编译原理
Complier Principles

Lecture 4
Syntax Analysis: Top-Down

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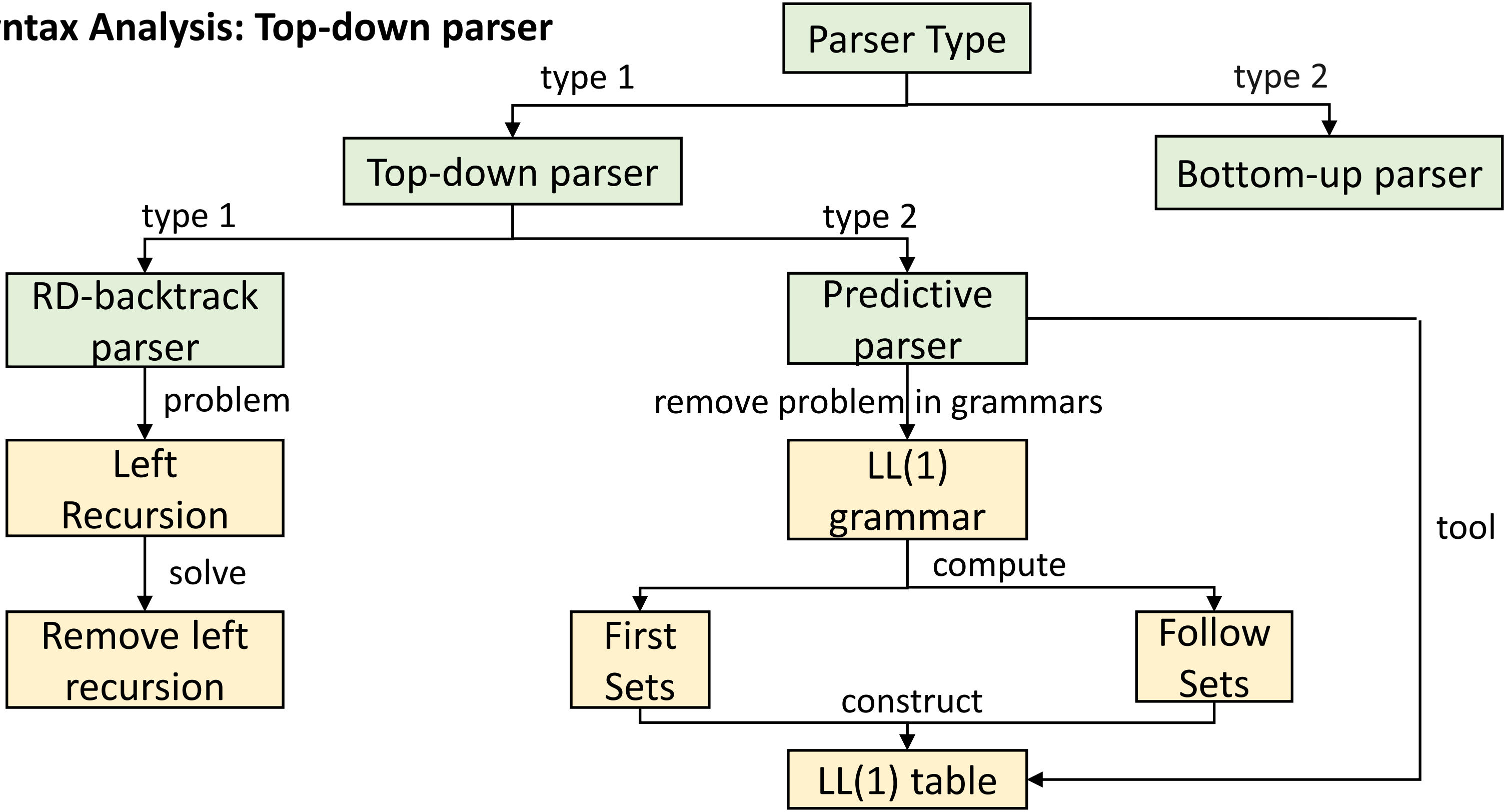
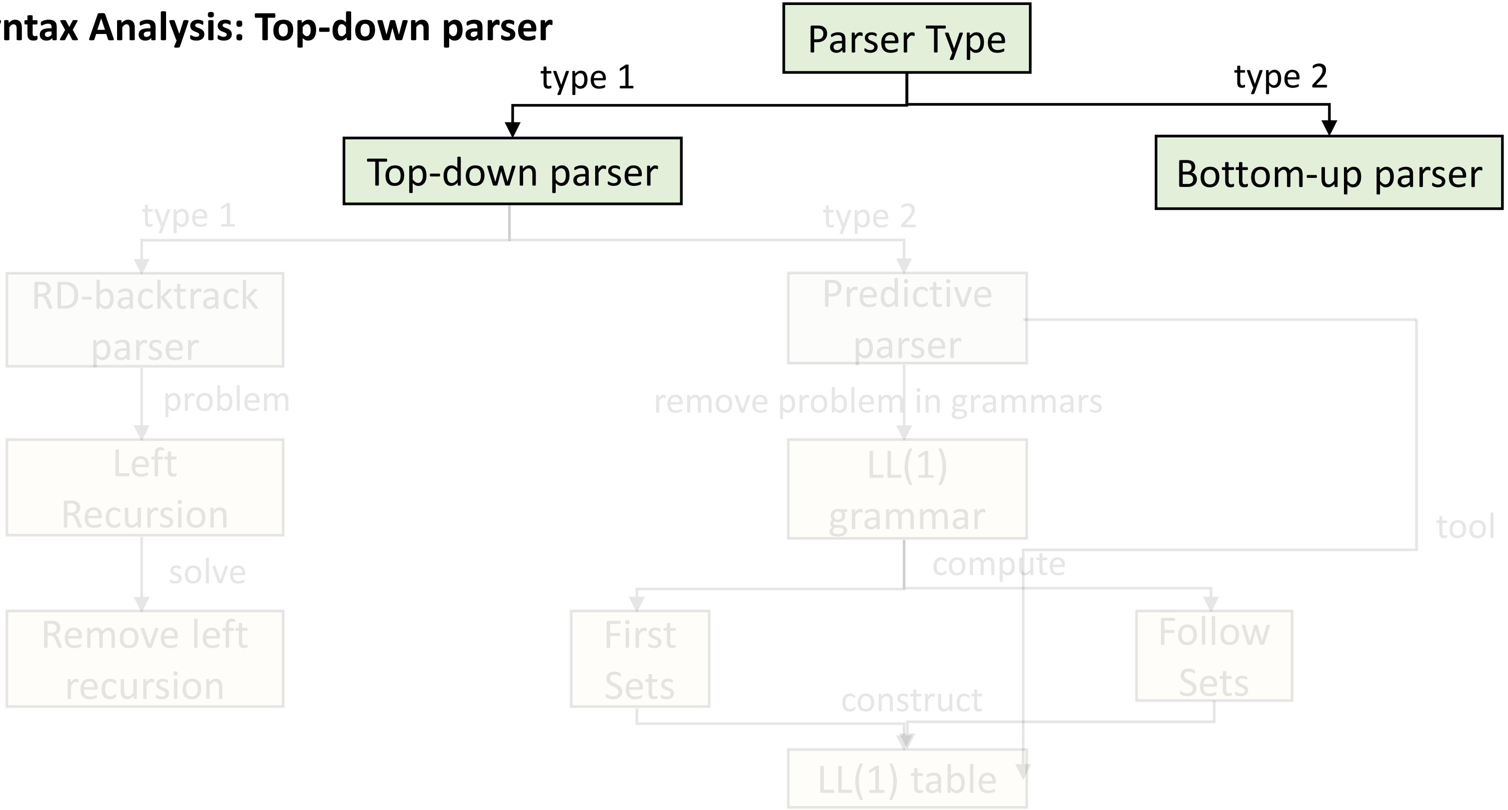
Mind Map[思维导图]

Syntax Analysis: Top-down parser



Mind Map[思维导图]

Syntax Analysis: Top-down parser



Parser Type[语法分析类型]

- Most compilers use either **Top-Down** or **Bottom-Up** parsers.
- Bottom-up parsing [自底向上分析]
  - Begin at the leaves (*the bottom*) and working up towards the root (*the top*).
  - Tries to reduce[规约] the input string to the start symbol.
  - Finds reverse order of the rightmost derivation[最右推导的逆过程即最左规约]
    规范推导 规范规约
  - Parser code structure nothing like grammar.
    - Very difficult to implement manually.
    - Automated tools exist to convert to code (e.g., Yacc, Bison).

Parser Type[语法分析类型]

- Top-Down parsing [自顶向下分析]
  - ◆ Starting from the root (*top*) and create the leaves (*down*) of the parse tree in a pre-defined order (depth-first) [深度优先,先根次序/前序].
  - ◆ Top-down parsing can be viewed as **finding a leftmost derivation**[寻求最左推导] for an input string. ***Why not rightmost or arbitrary derivation?***

◆ **Review:** *In each step of derivation*, the following choices need to be made:

- Choice of the non-terminal to be replaced. [替换哪个非终结符] **Leftmost!**
- Choice of the production to be applied for a non-terminal. [使用文法中哪个规则来替换] **Key Problem!**

◆ **Question:** *At each step of a top-down parse*, What is the key problem?

Parser Type[语法分析类型]

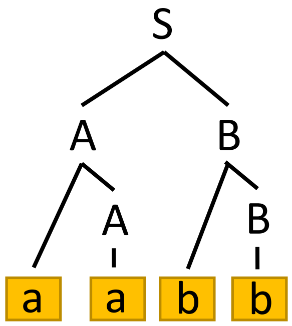
- Top-Down parsing [自顶向下分析]
  - Once a production is chosen, we try to match the terminal symbols in the production body with the input string.
  - Parser code structure closely mimics grammar.
    - Manually implementation is feasible.
    - Automated tools exist to convert to code. (e.g. ANTLR)
- Top-Down **vs.** Bottom-Up [对比]
  - Top-down: easier to understand and implement manually. (E.g. ANTLR)
  - Bottom-up: more powerful, can be implemented automatically. (E.g. YACC/Bison)

Parser Type[语法分析类型]

◆Example

- ◆Grammar G(S): S → AB ; A → aA | a ; B → bB | b ;
- ◆**Language?** **L(G)={$a^m b^n$|m,n≥1}**
- ◆Sentence: aabb;

S ⇒ AB	(5) *B* reduce to *S*.
⇒ AbB	(4) *bB* reduce to *B*.
⇒ Abb	(3) 2nd *b* reduce to *B*.
⇒ aAbb	(2) *aA* reduce to *A*.
⇒ aabb	(1) 2nd *a* reduce to *A*.



<u>Reverse of rightmost derivation!</u>

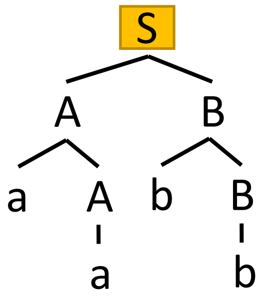
Parser Type[语法分析类型]

◆**Example**

◆Grammar G(S): S → AB ; A → aA | a ; B → bB | b ;

◆Sentence: aabb;

$S \Rightarrow AB$ (1)
$\Rightarrow aAB$ (2)
$\Rightarrow aaB$ (3)
$\Rightarrow aabB$ (4)
$\Rightarrow aabb$ (5)



$S \Rightarrow AB$ (5) *B* reduce to *S*.
$\Rightarrow AbB$ (4) *bB* reduce to *B*.
$\Rightarrow Abb$ (3) 2^{nd} *b* reduce to *B*.
$\Rightarrow aAbb$ (2) *aA* reduce to *A*.
$\Rightarrow aabb$ (1) 2^{nd} *a* reduce to *A*.

