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1 Clustering Coefficients

1.1

$k_{A,B} = \ell + 1$, $k_\ell = 2$, so:

$$\langle C \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{2L_i}{k_i(k_i - 1)}$$

$$= \frac{1}{N} \left(\underbrace{\sum_{A,B} \frac{2\ell}{(\ell+1)\ell}}_{\text{nodes } A,\ B} + \underbrace{\sum_{i=0}^{N-2} \frac{2 \cdot 1}{2 \cdot 1}}_{\text{nodes } l} \right)$$

$$= \frac{1}{N} \left(\frac{4}{\ell + 1} + N - 2 \right)$$

$$= \frac{1}{N} \left(\frac{4}{N - 1} + N - 2 \right)$$

$$= \frac{1}{N} \left(\frac{4 + (N-2)(N-1)}{N - 1} \right)$$

$$= \frac{1}{N} \left(\frac{4 + N^2 - 3N + 2}{N - 1} \right)$$

$$= \frac{N^2 - 3N + 6}{N^2 - N}.$$

1.2

According to hint:

$$C_\Delta = \frac{3 \cdot \#triangles}{\#triples}, \qquad \#triples = \sum_{i=1}^{N} \frac{k_i(k_i - 1)}{2}.$$

In the given graph, # triangles $= \ell$, therefore:

$$C_\Delta = \frac{3\ell}{\sum\limits_{i=1}^{N} \frac{k_i(k_i-1)}{2}}.$$

From problem 1.1: $k_{A,B} = \ell + 1$, $k_\ell = 2$, so:

$$C_\Delta = \frac{3\ell}{\sum\limits_{A,B} \frac{(\ell+1)\ell}{2} + \sum\limits_{i=0}^{N-2} \frac{2\cdot 1}{2}}$$

$$= \frac{3\ell}{\ell(\ell+1) + N - 2}$$

$$= \frac{3(N-2)}{(N-1)(N-2) + N - 2}$$

$$= \frac{3(N-2)}{N^2 - 3N + 2 + N - 2}$$

$$= \frac{3(N-2)}{N^2 - 2N}$$

$$= \frac{3(N-2)}{N(N-2)}$$

$$= \frac{3}{N}.$$

1.3

As $N \to \infty$, the average clustering coefficient $\langle C \rangle = (\frac{N^2-3N+6}{N^2-N})$ goes to 1. These reflects the fact that nodes A and B are connected to all others, and as the number of nodes goes to infinity, these connections dominate the coefficient. The global clustering coefficient $C_\Delta = \frac{3}{N}$ goes to 0, meaning that the number of triangles grows much more slowly than the number of triples. These limits show that in some cases, there is a very clear difference between the two formulations.

2 Watts-Strogatz Network Model

2.1

Minimum average clustering coefficient: when $p = 0$: $\langle C(0) \rangle = \frac{3(2m-2)}{4(2m-1)}$. As m grows, $\langle C(0) \rangle \rightarrow \frac{3}{4}$.
Maximum average clustering coefficient: when $p = 1$. The network becomes random: $\langle C(1) \rangle = \frac{2m}{N}$.

2.2

Minimum average shortest path: when $p = 0$: $\langle d(0) \rangle = \frac{N}{4m}$.
Maximum average shortest path: when $p = 1$: $\langle d(1) \rangle = \frac{\ln N}{\ln 2m}$.

2.3

Find N by counting the nodes: $N = 12$.
Find m by counting how many neighbouring nodes each node is connected to. While not all links will be present due to the random rewiring, a large portion of nodes are connected to the nodes right next to themselves, as well as the next nodes over, so $m = 2$ is a reasonable estimate.
Out of the 24 initial links, 21 are preserved and 3 have been rewired. Therefore we can estimate $p \approx \frac{3}{21} = \frac{1}{7}$.

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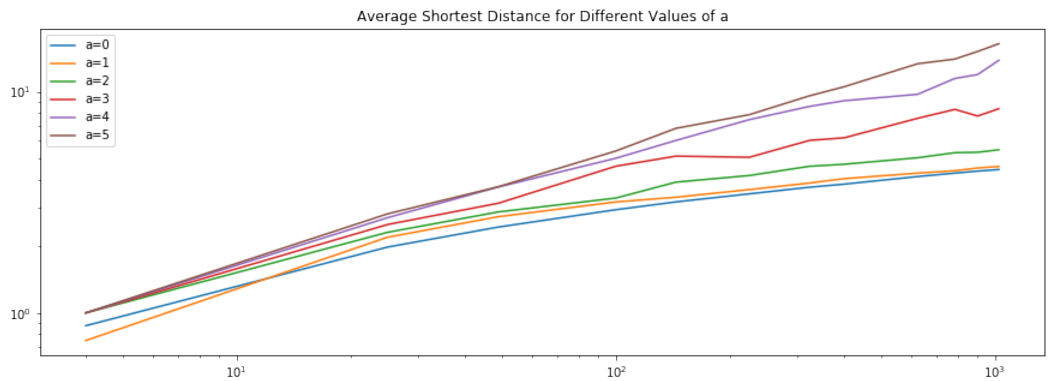
3 Kleinberg Network Model

Note: plots and discussion also given in asg03-3.ipynb.

3.1

– implemented in asg03-3.ipynb –

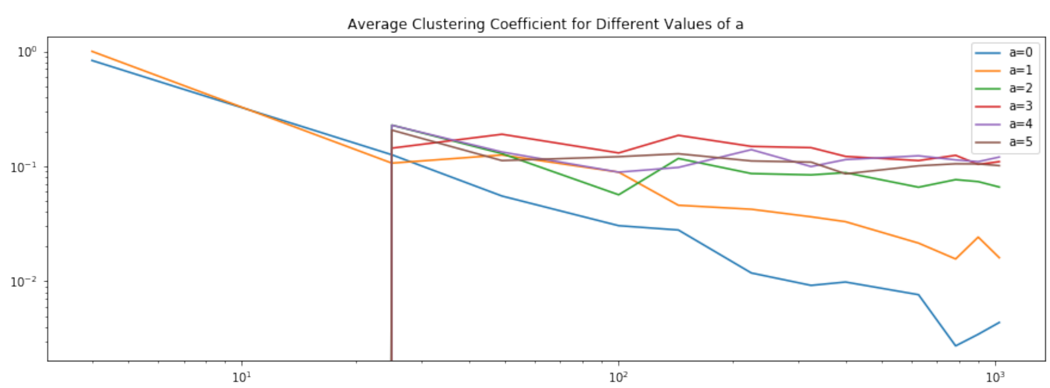
3.2



3.3

For higher values of a ($a = 4$, $a = 5$), the average shortest distance $< d(N) >$ grows approximately proportional to $log(N)$. This means, the Kleinberg model generates networks with small-world-behaviour for higher values of a

3.4



We can see here that the average clustering coefficient for the lower values of α, ($\alpha = 0, 1, 2$), the average clustering coefficient is inversely proportional to the $log(N)$

For the higher values of α, ($\alpha = 5, 4, 3$), the average clustering coefficient increase significantly between $N = 10$ and $N = 100$. But generally, the average clustering coefficient stays rather low for higher values α.
This reflects the weighted random selection of the extra link that would be added to each node. The neighbors y of a node x would have the distance $d(x, y) = 1$, which would give these neighbors the highest chances of being picked as target node of the extra link. As a result, the chances of the graph actually getting new links for high values of α is rather small. If no new link is created, then the average clustering coefficient would be 0 , or near to 0. (As we found in the previous assignment, a grid-like lattice network has the average clustering coefficient of 0)