

## Problem 7-1 Molloy-Reed Criterion

### 1.1

$$\text{Kronecker delta} = \delta_{ij} = \begin{cases} 0, & i \neq j \\ 1, & i = j \end{cases}$$

$$\langle k \rangle = \sum_{k=1}^3 k p_k = p_1 + 2p_2 + 3p_3$$

$$\langle k^2 \rangle = \sum_{k=1}^3 k^2 p_k = p_1 + 4p_2 + 9p_3$$

### 1.2

$$k = \frac{\langle k^2 \rangle}{\langle k \rangle} > 2$$

$$\iff \frac{p_1 + 4p_2 + 9p_3}{p_1 + 2p_2 + 3p_3} > 2$$

$$\iff p_1 + 9p_3 > 2p_1 + 6p_3$$

$$\iff 3p_3 > p_1$$

### 1.3

$$p_1 < 3p_3 \iff \frac{p_1}{3} < p_3$$

The probability that a node has degree 1 should be 3 times smaller than the probability that a node has degree 3. Therefore the network is dense, and most nodes are connected to multiple other nodes, few nodes with only one link.

$p_2$  is irrelevant because  $k = 2$  is the “critical regime”. Slide 8-20: for a network to have a giant component most nodes that belong to it must be connected to at least two other nodes.

## Problem 7-2 Random Failures in Uncorrelated Networks

$$f_c \approx 1 - \frac{1}{\frac{\langle k^2 \rangle}{\langle k \rangle} - 1}$$

### 2.1

$$\langle k \rangle = \mu, \langle k^2 \rangle = \mu^2 + \mu$$

$$f_c \approx 1 - \frac{1}{\frac{\mu^2 + \mu}{\mu} - 1} = 1 - \frac{1}{\mu + 1 - 1} = 1 - \frac{1}{\mu}$$

$f_c$  depends on parameter  $\mu$ :  $f_c \rightarrow \infty$  as  $\mu \rightarrow \infty$ .

### 2.2

Geometric distribution:  $\mathbf{p}(\kappa) = \mathbf{p}(1 - \mathbf{p})^\kappa$

Discrete exponential distribution:  $p_k = (1 - e^{-\lambda})e^{-\lambda k}, p = e^{-\lambda}$

$$\langle k \rangle = \frac{1 - p}{p}, \langle k^2 \rangle = \frac{1 - p}{p^2}$$

$$k = \frac{\langle k^2 \rangle}{\langle k \rangle} = \frac{\frac{1-p}{p^2}}{\frac{1-p}{p}} = \frac{p(1-p)}{p^2(1-p)} = \frac{1}{p}$$

$$f_c \approx 1 - \frac{1}{\frac{\langle k^2 \rangle}{\langle k \rangle} - 1} = 1 - \frac{1}{\frac{1}{e^{-\lambda} - 1}}$$

$$= 1 - \frac{1}{\frac{1 - e^{-\lambda}}{e^{-\lambda}}}$$

$$= 1 - \frac{e^{-\lambda}}{1 - e^{-\lambda}}$$

$f_c \rightarrow 1$  as  $\lambda \rightarrow \infty$ .

## 2.3

$$\langle k \rangle = \sum_{k=k_{min}}^{k_{max}} k p_k = k_0$$

$$\langle k \rangle = \sum_{k=k_{min}}^{k_{max}} k^2 p_k = k_0^2$$

$$f_c \approx 1 - \frac{1}{\frac{k_0^2}{k_0} - 1} = 1 - \frac{1}{k_0 - 1}$$

See: poisson distribution

## Problem 7-4 European Power Grid

For source code, see Problem\_7-4.ipynb

### 4.1

$$\kappa = 3.160386641199283$$

According to the Molloy-Reed Criterion, the given graph has a giant component, as  $\kappa > 2$ .

### 4.2

Absolute size of largest component = 13478

Relative size of largest component = 1.0271553642973734

The relative size of the largest component ( $\approx 1$ ) confirms that the given network has a giant component.

### 4.3

$$f_c = 0.5371198928332218$$

$$f_c^{ER} = 0.5993517393065926$$

About 54% of the nodes need to be removed to break the network into unconnected components. This network does not have enhanced robustness since  $f_c < f_c^{ER}$ .