Leftmost derivation

Top-Down

Leftmost reduction

Bottom-Up

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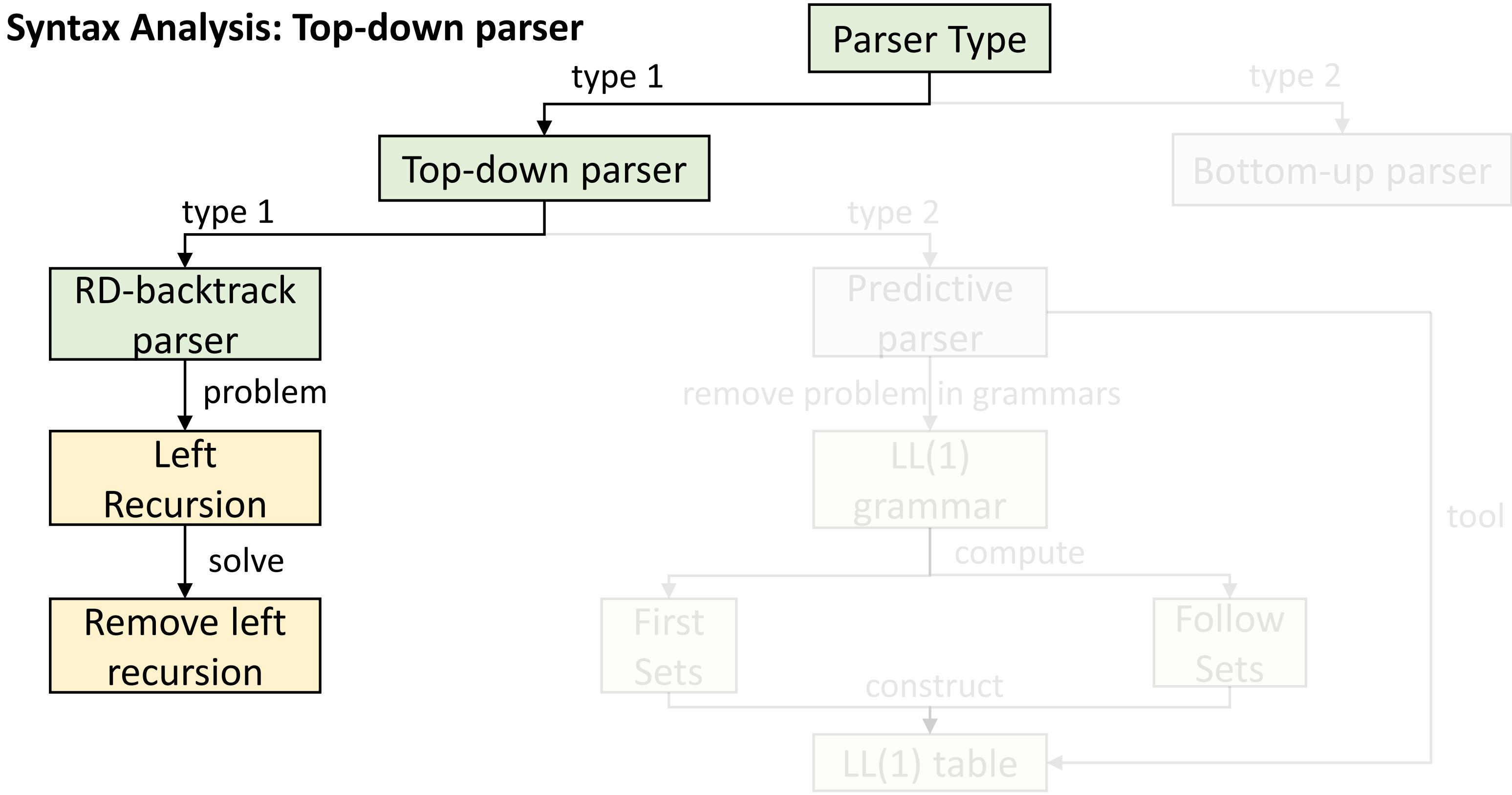
Mind Map[思维导图]



Top-down Parsing[自顶向下分析]

- **Recursive-descent parsing**[RDP, 递归下降语法分析]
  - A general form[通用形式] of top-down parsing.
  - A recursive-descent parsing program consists of a set of *procedures*, one for each non-terminal.
  - Execution begins with the procedure for the start symbol, which halts and announces success if its procedure body scans the entire input string.

```
void A() {
    Choose an A-production, A → X1 X2 ··· Xk;
    for ( i = 1 to k ) {
        if ( Xi is a nonterminal )
            call procedure Xi();
        else if ( Xi equals the current input symbol a )
            advance the input to the next symbol;
        else /* an error has occurred */;
    }
}
```

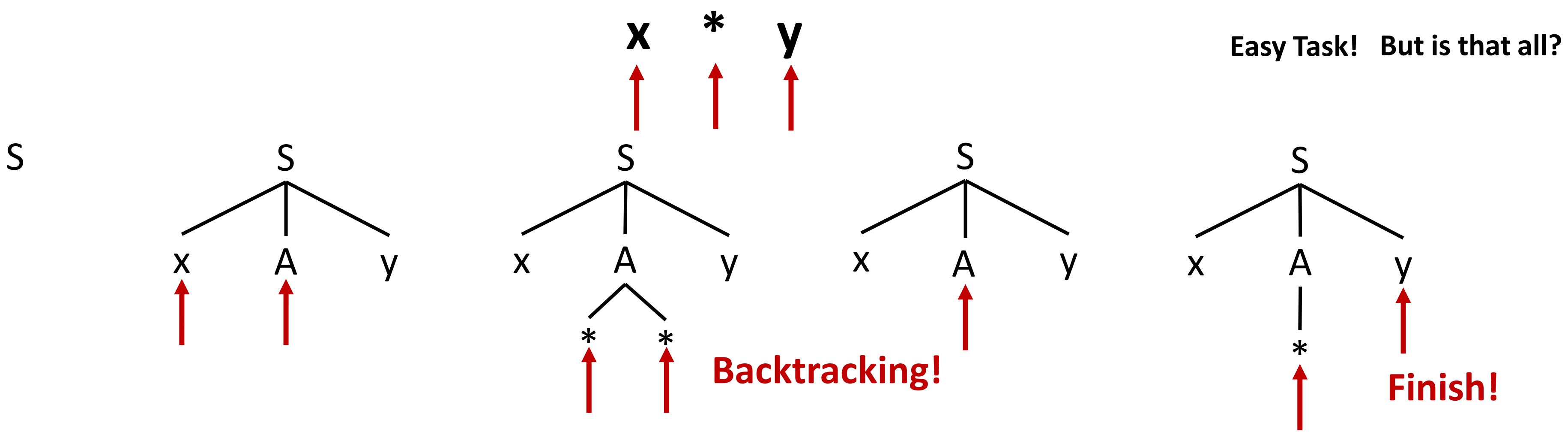
How to choose A-production?
It's not specified, so the pseudocode is nondeterministic[伪代码是不确定的].

RDP with backtracking[回溯]

- RDP may require ***backtracking***.
- **Approach**: for a non-terminal in the derivation, productions are tried in some order until
  - A production is found that generates a portion of the input, or
  - No production is found that generates a portion of the input, in which case backtrack to previous non-terminal.
- Terminals of the derivation are compared against input
  - Match: advance input, continue parsing
  - Mismatch: backtrack, or fail
- Parsing fails if no derivation generates the entire input.

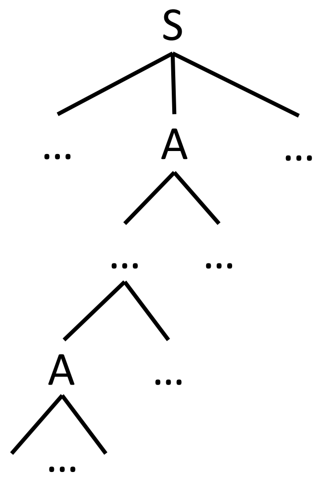
RDP with backtracking[回溯]

- In the analysis process, when a non-terminal is successfully matched with an alternative, the match may be temporary.
- If a mismatch occurs, backtracking[回溯] will be performed.
- G[S]: S → xAy; A → ** | *, whether the input string **x * y** is its sentence?



Left Recursion Problem[左递归问题]

- A grammar is **left recursive[左递归]** if it has a non-terminal **A** such that there is a derivation $\mathbf{A} \overset{+}{\Rightarrow} \mathbf{A\alpha}$.
- Sentence can grow infinitely without consuming input.(Into an infinite loop!)
- Top-down parsing methods cannot handle left-recursive problems.[自顶向下语法分析方法不能处理左递归的文法]



Left Recursion Problem[左递归问题]

- Immediate left recursion [直接/立即左递归]
  - There is a production A → Aα.
- Non-immediate left recursion [间接/非立即左递归]
  - Left recursion involving derivation of 2+ step.
  - A → Bβ; B → Aα.
- A transformation is needed to *eliminate left recursion*. [需要一个转换方法来消除左递归]
- Rewrite the grammar so that it is right recursive. [改为右递归]

Remove Left Recursion[消除左递归]

- Immediate left recursion[直接左递归的消除]
  - Grammar: $A \rightarrow A\alpha \mid \beta$ ($\alpha \neq \beta$, β doesn't start with A)
  - *rewrite the rule of A as the following form equivalently:*
  - Grammar: $A \rightarrow \beta A'$; $A' \rightarrow \alpha A' \mid \varepsilon$ **(right recursion)**

G[A]: $A \rightarrow A\alpha \mid \beta$

$A \Rightarrow A\alpha$

$\Rightarrow A\alpha\alpha$

$\Rightarrow A\alpha\alpha\alpha$

……

$\Rightarrow A\alpha\ldots\alpha$

$\Rightarrow \beta\alpha\ldots\alpha$

Remove Left Recursion

G[A]: $A \rightarrow \beta A'$

$A' \rightarrow \alpha A' \mid \varepsilon$

$A \Rightarrow \beta A'$

$\Rightarrow \beta\alpha A'$

$\Rightarrow \beta\alpha\alpha A'$

……

$\Rightarrow \beta\alpha\ldots\alpha A'$

$\Rightarrow \beta\alpha\ldots\alpha$

想办法把"Aαα.."变成"αα..A"

A → Aα; A → β
=> Aα => Aααα.. => βααα..

1. 避免对A的左递归
A -> βx, where x-> ααα..

2. 引入bridging production
A' –> αA' | ε
=> αA' => αα..A' => αα..α

3. 连接两个productions
A -> βA'; A' –> αA' | ε
=> βA' => βαα..A' => βαα..α

Remove Left Recursion[消除左递归]

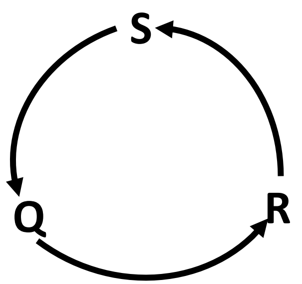
- **Immediate left recursion** can be eliminated by the following technique, which works for any number of A-productions.
  - First, group the productions as

    $\mathbf{A \rightarrow A\alpha_1 \mid A\alpha_2 \mid \dots \mid A\alpha_m \mid \beta_1 \mid \beta_2 \ \dots \mid \beta_n}$ where no β_i begins with an A.
  - Then, replace the A-productions by

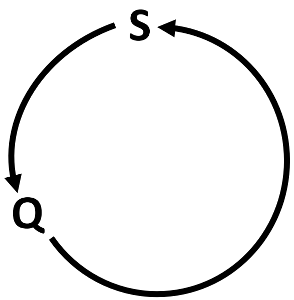
    $\mathbf{A \rightarrow \beta_1 A' \mid \beta_2 A' \mid \dots \mid \beta_n A'}$ **AND** $\mathbf{A' \rightarrow \alpha_1 A' \mid \alpha_2 A' \mid \dots \mid \alpha_m A' \mid \varepsilon}$
- Exercise-Remove Immediate left recursion.
  - $G_1[E]$: $E \rightarrow \underbrace{\boxed{E+T}}_{\alpha} \mid \underbrace{\boxed{T}}_{\beta}$; $T \rightarrow \underbrace{\boxed{T*F}}_{\alpha} \mid \underbrace{\boxed{F}}_{\beta}$; $F \rightarrow (E) \mid i$.
  - $G_2[E]$: $\mathbf{E \rightarrow TE'; E' \rightarrow +TE' \mid \varepsilon;}$ $\mathbf{T \rightarrow FT'; T' \rightarrow *FT' \mid \varepsilon;}$ $F \rightarrow (E) \mid i$.

Remove Left Recursion[消除左递归]

- **Non-Immediate left recursion**[非直接左递归的消除]
  - Grammar: **S → Qc | c ; Q → Rb | b ; R → Sa | a.**
  - Although there is no immediate left recursion, **S, Q and R are all left recursion.**

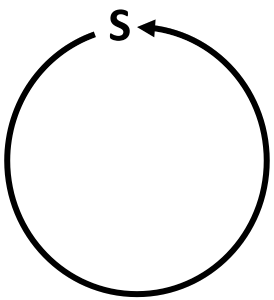


S → Qc | c
Q → Rb | b
R → Sa | a



S → Qc | c
Q → Sab | ab | b
R → Sa | a

Left Recursion!



S → Sabc | abc | bc | c
Q → Sab | ab | b
R → Sa | a

Remove Left Recursion[消除左递归]

- The following algorithm systematically eliminates left recursion from a grammar[直接/间接]. It is guaranteed to work if:
  - the grammar has no cycles (derivations of the form $A \overset{+}{\Rightarrow} A$)
  - the grammar has no ε–productions (productions of the form $A \rightarrow \varepsilon$).
  - These two can be eliminated systematically from a grammar.
- **Algorithm:** Eliminating left recursion.
  - **INPUT:** Grammar *G* with no cycles or ε–productions.
  - **OUTPUT:** An equivalent grammar with no left recursion.
  - **METHOD:** Apply the following 3 steps to G. Note that the resulting non-left-recursive grammar may have ε–productions.

Remove Left Recursion[消除左递归]

- **Step 1 :** Arrange all non-terminals of grammar G in some order $A_1, A_2, \ldots, A_n$;
- **Step 2 :** *Execute in order obtained in Step 1:*

FOR **i:=1** TO **n** DO

FOR **j:=1** TO **i-1** DO

Replace each production of the form $A_i \rightarrow A_j\gamma$ by the productions $A_i \rightarrow \delta_1\gamma \mid \delta_2\gamma \mid \ldots \delta_k\gamma$, where $A_j \rightarrow \delta_1 \mid \delta_2 \mid \ldots \delta_k$ are all current A_j-productions; [间接左递归问题 转换为 直接左递归问题]

eliminate the immediate left recursion among the A_i-productions;

[解决直接左递归问题]

$A \rightarrow A\alpha \mid \beta$

$A \rightarrow \beta A'; A' \rightarrow \alpha A' \mid \varepsilon$

END

END

- **Step 3 :** Simplify the grammar obtained from Step 2 --- remove the production rules of non-terminal that can never be reached from the start symbol.

