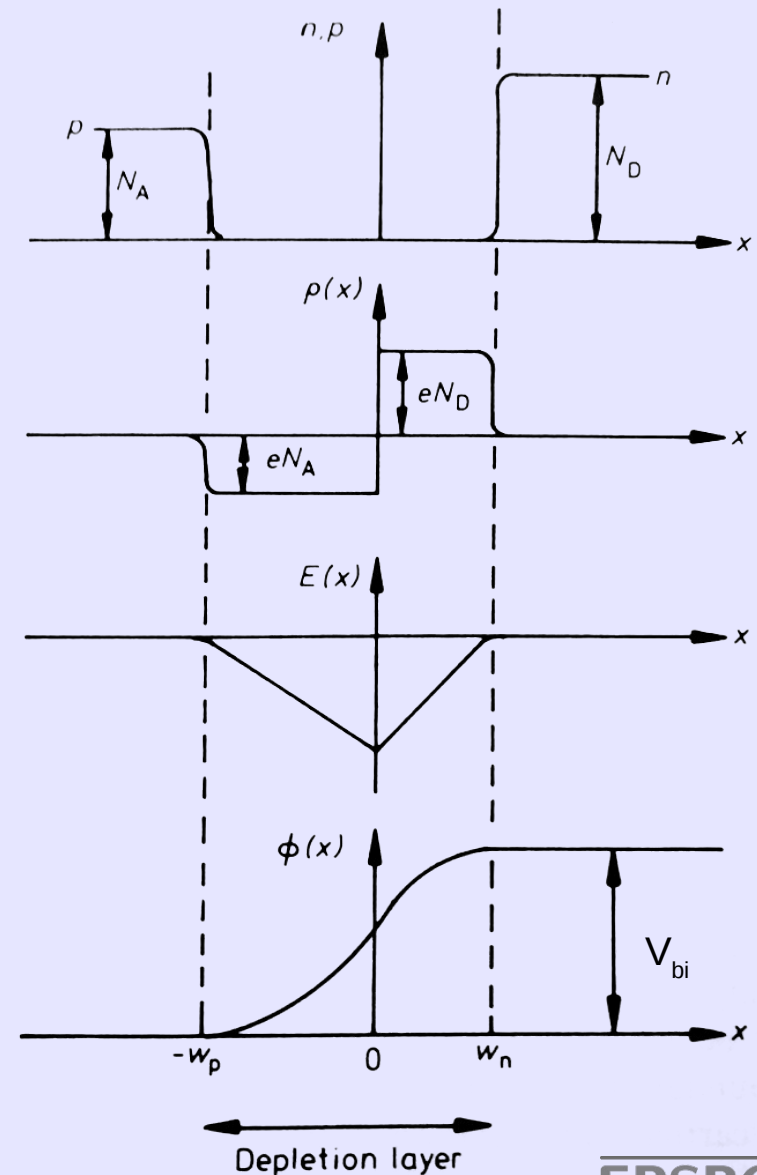
using result: $N_A w_p = N_D w_n$

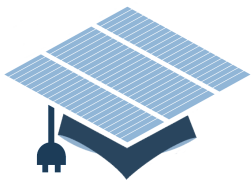
$$w_n = \sqrt{\frac{2\epsilon N_A V_{bi}}{e N_D (N_A + N_D)}}$$

$$w_p = \sqrt{\frac{2\epsilon N_D V_{bi}}{e N_A (N_A + N_D)}}$$

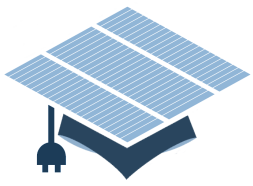
and with some jiggery pokery:

$$W = w_p + w_n = \sqrt{\frac{2\epsilon}{e} \left(\frac{N_A + N_D}{N_A N_D} \right) V_{bi}}$$

