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1 Clustering Coefficients

1.1

$$\langle C \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{2L_i}{k_i(k_i-1)}$$

$$= \frac{1}{N}(\sum_{i=0}^{N} \frac{2\ell}{(\ell+1)\ell} + \sum_{i=0}^{N} \frac{2\cdot 1}{2\cdot 1})$$

$$= \frac{1}{N}(\frac{4}{\ell+1} + N - 2)$$

$$= \frac{1}{N}(\frac{4}{N-1} + N - 2)$$

$$= \frac{1}{N}(\frac{4+(N-2)(N-1)}{N-1})$$

$$= \frac{1}{N}(\frac{4+N^2-3N+2}{N-1})$$

$$= \frac{N^2-3N+6}{N^2-N}.$$

1.2

$$C_\Delta = \frac{3 \cdot \#triangles}{\#triples}.$$

According to the hint,

$$\#triples = \sum_{i=1}^{N} \frac{k_i(k_i-1)}{2}$$

In the given graph, $\#triangles = \ell$, so

$$C_\Delta = \frac{3\ell}{\sum_{i=1}^{N} \frac{k_i(k_i-1)}{2}}$$

From problem 1.1: $k_{A,B} = \ell + 1, k_\ell = 2$, so

$$C_\Delta = \frac{3\ell}{\sum_{i=0}^{1} \frac{(\ell+1)\ell}{2} + \sum_{2}^{N} \frac{2\cdot 1}{2}}$$

$$= \frac{3\ell}{\ell(\ell+1)+N-2}$$

$$= \frac{3(N-2)}{(N-1)(N-2)+N-2}$$

$$= \frac{3(N-2)}{N^2-3N+2+N-2}$$

$$= \frac{3(N-2)}{N^2-2N}$$
$$= \frac{3(N-2)}{N(N-2)}$$
$$= \frac{3}{N}.$$

1.3

As $N \to \infty$, the average clustering coefficient $\langle C \rangle = (\frac{N^2-3N+6}{N^2-N})$ goes to 1. These reflects the fact that nodes A and B are connected to all others, and as the number of nodes goes to infinity, these connections dominate the coefficient. The global clustering coefficient $C_\Delta = \frac{3}{N}$ goes to 0, meaning that the number of triangles grows much more slowly than the number of triples. These limits show that in some cases, there is a very clear difference between the two formulations.