[去掉无效productions]

Remove Left Recursion[消除左递归]

- **Example:** Consider Grammar G(S): (**ILR = immediate left recursion**)

  R → Sa | a   Q → Rb | b   S → Qc | c

- **Step 1 :** non-terminal order: **R,Q,S**; (A_1, A_2, A_3) [顺序决定了哪个non-terminal可能会被消除(如果符合算法要求), 最后的non-terminal会留下]
- **Step 2 :** (i=1) For R, there is no **ILR**; [A_i=R]

  (i=2,j=1) Replace production Q → Rb by the productions R → Sa | a, which generates Q → Sab | ab | b; ***not ILR*** [A_i=Q, A_j=R]

  (i=3,j=1) No operations. [A_i=S, A_j=R]

  (i=3,j=2) Replace production S → Qc by the productions Q → Sab | ab | b, which generates S → Sabc | abc | bc | c; ***contain ILR;*** [A_i=S, A_j=Q]

  (continue...)

Remove Left Recursion[消除左递归]

(continue...)

(α) (β)

(i=3,j=2) S → Sabc | abc | bc | c contains ILR.

Eliminate ILR about S:

S → abcS' | bcS' | cS'

S' → abcS' | ε

Q → Sab | ab | b

R → Sa | a

- **Step 3 :** Simplify the grammar and get **Final Grammar**:

S → abcS' | bcS' | cS' S' → abcS' | ε

R → Sa | a

Q → Rb | b

S → Qc | c

(Q&R's production is included by S)

Remove Left Recursion[消除左递归]

- **Question: What will happen if the order in step 1 is different**
- Again, consider G(S): S → Qc | c   Q → Rb | b   R → Sa | a
- **Exercise:** If the order in step 1 is **S,Q,R**, the final grammar without ILR is?   (was R, Q, S in the previous example)

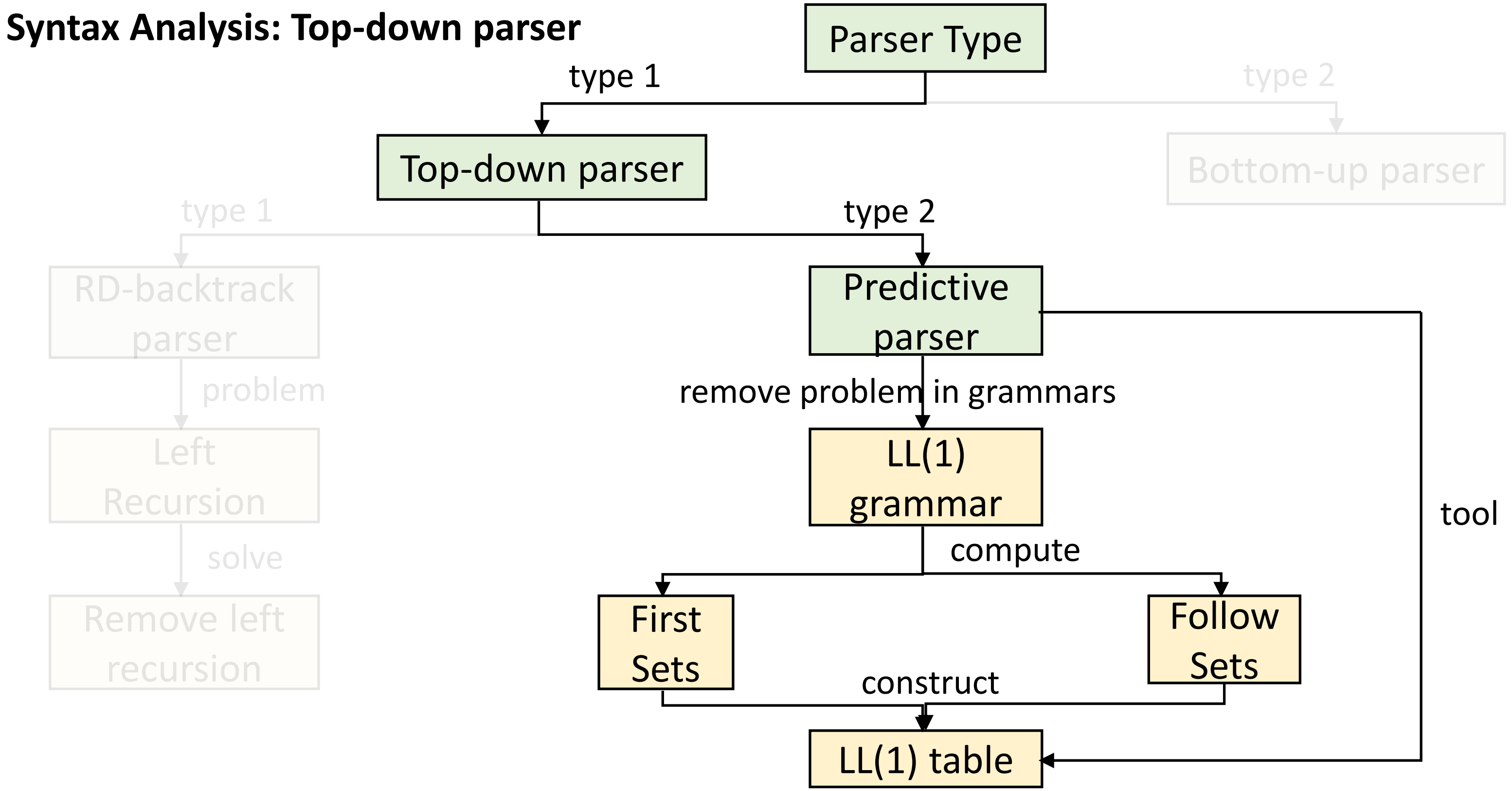
S,Q,R		R,Q,S
S → Qc | c Q → Rb | b R → bcaR' | caR' | aR' R' → bcaR' | ε	equivalent ⟺	S → abcS' | bcS' | cS' S' → abcS' | ε

- Different order in Step1 may cause the final grammar different in form[可能在形式上不同], but it is not difficult to prove that they are equivalent.

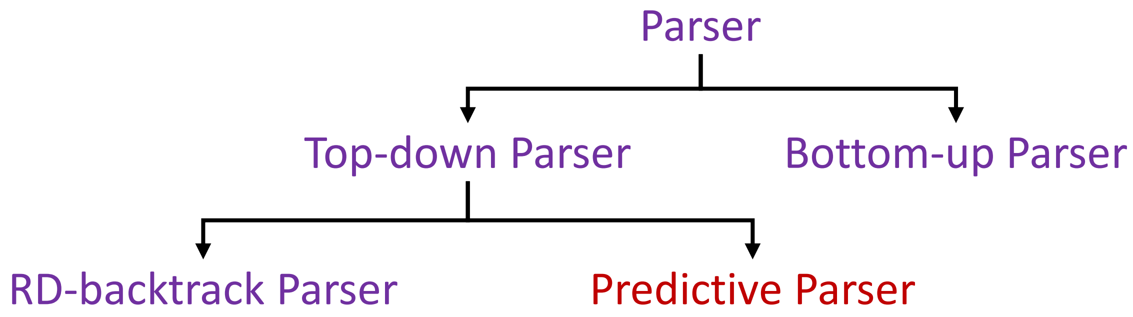
Recursive-descent parsing[递归下降语法分析]

- **Recursive-descent parsing** [RDP, 递归下降语法分析]
  - A general form of top-down parsing.
  - RDP is a simple and general parsing strategy.
    - Left-recursion must be eliminated first.(Can be eliminated automatically using some algorithm)
  - However, it is not popular because of **backtracking**.
    - Backtracking requires *re-parsing* the same string.
    - Tried to solve problem *in any possible ways*[穷尽一切可能的试探法] which is inefficient.(can take exponential time)
    - Also removing already added nodes in parse tree is troublesome.

Mind Map[思维导图]



Predictive Parsing[预测分析]



- **Predictive Parsing**[预测分析法]
  - ◆ A special case of recursive-descent parsing **without backtracking**[无回溯].
  - ◆ Predictive parsing chooses the correct production by looking ahead at the input a fixed number of symbols, typically we may look only at one. (that is, the next input symbol)
  - ◆ Need Restrictions on the grammar to avoid backtracking. [LL(k)]

Predictive Parsing[预测分析]

- **A parser with no backtracking** [无回溯]**:** select the correct alternative through given next input terminal(s)[下一个输入符号/终结符]
- A predictive parser chooses the production to apply solely on the basis of [选取产生式的依据]
  - Next input symbol(s).
  - Current non-terminal being processed.

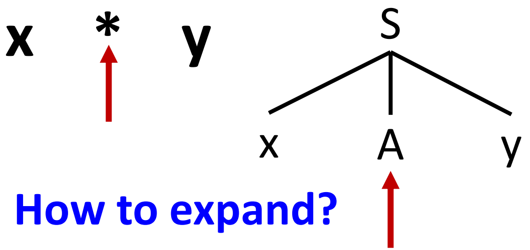
G(S): S → aBD | bBB; B → c | bce; D → d
parsing input “abced” requires no backtracking

What if S → aBD | aBB   **Common Prefix[共同前缀]**

Given input terminal(s) a, cannot choose between two rules.

Common Prefix[共同前缀]

- Given $G[S]$: $S \rightarrow xAy$; $A \rightarrow ** \mid *$, if the current input symbol is $*$, $A \rightarrow \alpha_1 \mid \alpha_2 \mid \ldots \mid \alpha_n$. How to choose the right A-production?



A → ** OR A → *?

1. If there is a unique α_i with * as the head, replace using this unique α_i.
2. If there are multiple α_i with * as the head, the substitution is not unique[替换是不唯一的], ***backtracking*** may happen.

- **Left factoring**[提取左公因子]: Rewrite the productions to ***defer the decision*** until enough of the input has been seen that we can make the right choice. [推后决定直至可选择]

Left factoring[提取公共左因子]

- For each non-terminal A, find *the longest common prefix α* to two or more of its alternatives.
  - **Change** A-productions $A \rightarrow \alpha\beta_1 \mid \alpha\beta_2 \mid \dots \mid \alpha\beta_n \mid \gamma$, where γ represents all alternatives that do not begin with α, to $A \rightarrow \alpha A' \mid \gamma$; $A' \rightarrow \beta_1 \mid \beta_2 \mid \dots \mid \beta_n$.
- *Repeatedly apply this transformation until no two alternatives for have a common prefix.*
- **Example:** G[S]: $S \rightarrow abc \mid abd \mid ae$;
  - **Step1:** $S \rightarrow aA'$; $A' \rightarrow bc \mid bd \mid e$;
  - **Step2:** $S \rightarrow aA'$; $A' \rightarrow bB' \mid e$; $B' \rightarrow c \mid d$;

Predictive Parsing[预测分析]

- Patterns in grammars that prevent predictive parsing [并非总是能预测分析] :
  - ◆ **Left Recursion Problem**[左递归问题]
    - ◻ Lookahead symbol changes **only** when a terminal is matched.
    - ◻ *Q: How to solve? A:* **Remove Left Recursion**.
  - ◆ **Common Prefix**[共同前缀] —Cause **Backtracking Problem**.
    - ◻ *Q: How to solve? A:* **Left factoring**.
  - ◆ **Question:** After left factoring, Can we *completely* avoid the backtracking and enable prediction? What if S → EBD | FBB; E → a ; F → a

FIRST && FOLLOW!

G(S): S → aBD | bBB; B → c | bce; D → d
parsing input “abced” requires no backtracking

FIRST[终结首符集]

- During top-down parsing, FIRST and FOLLOW allow us to choose which production to apply, based on the next input symbol.
- **FIRST(α)** [终结首符集] (α is *any string* of grammar symbols)
  - FIRST(α) is the set of terminals that are the beginning strings derived from α.
    FIRST(α) = $\{a \mid \alpha \overset{*}{\Rightarrow} a\ldots,\ a \in V_T\}$ [串的终结首符的集合]
  - AND if $\alpha \overset{*}{\Rightarrow} \varepsilon$, then ε is also in FIRST(α).
  - Consider two A-productions $A \to \alpha \mid \beta$, **where FIRST(α) and FIRST(β) are disjoint sets** [不相交的集合].
    - We can then choose between these A-productions by looking at the next input symbol a, which exists either FIRST(α) and FIRST(β), not both.

FOLLOW[后继终结符号集]

- **Question:** for an input symbol ***a*** and the non-terminal **A** ($A \rightarrow \alpha_1 \mid \alpha_2 \mid \ldots \mid \alpha_n$), ***a*** $\boldsymbol{\notin}$ **FIRST(α_i)**, in this case, how to choose α_i, or draw a conclusion of grammatical errors?
- **FOLLOW(A):** For a <u>*nonterminal*</u> A, the set of terminals *a* that can appear immediately to the right of A is FOLLOW(A) = $\{a \mid S \overset{*}{\Rightarrow} \ldots Aa\ldots,\ a \in V_T\}$.[紧跟在A右边的终结符号的集合]
  - If A is the rightmost symbol in some sentential forms, then $ is in FOLLOW(A); recall that $ is a special “endmarker” symbol[结束标记] that is assumed not to be a symbol of any grammar.