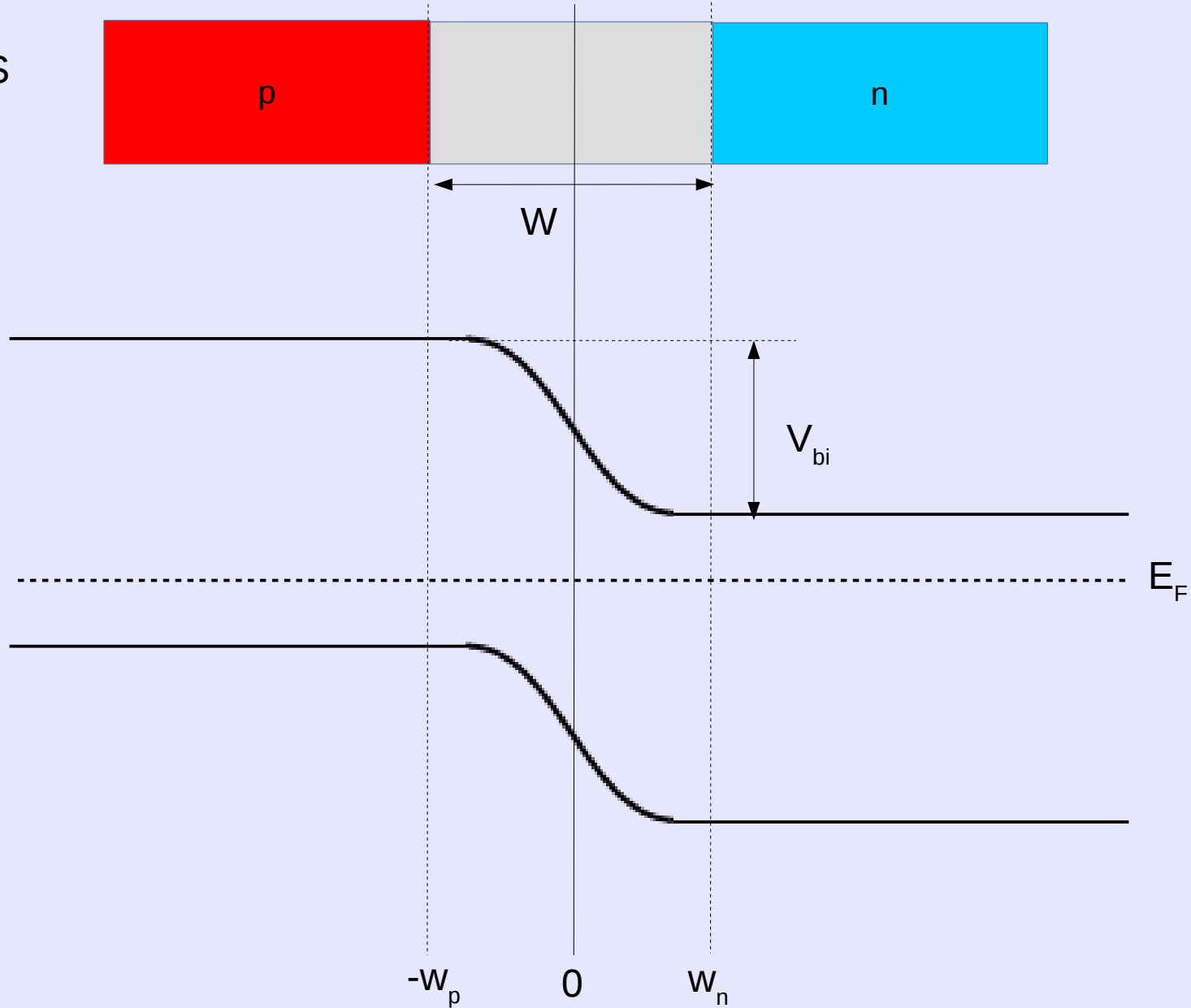


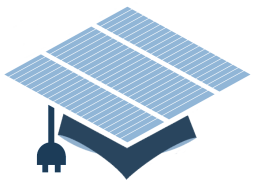


What happens to our ideal p-n junction if we put a **forward** or **reverse** bias across it?

