

1 Power Laws

1.1

(a) \rightarrow not scale-free, because the plot is curved, therefore $\log p_k$ does not depend linearly on $\log k$.

(b) \rightarrow scale-free, because $\log p_k$ depends linearly on $\log k$ (the plot is approximately a straight line).

1.2

$\log_{10} p_k \sim -\gamma \cdot \log_{10} k \Rightarrow \gamma = -\frac{\log_{10} p_k}{\log_{10} k}$. Sample several points on the graph and estimate the values in those points, then plug them in the formula:

- $k = 10, p_k = 10^{-2} \Rightarrow \gamma = 2$
- $k = 2, p_k = 10^{-1} \Rightarrow \gamma = 3.321$
- $k = 50, p_k = 10^{-3} \Rightarrow \gamma = 1.765$

We can estimate γ to be around 2.

1.3

$\gamma = 1 + N[\sum_{i=1}^N \ln \frac{K_i}{K_{min} - \frac{1}{2}}] = 1.756, \sigma = 0.16913$ (values calculated using a Python script, see 4-1.pynb).