FOLLOW[后继终结符号集]

- **Question:** for an input symbol ***a*** and the non-terminal **A** ($A \to \alpha_1 \mid \alpha_2 \mid \ldots \mid \alpha_n$), ***a*** **∉ FIRST($\alpha_i$),** in this case, how to choose α_i, or draw a conclusion of grammatical errors?
- **Answer:** if ε ∈ FIRST (α_i), then when *a* ∈ FOLLOW(A), select $A \to \alpha_i$, otherwise, report grammatical error.
- **Example:** G(S): S → aA | d ;   A → bAS | ε.   String : *abd*.

S	*abd*	FIRST(aA)= {a}	FOLLOW(A) ={$,a,d}
⇒ aA	*abd*	FIRST(d) = {d}	
⇒ abAS	*abd*	FIRST(bAS)= {b}	
⇒ abS	*abd*	FIRST(A) = {b, ε}	
⇒ abd		FIRST(S) = {a, d}	

HOW TO compute FIRST & FOLLOW?

Compute FIRST[计算FIRST集合的方法]

- To compute FIRST(X) for a grammar symbol X, **apply the following rules until no more terminals or ε can be added to any FIRST set:**
  - **Rule1:** If $X \in V_T$, then FIRST(X) = {X}. [X是终结符]
  - **Rule2:** If $X \in V_N$ and $X \rightarrow \varepsilon$ exists, then FIRST(X) = FIRST(X) ∪ ε. [非终结符,空式]
  - **Rule3:** If $X \in V_N$ and $X \rightarrow Y_1Y_2Y_3\ldots Y_k$, then [非终结符,非空式]
    - If for some Y_i and a terminal a:
      1. $\varepsilon \in \mathrm{FIRST}(Y_1), \ldots, \mathrm{FIRST}(Y_{i-1})$, i.e., $Y_1\ldots Y_{i-1} \overset{*}{\Rightarrow} \varepsilon$
      2. $a \in \mathrm{FIRST}(Y_i) \setminus \{\varepsilon\}$
    - Then, we have FIRST(X) = FIRST(X) ∪ a.
    - If $\varepsilon \in \mathrm{FIRST}(Y_i)$ for i=1,…,k, FIRST(X) = FIRST(X) ∪ ε.

Starting from Y_1

- Everything in $\mathrm{FIRST}(Y_1) \setminus \{\varepsilon\}$ is surely in FIRST(X).
- If Y_1 doesn't derive ε, then we add nothing more.
- But if $Y_1 \overset{*}{\Rightarrow} \varepsilon$, then we add $\mathrm{FIRST}(Y_2) \setminus \{\varepsilon\}$, and so on…

Compute FIRST[计算FIRST集合的方法]

- Next, we can compute FIRST for any string $\alpha = X_1X_2 \ldots X_n$ [符号串]
  - Add all non-ε symbols of FIRST(X_1) to FIRST(α).
  - If FIRST(X_1), ..., FIRST(X_{i-1}) all contain ε[前k-1个都包含空串], add non-ε symbols of FIRST(X_i), $2 \le i \le n$, to FIRST(α).
  - If FIRST(X_1), ..., FIRST(X_n) all contain ε, Add ε to FIRST(α).

Compute FIRST[计算FIRST集合的方法]

- **Example:** G[E]: E → TE’ ; E’ → +TE’ | ε ; T → FT’ ; T’ → *FT’ | ε ; F → (E) | **id , compute the FIRST set of each non-terminal.**
  - FIRST(E):{
  - FIRST(T):{
  - FIRST(E’):{ + , ε
  - FIRST(T’):{ * , ε
  - FIRST(F):{ (, **id**

Apply rules for the first time:

E → TE’: *FIRST(T) add to FIRST(E), FIRST(T) doesn’t contain ε*

E’ → +TE’: *FIRST(+) add to FIRST(E’), FIRST(+) doesn’t contain ε*

E’ → ε: *ε add to FIRST(E’)*

T → FT’: *FIRST(F) add to FIRST(T), FIRST(F) doesn’t contain ε*

T’ → *FT’: *FIRST(*) add to FIRST(T’), FIRST(*) doesn’t contain ε*

T’ → ε: *ε add to FIRST(T’)*

F → (E): *FIRST(‘(’) add to FIRST(F), FIRST(‘(’) doesn’t contain ε*

F → **id**: *FIRST(**id**) add to FIRST(F)*

Compute FIRST[计算FIRST集合的方法]

- **Example:** G[E]: E → TE' ; E' → +TE' | ε ; T → FT' ; T' → *FT' | ε ; F → (E) | **id** , **compute the FIRST set of each non-terminal.**
  - FIRST(E):{
  - FIRST(T):{ (, **id**
  - FIRST(E'):{ +, ε
  - FIRST(T'):{ *, ε
  - FIRST(F):{ (, **id**

It is necessary to determine whether the FIRST set has changed after each rule application. first time YES!

Apply rules for the second time:

E → TE': *FIRST(T) add to FIRST(E), FIRST(T) doesn't contain ε*

E' → +TE': *FIRST(+) add to FIRST(E'), FIRST(+) doesn't contain ε*

E' → ε: *ε add to FIRST(E')*

T → FT': *FIRST(F) add to FIRST(T), FIRST(F) doesn't contain ε*

T' → *FT': *FIRST(*) add to FIRST(T'), FIRST(*) doesn't contain ε*

T' → ε: *ε add to FIRST(T')*

F → (E): *FIRST('(') add to FIRST(F), FIRST('(') doesn't contain ε*

F → **id**: *FIRST(**id**) add to FIRST(F)*

Compute FIRST[计算FIRST集合的方法]

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- **Example:** G[E]: E → TE' ; E' → +TE' | ε ; T → FT' ; T' → *FT' | ε ; F → (E) | **id** , **compute the FIRST set of each non-terminal.**

  - FIRST(E):{ (, **id**
  - FIRST(T):{ (, **id**
  - FIRST(E'):{ +, ε
  - FIRST(T'):{ *, ε
  - FIRST(F):{ (, **id**

It is necessary to determine whether the FIRST set has changed after each rule application. second time YES!

Apply rules for the third time:

E → TE': *FIRST(T) add to FIRST(E), FIRST(T) doesn't contain ε*

E' → +TE': *FIRST(+) add to FIRST(E'), FIRST(+) doesn't contain ε*

E' → ε: *ε add to FIRST(E')*

T → FT': *FIRST(F) add to FIRST(T), FIRST(F) doesn't contain ε*

T' → *FT': *FIRST(*) add to FIRST(T'), FIRST(*) doesn't contain ε*

T' → ε: *ε add to FIRST(T')*

F → (E): *FIRST('(') add to FIRST(F), FIRST('(') doesn't contain ε*

F → **id**: *FIRST(**id**) add to FIRST(F)*

Compute FIRST[计算FIRST集合的方法]

- **Example:** G[E]: E → TE' ; E' → +TE' | ε ; T → FT' ; T' → *FT' | ε ; F → (E) | **id** , **compute the FIRST set of each non-terminal.**

  - FIRST(E):{ (, **id** }
  - FIRST(T):{ (, **id** }
  - FIRST(E'):{ +, ε }
  - FIRST(T'):{ *, ε }
  - FIRST(F):{ (, **id** }

It is necessary to determine whether the FIRST set has changed after each rule application. third time YES!

Apply rules for the 4th time:

E → TE': *FIRST(T) add to FIRST(E), FIRST(T) doesn't contain ε*

E' → +TE': *FIRST(+) add to FIRST(E'), FIRST(+) doesn't contain ε*

E' → ε: *ε add to FIRST(E')*

T → FT': *FIRST(F) add to FIRST(T), FIRST(F) doesn't contain ε*

T' → *FT': *FIRST(*) add to FIRST(T'), FIRST(*) doesn't contain ε*

T' → ε: *ε add to FIRST(T')*

F → (E): *FIRST('(') add to FIRST(F), FIRST('(') doesn't contain ε*

F → **id**: *FIRST(**id**) add to FIRST(F)*

It is necessary to determine whether the FIRST set has changed after each rule application. 4th time NO!

Compute FOLLOW[计算FOLLOW集合的方法]

- To compute FOLLOW(A) for all non-terminals A, **apply the following rules until nothing can be added to any FOLLOW set.**
  - **Rule1:** Place \$ in FOLLOW(S), where S is the start symbol.
  - **Rule2:** If there is a production $A \to \alpha B \beta$, then everything in FIRST(β) except ε is placed in FOLLOW(B).
  - **Rule3:** If there is a production $A \to \alpha B$, or a production $A \to \alpha B \beta$, where FIRST(β) contains ε, then everything in FOLLOW(A) is placed in FOLLOW(B).
    - for example, in bigger context "abc($A \to \alpha B$)abc"

Compute FOLLOW[计算FOLLOW集合的方法]

- G[E]: E → TE'; E' → +TE' | ε; T → FT'; T' → *FT' | ε; F → (E) | **id**;
  - FIRST(E):{ (, **id** }
  - FIRST(T):{ (, **id** };
  - FIRST(E'):{ +, ε };
  - FIRST(T'):{ *, ε };
  - FIRST(F):{ (, **id** }.
  - FOLLOW(E):{ $
  - FOLLOW(T):{ +, $
  - FOLLOW(E'):{ $
  - FOLLOW(T'):{
  - FOLLOW(F):{

Apply rules for the first time:

- Place $ in FOLLOW(E), since E is the start symbol.
- E → TE'
  - FIRST(E') except ε is in FOLLOW(T).
  - Everything in FOLLOW(E) is in FOLLOW(E').
  - Since FIRST(E') contains ε, then everything in FOLLOW(E) is in FOLLOW(T).
- E' → +TE' | ε
  - FIRST(E') except ε is in FOLLOW(T).
  - Everything in FOLLOW(E') is in FOLLOW(E').
  - Since FIRST(E') contains ε, then everything in FOLLOW(E') is in FOLLOW(T).

Compute FOLLOW[计算FOLLOW集合的方法]

- G[E]: E → TE'; E' → +TE' | ε; T → FT'; T' → *FT' | ε; F → (E) | **id**;
  - FIRST(E):{ (, **id** }
  - FIRST(T):{ (, **id** };
  - FIRST(E'):{ +, ε };
  - FIRST(T'):{ *, ε };
  - FIRST(F):{ (, **id** }.

  - FOLLOW(E):{ $
  - FOLLOW(T):{ +, $
  - FOLLOW(E'):{ $
  - FOLLOW(T'):{+, $
  - FOLLOW(F):{ *, +, $

Apply rules for the first time:

- T → FT'
  - FIRST(T') except ε is in FOLLOW(F).
  - Everything in FOLLOW(T) is in FOLLOW(T').
  - Since FIRST(T') contains ε, then everything in FOLLOW(T) is in FOLLOW(F).
- T' → *FT' | ε
  - FIRST(T') except ε is in FOLLOW(F).
  - Everything in FOLLOW(T') is in FOLLOW(T').
  - Since FIRST(T') contains ε, then everything in FOLLOW(T') is in FOLLOW(F).

Compute FOLLOW[计算FOLLOW集合的方法]

- G[E]: E → TE’; E’ → +TE’ | ε; T → FT’; T’ → *FT’ | ε; F → (E) | **id**;
  - FIRST(E):{ (, **id** }
  - FIRST(T):{ (, **id** };
  - FIRST(E’):{ +, ε };
  - FIRST(T’):{ *, ε };
  - FIRST(F):{ (, **id** }.
  - FOLLOW(E):{ $,)
  - FOLLOW(T):{ +, $,)
  - FOLLOW(E’):{ $,)
  - FOLLOW(T’):{ +, $
  - FOLLOW(F):{ *, +, $

Apply rules for the first time:

- F → (E) | **id**
  - FIRST(‘)’) except ε is in FOLLOW(E).

Since FOLLOW sets has changed at the first time,

Apply rules for the second time:

- E → TE’
  - FIRST(E’) except ε is in FOLLOW(T).
  - Everything in FOLLOW(E) is in FOLLOW(E’).
  - Since FIRST(E’) contains ε, then everything in FOLLOW(E) is in FOLLOW(T).

Compute FOLLOW[计算FOLLOW集合的方法]

- G[E]: E → TE'; E' → +TE' | ε; T → FT'; T' → *FT' | ε; F → (E) | **id**;
  - FIRST(E):{ (, **id** }
  - FIRST(T):{ (, **id** };
  - FIRST(E'):{ +, ε };
  - FIRST(T'):{ *, ε };
  - FIRST(F):{ (, **id** }.
  - FOLLOW(E):{ $,) }
  - FOLLOW(T):{ +, $,) }
  - FOLLOW(E'):{ $,) }
  - FOLLOW(T'):{ +, $,) }
  - FOLLOW(F):{ *, +, $,) }

- E' → +TE' | ε
  - FIRST(E') except ε is in FOLLOW(T).
  - Everything in FOLLOW(E') is in FOLLOW(E').
  - Since FIRST(E') contains ε, then everything in FOLLOW(E') is in FOLLOW(T).
- T → FT'
  - FIRST(T') except ε is in FOLLOW(F).
  - Everything in FOLLOW(T) is in FOLLOW(T').
  - Since FIRST(T') contains ε, then everything in FOLLOW(T) is in FOLLOW(F).

