Probability of 1 ticket to win = $p$

Probability of 1 out of N tickets to win => $\mathbb{P}(N) = 1-(1-p)^N$

Probability to win at the k-th round is:

$$P(\text{round } k) = (1-\mathbb{P}(N))^{k-1} \times \mathbb{P}(N)$$

Expectation is:

$$\mathbb{E}(\text{win}) = \sum_k k\, P(\text{round } k) = \sum_{k=1}^{\infty} k\,(1-\mathbb{P}(N))^{k-1}\,\mathbb{P}(N) =$$

$$= \mathbb{P}(N)\left[\sum_{k=1}^{\infty} k\,(1-\mathbb{P}(N))^{k-1}\right] \underset{\phantom{x}}{\overset{x=1-\mathbb{P}(N)}{=\!=\!=}} \mathbb{P}(N)\left[\sum_{k=1} k\,x^{k-1}\right] =$$

$$= \mathbb{P}(N)\left(\sum_{k=1} x^k\right)' = \mathbb{P}(N)\left(\frac{1}{1-x}-1\right)' = \mathbb{P}(N) \times \frac{1}{(1-x)^2} =$$

$$= \mathbb{P}(N) \times \frac{1}{(1-(1-\mathbb{P}(N)))^2} = \frac{1}{\mathbb{P}(N)} \quad \text{or} \quad \boxed{\mathbb{E}(\text{win}) = \frac{1}{\mathbb{P}(N)}}$$

$$\left(\frac{1}{1-x}-1\right)' = \left(\frac{x}{1-x}\right)' = \frac{x'(1-x)-x(1-x)'}{(1-x)^2} = \frac{1-x+x}{(1-x)^2} = \frac{1}{(1-x)^2}$$

$$\sum_{k=1}^{\infty} x^k = \left(\sum_{k=0}^{\infty} x^k\right) - 1 = \frac{1}{1-x} - 1 \;,\; |x|<1$$

For $\mathbb{P}(N) \geq \frac{1}{2} \iff 1-(1-p)^N \geq \frac{1}{2} \iff -(1-p)^N \geq -\frac{1}{2} \iff$

$$(1-p)^N \leq \frac{1}{2} \iff N \times \log(1-p) \leq \log\left(\frac{1}{2}\right) < 0 \implies$$

$$\boxed{N \geq \frac{\log\left(\frac{1}{2}\right)}{\log(1-p)}}$$