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Integrated selection of suppliers and scheduling of customer orders in the presence of supply chain disruption risks

Tadeusz Sawik[a]
[a] Department of Operations Research and Information Technology, AGH University of Science & Technology, Kraków, Poland.
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Integrated selection of suppliers and scheduling of customer orders in the presence of supply chain disruption risks

Tadeusz Sawik*

Department of Operations Research and Information Technology, AGH University of Science & Technology, Kraków, Poland

This paper presents a new stochastic mixed integer programming approach to integrated supplier selection, order quantity allocation and customer order scheduling in the presence of supply chain disruption risks. Given a set of customer orders for products, the decision maker needs to decide from which supplier to purchase parts required to complete the orders, how to allocate the demand for parts among the selected suppliers, and how to schedule the customer orders over the planning horizon to minimize total cost of ordering and purchasing of parts plus penalty cost of delayed and unfulfilled customer orders and to mitigate the impact of disruption risks. The risk-neutral and risk-averse solutions that optimize, respectively average and worst-case performance of a supply chain are compared for both single and multiple sourcing strategy. Numerical examples are presented and computational results are reported.

Keywords: supply chain operations; disruption risk; supplier selection; production and supply scheduling; stochastic mixed integer programming

1. Introduction

Modern supply chains of high technology products have global characteristics: include many suppliers located in different geographic regions, many manufacturers and finished products distribution centers, and many end-users of these products. The optimal functioning of such complex networks requires solving diverse, multi-criteria, interconnected decision problems, which generally speaking include the control and optimization of material flows in these networks. Different types of flows (e.g. flows of parts from suppliers to producers, flows of semi-finished products at producers, flows of finished products from producers to distribution centers and from the distribution centers to customers) should be coordinated in an efficient manner, in particular by the integrated scheduling of supplies of parts, scheduling of production and customer orders for finished products, and scheduling of deliveries to customers. The coordinated scheduling is much more important in view of supply disruption risks due to unexpected natural or man-made disasters such as earthquakes, fires, floods, hurricanes or labor strikes, economic crisis, terrorist attack, etc. The probability of such disaster events is very low, however their business impact can be very high, as evidenced by the diverse real-world consequences of their occurrence. For example, the recent disruptions in the electronics supply chains due to the great East Japan earthquake of 11 March 2011 and then the catastrophic Thailand flooding of October 2011, where many component manufacturers were concentrated, resulted in huge losses of many Japanese companies, e.g. Fuller (2012), Park, Hong, and Roh (2013).

The supplier selection and order quantity allocation problem is a complex stochastic optimization problem with a common risk-neutral objective of optimizing average performance of a supply chain replaced with a new risk-averse objective of minimizing the potential worst-case losses. The literature on supplier selection and order quantity allocation problems is abundant and a variety of different methods can be applied for the problem solution. The various problems of suppliers selection, different selection criteria and the existing methods to solve these problems were discussed by many authors, for example by Aissaoui, Haouari, and Hassini (2007), Dotoli and Falagario (2012), Jain, Benyoucef, and Deshmukh (2009), Kuo and Lin (2012), Parthiban, Zubar, and Katakar (2013).

On the other hand, the research on supplier selection under disruption risks is very limited, e.g. Simangunsong, Hendry, and Stevenson (2012). For example, risks associated with a supplier network was studied by Berger, Gerstenfeld, and Zeng (2004), who considered catastrophic super events that affect all suppliers, as well as unique events that impact only one single supplier, and then a decision-tree based model was presented to help determine the optimal number of suppliers needed for the buying firm. Ruiz-Torres and Mahmoodi (2007) considered unequal failure probabilities for all the suppliers. Berger and Zeng (2006) studied the optimal supply size in a single or multiple sourcing strategy context, under a number of scenarios that are determined by various financial loss functions, the operating cost functions and the probabilities of all the suppliers

*Email: ghsawik@cyf-kr.edu.pl

being down. Yu, Zeng, and Zhao (2009) considered the impacts of supply disruption risks on the choice between the single and dual sourcing methods in a two-stage supply chain with a non-stationary and price-sensitive demand. Ravindran et al. (2010) developed multi-criteria supplier selection models incorporating supplier risks. In the multi-objective formulation, price, lead-time, disruption risk due to natural event and quality risk are explicitly considered as four conflicting objectives that have to be minimized simultaneously. Four different variants of goal programming were used to solve the multi-objective optimization problem. A stochastic lot-sizing problem with multiple suppliers and quantity discounts to minimize total costs, where the costs include ordering cost, holding cost, purchase cost and shortage cost, and to maximize service level of the supply chain was considered by Kang and Lee (2013). The stochastic lot-sizing problem was formulated as a multi-objective mixed integer program. Ould-Louly and Dolgui (2002) and Louly, Dolgui, and Hnaien (2008) considered assembly systems under lead time uncertainties. The authors generalized the well-known discrete newsboy model to calculate the safety lead times and developed an efficient exact model to calculate planned lead times when the component procurement times are random. Merzifonluoglu and Feng (forthcoming) considered a newsvendor problem with multiple unreliable suppliers and developed an exact algorithm to solve the problem optimally and a heuristic algorithm to solve the problem efficiently. Through structural properties of the optimal solution and a numerical study, they provided useful managerial implications regarding optimal sourcing strategies in complex supply chains. Sawik (2011a, 2013) proposed a portfolio approach for the supplier selection and order quantity allocation under disruption risks and applied the two popular in financial engineering percentile measures of risk, Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) (e.g. Sarykalin, Serraino, and Uryasev 2008; Sawik 2012a,b) for managing the risk of supply disruptions. The two different types of disruption scenarios were considered: scenarios with independent local disruptions of each supplier and scenarios with local and global disruptions that may result in all suppliers disruption simultaneously. The resulting scenario-based optimization problem under uncertainty was formulated as a single- or bi-objective mixed integer program.