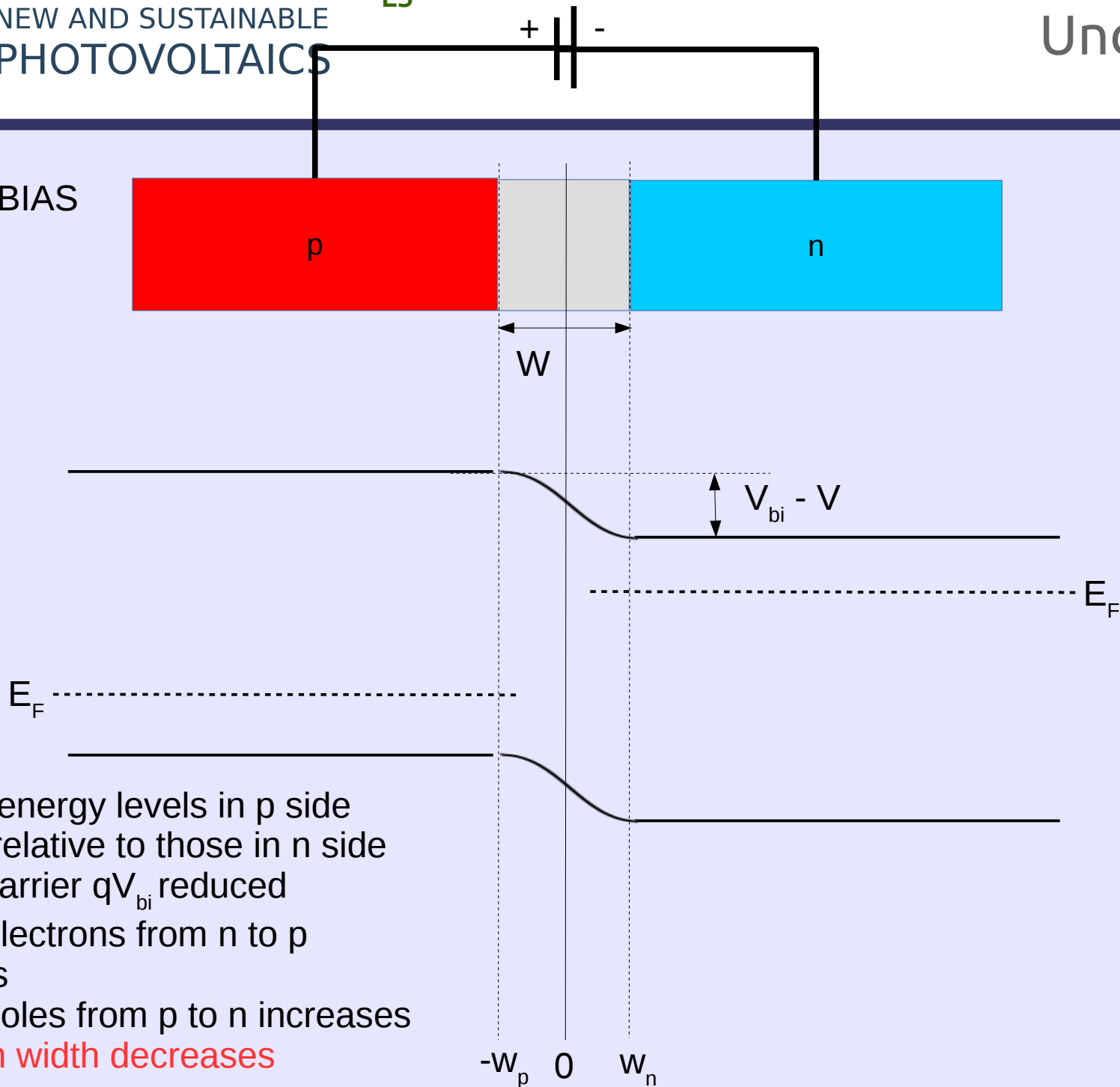


ZERO BIAS

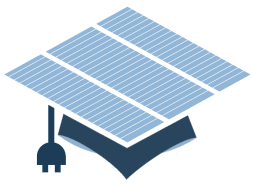




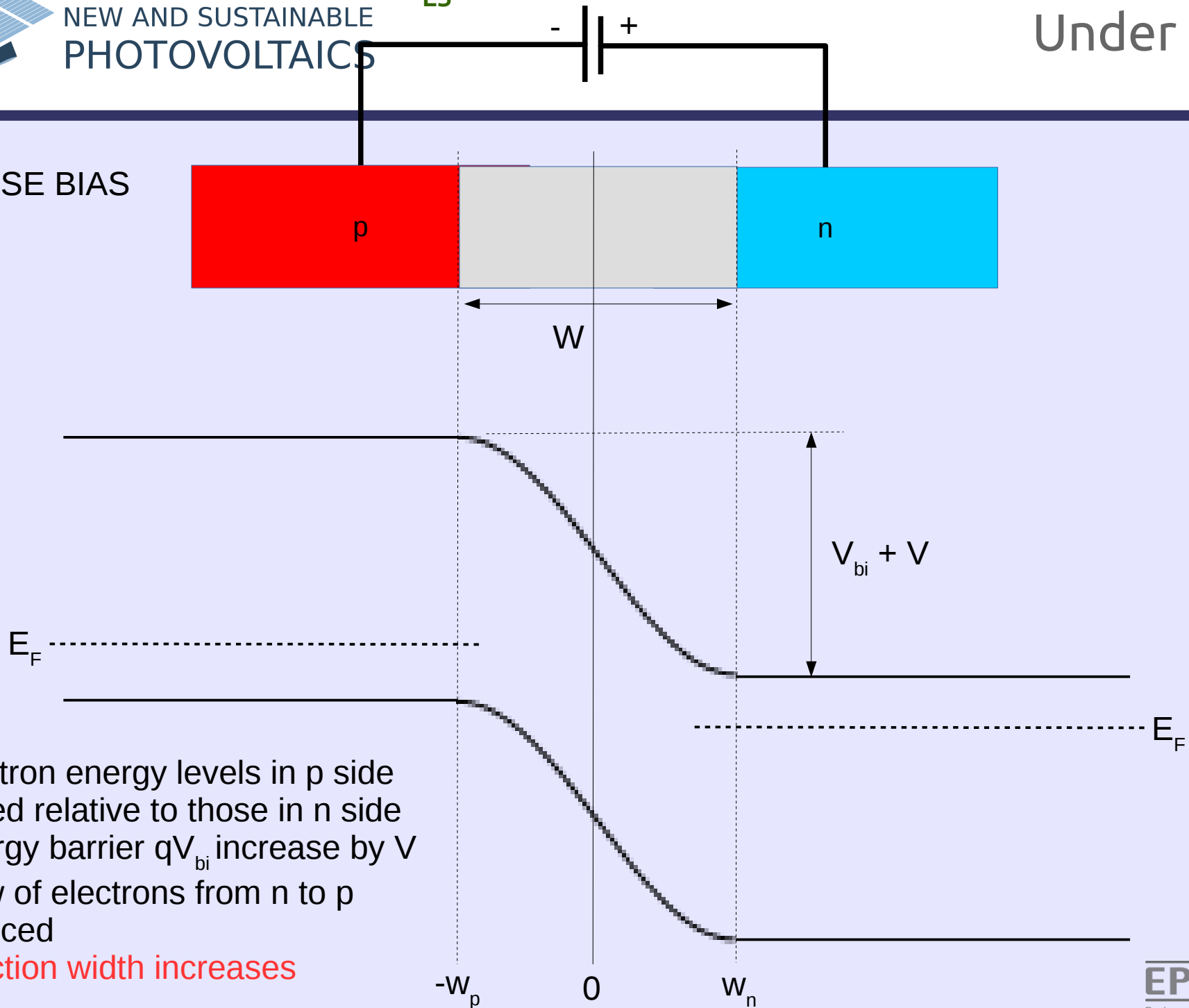
FORWARD BIAS



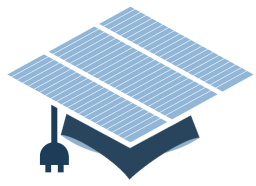
- Electron energy levels in p side lowered relative to those in n side
- Energy barrier qV_{bi} reduced
- Flow of electrons from n to p increases
- Flow of holes from p to n increases
- Depletion width decreases



