CNA-Blatt 7 $\boxed{7.1}$ 1) Kronecker delta = $\delta_{ij} = \begin{cases} 0 & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases}$

$\langle k \rangle = \sum_{k=1}^{3} k p_k = p_1 + 2p_2 + 3p_3$

$\langle k^2 \rangle = \sum_{k=1}^{3} k^2 p_k = p_1 + 4p_2 + 9p_3$

2) $\kappa = \frac{\langle k^2 \rangle}{\langle k \rangle} > 2$

$\Leftrightarrow \frac{p_1 + 4p_2 + 9p_3}{p_1 + 2p_2 + 3p_3} > 2$

$\Leftrightarrow p_1 + \cancel{4p_2} + 9p_3 > 2p_1 + \cancel{4p_2} + 6p_3$

$3p_3 > p_1$

3) $p_1 < 3p_3 \Leftrightarrow \frac{p_1}{3} < p_3$

The probability that a node has degree 1 should be 3 times smaller than the probability that a node has degree 3. => The network is dense, and most nodes are connected to multiple other nodes, few nodes with only one link.

p_2 is irrelevant because $k=2$ is the "critical regime". (Slide 8-20): For a network to have a giant component most nodes that belong to it must be connected to at least two other nodes.

$\boxed{7.3}$ $f_c \approx 1 - \frac{1}{\frac{\langle k^2 \rangle}{\langle k \rangle} - 1}$

1) $\langle k \rangle = \mu$, $\langle k^2 \rangle = \mu^2 + \mu$

$f_c \approx 1 - \frac{1}{\frac{\mu^2+\mu}{\mu} - 1} = 1 - \frac{1}{\mu + 1 - 1} = 1 - \frac{1}{\mu}$

f_c depends on parameter μ, $\mu \to \infty$ $f_c \to 1$.

2) Geometric distribution: $P(k) = p(1-p)^k$

$\updownarrow$

Discrete exponential distribution: $p_k = (1 - e^{-\lambda}) e^{-\lambda k}$

$p = e^{-\lambda}$

$\langle k \rangle = \frac{1-p}{p}$, $\langle k^2 \rangle = \frac{1-p}{p^2}$

$\kappa = \frac{\langle k^2 \rangle}{\langle k \rangle} = \frac{\frac{1-p}{p^2}}{\frac{1-p}{p}} = \frac{p(1-p)}{p^2(1-p)} = \frac{1}{p}$

$$f_c \approx 1 - \frac{1}{\frac{\langle k^2 \rangle}{\langle k \rangle} - 1} = 1 - \frac{1}{\frac{1}{e^{-d}} - 1}$$

$$= 1 - \frac{1}{\frac{1-e^{-d}}{e^{-d}}} = 1 - \frac{e^{-d}}{1-e^{-d}}$$

$f_c \to 1$ as $d \to \infty$.

$p_k = 1$ only if $k = k_0$
rest of terms are 0

3) $\langle k \rangle = \sum_{k=k_{min}}^{k_{max}} k p_k = k_0$

$\langle k^2 \rangle = \sum_{k=k_{min}}^{k_{max}} k^2 p_k = k_0^2$

$$f_c \approx 1 - \frac{1}{\frac{k_0^2}{k_0} - 1} = 1 - \frac{1}{k_0 - 1}$$

See poisson distribution