In a make-to-order manufacturing, the supplier selection and order quantity allocation decisions and the resulting schedule of material supplies determine the material availability constraints that must be satisfied by any feasible schedule of customer orders at the producer of finished products. On the other hand, the schedule of customer orders implicitly determines the subset of accepted orders, scheduled before, on or after the customer requested due dates as well as the subset of rejected customer orders, unscheduled during the planning horizon because of material shortage or limited manufacturing capacity. Owing to limited available capacity of both the part suppliers and the producers of finished products, in order to complete customer orders by the customer required shipping dates, the supply schedules for each supplier of parts and the customer order schedule for each producer should be coordinated in an efficient manner to achieve a high customer service level at a low cost, e.g. Dolgui and Proth (2010). Various perspectives on supply chain coordination issues were reported and reviewed by Arshinder, Kanda, and Deshmukh (2008) and the gaps existing in the literature were identified. Li and Wang (2007) reviewed coordination mechanisms of supply chain systems in a framework that was based on supply chain decision structure and nature of demand. A review of methods and literature on supply chain coordination through contracts was provided by Hezarkhani and Kubiak (2010).

Most work on coordinated supply chain scheduling focuses on coordinating the flows of supply and demand to minimize the inventory, transportation and shortage costs. For example, Chen and Vairaktarakis (2005) and Chen and Pundoor (2006) studied simplified models for integrated scheduling of production and distribution operations. The authors have analyzed computational complexity of various cases of the problem and have developed heuristics for NP-hard cases. The issues of conflict and cooperation between the suppliers and the producer were studied by Chen and Hall (2007), where classical scheduling objectives were considered: minimization of the total completion time and of the maximum lateness. Wang and Lei (2012) considered the problem of operations scheduling for a capacitated multi-echelon shipping network with delivery deadlines, where semi-finished goods are shipped from suppliers to customers through processing centers, with the objective of minimizing the shipping and penalty cost. The three polynomial-time solvable cases of this problem were reported: with identical order quantities; with designated suppliers; and with divisible customer order sizes. An integrated approach to deterministic coordinated supply chain scheduling was proposed by Sawik (2009) to simultaneously schedule manufacturing and supply of parts and assembly of finished products. Given a set of part suppliers and a set of customer orders for finished products, the problem objective is to determine which orders are provided with parts by each supplier, to schedule manufacturing of parts at each supplier and delivery of parts from each supplier to the producer, and to schedule customer orders at the producer, such that a high customer service level is achieved and the total cost is minimized. The selection of part supplier for each customer order is combined with a due date setting for some orders to maximize the number of orders that can be completed by customer requested due dates. A monolithic mixed integer programming model is presented and compared with a hierarchy of mixed integer programs for a sequential selection of suppliers and scheduling of manufacturing and delivery of parts and assembly of products. Various enhancements of the above mixed integer programming approach for the coordinated scheduling in multi-stage supply chains were presented in Sawik (2011b).

Coordinated selection of part suppliers and allocation of order quantities and scheduling of customer orders may particularly help to optimize performance of a multi-stage supply chain under disruption risks. However, the research on quantitative approaches to the coordinated supplier selection and customer order scheduling in the presence of supply chain disruption risks is very limited. Sawik (forthcoming) developed a stochastic mixed integer programming approach to joint supplier selection and customer order scheduling under disruption risks, for a single or dual sourcing strategy. The suppliers are assumed to be located in two different geographical regions: in the producer's region (domestic suppliers) and outside the producer's region (foreign suppliers). The supplies are subject to independent random local disruptions that are uniquely associated with a particular supplier and to random semi-global (regional) disruptions that may result in disruption of all suppliers in the same geographical region simultaneously. The problem objective is either to minimize total cost or to maximize customer service level.

The major contribution of this paper, which continues the idea presented in Sawik (forthcoming), is that it proposes a new stochastic mixed integer programming approach to integrated supplier selection, order quantity allocation and customer orders scheduling in a multi-stage supply chain under disruption risks. Risk-neutral and risk-averse solutions that optimize, respectively average and worst-case performance of a supply chain and the total cost objective function, are compared for both single and multiple sourcing strategy.

The paper is organized as follows. In Section 2 description of the integrated selection of suppliers and scheduling of customer orders in the presence of supply chain disruption risks is provided. The mixed integer programs for risk-neutral and risk averse solutions for both single and multiple sourcing are developed in Section 3. Numerical examples and some computational results are provided in Section 4, and final conclusions are made in the last section.

2. Problem description

Consider a three-stage customer driven supply chain in which various types of products are assembled by a single producer to meet customer orders, using different part types purchased from multiple suppliers (for notation used, see Table 1).

Table 1. Notation.