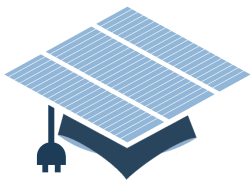
REVERSE BIAS



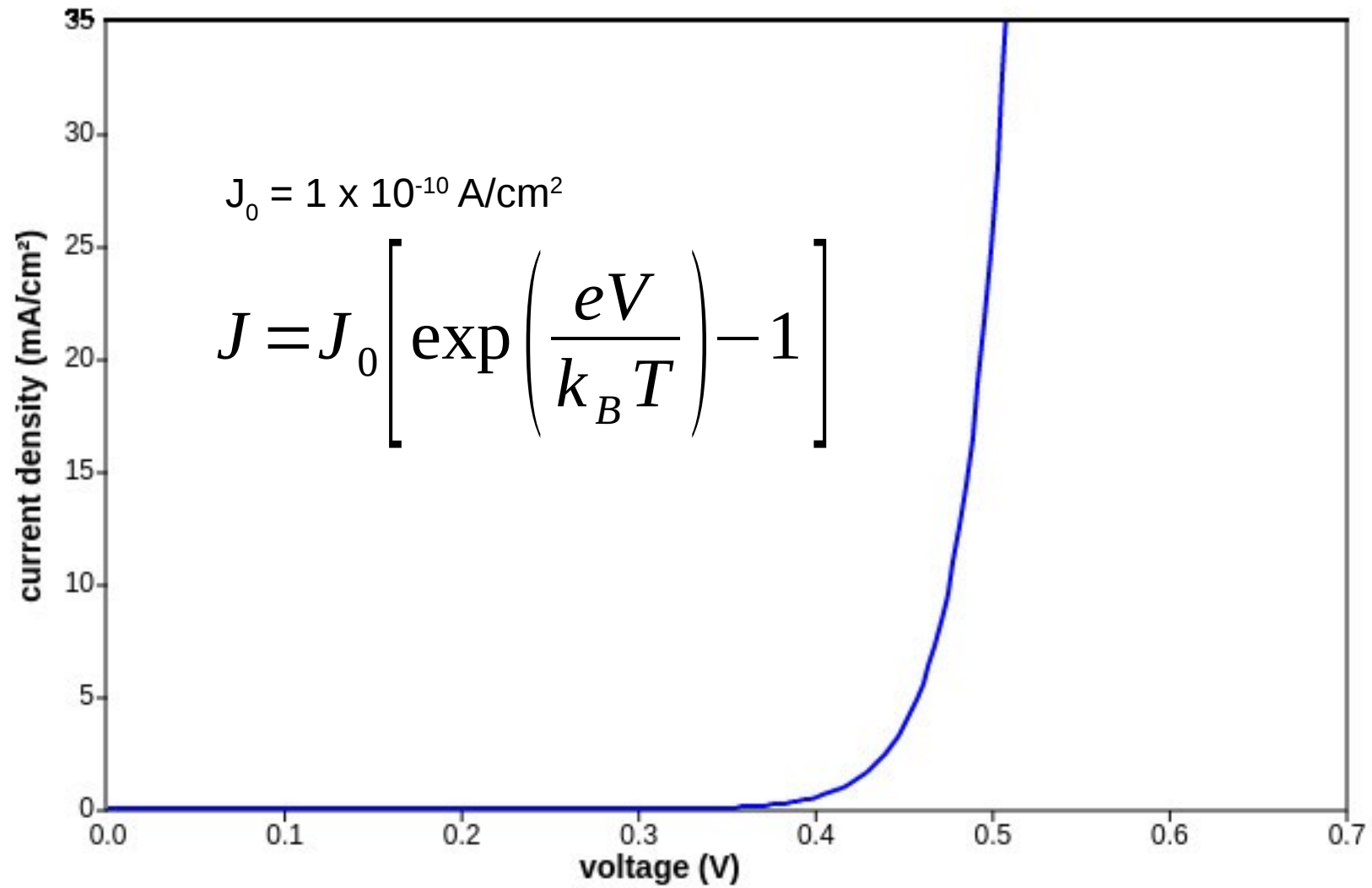
- Electron energy levels in p side raised relative to those in n side
- Energy barrier qV_{bi} increase by V
- Flow of electrons from n to p reduced
- Junction width increases

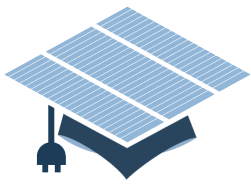


Of course, you already know all about this:

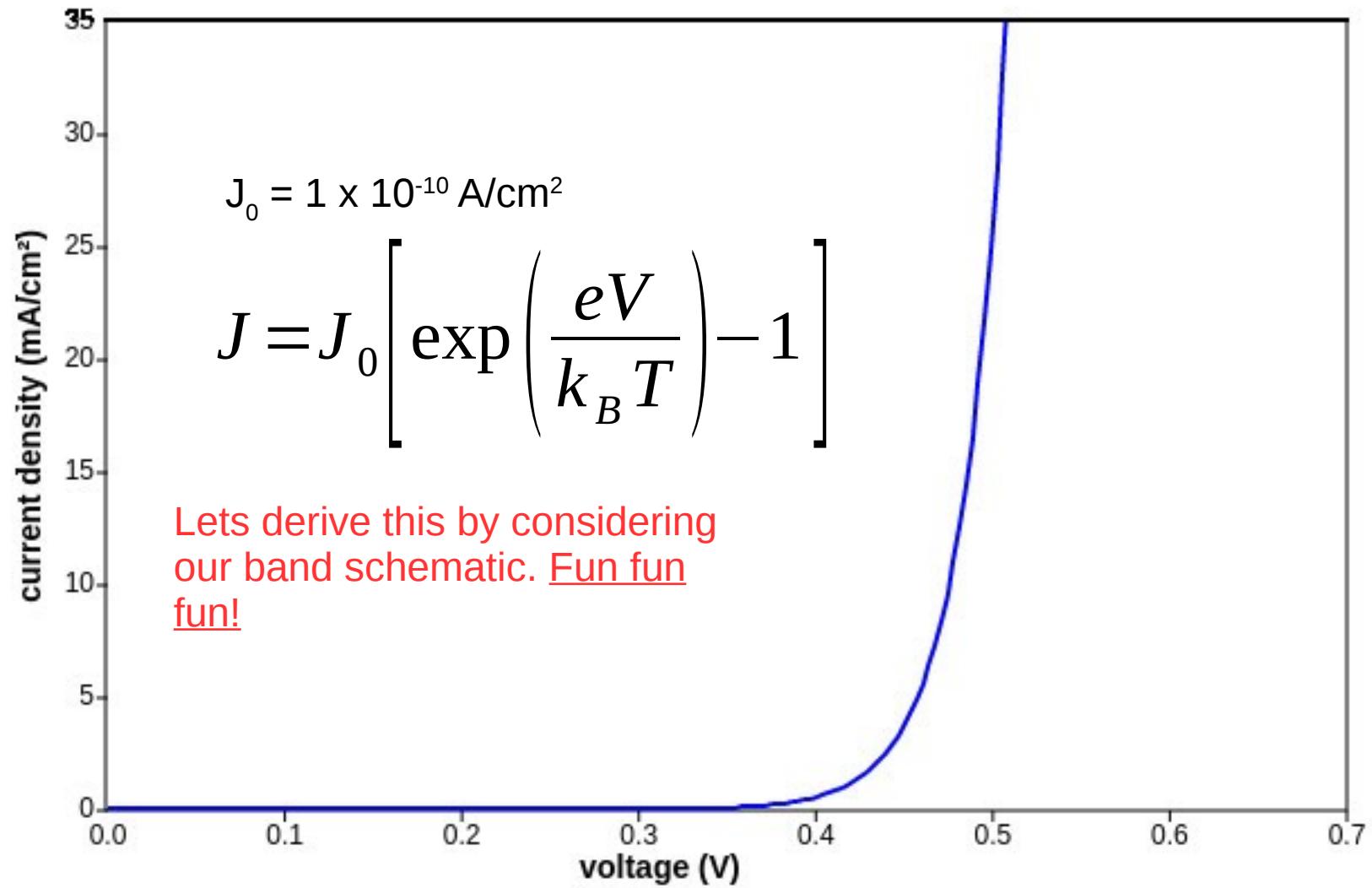


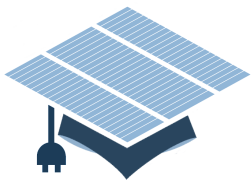
Ideal dark JV curve





Ideal dark JV curve



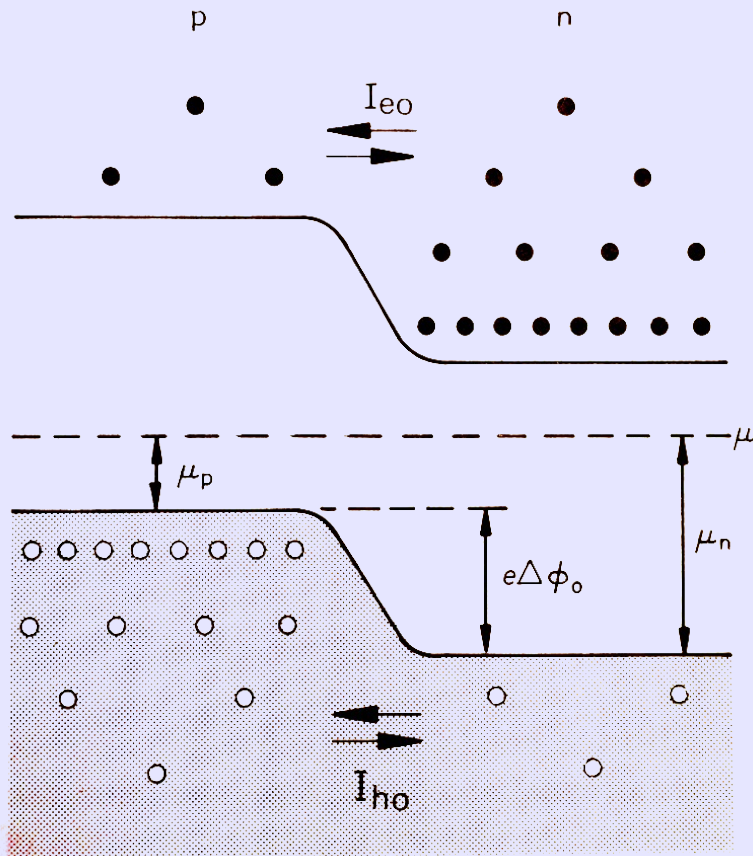


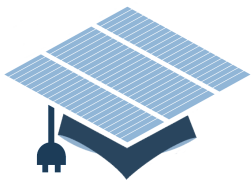
Deriving Shockley Equation

Steady State again (i.e. no bias)

Consider electron drift current

– I_{eo} from p to n



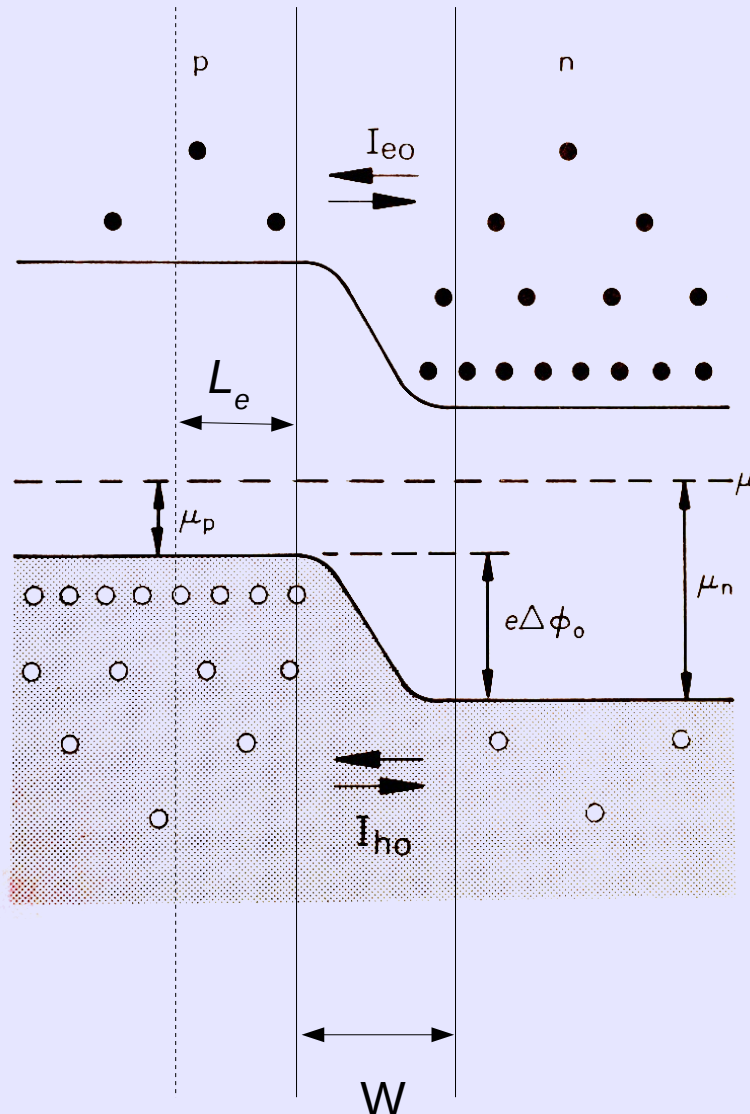


Deriving Shockley Equation

Steady State again (i.e. no bias)

