Maria Kagkeli
Maria Regina Lily
Mihai Verzan

Problem 7-1 Molloy-Reed Criterion

1.1

Kronecker delta $= \delta_{ig} = \begin{cases} 0,\ i \neq j \\ 1,\ i = j \end{cases}$

$$\langle k \rangle = \sum_{k=1}^{3} k p_k \;\; = p_1 + 2p_2 + 3p_3$$
$$\langle k^2 \rangle = \sum_{k=1}^{3} k^2 p_k = p_1 + 4p_2 + 9p_3$$

1.2

$$k = \frac{\langle k^2 \rangle}{\langle k \rangle} > 2$$

$$\iff \frac{p_1 + 4p_2 + 9p_3}{p_1 + 2p_2 + 3p_3} > 2$$

$$\iff p_1 + 9p_3 > 2p_1 + 6p_3$$

$$\iff 3p_3 > p_1$$

1.3

$p_1 < 3p_3 \iff \frac{p_1}{3} < p_3$

The probability that a node has degree 1 should be 3 times smaller than the probability that a node has degree 3. Therefore the network is dense, and most nodes are connected to multiple other nodes, few nodes with only one link.

p_2 is irrelevant because $k = 2$ is the "critical regime". Slide 8-20: for a network to have a giant component most nodes that belong to it must be connected to at least two other nodes.

Maria Kagkeli
Maria Regina Lily
Mihai Verzan

Problem 7-2 Xalvi-Brunet and Sokolov Algorithm

text

Problem 7-3 Random Failures in Uncorrelated Networks

$$f_c \approx 1 - \frac{1}{\frac{\langle k^2 \rangle}{\langle k \rangle} - 1}$$

3.1

$$\langle k \rangle = \mu, \langle k^2 \rangle = \mu^2 + \mu$$

$$f_c \approx 1 - \frac{1}{\frac{\mu^2+\mu}{\mu} - 1} = 1 - \frac{1}{\mu + 1 - 1} = 1 - \frac{1}{\mu}$$

f_c depends on parameter μ: $f_c \to \infty$ as $\mu \to \infty$.

3.2

Geometric distribution: $\mathbf{p}(\kappa) = \mathbf{p}(\mathbf{1} - \mathbf{p})^{\kappa}$
Discrete exponential distribution: $p_k = (1 - e^{-\lambda})e^{-\lambda\kappa}, p = e^{-\lambda}$

$$\langle k \rangle = \frac{1-p}{p}, \langle k^2 \rangle = \frac{1-p}{p^2}$$

$$k = \frac{\langle k^2 \rangle}{\langle k \rangle} = \frac{\frac{1-p}{p^2}}{\frac{1-p}{p}} = \frac{p(1-p)}{p^2(1-p)} = \frac{1}{p}$$

$$f_c \approx 1 - \frac{1}{\frac{\langle k^2 \rangle}{\langle k \rangle} - 1} = 1 - \frac{1}{\frac{1}{e^{-\lambda} - 1}}$$

$$= 1 - \frac{1}{\frac{1-e^{-\lambda}}{e^{-\lambda}}}$$

$$= 1 - \frac{e^{-\lambda}}{1 - r^{-\lambda}}$$

$f_c \to 1$ as $\lambda \to \infty$.

Maria Kagkeli
Maria Regina Lily
Mihai Verzan

3.3

$$\langle k \rangle = \sum_{k=k_{min}}^{k_{max}} k p_k \;\; = k_0$$
$$\langle k \rangle = \sum_{k=k_{min}}^{k_{max}} k^2 p_k = k_0^2$$

$f_c \approx 1 - \frac{1}{\frac{k_0^2}{k_0} - 1} = 1 - \frac{1}{k_0 - 1}$

See: poisson distribution

Maria Kagkeli
Maria Regina Lily
Mihai Verzan

Problem 7-4 European Power Grid

text