		Indices
i	=	supplier, $i \in I = \{1, \ldots, m\}$
j	=	customer order, $j \in J = \{1, \ldots, n\}$
k	=	part type, $k \in K = \{1, \ldots, q\}$
s	=	disruption scenario, $s \in S = \{1, \ldots, r\}$
t	=	planning period, $t \in T = \{1, \ldots, h\}$
		Input Parameters
a_{jk}	=	per unit requirement for part type k of each product in customer order j
b_j	=	size (number of products) of customer order j
A_k	=	demand for part type k
A	=	total demand for parts
B	=	total demand for products
c_{ik}	=	per unit capacity consumption of supplier i for part type k
C_i	=	capacity of supplier i
c^j	=	per unit capacity consumption of producer for customer order j
C^t	=	capacity of producer in period t
d_j	=	due date for customer order j
e_j	=	per unit and per period (e.g. daily) penalty cost of delayed customer order j
f_j	=	per unit penalty cost of unfulfilled customer order j
g_i	=	fixed cost of ordering parts from supplier i
I_k	=	the subset of suppliers capable of manufacturing part type k
J_k	=	the subset of customer orders for products requiring part type k
p_{ik}	=	per unit price of part type k purchased from supplier i
α	=	confidence level
ν	=	the minimum order size
π_i	=	the local disruption probability for supplier i
π^*	=	the global disruption probability for all suppliers
σ_i	=	shipping time from supplier i

Let $i \in I = \{1, \ldots, m\}$ be the set of m suppliers, $J = \{1, \ldots, n\}$ the set of n customer orders for products and $K = \{1, \ldots, q\}$ the set of q part types required to assemble the products. The planning horizon consists of h planning periods (e.g. days) and denote by $T = \{1, \ldots, h\}$ the index set of all periods.

The suppliers manufacture and deliver parts to the producer and all part types ordered from a supplier are shipped together in a single delivery. The transportation time of a shipment from supplier $i \in I$ to the producer is constant and equals to σ_i periods so that the parts ordered from supplier $i \in I$ are delivered in period σ_i and then can be used for the assembly of products in period $\sigma_i + 1$, at the earliest.

Denote by b_j and d_j, respectively the size and the due date of customer order $j \in J$, i.e. the number units of ordered product type and the latest period of their completion required to deliver the products to the customer by requested date. The customer orders are single-period orders (e.g. Sawik 2007) such that each order can be completed in one planning period. The total demand for all products is $B = \sum_{j \in J} b_j$.

Let a_{jk} be the unit requirement for part type $k \in K$ of each product in customer order $j \in J$. The demand for each part type $k \in K$ is $A_k = \sum_{j \in J} a_{jk} b_j$, and the total demand for all parts is $A = \sum_{k \in K} A_k$.

The suppliers have different limited capacity and, in addition, differ in price of offered parts. Let C_i be the capacity of supplier $i \in I$, c_{ik} – unit capacity consumption of supplier i for part type k and p_{ik} – unit purchasing price, including shipping cost of part type $k \in K$ from supplier $i \in I$. The fixed cost of ordering parts from supplier $i \in I$ is denoted by g_i.

Denote by $I_k \subset I$, the subset of suppliers capable of manufacturing part type $k \in K$. Assume that for suppliers incapable of providing some part types, the corresponding unit capacity consumption and unit price are very large numbers.

Let C^t be the capacity of producer available in planning period $t \in T$, and c^j – unit capacity consumption of producer for each product in customer order $j \in J$.

The supplies of parts are subject to random local disruptions that are uniquely associated with a particular supplier, which may arise from equipment breakdowns, local labor strike, bankruptcy, terrorist attack, from local natural disasters such as earthquakes, fires, floods, hurricanes, etc. Denote by π_i the local disruption probability for supplier i, i.e. the parts ordered from supplier i are delivered without disruptions with probability $(1 - \pi_i)$, or not at all with probability π_i.

In addition to independent local disruptions of each supplier, there are potential global disasters that may result in all suppliers disruption simultaneously. For example, such global super events may include economic crisis, widespread labor strike in a transportation sector, etc. Although the probability of such disaster events usually is very low, their consequences may be very high. Denote by π^* the probability of simultaneous global disruption of all suppliers due to some disaster super event.

Denote by P_s the probability that disruption scenario s is realized, where each scenario $s \in S$ is comprised of a unique subset $I^s \subset I$ of suppliers who deliver parts without disruptions, and $S = \{1, \ldots, r\}$ is the index set of all disruption scenarios (note that there are a total of $r = 2^m$ potential disruption scenarios).

The global disaster and the local disasters at each supplier are assumed to be independent events, therefore the probability P_s of each disruption scenario $s \in S$ under the risks of both type of events is

$$P_s = \begin{cases} (1 - \pi^*) Q_s & \text{if } I^s \neq \emptyset \\ \pi^* + (1 - \pi^*) \prod_{i \in I} \pi_i & \text{if } I^s = \emptyset, \end{cases}$$

where Q_s is the probability of disruption scenario s in the presence of independent local disaster events only

$$Q_s = \prod_{i \in I^s} (1 - \pi_i) \cdot \prod_{i \notin I^s} \pi_i.$$

If the probability of global disruption $\pi^* = 0$, then the probability P_s reduces to Q_s for independent local disaster events.

The producer can be charged with a contractual penalty cost for delayed or unfulfilled customer orders, caused by the shortage of parts, that are delivered late or not at all due to supply disruptions. Let e_j and f_j be, respectively the per unit daily penalty cost of delayed and the per unit total penalty cost of unfulfilled customer order $j \in J$. In addition to the contractual penalty costs, the unfulfillment of customer orders results in loss of the producer profits. The lost profits are not explicitly considered in the models proposed in this paper and are assumed to be included in the penalty costs f_j. Therefore, the unit penalty costs e_j and f_j are selected in such a way that each product in an unfulfilled order is penalized much higher than the corresponding product in a delayed order, i.e. $f_j \gg e_j, \; j \in J$.

