

Problem 5-1: Role of Preferential Attachment

1

$$\frac{dk_i}{dt} \approx m \Pi k_i \approx m \cdot \frac{1}{m_0 + k - 1} = \frac{m}{m_0 + t - 1} \quad (5.12)$$

2

$$K_i(t) = m \left[1 + \log \left(\frac{m_0 + t - 1}{m_0 + t_i - 1} \right) \right]$$

3

$$\begin{aligned} & m \left[1 + \log \frac{m_0 + t - 1}{m_0 + 1 - m_0 + (m_0 + t - 1) \exp \left(1 - \frac{k}{m} \right)} \right] \\ &= m \left[1 + \log(m_0 + t - 1) - \log(m_0 + t - 1) \exp \left(1 - \frac{k}{m} \right) \right] \\ &= m \left[1 + \log(m_0 + t - 1) - (\log m_0 + t - 1) - \log \left(\exp \left(1 - \frac{k}{m} \right) \right) \right] \\ &= m \left[1 - 1 + \frac{k}{m} \right] \\ &= k \end{aligned}$$

4

$$P(k) = 1 - \exp \left(1 - \frac{k}{m} \right)$$

5

$$P(k) = \frac{dP(k)}{dk} = \frac{e^{1 - \frac{k}{m}}}{m}$$

Problem 5-2: Friendship Paradox

Problem 5-3: Barabási-Albert Model