$$\frac{\partial N_1}{\partial t} = k_1 N_1 \left(1 - \frac{N_1}{\theta_1 - \alpha_1 M} - \frac{N_2}{\theta_2 - \alpha_2 M} \right) + \nabla \cdot \left(D_{N_1} \left(1 - \frac{N_1}{\theta_1 - \alpha_1 M} - \frac{N_2}{\theta_2 - \alpha_2 M} \right) \nabla N_1 \right) - f_1(H) N_1$$

$$\underline{\text{set arbitrary variable}} \ \mathbf{z} = \left(1 - \frac{N_1}{\theta_1 - \alpha_1 M} - \frac{N_2}{\theta_2 - \alpha_2 M} \right)$$

$$\frac{\partial N_1}{\partial t} = k_1 N_1 z + \nabla \cdot \left(D_{N_1} z \nabla N_1 \right) - f_1(H) N_1$$

$$\frac{\partial N_1}{\partial t} = k_1 N_1 z + D_{N_1} \frac{\partial}{\partial x} \left(z \frac{\partial N_1}{\partial x} + z \frac{\partial N_1}{\partial y} \right) + D_{N_1} \frac{\partial}{\partial y} \left(z \frac{\partial N_1}{\partial x} + z \frac{\partial N_1}{\partial y} \right) - f_1(H) N_1$$

Product rule:

$$\frac{\partial N_1}{\partial t} = k_1 N_1 z + D_{N_1} \left(\frac{\partial z}{\partial x} \frac{\partial N_1}{\partial x} + z \frac{\partial^2 N_1}{\partial x^2} + \frac{\partial z}{\partial x} \frac{\partial N_1}{\partial y} + z \frac{\partial^2 N_1}{\partial x} + z \frac{\partial^2 N_1}{\partial x \partial y} + \frac{\partial z}{\partial y} \frac{\partial N_1}{\partial x} + z \frac{\partial^2 N_1}{\partial x \partial y} + \frac{\partial z}{\partial y} \frac{\partial N_1}{\partial y} + z \frac{\partial^2 N_1}{\partial y^2} \right) - f_1(H) N_1$$