Summarizing, the problem of integrated supplier selection and customer order scheduling can be formulated as follows. Given a set of customer orders for products, the decision maker needs to decide from which supplier to purchase parts required to complete the orders, how to allocate the demand for parts among the selected suppliers, and how to schedule the customer orders over the planning horizon to achieve a minimum total cost of ordering and purchasing of parts plus penalty cost of delayed and unfulfilled customer orders, and to mitigate the impact of disruption risks. The resulting supply portfolio (the allocation of total demand for parts among the suppliers) and the schedule of customer orders for each disruption scenario, are determined ahead of time to minimize the potential average or worst-case cost.

3. Supplier selection and customer order scheduling

In this section the four time-indexed stochastic MIP models are proposed for the integrated supplier selection and customer order scheduling to optimize average (models **Es** and **Em**) or worst-case (models **CVs** and **CVm**) performance of a supply chain in the presence of disruption risks.

The following two different versions of the supplier selection problem will be considered:

- Single sourcing ('s' in the model name), where the total demand for each part type is assigned to exactly one supplier, capable of manufacturing this part type,
- Multiple sourcing ('m' in the model name), where the total demand for each part type is allocated among one or more suppliers, capable of manufacturing this part type.

The following four basic decision variables are introduced in the proposed MIP models.

- Supplier selection variable: $u_i = 1$, if supplier i is selected; otherwise $u_i = 0$,
- Order-to-period assignment variable: $x^s_{jt} = 1$, if under disruption scenario s customer order j is assigned to planning period t; otherwise $x^s_{jt} = 0$,
- Part type-to-supplier assignment variable (single sourcing): $y_{ik} = 1$, if all required parts of type k are ordered from supplier i; otherwise $y_{ik} = 0$,
- Part type demand allocation variable (multiple sourcing): $z_{ik} \in [0, 1]$ is the fraction of total demand for part type k ordered from supplier i.

3.1 *Risk-neutral models*

In this subsection two MIP models are presented for the selection of supply portfolio and scheduling of customer orders in a risk-neutral supply chain environment for single sourcing (model **Es**) and for multiple sourcing (model **Em**).

For a single sourcing in the risk-neutral operating conditions, the overall quality of the supply portfolio and customer orders schedule can be measured by the expected cost per product, (1), of parts ordering, $\sum_{i\in I} g_i u_i/B$, and purchasing, $\sum_{s\in S} P_s \left(\sum_{k\in K}\sum_{i\in I_k \bigcap I^s} A_k p_{ik} y_{ik}\right)/B$, where the producer is not charged with ordered and undelivered parts, plus penalty cost of delayed and unfulfilled (rejected) customer orders due to delays and disruptions of part supplies, $\sum_{s\in S} P_s \left(\sum_{j\in J}\sum_{t\in T:t>d_j} e_j b_j (t-d_j) x^s_{jt}\right)/B + \sum_{s\in S} P_s \left(\sum_{j\in J} f_j b_j \left(1-\sum_{t\in T} x^s_{jt}\right)\right)/B$.

The binary program **Es** for a single sourcing selection of supply portfolio and scheduling of customer orders to optimize an average performance of supply chain in a risk-neutral environment is formulated below.

Model Es: *Risk-neutral supplier selection and customer order scheduling to minimize*

E*xpected cost:* **s***ingle sourcing*

Minimize Expected Cost per Product

$$\sum_{i\in I} g_i u_i/B + \sum_{s\in S} P_s \left(\sum_{k\in K}\sum_{i\in I_k \bigcap I^s} A_k p_{ik} y_{ik}\right) \Big/ B$$
$$+\sum_{s\in S} P_s \left(\sum_{j\in J}\sum_{t\in T:t>d_j} e_j b_j (t-d_j) x^s_{jt}\right) \Big/ B + \sum_{s\in S} P_s \left(\sum_{j\in J} f_j b_j \left(1-\sum_{t\in T} x^s_{jt}\right)\right) \Big/ B \qquad (1)$$

subject to

(1) Part type-to-supplier assignment constraints:

- each part type can be assigned to exactly one selected supplier, capable of manufacturing this part type,
- no part type can be assigned to non-selected suppliers and at least one part type must be assigned to each selected supplier,

$$\sum_{i \in I_k} y_{ik} = 1; \quad k \in K \tag{2}$$

$$y_{ik} \le u_i; \quad k \in K, i \in I_k \tag{3}$$

$$\sum_{k \in K} y_{ik} \ge u_i; \quad i \in I \tag{4}$$

(2) Order-to-period assignment constraints:

- for each disruption scenario s, each customer order j is either scheduled during the planning horizon ($\sum_{t \in T} x_{jt}^s = 1$), or unscheduled and rejected ($\sum_{t \in T} x_{jt}^s = 0$),
- for each disruption scenario s and each part type k, all customer orders for products requiring this part type (such that $a_{jk} \ge 1$) can be scheduled only after the delivery of the required part type, purchased from a non disrupted supplier i, ($i \in I^s$), capable of manufacturing this part type ($i \in I_k$),

$$\sum_{t \in T} x_{jt}^s \le 1; \quad j \in J, s \in S \tag{5}$$

$$\sum_{j \in J_k} \sum_{\tau \in T: \tau \le t} x_{j\tau}^s \le |J_k| \sum_{i \in I_k \bigcap I^s : \sigma_i \le t-1} y_{ik}; \quad k \in K, t \in T, s \in S \tag{6}$$

where $J_k = \{j \in J : a_{jk} \ge 1\}$ is the subset of customer orders for products requiring part type $k \in K$,