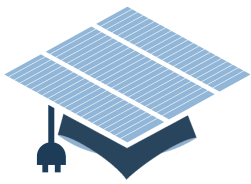
Consider electron drift current

– I_{eo} from p to n



$$I_{e0} = e \times (\text{generation rate / volume}) \times (\text{volume within } L_e \text{ of depletion zone})$$

$$J_{e0} = e \left(\frac{n_p}{\tau_p} \right) L_e$$

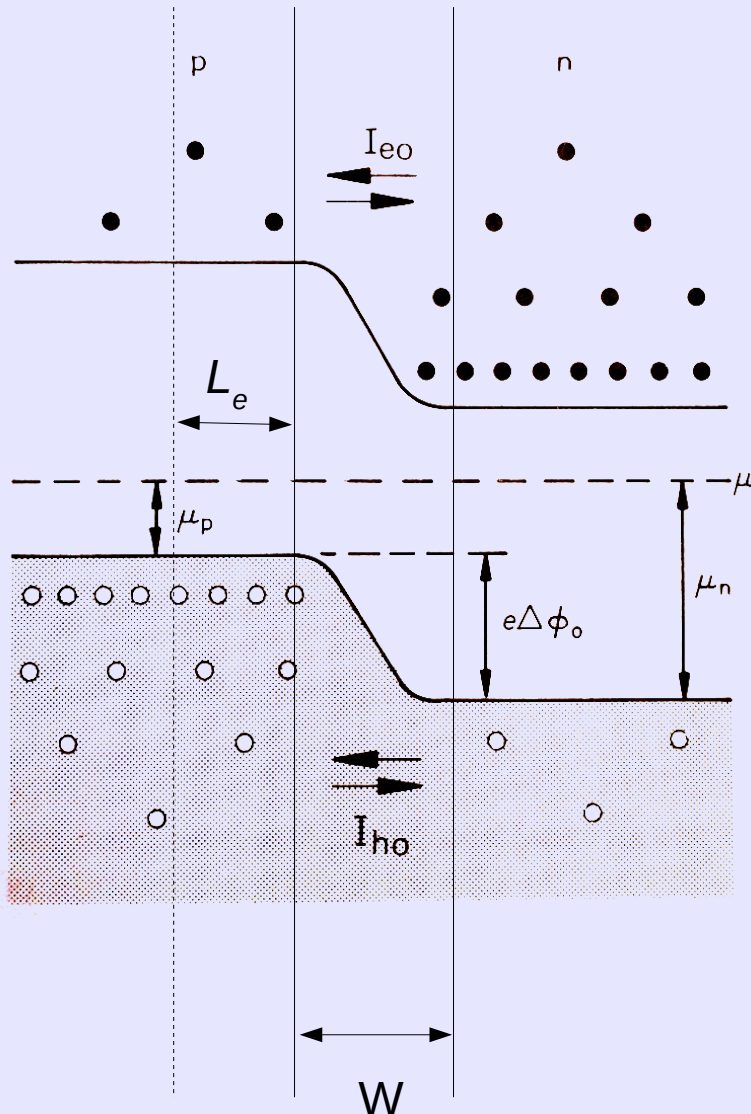


Deriving Shockley Equation

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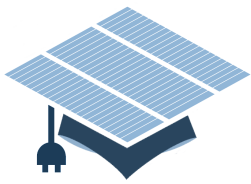
number of electrons on p side

$$J_{e0} = e \left(\frac{n_p}{\tau_p} \right) L_e$$

electron lifetime

diffusion length

$$L_e = \sqrt{D_e \tau_p}$$



Deriving Shockley Equation

Steady State again (i.e. no bias)

