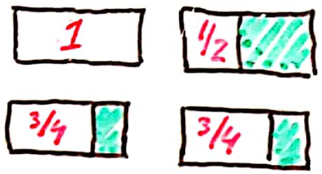
NS&I 1:34500

£4042 → 0.11
£5263 → 0.14
£6077 → 0.16
£10132 → 0.25
£15809 → 0.368

Lottery 1 Lottery 2



$P(win) = \frac{1}{2}$

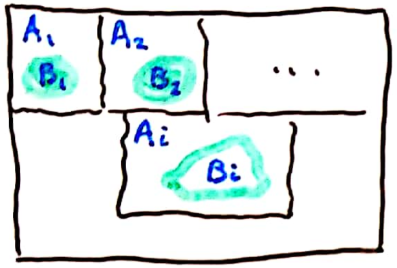
$P(win) = \frac{1}{4} + \frac{1}{4} - \frac{1}{4} \times \frac{1}{4} = \frac{7}{16} < \frac{1}{2}$

$P(win) = P(W_{L_1}) + P(W_{L_2}) - P(W_{L_1} \cap W_{L_2})$

M – total tickets
N – purchased
n – no. of draws

$P(win) = 1 - P(lose)$

$$P(lose) = \frac{M-N}{M} \times \frac{M-1-N}{M-1} \times \dots \times \frac{M-(n-1)-N}{M-(n-1)} = \frac{\binom{M-N}{n}}{\binom{M}{n}} \Rightarrow P(win) = 1 - \frac{\binom{M-N}{n}}{\binom{M}{n}}$$



$(A_i)_i$ – partition

$\sum_i P(A_i) = 1$

B_i – bought

$B_i \subset A_i$ and $A_i \cap B_i = B_i$

$$(B_i) = \underset{(B_i)}{\arg\max}\, P\left(win \mid (B_i)_i \ \& \text{ unifor random}\right) - ?$$

$R = \{\text{random values}\} = \cup R_i$

$R_i \subset A_i$ and $R_i \cap A_i = R_i$

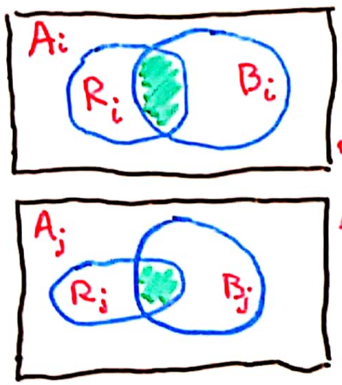
$R_j \cap B_i = \emptyset,\ j \neq i$

$$WIN = R \cap \left(\bigcup_i B_i\right) \neq \emptyset$$

$$Win = \bigcup_i (R \cap B_i) = \bigcup_i \left(\left(\bigcup_j R_j\right) \cap B_i\right) = \bigcup_i \left(\bigcup_j (R_j \cap B_i)\right) =$$

$$= \bigcup_i (R_i \cap B_i) \leftarrow \text{if } A_{i_0} = B_{i_0} \text{ for some } i_0, \text{ then guaranteed win}$$

Induction style



$$P(WIN) = 1 - P(R_i \cap B_i = \emptyset) = 1 - P(R_i \subset A_i \setminus B_i)$$

$$P(WIN_{ij}) = 1 - P(\overline{W_j} \cap \overline{W_i}) = 1 - P(R_j \subset A_j \setminus B_j \cap R_i \subset A_i \setminus B_i)$$

Generally: $P(WIN) = P\left(\bigcup_i WIN_i\right) = 1 - P\left(\bigcap_i \overline{WIN_i}\right) = 1 - P\left(\bigcap_i R_i \subset A_i \setminus B_i\right)$

Assuming independence:

$$P(WIN) = 1 - \prod_j P(R_j \subset A_j \setminus B_j)$$