(3) Suppliers capacity constraints:

- for each selecteed supplier, the total capacity consumption required for ordered parts cannot exceed supplier's capacity,

$$\sum_{k \in K} A_k c_{ik} y_{ik} \le C_i u_i; \quad i \in I \tag{7}$$

(4) Producer capacity constraints:

- for any period t and each disruption scenario s, the total demand on capacity of all orders scheduled in period t must not exceed the producer capacity available in this period,

$$\sum_{j \in J} b_j c^j x_{jt}^s \le C^t; \quad t \in T, s \in S \tag{8}$$

(5) Integrality conditions:

$$u_i \in \{0, 1\}; \quad i \in I \tag{9}$$

$$x_{jt}^s \in \{0, 1\}; \quad j \in J, t \in T, s \in S \tag{10}$$

$$y_{ik} \in \{0, 1\}; \quad k \in K, i \in I_k. \tag{11}$$

A similar model can be constructed for a multiple sourcing. For the multiple sourcing in a risk-neutral operating conditions, the overall quality of the supply portfolio and the schedule of customer orders can be measured by the expected cost per product, (12), of parts ordering, parts purchasing ($\sum_{s \in S} P_s (\sum_{k \in K} \sum_{i \in I_k \bigcap I^s} A_k p_{ik} z_{ik})/B$), and penalty cost of delayed and unfulfilled customer orders due to delays and disruptions of part supplies.

The mixed integer program **Em** for a multiple sourcing selection of supply portfolio and scheduling of customer orders in a risk-neutral supply chain environment is formulated below.

Model Em: *Risk-neutral supplier selection and customer order scheduling to minimize*

E*xpected cost:* **m***ultiple sourcing*

Minimize Expected Cost per Product

$$\sum_{i \in I} g_i u_i / B + \sum_{s \in S} P_s \left(\sum_{k \in K} \sum_{i \in I_k \bigcap I^s} A_k p_{ik} z_{ik} \right) \Big/ B$$
$$+ \sum_{s \in S} P_s \left(\sum_{j \in J} \sum_{t \in T: t > d_j} e_j b_j (t - d_j) x^s_{jt} \right) \Big/ B + \sum_{s \in S} P_s \left(\sum_{j \in J} f_j b_j \left(1 - \sum_{t \in T} x^s_{jt} \right) \right) \Big/ B \tag{12}$$

subject to

(1) Part type demand allocation constraints:

- the demand for each part type must be fully allocated among the selected suppliers, capable of manufacturing this part type,
- no order for parts can be assigned to non-selected suppliers and at least a minimum order size must be assigned to each selected supplier,

$$\sum_{i \in I_k} z_{ik} = 1; \quad k \in K \tag{13}$$
$$z_{ik} \le u_i; \quad i \in I_k, k \in K \tag{14}$$
$$\sum_{k \in K} A_k z_{ik} \ge \nu u_i; \quad i \in I \tag{15}$$

(2) Order-to-period assignment constraints: (5) and

- for each disruption scenario s and each planning period t, the cumulative demand for each part type k of all customer orders scheduled in periods 1 through t cannot exceed the cumulative deliveries of this part type in periods 1 through $t-1$, from the non disrupted suppliers $i \in I^s$, capable of manufacturing part type k,

$$\sum_{j \in J} \sum_{\tau \in T: \tau \le t} a_{jk} b_j x^s_{j\tau} \le A_k \sum_{i \in I_k \bigcap I^s: \sigma_i \le t-1} z_{ik}; \quad k \in K, t \in T, s \in S \tag{16}$$

(3) Suppliers capacity constraints:

- for each selected supplier, the total capacity consumption required for ordered parts cannot exceed supplier's capacity,

$$\sum_{k \in K} A_k c_{ik} z_{ik} \le C_i u_i; \quad i \in I \tag{17}$$

(4) Producer capacity constraints: (8)
(5) Non-negativity and integrality conditions: (9), (10) and

$$z_{ik} \in [0, 1]; \quad k \in K, i \in I_k. \tag{18}$$

3.2 *Risk-averse models*

The MIP models **Es** and **Em** presented in Section 3.1 will be used to compare the risk-neutral results with those obtained by applying the corresponding risk aversive decision making models described in this subsection.

To control the risk of supply disruptions, the following two, popular in financial engineering, percentile measures of risks will be applied (Rockafellar and Uryasev 2000; Sarykalin, Serraino, and Uryasev 2008; Uryasev 2000).

- Value-at-Risk (VaR) at a $100\alpha\%$ confidence level is the targeted cost of the supply portfolio such that for $100\alpha\%$ of the scenarios, the outcome will not exceed VaR. In other words, VaR is a decision variable based on the α-percentile of costs, i.e. in $100(1-\alpha)\%$ of the scenarios, the outcome may exceed VaR.

- Conditional Value-at-Risk (CVaR) at a $100\alpha\%$ confidence level is the expected cost (under certain conditions, Uryasev 2000) of the supply portfolio in the worst $100(1-\alpha)\%$ of the cases. In other words, we allow $100(1-\alpha)\%$ of the outcomes to exceed VaR, and the mean value of these outcomes is represented by CVaR.

