

1

1.1

$$\text{Kronecker delta} = \delta_{ij} = \begin{cases} 0, & i \neq j \\ 1, & i = j \end{cases}$$

$$\begin{aligned} \langle k \rangle &= \sum_{k=1}^3 k p_k = p_1 + 2p_2 + 3p_3 \\ \langle k^2 \rangle &= \sum_{k=1}^3 k^2 p_k = p_1 + 4p_2 + 9p_3 \end{aligned}$$

1.2

$$k = \frac{\langle k^2 \rangle}{\langle k \rangle} > 2$$

$$\iff \frac{p_1 + 4p_2 + 9p_3}{p_1 + 2p_2 + 3p_3} > 2$$

$$\iff p_1 + 9p_3 > 2p_1 + 6p_3$$

$$\iff 3p_3 > p_1$$

1.3

$$p_1 < 3p_3 \iff \frac{p_1}{3} < p_3$$

The probability that a node has degree 1 should be 3 times smaller than the probability that a node has degree 3. Therefore the network is dense, and most nodes are connected to multiple other nodes, few nodes with only one link.

p_2 is irrelevant because $k = 2$ is the “critical regime”. Slide 8-20: for a network to have a giant component most nodes that belong to it must be connected to at least two other nodes.

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