Consider electron drift current

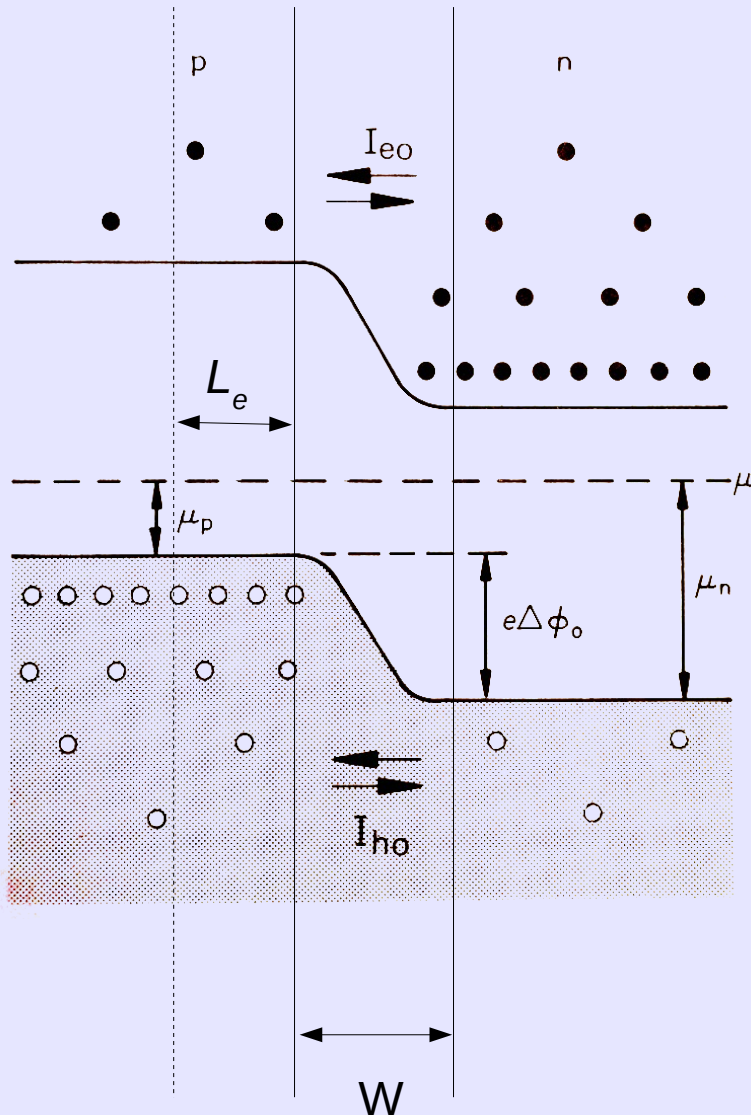
– I_{eo} from p to n

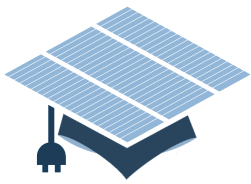
By assuming that all acceptors on p side are ionized:

$$n_p = \frac{n_i^2}{p_p} = \frac{n_i^2}{N_A}$$

can re-write J_{e0} as:

$$J_{e0} = \frac{eD_e n_i^2}{L_e N_A}$$





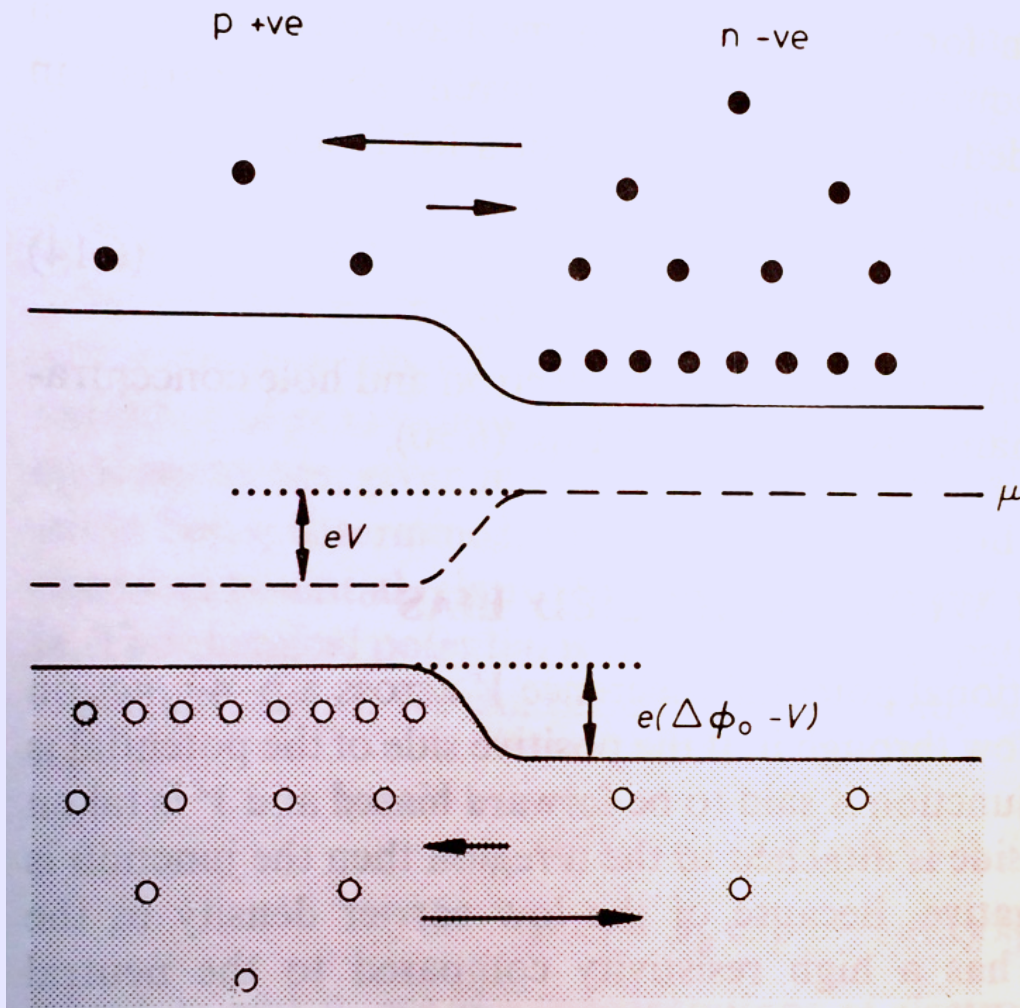
Deriving Shockley Equation

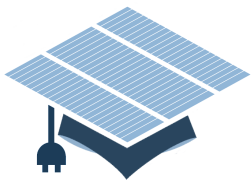
What happens **under forward bias**?