Compute FOLLOW[计算FOLLOW集合的方法]

- G[E]: E → TE’; E’ → +TE’ | ε; T → FT’; T’ → *FT’ | ε; F → (E) | **id**;
  - FIRST(E):{ (, **id** }
  - FIRST(T):{ (, **id** };
  - FIRST(E’):{ +, ε };
  - FIRST(T’):{ *, ε };
  - FIRST(F):{ (, **id** }.

  - FOLLOW(E):{ $,) }
  - FOLLOW(T):{ +, $,) }
  - FOLLOW(E’):{ $,) }
  - FOLLOW(T’):{ +, $,) }
  - FOLLOW(F):{ *, +, $,) }

- T’ → *FT’ | ε
  - FIRST(T’) except ε is in FOLLOW(F).
  - Everything in FOLLOW(T’) is in FOLLOW(T’).
  - Since FIRST(T’) contains ε, then everything in FOLLOW(T’) is in FOLLOW(F).
- F → (E) | **id**
  - FIRST(‘)’) except ε is in FOLLOW(E).

Since FOLLOW sets has changed at the second time,

Apply rules for the third time:

- **EXERCISE!**
- **Ans: no FOLLOW set will be changed.**

LL(1) Grammar[LL(1)文法]

- *Predictive parsers*, that is, recursive-descent parsers needing no backtracking, can be constructed for a class of grammars called LL(1).
- **Definition:** A grammar G is LL(1) **if and only if** each productions of G, $A \rightarrow \alpha \mid \beta$, meet the following conditions:
  - For no terminal *a* do both α and β derive strings beginning with *a*.
  - At most one of α and β can derive the empty string.
  - If $\beta \overset{*}{\Rightarrow} \varepsilon$, then α does not derive any string beginning with a terminal in FOLLOW(A). Likewise, if $\alpha \overset{*}{\Rightarrow} \varepsilon$, then β does not derive any string beginning with a terminal in FOLLOW(A).

LL(1) Grammar[LL(1)文法]

- The first two conditions are equivalent to the statement that FIRST(α) and FIRST(β) are disjoint sets [不相交的集合].

  $\mathrm{FIRST}(\alpha) \cap \mathrm{FIRST}(\beta) = \emptyset$.

- The third condition is equivalent to stating that if ε is in FIRST(β), then FIRST(α) and FOLLOW(A) must be disjoint sets, and likewise if ε is in FIRST(α).

  If $\beta \overset{*}{\Rightarrow} \varepsilon$, then $\mathrm{FIRST}(\alpha) \cap \mathrm{FOLLOW}(A) = \emptyset$.

  Otherwise, we cannot choose α or β.
  $(A \rightarrow \alpha \mid \beta)$

- LL(1) Grammar does not contain left recursion. [LL(1)文法不含左递归]
- LL(1) Grammar is not ambiguous. [LL(1)文法不是二义的]

LL(1)/LL(k) Grammar[LL(1)/LL(k)文法]

- LL (1) grammar.
  - L: The first "L" in LL(1) stands for scanning the input from left to right.
  - L: The second "L" for producing a leftmost derivation.
  - 1: The "1" for using one input symbol of lookahead at each step to make parsing action decisions.
- LL (k) grammar.
  - k: using **k** input symbols of lookahead at each step to make parsing action decisions.
- Is LL(0) useful at all?
  - Grammar where rules can be predicted with no lookahead.
  - Meaning there can only be one rule per non-terminal.
  - Meaning this language can have only one string.

LL(1) Parser Implementation[实现]

- *Recursive* LL(1) parser for G[S]: S → A | B; A → a; B → b.
  - maintaining a stack implicitly. [隐式维护栈]

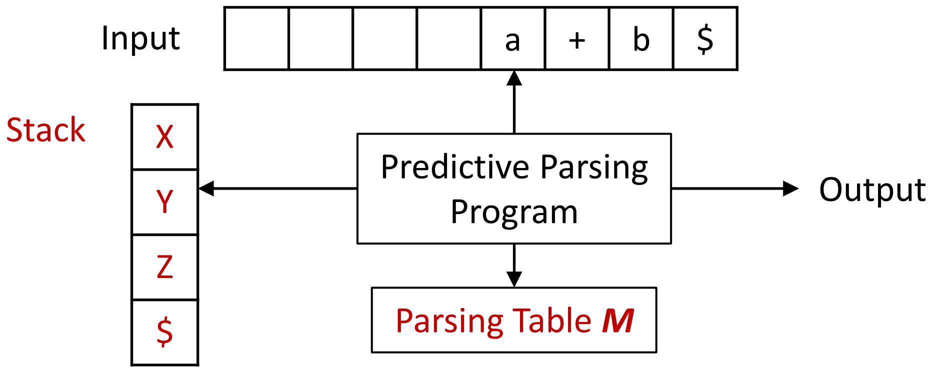
```
void S(){
        token = Next(); // lookahead
        if(token == a) // 'A' starts with 'a'
                A(); // call procedure A()
        else if (token == b) // 'B' starts with 'b'
                B(); // call procedure B()
        else
                return; // error, reject.
}
```

- Is there a way to express above code more concisely? [简洁]

- *Non-recursive* LL(1) parsers
  - Use a predictive parsing table. [预测分析表]
  - maintaining a stack explicitly. [显式维护栈]
  - Table-driven parser. [表驱动]

Non-recursive LL(1) Parser [非递归]

- **Input buffer**: contains the string to be parsed, followed by the endmarker $.
- **Stack:** holds a sequence of grammar symbols and the symbol $ to mark the bottom of the stack. The stack may contain:
  - **Terminals** that have yet to be matched against the input symbol.
  - **Non-terminals** that have yet to be expanded.



- **Parsing Table M[A,a]:** an entry containing production “A→…” or error.
- **Predictive Parsing Program:** Execute the action according to **<stack top, current input symbol>**

LL(1) Parse Table [预测分析表]

G[E]:
E → TE'
E' → +TE' | ε
T → FT'
T' → *FT' | ε
F → (E) | **id**

	id	+	*	(	)	$
E	E → TE'			E → TE'		
E'		E' → +TE'			E' → ε	E' → ε
T	T → FT'			T → FT'		
T'		T' → ε	T' → *FT'		T' → ε	T' → ε
F	F → **id**			F → (E)		

Input symbol, lists all possible terminals and $

all non-terminals in the grammar

One action for each <non-terminal, next input>, it "predicts" the correct action based on one lookahead

- Reject on reaching error state
- Accept on end of input & empty stack

FIRST(E):{ (, id } FOLLOW(E):{ $,)}

FIRST(T):{ (, id } FOLLOW(T):{ +, $,)}

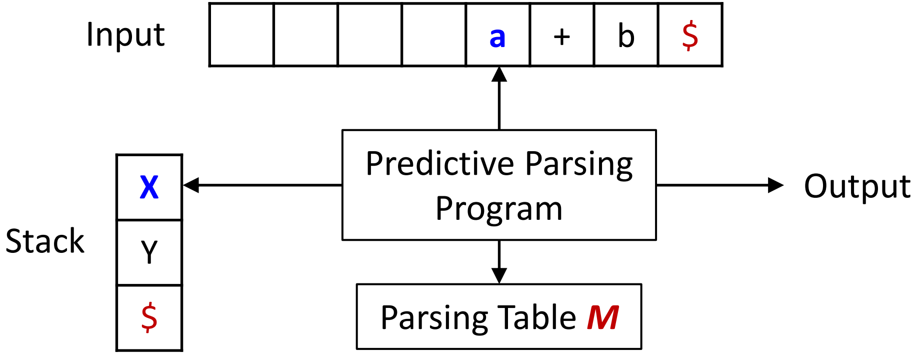
FIRST(E'):{ +, ε } FOLLOW(E'):{ $,)}

FIRST(T'):{ *, ε } FOLLOW(T'):{ +, $,)}

FIRST(F):{ (, id }. FOLLOW(F):{ *, +, $,)}

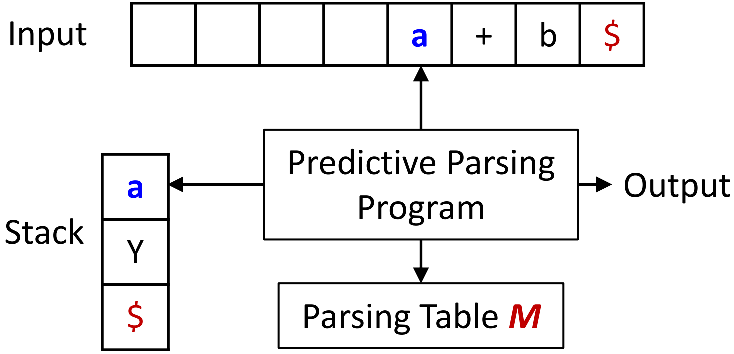
LL(1) Parsing Algorithm [非递归算法]

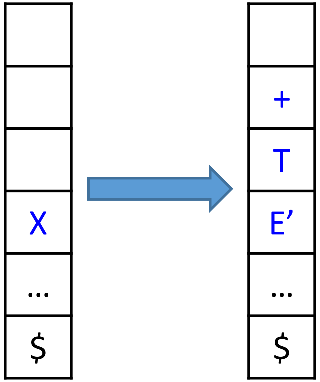
- Initial state [初始态]
  - **Input:** A string ***w*** and a parsing table ***M*** for grammar ***G***. **Input Buffer**: ***w$***.
  - **Stack:** start symbol followed by '**$**' at bottom.
  - Assume **X:** symbol at the top of the stack, **a:** current input symbol.
- General idea [总体思路] : repeat one of two actions
  - **Expand** symbol at top of stack by applying a production
  - **Match** terminal symbol at top of stack with input token



LL(1) Parsing Algorithm [非递归算法]

- Algorithm Step-by-Step based on **<X, a>**:
  - X: symbol at the top of the stack
  - a: current input token
  - If $X \in V_T$, [栈顶符号为终结符] and
    - X == a == \$, declare **SUCCESS**, stop parsing.
    - X == a != \$, **pop X from stack** and move the current input symbol forward one.
    - X != a, declare **ERROR**, input is rejected, stop parsing.
  - If $X \in V_N$, [栈顶符号为非终结符] and
    - M[X, a] has a production about X, pop X and ***push right side of production to stack.***
    - M[X, a] == empty, declare **ERROR**, input is rejected, stop parsing.





If M[X,a] = “X → +TE’”

Push to stack in Reverse Order

[逆序入栈]

If M[X,a] = “X → ε”

Only pop and push nothing

Use the Parse Table [使用表进行预测分析]

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- When using parse table for predictive parsers, if the grammar does not conform to the specification of LL (1) grammar:
  - the grammar should be rewritten into LL (1) grammar by removing left recursion and backtracking, then the parse table should be constructed.

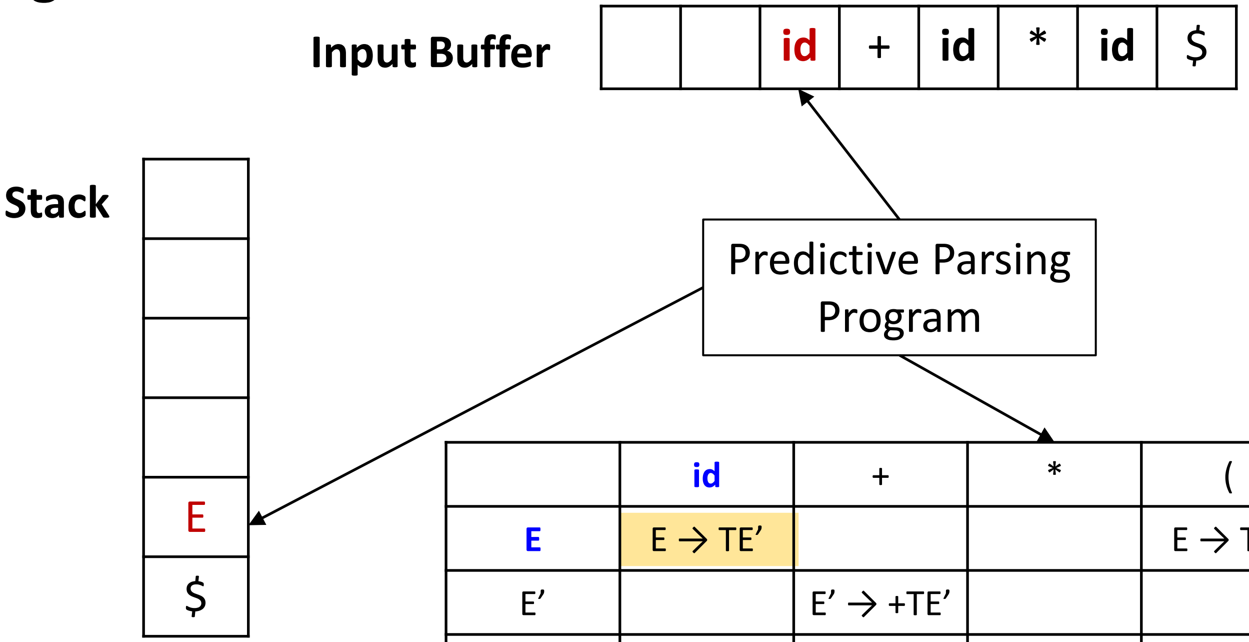
- **LL(1) parser example:** consider G[E] to recognize "**id** + **id** * **id**"

