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1 Degree Correlation Coefficient

For all the computations, please see `Problem 6-1.ipynb`

1.1 Degree Correlation Matrix

$$E = \begin{bmatrix} 0 & 0 & 1/8 \\ 0 & 1/4 & 1/4 \\ 1/8 & 1/4 & 0 \end{bmatrix}$$

1.2 Probabilities q_k

$$\begin{aligned} q_1 &= 0 + 0 + 1/8 = 1/8 \\ q_2 &= 0 + 1/4 + 1/4 = 1/2 \\ q_3 &= 1/8 + 1/4 + 0 = 3/8 \end{aligned}$$

1.3 Degree Correlation Coefficient

$r = -0.7142857142857143$

Based on r, the given network is disassortative ($r < 0$)

2 Degree Correlations in Random Graphs

2.1

2.2

2.3

$$\begin{aligned}
\frac{P\left(a_{ij}=1|a_{xy}=0\right)}{P\left(a_{ij}=0\right)} &= \frac{L}{\frac{N(N-1)}{2}-1}\cdot\frac{\frac{N(N-1)}{2}}{L} \\
&= \frac{2}{N(N-1)-2}\cdot\frac{N(N-1)}{2} \\
&= \frac{N(N-1)}{N(N-1)-2}
\end{aligned}$$

$$\lim_{N\to\infty}\frac{N(N-1)}{N(N-1)-2}=1$$

$$\begin{aligned}
\frac{P\left(a_{ij}=1|a_{xy}=1\right)}{P\left(a_{ij}=0\right)} &= \frac{L-1}{\frac{N(N-1)}{2}-1}\cdot\frac{\frac{N(N-1)}{2}}{L} \\
&= \frac{L-1}{\frac{N(N-1)-2}{2}}\cdot\frac{N(N-1)}{2L} \\
&= \frac{N(L-1)(N-1)}{\frac{2LN(N-1)-2}{2}} \\
&= \frac{N(L-1)(N-1)}{LN(N-1)-2}
\end{aligned}$$

$$\lim_{N\to\infty}\frac{N(L-1)(N-1)}{LN(N-1)-2}=\frac{L-1}{L}$$

2.4

for $G(N,p)$ model, $r'_0=r'_1=1$, in other words, the conditional probabilities don't change and is always the same as $p(a_{ij}=1)$. This is the because the probability p that an edge exist in a $G(N,p)$ model is fixed, unlike in an $G(N,L)$ model

2.5

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3 Degree Correlations and Assortativity