Name: _____

Consider the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

Let $T(\mathbf{x}) = A\mathbf{x}$ where $A = \begin{bmatrix} 2 & 0 & 1 \\ 4 & -2 & -4 \\ 0 & 0 & 3 \end{bmatrix}$.

1. Explain why $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a basis for $\mathbb{R}^3$.

2. If $S = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 \end{bmatrix}$, find S^{-1}.

3. Find the $\mathcal{B}$-matrix for T by using the formula

$$B = SAS^{-1}$$

4. Compute $T(\mathbf{v}_1), T(\mathbf{v}_2)$ and $T(\mathbf{v}_3)$.

5. Write each of $T(\mathbf{v}_1), T(\mathbf{v}_2)$ and $T(\mathbf{v}_3)$ as a linear combination of $\mathbf{v}_1, \mathbf{v}_2$ and $\mathbf{v}_3$.

6. Find the coordinate vectors $[T(\mathbf{v}_1)]_{\mathcal{B}}, [T(\mathbf{v}_2)]_{\mathcal{B}}, [T(\mathbf{v}_3)]_{\mathcal{B}}$.

7. Find the $\mathcal{B}$-matrix for T by the formula
$$B = \begin{bmatrix} [T(\mathbf{v}_1)]_{\mathcal{B}} & [T(\mathbf{v}_2)]_{\mathcal{B}} & [T(\mathbf{v}_3)]_{\mathcal{B}} \end{bmatrix}.$$