G[E]:
E → TE'
E' → +TE' | ε
T → FT'
T' → *FT' | ε
F → (E) | **id**

	id	+	*	(	)	$
E	E → TE'			E → TE'		
E'		E' → +TE'			E' → ε	E' → ε
T	T → FT'			T → FT'		
T'		T' → ε	T' → *FT'		T' → ε	T' → ε
F	F → **id**			F → (E)		

Use the Parse Table [使用表进行预测分析]

- recognize “**id + id * id**”



G[E]:

E → TE’

E’ → +TE’ | ε

T → FT’

T’ → *FT’ | ε

F → (E) | **id**

	id	+	*	(	)	$
E	E → TE’			E → TE’		
E’		E’ → +TE’			E’ → ε	E’ → ε
T	T → FT’			T → FT’		
T’		T’ → ε	T’ → *FT’		T’ → ε	T’ → ε
F	F → **id**			F → (E)		

Use the Parse Table [使用表进行预测分析]

- recognize “**id + id * id**”

Input Buffer

		id	+	id	*	id	$

Stack

T
E’
$

Predictive Parsing Program

G[E]:

E → TE’
E’ → +TE’ | ε
T → FT’
T’ → *FT’ | ε
F → (E) | **id**

	id	+	*	(	)	$
E	E → TE’			E → TE’		
E’		E’ → +TE’			E’ → ε	E’ → ε
T	T → FT’			T → FT’		
T’		T’ → ε	T’ → *FT’		T’ → ε	T’ → ε
F	F → **id**			F → (E)		

Use the Parse Table [使用表进行预测分析]

- recognize "**id + id * id**"

Input Buffer

		id	+	**id**	*	**id**	$

Stack

F
T'
E'
$

Predictive Parsing Program

	id	+	*	(	)	$
E	E → TE'			E → TE'		
E'		E' → +TE'			E' → ε	E' → ε
T	T → FT'			T → FT'		
T'		T' → ε	T' → *FT'		T' → ε	T' → ε
F	F → **id**			F → (E)		

G[E]:

E → TE'

E' → +TE' | ε

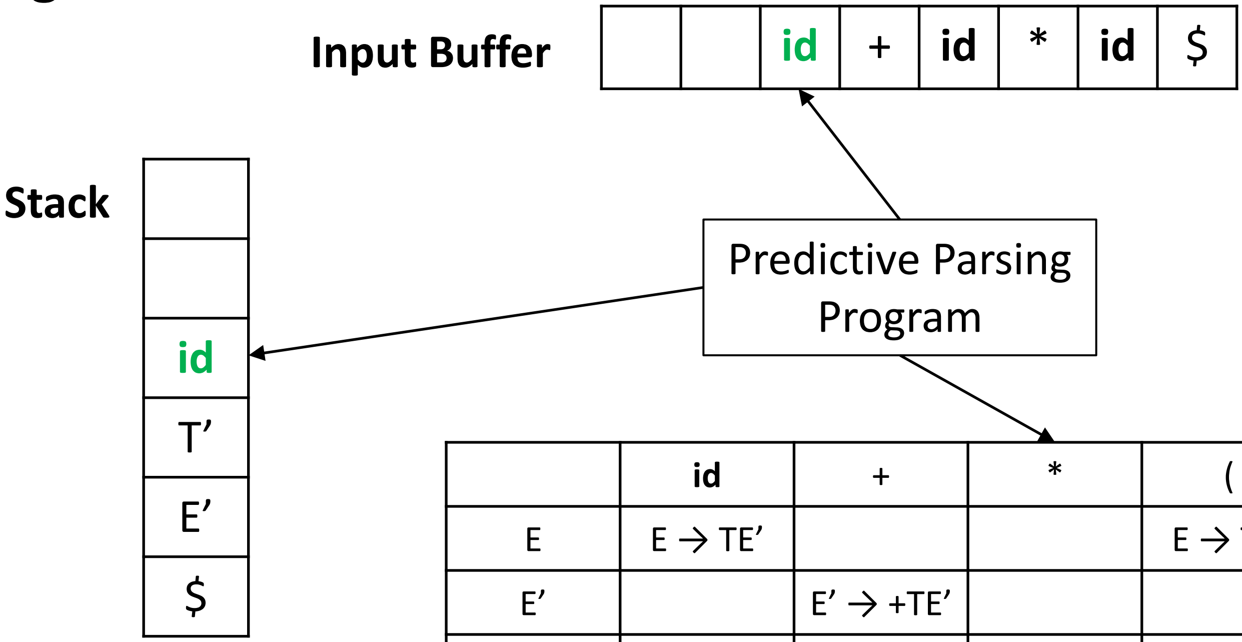
T → FT'

T' → *FT' | ε

F → (E) | **id**

Use the Parse Table [使用表进行预测分析]

- recognize "**id + id * id**"



G[E]:

E → TE'

E' → +TE' | ε

T → FT'

T' → *FT' | ε

F → (E) | **id**

	id	+	*	(	)	$
E	E → TE'			E → TE'		
E'		E' → +TE'			E' → ε	E' → ε
T	T → FT'			T → FT'		
T'		T' → ε	T' → *FT'		T' → ε	T' → ε
F	F → **id**			F → (E)		

Use the Parse Table [使用表进行预测分析]

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- recognize "**id** + **id** * **id**"

Input Buffer

		id	+	id	*	id	$

Stack

T'
E'
$

Predictive Parsing Program

	id	+	*	(	)	$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \rightarrow \varepsilon$	$E' \rightarrow \varepsilon$
T	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T' \rightarrow \varepsilon$	$T' \rightarrow *FT'$		$T' \rightarrow \varepsilon$	$T' \rightarrow \varepsilon$
F	$F \rightarrow$ **id**			$F \rightarrow (E)$		

G[E]:

$E \rightarrow TE'$

$E' \rightarrow +TE' \mid \varepsilon$

$T \rightarrow FT'$

$T' \rightarrow *FT' \mid \varepsilon$

$F \rightarrow (E) \mid$ **id**

Use the Parse Table [使用表进行预测分析]

- recognize "**id** + **id** * **id**"

G[E]:

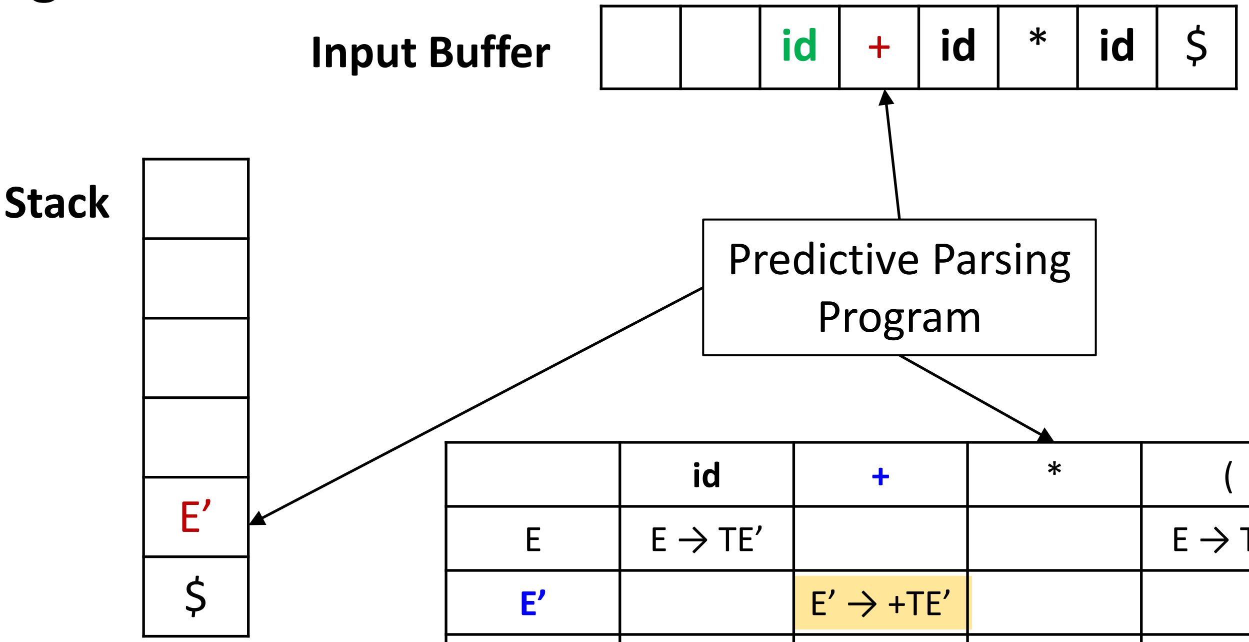
E → TE'

E' → +TE' | ε

T → FT'

T' → *FT' | ε

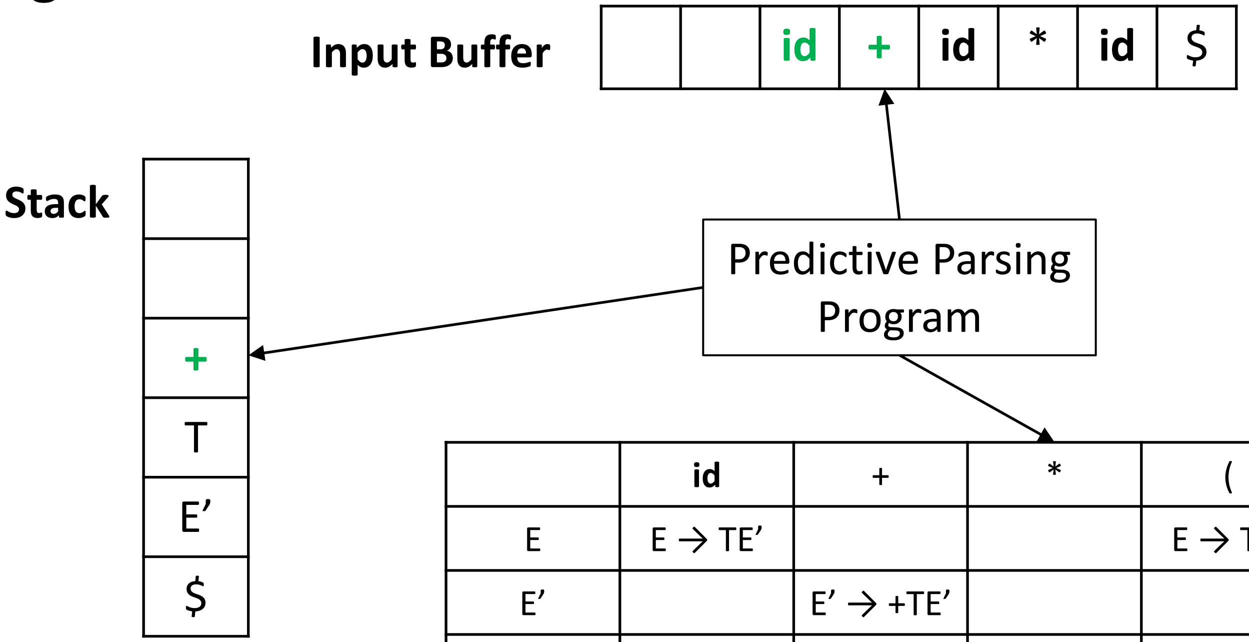
F → (E) | **id**



	id	+	*	(	)	$
E	E → TE'			E → TE'		
E'		E' → +TE'			E' → ε	E' → ε
T	T → FT'			T → FT'		
T'		T' → ε	T' → *FT'		T' → ε	T' → ε
F	F → **id**			F → (E)		

Use the Parse Table [使用表进行预测分析]

- recognize "**id** + **id** * **id**"



G[E]:

E → TE'

E' → +TE' | ε

T → FT'

T' → *FT' | ε

F → (E) | **id**

	id	+	*	(	)	$
E	E → TE'			E → TE'		
E'		E' → +TE'			E' → ε	E' → ε
T	T → FT'			T → FT'		
T'		T' → ε	T' → *FT'		T' → ε	T' → ε
F	F → **id**			F → (E)		

Use the Parse Table [使用表进行预测分析]

- recognize "**id** + **id** * **id**"

Input Buffer

		id	+	id	*	id	$

Stack

T
E'
$

Predictive Parsing Program

	id	+	*	(	)	$
E	E → TE'			E → TE'		
E'		E' → +TE'			E' → ε	E' → ε
T	T → FT'			T → FT'		
T'		T' → ε	T' → *FT'		T' → ε	T' → ε
F	F → **id**			F → (E)		

G[E]:

E → TE'

E' → +TE' | ε

T → FT'

T' → *FT' | ε

F → (E) | **id**

Use the Parse Table [使用表进行预测分析]

- recognize “**id** + **id** * **id**”

G[E]:

E → TE’

E’ → +TE’ | ε

T → FT’