In other words, VaR is the acceptable cost level above which we want to minimize the number of outcomes and CVaR considers those portfolio outcomes, where costs exceed VaR.

In the selection of supply portfolio and scheduling of customer orders under disruption risks, the decision maker controls the risk of high losses due to supply disruptions by choosing the confidence level α. We assume that the decision maker is willing to accept only portfolios for which the the total probability of scenarios with costs greater than VaR is not greater than $1-\alpha$. The greater the confidence level α, the more risk aversive is the decision maker and the smaller percent of the highest cost outcomes is focused on.

Define $\mathcal{T}_s$ as the tail cost for scenario s, where tail cost is defined as the amount by which costs in scenario s exceed VaR. The supply portfolio and the production schedule will be optimized by calculating VaR and minimizing CVaR simultaneously. By measuring CVaR, the magnitude of the tail costs is considered to achieve a more accurate estimate of the risks of minimizing cost. In the proposed model, CVaR is represented by an auxiliary function (19) introduced by Rockafellar and Uryasev (2000).

The mixed integer programs **CVs** and **CVm** for supplier selection and customer order scheduling to optimize worst-case performance of supply chain and reduce the risk of high costs, respectively for single and for multiple sourcing, are formulated below.

Model CVs: *Risk-averse supplier selection and customer order scheduling to minimize*

CV*aR:* **s***ingle sourcing*

Minimize Expected Worst-Case Cost per Product (CVaR)

$$CVaR = VaR + (1-\alpha)^{-1}\sum_{s\in S} P_s\mathcal{T}_s \tag{19}$$

subject to

(1) Part type-to-supplier assignment constraints: (2)–(4)
(2) Order-to-period assignment constraints: (5) and (6)
(3) Suppliers capacity constraints: (7)
(4) Producer capacity constraints: (8)
(5) Risk constraints:

- the tail cost for scenario s is defined as the nonnegative amount by which cost in scenario s exceeds VaR,

$$\begin{aligned}\mathcal{T}_s \geq & \sum_{i\in I} g_i u_i/B + \sum_{k\in K}\sum_{i\in I_k\bigcap I^s} A_k p_{ik} y_{ik}/B \\ & + \sum_{j\in J}\sum_{t\in T: t>d_j} e_j b_j (t-d_j) x^s_{jt}/B + \sum_{j\in J} f_j b_j\left(1-\sum_{t\in T} x^s_{jt}\right)\Big/ B - VaR;\ s\in S\end{aligned} \tag{20}$$

(6) Non-negativity and integrality conditions: (9)–(11) and

$$\mathcal{T}_s \geq 0; \qquad s\in S. \tag{21}$$

Model CVm: *Risk-averse supplier selection and customer order scheduling to minimize*

CV*aR:* **m***ultiple sourcing*

Minimize Expected Worst-Case Cost per Product (CVaR): (19)
subject to

(1) Part type demand allocation constraints: (13)–(15)
(2) Order-to-period assignment constraints: (5) and (16)
(3) Suppliers capacity constraints: (17)
(4) Producer capacity constraints: (8)
(5) Risk constraints:

- the tail cost for scenario s is defined as the nonnegative amount by which cost in scenario s exceeds VaR,

$$\mathcal{T}_s \geq \sum_{i\in I} g_i u_i/B + \sum_{k\in K}\sum_{i\in I_k\bigcap I^s} A_k p_{ik} z_{ik}/B + \sum_{j\in J}\sum_{t\in T:t>d_j} e_j b_j (t-d_j) x^s_{jt}/B + \sum_{j\in J} f_j b_j \left(1-\sum_{t\in T} x^s_{jt}\right)\Big/ B - VaR;\ s\in S \tag{22}$$

(6) Non-negativity and integrality conditions: (9), (10), (18), (21).

The solution to supplier selection and customer order scheduling problem determines an optimal supply portfolio, that is, an m-dimensional vector of optimal allocation of total demand for parts among the suppliers, in which each ith component is the fraction of total demand for parts ordered from supplier $i \in I$, $\sum_{k\in K} A_k y_{ik}/A$ – for a single sourcing and $\sum_{k\in K} A_k z_{ik}/A$ – for a multiple sourcing. Simultaneously, for each disruption scenario $s \in S$, aggregate production schedule $\{\sum_{j\in J} b_j x^s_{jt};\ t\in T\}$ and the corresponding fill rate, $(\sum_{j\in J}\sum_{t\in T} x^s_{jt}/n)100\%$, (percent of fulfilled customer order) are found.

In addition, for each disruption scenario $s \in S$, the subset of customer orders accepted with the requested due dates d_j, $\{j \in J : 1 + \min_{i\in I}\sigma_i \leq \sum_{t\in T} t x^s_{jt} \leq d_j\}$, the subset of delayed customer orders, $\{j \in J : \sum_{t\in T} t x^s_{jt} > d_j\}$, and the subset of rejected customer orders, $\{j \in J : \sum_{t\in T} x^s_{jt} = 0\}$, are determined.

3.3 *Cutting constraints*

The proposed MIP models can be further strengthened by the addition of some cutting constraints. Examples of such cuts, that consider schedulability of customer orders, are presented below.

