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1 Clustering Coefficients

1.1

$k_{A,B} = \ell + 1$, $k_\ell = 2$, so:

$$\langle C \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{2L_i}{k_i(k_i - 1)}$$

$$= \frac{1}{N} \left(\underbrace{\sum_{A,B} \frac{2\ell}{(\ell+1)\ell}}_{\text{nodes } A,\ B} + \underbrace{\sum_{i=0}^{N-2} \frac{2 \cdot 1}{2 \cdot 1}}_{\text{nodes } l} \right)$$

$$= \frac{1}{N} \left(\frac{4}{\ell + 1} + N - 2 \right)$$

$$= \frac{1}{N} \left(\frac{4}{N - 1} + N - 2 \right)$$

$$= \frac{1}{N} \left(\frac{4 + (N-2)(N-1)}{N - 1} \right)$$

$$= \frac{1}{N} \left(\frac{4 + N^2 - 3N + 2}{N - 1} \right)$$

$$= \frac{N^2 - 3N + 6}{N^2 - N}.$$

1.2

According to hint:

$$C_\Delta = \frac{3 \cdot \#triangles}{\#triples}, \qquad \#triples = \sum_{i=1}^{N} \frac{k_i(k_i - 1)}{2}.$$

In the given graph, # triangles $= \ell$, therefore:

$$C_\Delta = \frac{3\ell}{\sum\limits_{i=1}^{N} \frac{k_i(k_i-1)}{2}}.$$

From problem 1.1: $k_{A,B} = \ell + 1$, $k_\ell = 2$, so:

$$C_\Delta = \frac{3\ell}{\sum\limits_{A,B} \frac{(\ell+1)\ell}{2} + \sum\limits_{i=0}^{N-2} \frac{2\cdot 1}{2}}$$

$$= \frac{3\ell}{\ell(\ell+1) + N - 2}$$

$$= \frac{3(N-2)}{(N-1)(N-2) + N - 2}$$

$$= \frac{3(N-2)}{N^2 - 3N + 2 + N - 2}$$

$$= \frac{3(N-2)}{N^2 - 2N}$$

$$= \frac{3(N-2)}{N(N-2)}$$

$$= \frac{3}{N}.$$

1.3

As $N \to \infty$, the average clustering coefficient $\langle C \rangle = (\frac{N^2-3N+6}{N^2-N})$ goes to 1. These reflects the fact that nodes A and B are connected to all others, and as the number of nodes goes to infinity, these connections dominate the coefficient. The global clustering coefficient $C_\Delta = \frac{3}{N}$ goes to 0, meaning that the number of triangles grows much more slowly than the number of triples. These limits show that in some cases, there is a very clear difference between the two formulations.