proliferation carrying capacity

matrix limited

Use initial conditions to introduce

B into system instead

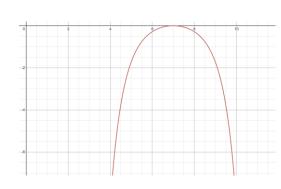
slower growth/death

healthy cells

tumor cells

$$\frac{\partial N_1}{\partial t} = k_1 N_1 \left( 1 - \frac{N_1}{\theta_1} - \frac{N_2}{\theta_2} \right) \left( 1 - \frac{M}{M_o} \right) + \nabla \cdot \left( D_{N_1} \left( 1 - \frac{N_1}{\theta_1} - \frac{N_2}{\theta_2} \right) \left( 1 - \frac{M}{M_o} \right) \nabla N_1 \right) - f_1(H) N_1$$

$$\frac{\partial N_2}{\partial t} = k_2 N_2 \left( 1 - \frac{N_1}{\theta_1} - \frac{N_2}{\theta_2} \right) \left( 1 - \frac{M}{M_o} \right) + \nabla \cdot \left( D_{N_2} \left( 1 - \frac{N_1}{\theta_1} - \frac{N_2}{\theta_2} \right) \left( 1 - \frac{M}{M_o} \right) \nabla N_2 \right) - f_2(H) N_2$$
from non-optimal pH



$$f_1(H) = d_1 \left( 1 - e^{\left(\frac{H - H_1^{opt}}{H_1^{width}}\right)^2} \right)$$

$$f_2(H) = d_2 \left( 1 - e^{\left(\frac{H - H_2^{opt}}{H_2^{width}}\right)^2} \right)$$

optimal pH curve

protons 
$$\frac{\partial H}{\partial t} = D_H \nabla^2 H + k_{acid} N_2 - d_H (H - H_0) - k_{neut} HB$$

bicarbonate  $\frac{\partial B}{\partial t} = D_B \nabla^2 B + k_R - k_{neut} HB$ 

diffusion bicarbonate introduction by surrounding tissue acid-base neutralization  $A_{neut} = A_{neut} + A_{neut}$ 

MMP degradation MMPs  $\frac{\partial P}{\partial t} = D_P \nabla^2 P + k_P \frac{N_2}{MP} - d_p P$ diffusion MMP production by tumor cells pH-dependent MMP activity