Customer order schedulability constraints: single sourcing

- each customer order j can be scheduled during the planning horizon under disruption scenario s only, if every required part type $k \in K_j$ is ordered from a non-disrupted supplier, capable of manufacturing this part type,

$$\sum_{t\in T} x^s_{jt} \leq \sum_{i\in I_k\bigcap I^s}^{H} y_{ik};\qquad j\in J, k\in K_j, s\in S, \tag{23}$$

where $K_j = \{k \in K : a_{jk} \geq 1\}$ is the subset of part types required for customer orders $j \in J$.

Customer orders schedulability constraints: multiple sourcing

- for each part type k and disruption scenario s, the fraction of total demand for part type k used by customer orders scheduled during the planning horizon cannot be greater than the fraction of total demand for this part type supplied by non-disrupted suppliers, capable of manufacturing part type k,

$$\sum_{j\in J_k} \lambda_{jk}\left(\sum_{t\in T} x^s_{jt}\right) \leq \sum_{i\in I_k\bigcap I^s} z_{ik};\qquad k\in K, s\in S, \tag{24}$$

where $\lambda_{jk} = a_{jk} b_j / A_k$, $j \in J_k$, $k \in K$, $\left(\sum_{j\in J_k}\lambda_{jk} = 1, k\in K\right)$.

Constraints (23) and (24) can be added ahead of time to reduce computational time required to find proven optimal solutions of the corresponding MIP models for single and multiple sourcing.

4. Computational examples

In this section some computational examples are presented to illustrate possible applications of the proposed mixed integer programming approach for the selection of suppliers, order quantity allocation and customer orders scheduling in a supply chain with disruption risks. The following parameters have been used for the example problems:

- m, the number of suppliers, was equal to 10 and the number of disruption scenarios, was equal to the total number of all potential scenarios $r = 2^m = 1024$;
- n, the number of customer orders, was equal to 25;
- q, the number of part types, was equal to 50;

- h, the number of planning periods, was equal to 7 (days);
- a_{jk}, the unit requirements for part types of products in customer orders were integers in {0, 1, 2, 3} drawn from int(U[0;3]) distribution, for all orders j and part types k;
- b_j, the size of customer orders (required numbers of products), were integers in {500, 1000, 1500, 2000, 2500, 3000, 3500, 4000, 4500, 5000} drawn from 500int(U[1;10]) distribution, for all customer orders j;
- c_{ik}, the unit capacity consumptions of suppliers, were integers in {1, 2, 3} drawn from int(U[1;3]) distribution, for all part types k and suppliers $i \in I_k$;
- c^j, the unit capacity consumptions of producer, were integers in {1, 2, 3} drawn from int(U[1;3]) distribution, for all customer orders j;
- C_i, the capacity of each supplier i, was integer drawn from $1000\lceil(\sum_{k\in K} A_k c_{ik})U[0.75; 1.25]/1000\rceil$ distribution ($\lceil\cdot\rceil$ denotes the smallest integer not less than $\cdot$), i.e. each supplier capacity was from 75% to 125% of the total capacity required to manufacture all required parts;
- C^t, the capacity of producer in each period t, was integer drawn from $1000\lceil(2\sum_{j\in J} b_j c^j/(h - \max_{i\in I}\sigma_i))$ $U[0.75; 1.25]/1000\rceil$ distribution, i.e. in each period the producer capacity was from 75% to 125% of the double capacity required to complete all customer orders during the planning horizon, after the latest delivery of parts;
- σ_i, the shipping times from supplier i, were integers in {1, 2, 3} drawn from int(U[1;3]) distribution, for all suppliers i;
- d_j, the due dates for customer orders, were integers in $\{1 + \min_{i\in I}(\sigma_i), \dots, h\}$ drawn from int(U[2;7]) distribution, for all customer orders j;
- e_j, the unit daily penalty cost of delayed customer orders, was equal to $\lceil\sum_{k\in K} a_{jk} \max_{i\in I_k}(p_{ik})/350\rceil$ for all orders j, i.e. was approximately 0.28% of the maximum unit price of required parts;
- f_j, the unit penalty cost of unfulfilled customer orders, was equal to $\lceil\sum_{k\in K} a_{jk} \max_{i\in I_k}(p_{ik})/10\rceil$ for all orders j, i.e. was approximately 10% of the maximum unit price of required parts;
- g_i, the cost of ordering parts, were integers in {5000, 6000, 7000, 8000, 9000, 10,000} drawn from 1000int(U[5;10]) distribution, for all suppliers i;
- p_{ik}, the unit price (including shipping cost) of part type k purchased and transported from each supplier i, was uniformly distributed over [10,15], i.e. drawn from U[10;15];
- α, the confidence level, was equal to 0.50, 0.75, 0.90, 0.95 or 0.99.
- ν, the minimum order size, was equal to 500;
- π_i, the local disruption probability was uniformly distributed over [0,0.05] or over [0.05,0.15], i.e. the disruption probabilities were drawn independently from U[0;0.05] or from U[0.05;0.15]. The global disruption probability π^*, was either 0.005 or 0.01, respectively.

The computational experiments were performed for the same replication of the above input data set. For all test examples, the resulting total demand for parts and products is $A = 4{,}998{,}000$ and $B = 66{,}000$, respectively.