T’ → *FT’ | ε

F → (E) | **id**

Input Buffer

		id	+	id	*	id	$

Stack

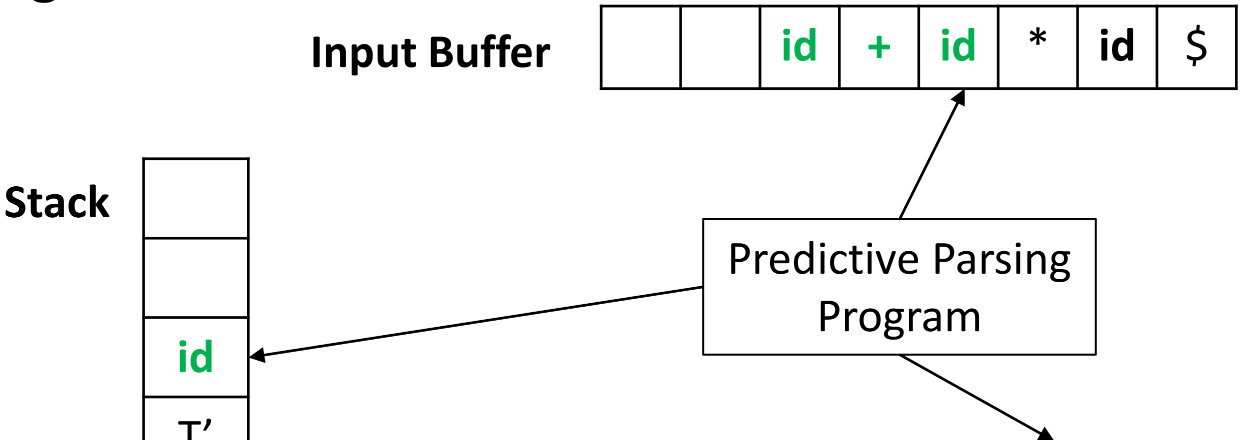
F
T’
E’
$

Predictive Parsing Program

	id	+	*	(	)	$
E	E → TE’			E → TE’		
E’		E’ → +TE’			E’ → ε	E’ → ε
T	T → FT’			T → FT’		
T’		T’ → ε	T’ → *FT’		T’ → ε	T’ → ε
F	F → **id**			F → (E)		

Use the Parse Table [使用表进行预测分析]

- recognize “**id + id * id**”



G[E]:
E → TE’
E’ → +TE’ | ε
T → FT’
T’ → *FT’ | ε
F → (E) | **id**

	id	+	*	(	)	$
E	E → TE’			E → TE’		
E’		E’ → +TE’			E’ → ε	E’ → ε
T	T → FT’			T → FT’		
T’		T’ → ε	T’ → *FT’		T’ → ε	T’ → ε
F	F → **id**			F → (E)		

Use the Parse Table [使用表进行预测分析]

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- recognize “**id + id * id**”

Input Buffer

		id	+	id	*	id	$

Stack

T’
E’
$

Predictive Parsing Program

G[E]:

E → TE’

E’ → +TE’ | ε

T → FT’

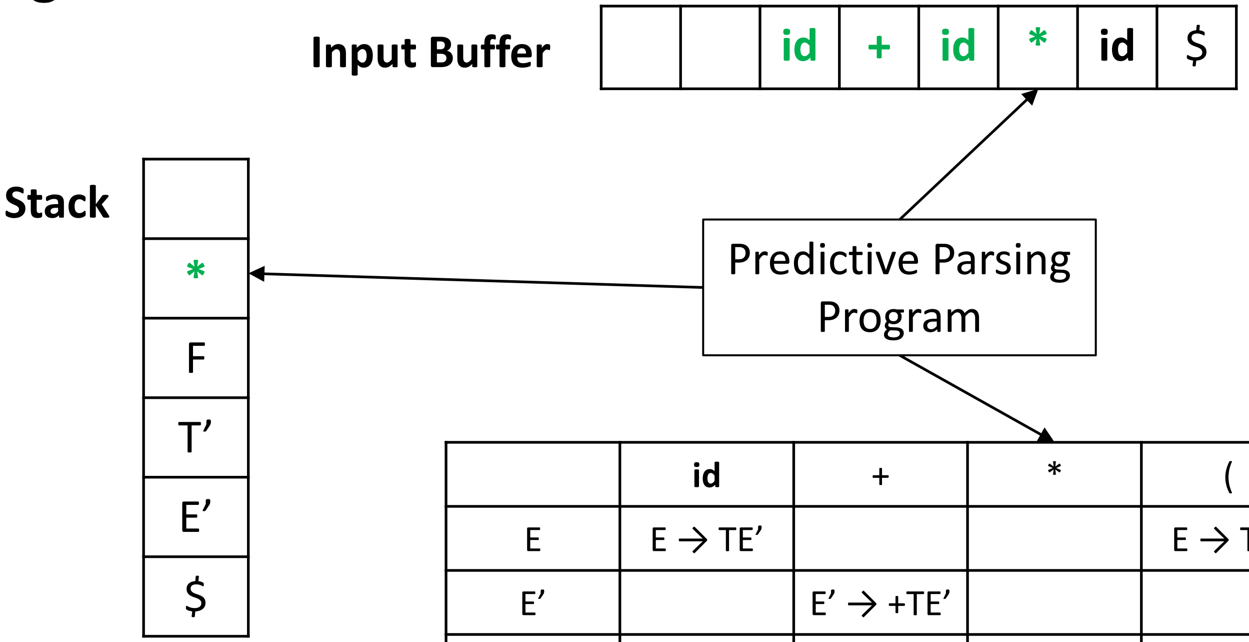
T’ → *FT’ | ε

F → (E) | **id**

	id	+	*	(	)	$
E	E → TE’			E → TE’		
E’		E’ → +TE’			E’ → ε	E’ → ε
T	T → FT’			T → FT’		
T’		T’ → ε	T’ → *FT’		T’ → ε	T’ → ε
F	F → **id**			F → (E)		

Use the Parse Table [使用表进行预测分析]

- recognize "**id + id * id**"



G[E]:

E → TE'

E' → +TE' | ε

T → FT'

T' → *FT' | ε

F → (E) | **id**

	id	+	*	(	)	$
E	E → TE'			E → TE'		
E'		E' → +TE'			E' → ε	E' → ε
T	T → FT'			T → FT'		
T'		T' → ε	T' → *FT'		T' → ε	T' → ε
F	F → **id**			F → (E)		

Use the Parse Table [使用表进行预测分析]

- recognize “**id + id * id**”

Input Buffer

		id	+	id	*	id	$

Stack

F
T’
E’
$

Predictive Parsing Program

	id	+	*	(	)	$
E	E → TE’			E → TE’		
E’		E’ → +TE’			E’ → ε	E’ → ε
T	T → FT’			T → FT’		
T’		T’ → ε	T’ → *FT’		T’ → ε	T’ → ε
F	F → **id**			F → (E)		

G[E]:

E → TE’

E’ → +TE’ | ε

T → FT’

T’ → *FT’ | ε

F → (E) | **id**

Use the Parse Table [使用表进行预测分析]

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- recognize "**id** + **id** * **id**"

Input Buffer

		id	+	id	*	id	$

Stack

id
T'
E'
$

Predictive Parsing Program

G[E]:

E → TE'

E' → +TE' | ε

T → FT'

T' → *FT' | ε

F → (E) | **id**

	id	+	*	(	)	$
E	E → TE'			E → TE'		
E'		E' → +TE'			E' → ε	E' → ε
T	T → FT'			T → FT'		
T'		T' → ε	T' → *FT'		T' → ε	T' → ε
F	F → **id**			F → (E)		

Use the Parse Table [使用表进行预测分析]

- recognize “**id** + **id** * **id**”

G[E]:

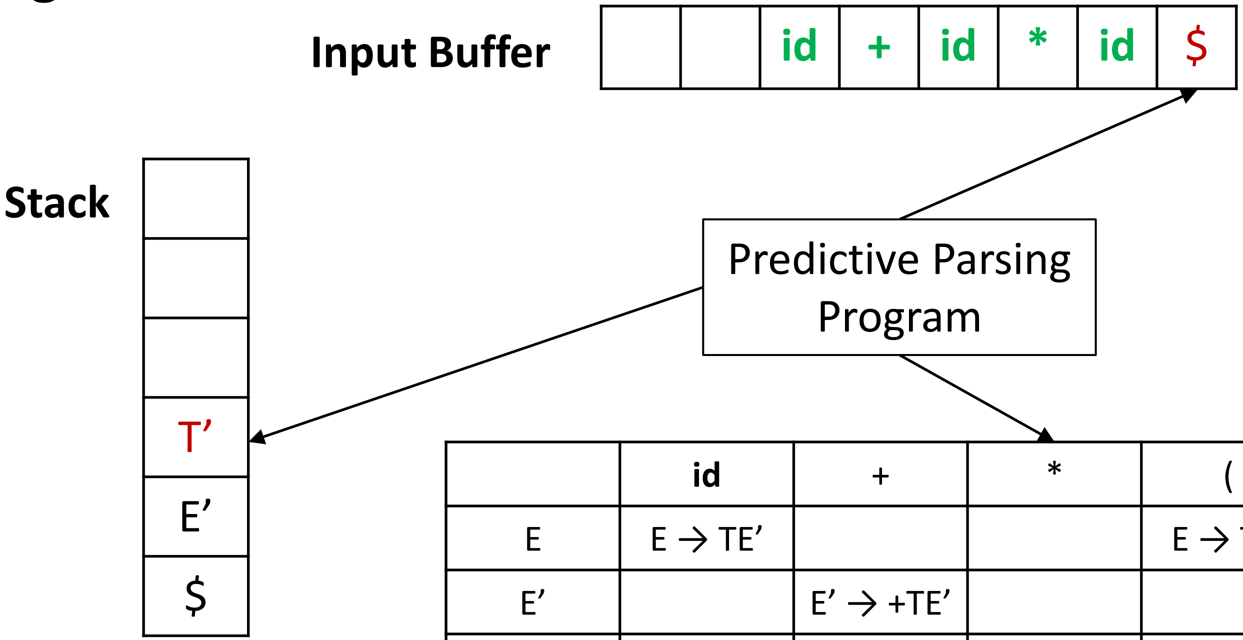
E → TE’

E’ → +TE’ | ε

T → FT’

T’ → *FT’ | ε

F → (E) | **id**



	id	+	*	(	)	**$**
E	E → TE’			E → TE’		
E’		E’ → +TE’			E’ → ε	E’ → ε
T	T → FT’			T → FT’		
T’		T’ → ε	T’ → *FT’		T’ → ε	T’ → ε
F	F → **id**			F → (E)		

Use the Parse Table [使用表进行预测分析]

- recognize "**id + id * id**"

Input Buffer

		id	+	id	*	id	$

Stack

E'
$

Predictive Parsing Program

G[E]:

E → TE'

E' → +TE' | ε

T → FT'

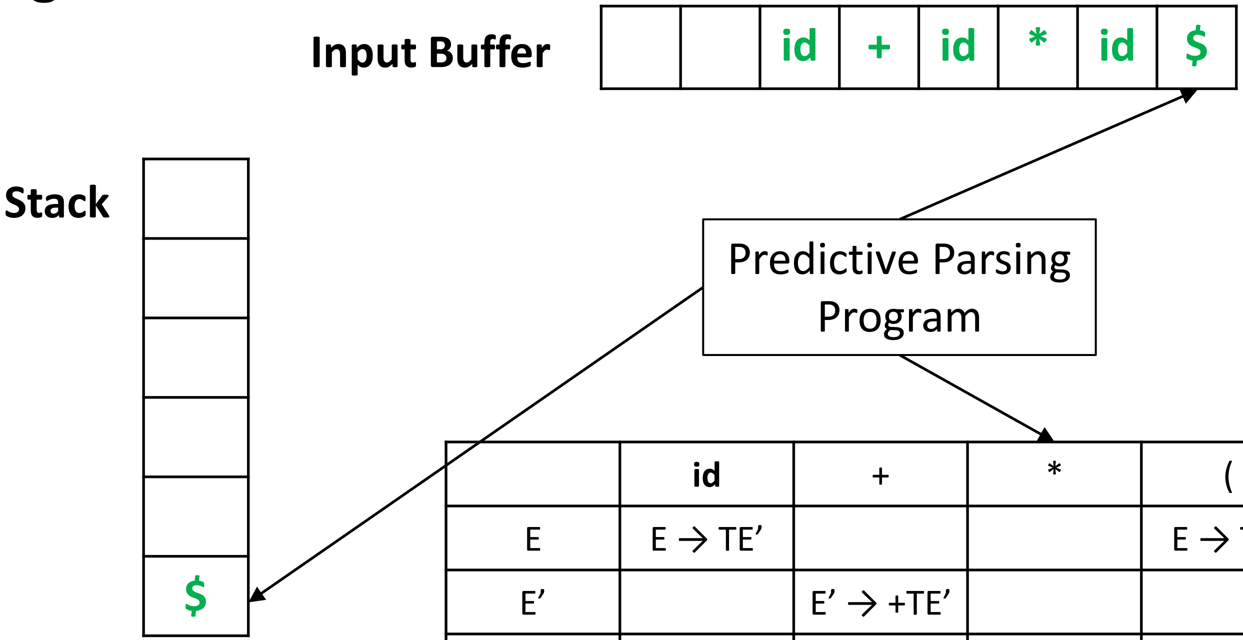
T' → *FT' | ε

F → (E) | **id**

	id	+	*	(	)	$
E	E → TE'			E → TE'		
E'		E' → +TE'			E' → ε	E' → ε
T	T → FT'			T → FT'		
T'		T' → ε	T' → *FT'		T' → ε	T' → ε
F	F → **id**			F → (E)		

Use the Parse Table [使用表进行预测分析]

- recognize “**id** + **id** * **id**”



ACCEPT!!

