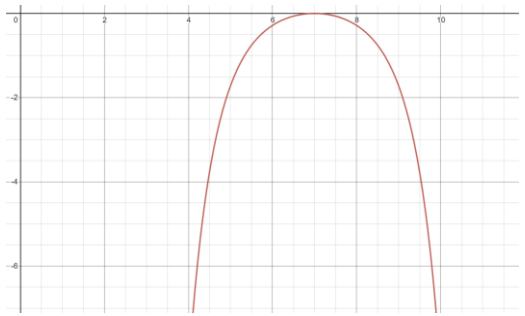
proliferation
carrying capacity is lowered by presents of matrix

diffusion
limited by presence of matrix

slower growth/death
from non-optimal pH

healthy cells $$\frac{\partial N_1}{\partial t} = k_1 N_1 \left(1 - \frac{N_1}{\theta_1 - \alpha_1 M} - \frac{N_2}{\theta_2 - \alpha_2 M}\right) + \nabla \cdot \left(D_{N_1}\left(1 - \frac{N_1}{\theta_1 - \alpha_1 M} - \frac{N_2}{\theta_2 - \alpha_2 M}\right)\nabla N_1\right) - f_1(H)N_1$$

tumor cells $$\frac{\partial N_2}{\partial t} = k_2 N_2 \left(1 - \frac{N_1}{\theta_1 - \alpha_1 M} - \frac{N_2}{\theta_2 - \alpha_2 M}\right) + \nabla \cdot \left(D_{N_2}\left(1 - \frac{N_1}{\theta_1 - \alpha_1 M} - \frac{N_2}{\theta_2 - \alpha_2 M}\right)\nabla N_2\right) - f_2(H)N_2$$



$$f_1(H) = d_1\left(1 - e^{\left(\frac{H - H_1^{opt}}{H_1^{width}}\right)^2}\right)$$

$$f_2(H) = d_2\left(1 - e^{\left(\frac{H - H_2^{opt}}{H_2^{width}}\right)^2}\right)$$

optimal pH curve

generalized extracellular matrix $$\frac{\partial M}{\partial t} = -f_{MMP}(H)PM$$

matrix degradation by MMPs

MMPs $$\frac{\partial P}{\partial t} = D_P \nabla^2 P + k_P N_2 - d_p P$$

diffusion | MMP production by tumor cells | MMP degradation

protons $$\frac{\partial H}{\partial t} = D_H \nabla^2 H + k_{acid} N_2 - d_H(H - H_0) - k_{neut} HB$$

diffusion | acid production by tumor cells | hydrogen uptake by surrounding tissue | acid-base neutralization

bicarbonate treatment $$\frac{\partial B}{\partial t} = D_B \nabla^2 B + \cancel{d_B} - k_{neut} HB$$

diffusion | bicarbonate introduction | acid-base neutralization

Use initial conditions to introduce B into system instead

$$f_{MMP}(H) =$$

pH-dependent MMP activity