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Problem 5-1: Role of Preferential Attachment

1

$$\frac{dk_i}{dt} \approx m\Pi k_i \approx m \cdot \frac{1}{m_0 + k - 1} = \frac{m}{m_0 + t - 1} \tag{5.12}$$

3

$$m\left[1 + \log \frac{m_0 + t - 1}{m_0 + 1 - m_0 + (m_0 + t - 1)\exp\left(1 - \frac{k}{m}\right)}\right]$$

$$= m\left[1 + \log(m_0 + t - 1) - \log(m_0 + t - 1)\exp\left(1 - \frac{k}{m}\right)\right]$$

$$= m\left[1 + \log(m_0 + t - 1) - (\log m_0 + t - 1) - \log\left(\exp\left(1 - \frac{k}{m}\right)\right)\right]$$

$$= m\left[1 - 1 + \frac{k}{m}\right]$$

$$= k$$

5

$$P(k) = \frac{dP(k)}{dk} = \frac{e^{1-\frac{k}{m}}}{m}$$

Problem 5-2: Friendship Paradox

1

$\sum q_k = 1, c = \frac{1}{\langle k \rangle}$

$p_k = C_k^{-\gamma}$

$\sum_{k=1}^{infty} p_k = 1$

$\int_{k_{min}}^{\infty} p(k)dk = 1 \Rightarrow c = \frac{1}{\int_{k_{min}}^{\infty} k^{\gamma} dk} = (\gamma - 1)k_{min}^{\gamma-1}$

(Slide 4.12)

4

Using the analogy of links as friendships: very social people have a lot of friends, therefore you are more likely to be friends with them than you are to be friends with someone with very few friends. As a result, your friends will (on average) tend to be more social than the average person and therefore (on average) more social than you.

In the language of graph theory: every individual node is more likely to be connected to nodes that have a lot of neighbours specifically because those nodes have a lot of neighbours, so the neighbours of any given node are more likely to be nodes with lots of neighbours.

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Problem 5-3: Barabási-Albert Model

For source code, see `asg005.ipynb`

2

	Practical Results	Analytical Results
Number of Nodes	105	105
Number of Edges	310	310
Sum of Node Degree	620	-

Based on these values, we can confirm that our `barabasi_albert` function is correct

3

	Practical Results	Analytical Results
Average Clustering Coefficient	0.0512	0.0475
Diameter	5	3.575
γ	2.763	3

We obtained a higher value for the diameter of our generated network than the expected value according the lecture slides. However it is not too far off, and the formula presented in the lecture is for the average diameter of all possible Barabási-Albert networks based on their size, so it is OK to deviate slightly from this average.