G[E]:

E → TE’

E’ → +TE’ | ε

T → FT’

T’ → *FT’ | ε

F → (E) | **id**

	id	+	*	(	)	$
E	E → TE’			E → TE’		
E’		E’ → +TE’			E’ → ε	E’ → ε
T	T → FT’			T → FT’		
T’		T’ → ε	T’ → *FT’		T’ → ε	T’ → ε
F	F → **id**			F → (E)		

Use the Parse Table [使用表进行预测分析]

Matched	Stack	Input	Action
	E$	**id** + **id** * **id**$	E → TE’
	TE’$	**id** + **id** * **id**$	T → FT’
	FT’E’$	**id** + **id** * **id**$	F → **id**
	idT’E’$	**id** + **id** * **id**$	match **id**
id	T’E’$	+ **id** * **id**$	T’ → ε
id	E’$	+ **id** * **id**$	E’ → +TE’
id	+TE’$	+ **id** * **id**$	match +
id +	TE’$	**id** * **id**$	T → FT’
id +	FT’ E’$	**id** * **id**$	F → **id**
id +	**id**T’ E’$	**id** * **id**$	match **id**
id + **id**	T’ E’$	* **id**$	T’ → *FT’
id + **id**	*FT’E’$	* **id**$	match *

Matched	Stack	Input	Action
id + **id** *	FT’E’$	**id**$	F → **id**
id + **id** *	**id**T’E’$	**id**$	match **id**
id + **id** * **id**	T’E’$	$	T’ → ε
id + **id** * **id**	E’$	$	E’ → ε
id + **id** * **id**	$	$	**ACCEPT**

The parser mimics a leftmost derivation.

How to construct LL(1) Parse Table?

Construct LL(1) Parse Table [构建预测分析表]

- Use **FIRST** and **FOLLOW** sets for a predictive parsing table M[A,a], and the algorithm is based on the following idea:
  - The production $A \to \alpha$ is chosen if the next input symbol *a* is in FIRST (α).
  - The only complication occurs when $\alpha = \varepsilon$ or, more generally, $\alpha \overset{*}{\Rightarrow} \varepsilon$:
    - we should again choose $A \to \alpha$, if (1) a is in FOLLOW(A) or (2) the \$ on the input has been reached and \$ is in FOLLOW (A).

Construct LL(1) Parse Table [构建预测分析表]

- ***Algorithm***: For each production A → α of the grammar:
  - For each terminal *a* ∈ FIRST(α), add A → α to M[A, a].

G[E]: **E → TE'** **E' → +TE' | ε** **T → FT'** **T' → *FT' | ε** **F → (E) | id**

	id	+	*	(	)	$
E	E → TE'			E → TE'		
E'		E' → +TE'				
T	T → FT'			T → FT'		
T'			T' → *FT'			
F	F → id			F → (E)		

Production	FIRST(α)
E → TE'	(, id
E' → +TE'	+
T → FT'	(, id
T' → *FT'	*
F → (E)	(
F → id	id
E' → ε	FOLLOW
T' → ε	FOLLOW

Construct LL(1) Parse Table [构建预测分析表]

- ***Algorithm***: For each production A → α of the grammar:
  - If ε ∈ FIRST(α), then for each terminal *b* in FOLLOW(A), add A → α to M[A,b]. If ε ∈ FIRST(α) and $ ∈ FOLLOW(A), add A → α to M[A, $] as well.

G[E]: E → TE'　　E' → +TE' | ε　　T → FT'　　T' → *FT' | ε　　F → (E) | id

	id	+	*	(	)	$
E	E → TE'			E → TE'		
E'		E' → +TE'			E' → ε	E' → ε
T	T → FT'			T → FT'		
T'		T' → ε	T' → *FT'		T' → ε	T' → ε
F	F → id			F → (E)		

Production	FIRST(α)
E' → ε	FOLLOW
T' → ε	FOLLOW

Symbol	FOLLOW(A)
E'	$,)
T'	+, $,)

Construct LL(1) Parse Table [构建预测分析表]

G[E]:
E → TE'
E' → +TE' | ε
T → FT'
T' → *FT' | ε
F → (E) | **id**

Symbol	FIRST	FOLLOW
E	(, i	$,)
E'	+, ε	$,)
T	(, i	+, $,)
T'	*, ε	+, $,)
F	(, i	*, +, $,)

Production	FIRST(α)
E → TE'	(, i
E' → +TE'	+
T → FT'	(, i
T' → *FT'	*
F → (E)	(
F → **id**	**id**
E' → ε	FOLLOW
T' → ε	FOLLOW

	id	+	*	(	)	$
E	E → TE'			E → TE'		
E'		E' → +TE'			E' → ε	E' → ε
T	T → FT'			T → FT'		
T'		T' → ε	T' → *FT'		T' → ε	T' → ε
F	F → **id**			F → (E)		

Determine If Grammar is LL(1)[判断LL(1)文法]

- Observation [直观依据]
  - If a grammar is LL(1), each of its LL(1) table entry contains at most one rule.
  - Otherwise, it is not LL(1).

	id	+	*	(	)	$
E	E → TE'			E → TE'		
E'		E' → +TE'			E' → ε	E' → ε
T	T → FT'			T → FT'		
T'		T' → ε	T' → *FT'		T' → ε	T' → ε
F	F → id			F → (E)		

- Two methods to determine if a grammar is LL(1) or not
  - Construct LL(1) table, and check if there is a multi-rule entry.
  - Check each rule as if the table is getting constructed, grammar G is LL(1) if and only if for any two distinct productions $A \rightarrow \alpha|\beta$:
    - $FIRST(\alpha) \cap FIRST(\beta) = \phi$
    - If $\varepsilon \in FIRST(\beta)$, $FIRST(\alpha) \cap FOLLOW(A) = \phi$ (Mentioned before)

Non-LL(1) Grammar [非LL(1)文法]

- Assume that a grammar is not LL(1). How to solve?
  - Case1- the language may still be LL(1)
    - Try to rewrite grammar to LL(1) grammar [*remove left-recursion & left-factoring*]
    - Try to remove ambiguity in grammar.
  - Case2- If Case-1 fails, language may not be LL(1)
    - It's impossible to resolve conflict at the grammar level.
    - Programmer chooses which rule to use for conflicting entry (if choosing that rule is always semantically correct)
    - Otherwise, use a more powerful parser (e.g. LL(k), LR(1))

LL(1) Time and Space Complexity[复杂度]

- **Linear** time and space relative to the length of input.
- **Time:** each input symbol is consumed within a constant number of steps.
  - If symbol at top of stack is a terminal: Matched immediately in **one step.**
  - If symbol at top of stack is a non-terminal:
    - Matched in at most N steps, where N = number of rules.
    - Since LL(1) is not left-recursive, we do not apply same rule twice without consuming input.

LL(1) Time and Space Complexity[复杂度]

- **Space:** smaller than input (after removing $X \rightarrow \varepsilon$)
  - Right side of production is always longer or equal to left side of production
    - Derivation string expands monotonically.
    - Derivation string is always shorter than final input string.
  - Stack is a subset of derivation string (unmatched portion)
- LL(k)'s size of parse table = $O(|N| * |T|^k)$[prevent LL(2) ... LL(k) from wide usage]
  - N = number of non-terminals, T = number of terminals

Summary

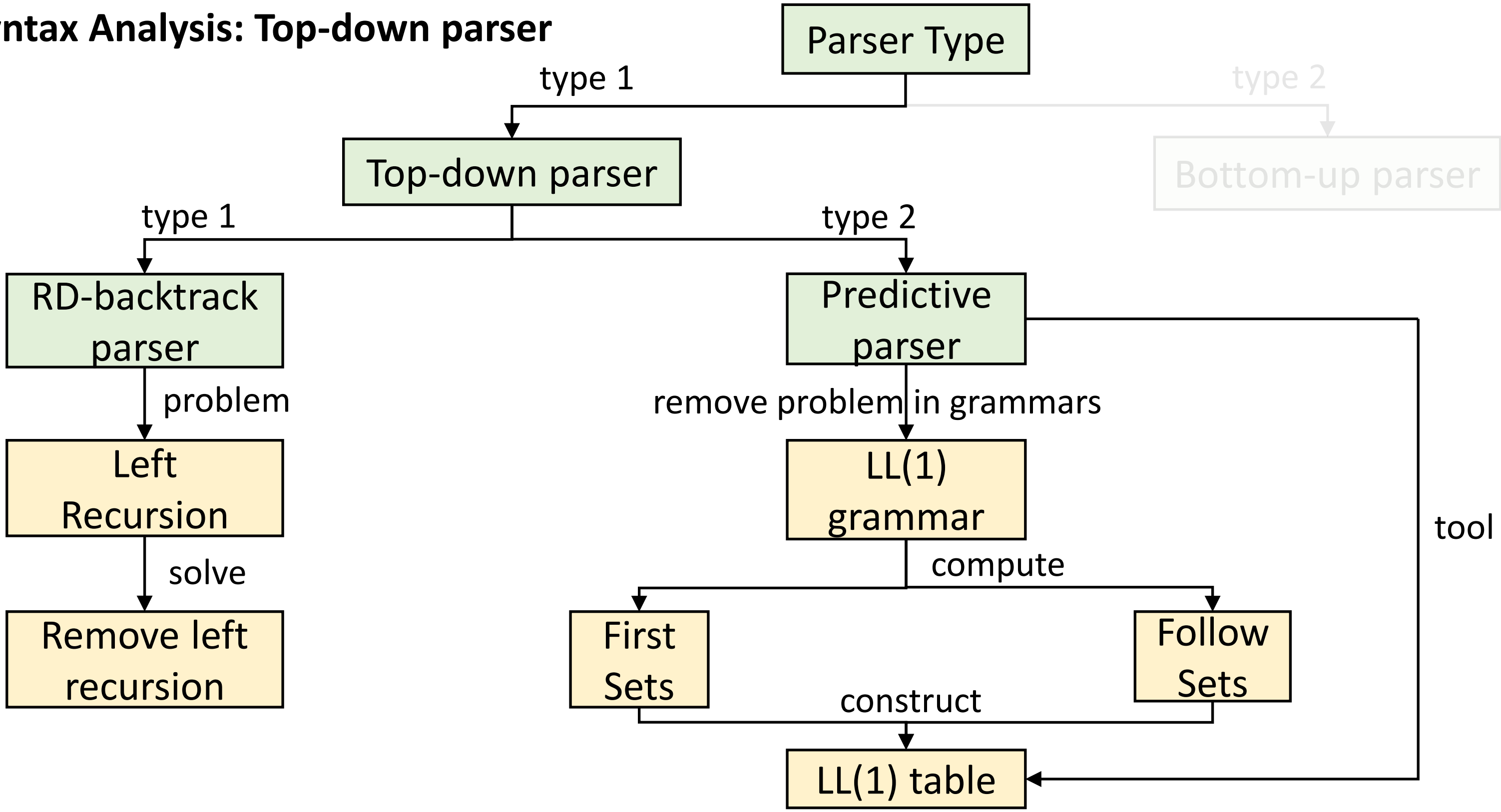
- Top-down Parsing; RDP with backtracking, predictive parsing.
- (Immediate / Non-immediate) Left Recursion and how to resolve
- Left-factoring.
- FIRST, FOLLOW.
- LL(1)/LL(k) Grammar.
- Recursive / Non-recursive LL(1) parser implementation.
- The use of LL(1) Parse Table.
- The construction of LL(1) Parse Table.

Summary [自顶向下分析]

- RDP [递归下降分析]
  - left recursion [左递归]
    - Remove Left Recursion [消除直接/间接左递归]
  - Backtracking [回溯]
    - left factoring [提取左公因子]
- Predictive Parsing [预测分析]
  - FIRST & FOLLOW [终结首符集 & 后继终结符号集]
  - Definition of LL(1)/LL(k) [LL(1)/LL(k)文法的定义]
  - LL(1) Parse Table [LL(1)的分析表]
    - The use of the Parse Table [分析表的使用]
    - The Construction of the Parse Table [分析表的构建]
  - Table-driven LL(1) Parser Implementation [分析表驱动的LL(1)语法分析实现]
  - Complexity of LL(1) [LL(1)的时间与空间复杂度]

Mind Map[思维导图]

Syntax Analysis: Top-down parser



Further Reading

- Dragon Book
  - Comprehensive Reading:
    - Section 4.1.2 and 4.4.1 for the introduction to top-down parsing.
    - Section 4.4.2 for function FIRST and FOLLOW.
    - Section 4.1.4, 4.4.3–4.4.4, and 4.4.5 for LL(1) parsing and error recovery.
  - Skip Reading:
    - Section 4.1.3–4.1.4 for error recovery in top-down parsing.
    - Section 4.2.7 for differences between CFGs and regular expressions.