$$\frac{\partial N_1}{\partial t} = k_1 N_1 \left(1 - \frac{N_1}{\theta_1 - \alpha_1 M} - \frac{N_2}{\theta_2 - \alpha_2 M}\right) + \nabla \cdot \left(D_{N_1}\left(1 - \frac{N_1}{\theta_1 - \alpha_1 M} - \frac{N_2}{\theta_2 - \alpha_2 M}\right)\nabla N_1\right) - f_1(H)N_1$$

<u>set arbitrary variable</u> $\mathrm{a} = \left(1 - \frac{N_1}{\theta_1 - \alpha_1 M} - \frac{N_2}{\theta_2 - \alpha_2 M}\right)$

$$\frac{\partial N_1}{\partial t} = k_1 N_1 a + \nabla \cdot \left(D_{N_1}\mathrm{a}\nabla N_1\right) - f_1(H)N_1$$

$$\frac{\partial N_1}{\partial t} = k_1 N_1 a + D_{N_1}\frac{\partial}{\partial x}\left(\mathrm{a}\frac{\partial N_1}{\partial x} + \mathrm{a}\frac{\partial N_1}{\partial y}\right) + D_{N_1}\frac{\partial}{\partial y}\left(\mathrm{a}\frac{\partial N_1}{\partial x} + \mathrm{a}\frac{\partial N_1}{\partial y}\right) - f_1(H)N_1$$

<u>Product rule</u>:

$$\frac{\partial N_1}{\partial t} = k_1 N_1 a + D_{N_1}\left(\frac{\partial a}{\partial x}\frac{\partial N_1}{\partial x} + a\frac{\partial^2 N_1}{\partial x^2} + \frac{\partial a}{\partial x}\frac{\partial N_1}{\partial y} + a\frac{\partial^2 N_1}{\partial x \partial y} + \frac{\partial a}{\partial y}\frac{\partial N_1}{\partial x} + a\frac{\partial^2 N_1}{\partial x \partial y} + \frac{\partial a}{\partial y}\frac{\partial N_1}{\partial y} + a\frac{\partial^2 N_1}{\partial y^2}\right) - f_1(H)N_1$$

$$\frac{\partial N_1}{\partial t} = k_1 N_1 a + D_{N_1}\left(a\left(\frac{\partial^2 N_1}{\partial x^2} + 2\frac{\partial^2 N_1}{\partial x \partial y} + \frac{\partial^2 N_1}{\partial y^2}\right) + \frac{\partial a}{\partial x}\left(\frac{\partial N_1}{\partial x} + \frac{\partial N_1}{\partial y}\right) + \frac{\partial a}{\partial y}\left(\frac{\partial N_1}{\partial x} + \frac{\partial N_1}{\partial y}\right)\right) - f_1(H)N_1$$

<u>Finite Difference:</u>

$$\frac{N_1(t) - N_1(t-1)}{\Delta t} = k_1 N_1(t-1)a(t-1) + D_{N_1}\left(a(t-1)\left(\frac{N_1(x-1,t-1) - 2N_1(x,t-1) + N_1(x+1,t-1)}{\Delta x^2}\right.\right.$$
$$+ \frac{1}{\Delta x}\left(\frac{N_1(x+1,y+1,t-1) - N_1(x+1,y-1,t-1)}{2\Delta y} - \frac{N_1(x-1,y+1,t-1) - N_1(x-1,y-1,t-1)}{2\Delta y}\right)$$
$$\left. + \frac{N_1(y-1,t-1) - 2N_1(y,t-1) + N_1(y+1,t-1)}{\Delta y^2}\right)$$
$$+ \frac{a(x+1,t-1) - a(x-1,t-1)}{\Delta x}\left(\frac{N_1(x+1,t-1) - N_1(x-1,t-1)}{\Delta x} + \frac{N_1(y+1,t-1) - N_1(y-1,t-1)}{\Delta y}\right)$$
$$\left. + \frac{a(y+1,t-1) - a(y-1,t-1)}{\Delta y}\left(\frac{N_1(x+1,t-1) - N_1(x-1,t-1)}{\Delta x} + \frac{N_1(y+1,t-1) - N_1(y-1,t-1)}{\Delta y}\right)\right) - f_1(H)N_1(t-1)$$