- **Drift (p to n)? - Nothing.** No potential barrier in this direction
- **Diffusion (n to p)? - Increases** by exponential factor:

$$\exp\left(\frac{eV}{k_B T}\right)$$

If we assume occupation of states in CB is given by Boltzmann distribution





Deriving Shockley Equation

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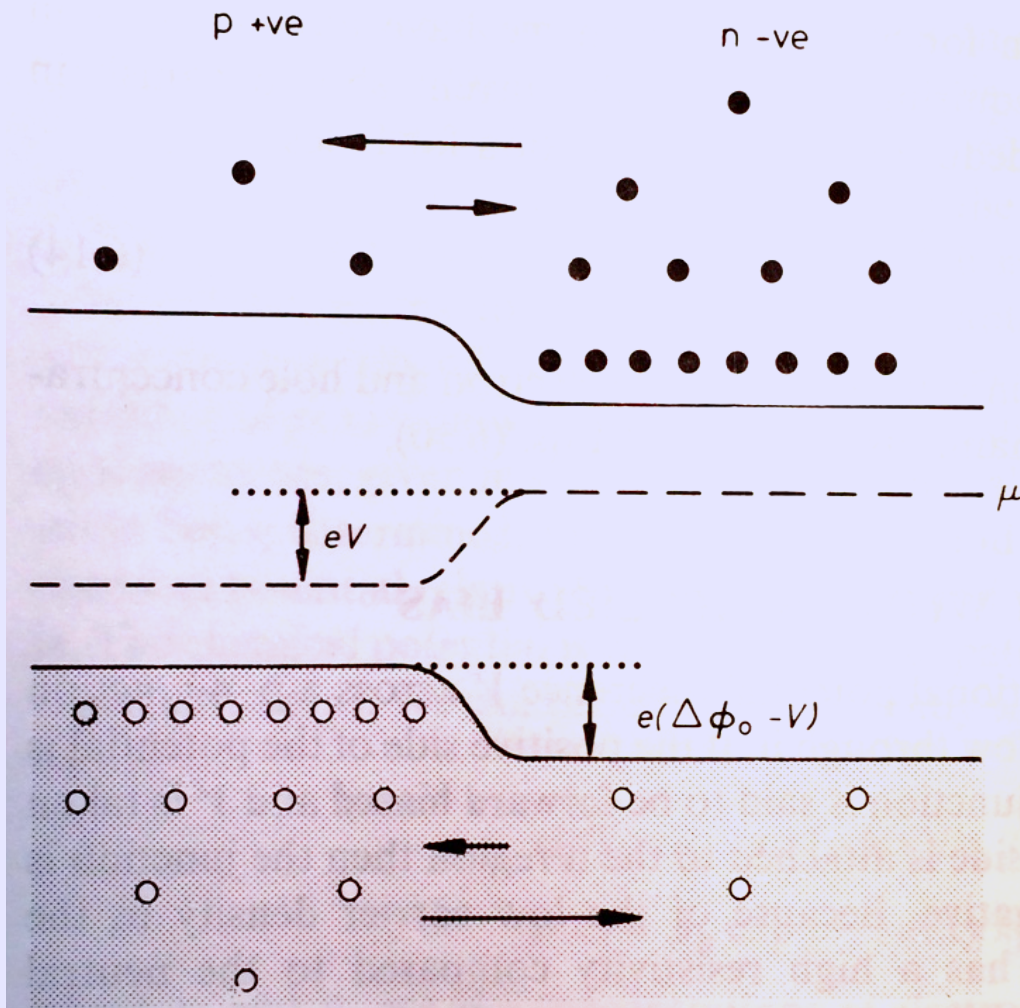
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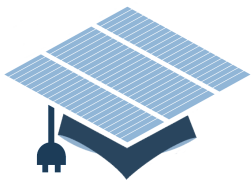
$$\exp\left(\frac{eV}{k_B T}\right)$$

If we assume occupation of states in CB is given by Boltzmann distribution

Net electron current:

$$\begin{aligned} J_e &= -J_{e0} + J_{e0} \exp\left(\frac{eV}{k_B T}\right) \\ &= J_{e0} \left[\exp\left(\frac{eV}{k_B T}\right) - 1 \right] \end{aligned}$$

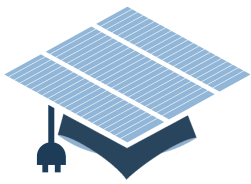




Deriving Shockley Equation

Can do the same for the net **hole** current:

$$J_h = J_{h0} \left[\exp\left(\frac{eV}{k_B T}\right) - 1 \right] \quad \text{where} \quad J_{h0} = \frac{eD_H n_i^2}{L_h N_D}$$



Deriving Shockley Equation

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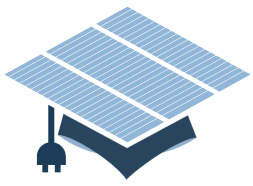
Therefore: **Net Current!**

$$J = J_e + J_h = J_0 \left[\exp\left(\frac{eV}{k_B T}\right) - 1 \right]$$

ideal diode equation

$$J_0 = J_{e0} + J_{h0} = en_i^2 \left(\frac{D_e}{L_n N_A} + \frac{D_h}{L_h N_A} \right)$$

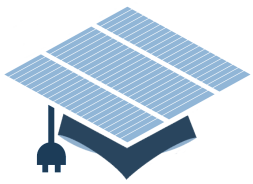
reverse saturation current,
a.k.a “dark current”



Deriving Shockley Equation

Previous derivation ignores the **recombination** and **generation** of carriers **within the depletion layer** itself!

$$J = J_0 \left[\exp \left(\frac{eV}{n k_B T} \right) - 1 \right]$$



Deriving Shockley Equation

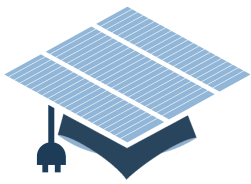
Previous derivation ignores the **recombination** and **generation** of carriers **within the depletion layer** itself!

$$J = J_0 \left[\exp \left(\frac{eV}{n k_B T} \right) - 1 \right]$$

ideality factor – a complete FUDGE

Worth reading up about – but don't stress out about it.

- S. M. Sze, “*Semiconductor Devices*”, 2nd ed. Chap 4, pp 109 – 113
 - Hook & Hall “*Solid State Physics*”, 2nd ed. Chap 6, pp 178 – 179
- I have these books if you want them.



- Drawing Junctions
- Junction Formation – A Physical Picture
- Abrupt Junction
- Deriving Important Junction parameters
- Deriving diode equation
- Ideality Factor - Recombination