$$\frac{\partial N_1}{\partial t} = k_1 N_1 z + D_{N_1} \left(z \left(\frac{\partial^2 N_1}{\partial x^2} + 2 \frac{\partial^2 N_1}{\partial x \partial y} + \frac{\partial^2 N_1}{\partial y^2} \right) + \frac{\partial z}{\partial x} \left(\frac{\partial N_1}{\partial x} + \frac{\partial N_1}{\partial y} \right) + \frac{\partial z}{\partial y} \left(\frac{\partial N_1}{\partial x} + \frac{\partial N_1}{\partial y} \right) \right) - f_1(H) N_1 \left(z \left(\frac{\partial^2 N_1}{\partial x^2} + 2 \frac{\partial^2 N_1}{\partial x \partial y} + \frac{\partial^2 N_1}{\partial y^2} \right) + \frac{\partial z}{\partial x} \left(\frac{\partial N_1}{\partial x} + \frac{\partial N_1}{\partial y} \right) \right) - f_1(H) N_1 \left(z \left(\frac{\partial^2 N_1}{\partial x^2} + 2 \frac{\partial^2 N_1}{\partial x \partial y} + \frac{\partial^2 N_1}{\partial y^2} \right) \right) - f_2(H) N_1 \left(z \left(\frac{\partial^2 N_1}{\partial x^2} + 2 \frac{\partial^2 N_1}{\partial x \partial y} + \frac{\partial^2 N_1}{\partial y^2} \right) \right) - f_2(H) N_1 \left(z \left(\frac{\partial^2 N_1}{\partial x^2} + 2 \frac{\partial^2 N_1}{\partial x \partial y} + \frac{\partial^2 N_1}{\partial y^2} \right) \right) - f_2(H) N_1 \left(z \left(\frac{\partial^2 N_1}{\partial x} + \frac{\partial^2 N_1}{\partial x^2} + 2 \frac{\partial^2 N_1}{\partial x^2} \right) \right) - f_2(H) N_1 \left(\frac{\partial^2 N_1}{\partial x} + \frac{\partial^2 N_1}{\partial y} \right) - f_2(H) N_1 \left(\frac{\partial^2 N_1}{\partial x} + \frac{\partial^2 N_1}{\partial y} \right) \right) - f_2(H) N_2 \left(\frac{\partial^2 N_1}{\partial x} + \frac{\partial^2 N_1}{\partial y} \right) - f_2(H) N_2 \left(\frac{\partial^2 N_1}{\partial x} + \frac{\partial^2 N_1}{\partial y} \right) + \frac{\partial^2 N_1}{\partial y} \left(\frac{\partial^2 N_1}{\partial x} + \frac{\partial^2 N_1}{\partial y} \right) \right) - f_2(H) N_1 \left(\frac{\partial^2 N_1}{\partial x} + \frac{\partial^2 N_1}{\partial y} \right) + \frac{\partial^2 N_1}{\partial y} \left(\frac{\partial^2 N_1}{\partial y} + \frac{\partial^2 N_1}{\partial y} \right) + \frac{\partial^2 N_1}{\partial y} \left(\frac{\partial^2 N_1}{\partial y} + \frac{\partial^2 N_1}{\partial y} \right) \right) - f_2(H) N_1 \left(\frac{\partial^2 N_1}{\partial y} + \frac{\partial^2 N_1}{\partial y} \right) + \frac{\partial^2 N_1}{\partial y} \left(\frac{\partial^2 N_1}{\partial y} + \frac{\partial^2 N_1}{\partial y} \right) + \frac{\partial^2 N_1}{\partial y} \left(\frac{\partial^2 N_1}{\partial y} + \frac{\partial^2 N_1}{\partial y} \right) + \frac{\partial^2 N_1}{\partial y} \left(\frac{\partial^2 N_1}{\partial y} + \frac{\partial^2 N_1}{\partial y} \right) + \frac{\partial^2 N_1}{\partial y} \left(\frac{\partial^2 N_1}{\partial y} + \frac{\partial^2 N_1}{\partial y} \right) + \frac{\partial^2 N_1}{\partial y} \left(\frac{\partial^2 N_1}{\partial y} + \frac{\partial^2 N_1}{\partial y} \right) + \frac{\partial^2 N_1}{\partial y} \left(\frac{\partial^2 N_1}{\partial y} + \frac{\partial^2 N_1}{\partial y} \right) + \frac{\partial^2 N_1}{\partial y} \left(\frac{\partial^2 N_1}{\partial y} + \frac{\partial^2 N_1}{\partial y} \right) + \frac{\partial^2 N_1}{\partial y} \left(\frac{\partial^2 N_1}{\partial y} + \frac{\partial^2 N_1}{\partial y} \right) + \frac{\partial^2 N_1}{\partial y} \left(\frac{\partial^2 N_1}{\partial y} + \frac{\partial^2 N_1}{\partial y} \right) + \frac{\partial^2 N_1}{\partial y} \left(\frac{\partial^2 N_1}{\partial y} + \frac{\partial^2 N_1}{\partial y} \right) + \frac{\partial^2 N_1}{\partial y} \left(\frac{\partial^2 N_1}{\partial y} + \frac{\partial^2 N_1}{\partial y} \right) + \frac{\partial^2 N_1}{\partial y} \left(\frac{\partial^2 N_1}{\partial y} + \frac{\partial^2 N_1}{\partial y} \right) + \frac{\partial^2 N_1}{\partial y} \left(\frac{\partial^2 N_1}{\partial y} + \frac{\partial^2 N_1}{\partial y} \right) + \frac{\partial^2 N_1}{\partial y} \left(\frac{\partial^2 N_1}{\partial y} + \frac{\partial^2 N_1}{\partial y} \right) + \frac{\partial^2 N_1}{\partial y} \left(\frac{\partial^2 N$$

Finite Difference:

$$\frac{N_{1}(t) - N_{1}(\overline{t-1})}{\Delta t} = k_{1}N_{1}(t-1)z(t-1)$$

$$+D_{N_{1}}\left(z(t-1)\left(\frac{N_{1}(x-1,t-1) - 2N_{1}(x,t-1) + N_{1}(x+1,t-1)}{\Delta x^{2}}\right) + \frac{1}{\Delta x}\left(\frac{N_{1}(x+1,y+1,t-1) - N_{1}(x+1,y-1,t-1)}{2\Delta y} - \frac{N_{1}(x-1,y+1,t-1) - N_{1}(x-1,y-1,t-1)}{2\Delta y}\right) + \frac{N_{1}(y-1,t-1) - 2N_{1}(y,t-1) + N_{1}(y+1,t-1)}{\Delta x}\left(\frac{N_{1}(x+1,t-1) - N_{1}(x-1,t-1)}{\Delta x} + \frac{N_{1}(y+1,t-1) - N_{1}(y-1,t-1)}{\Delta y}\right) + \frac{z(y+1,t-1) - z(y-1,t-1)}{\Delta y}\left(\frac{N_{1}(x+1,t-1) - N_{1}(x-1,t-1)}{\Delta x} + \frac{N_{1}(y+1,t-1) - N_{1}(y-1,t-1)}{\Delta y}\right)\right) - f_{1}(H)N_{1}(t-1)$$