The solution results for the two different ranges of disruption probability ($\pi_i \in [0, 0.05], i \in I, \pi^* = 0.005$ and $\pi_i \in [0.05, 0.15], i \in I, \pi^* = 0.01$) are presented in Tables 2–4, respectively for the risk-neutral models **Es**, **Em** and the risk-averse models **CVs**, **CVm** with different confidence levels. The confidence level α is set at five levels of 0.5, 0.75, 0.90, 0.95, and 0.99, which means that focus is on minimizing the highest 50%, 25%, 10%, 5%, and 1% of all scenario outcomes, i.e. costs per product. The size of the corresponding mixed integer programs (with cutting constraints (23) and (24) included) is represented by the total number of variables, Var., number of binary variables, Bin., number of constraints, Cons, and number of nonzero coefficients in the constraint matrix, Nonz. In addition to the optimal solution values, Tables 2–4 present the number of selected suppliers ($\sum_{i\in I} u_i$). Tables 3 and 4 also present the expected percent of fulfilled customer orders ($\sum_{s\in S} P_s \sum_{j\in J}\sum_{t\in T} x^s_{jt}/n$)100% as well as the expected cost, associated with the optimal risk-averse supply portfolio.

A general comparison of the corresponding results for single and multiple sourcing indicates that for both risk-neutral and risk-averse solutions, for a multiple sourcing greater number of suppliers is selected and lower costs are achieved. (Note that total costs can be calculated multiplying the corresponding costs per product by the total number of products, $B = 66{,}000$.) Furthermore, for the risk-averse models with different confidence levels α, the corresponding optimal solution values are very close to each other, in particular for a multiple sourcing, which indicates that worst-case costs per product are not varying much with α. The latter results are mainly due to the parameter settings for which the purchasing costs of parts dominate the penalty costs of unfulfilled customer orders due to disrupted supplies of parts. The maximum unit cost of parts required for each product in customer order j, $\sum_{k\in K} a_{jk} \max_{i\in I_k}(p_{ik})$, is approximately ten times greater than the unit penalty cost for unfulfilled customer order j, $f_j = \lceil\sum_{k\in K} a_{jk} \max_{i\in I_k}(p_{ik})/10\rceil$.

Comparison of the corresponding risk-averse solutions for different range of disruption probability demonstrates that for a higher disruption probability (for all unreliable suppliers with $\pi_i \in [0.05, 0.15], i \in I$ and $\pi^* = 0.01$), a smaller

Table 2. Solution results for the risk neutral models.

Disruption probability	$\pi^* = 0.005$, $\pi_i \in [0, 0.05]$, $i \in I$	$\pi^* = 0.01$, $\pi_i \in [0.05, 0.15]$, $i \in I$
Model **Es** (single sourcing): Var. = 144,410, Bin. = 144,410, Cons. = 1,281,321, Nonz. = 29,824,680[a]		
Expected cost	770.935	743.606
No. of suppliers selected	8	4
Expected percent of fulfilled orders[b]	82.96%	63.09%
CPU[c]	463	12,148
Model **Em** (multiple sourcing): Var. = 144,410, Bin. = 143,910, Cons. = 319,691, Nonz. = 19,600,730[a]		
Expected cost	770.263	737.842
No. of suppliers selected	10	10
Expected percent of fulfilled orders[b]	84.21%	59.88%
CPU[c]	2364	11762

[a] Var. = number of variables, Bin. = number of binary variables, Cons. = number of constraints, Nonz. = number of nonzero coefficients.
[b] $(\sum_{s\in S} P_s \sum_{j\in J} \sum_{t\in T} x^s_{jt}/n)100\%$.
[c] CPU seconds for proving optimality on a MacBookPro6.2, Intel Core i7, 2.66 GHz, RAM 8 GB/Gurobi 5.0.0.

Table 3. Solutions results for the risk-averse model **CVs** (single sourcing).

Confidence level α	0.50	0.75	0.90	0.95	0.99
	Var. = 145,435, Bin. = 144,410, Cons. = 1,282,345, Nonz. = 30,236,868[a]				
	$\pi^* = 0.005$, $\pi_i \in U[0; 0.05]$				
CVaR	779.479	780.193	780.871	780.885	781.000
VaR	778.258	779.523	780.856	780.856	780.856
Expected cost	773.875	774.421	775.736	775.736	775.736
No. of suppliers selected	8	7	7	7	7
Expected percent of fulfilled orders[b]	82.90%	83.32%	83.32%	83.32%	83.32%
CPU[c]	451	496	853	812	2451
	$\pi^* = 0.01$, $\pi_i \in U[0.05; 0.15]$				
CVaR	780.311	780.890	780.940	781.025	781.121
VaR	779.523	780.856	780.856	780.856	781.121
Expected cost	752.757	753.382	753.382	753.382	753.398
No. of suppliers selected	7	7	7	7	7
Expected percent of fulfilled orders[b]	47.26%	47.26%	47.26%	47.26%	47.26%
CPU[c]	1099	849	1606	1224	1247

[a] Var. = number of variables, Bin. = number of binary variables, Cons. = number of constraints, Nonz. = number of nonzero coefficients.
[b] $(\sum_{s\in S} P_s \sum_{j\in J} \sum_{t\in T} x^s_{jt}/n)100\%$.
[c] CPU seconds for proving optimality on a MacBookPro6.2, Intel Core i7, 2.66 GHz, RAM 8 GB/Gurobi 5.0.0.

expected percent of fulfilled customer orders is achieved, while the expected worst-case costs are similar. In contrast, for the risk-neutral solutions and all unreliable suppliers, the expected costs are smaller. For a higher disruption probability the expected number of parts undelivered due to supply disruption is greater, which leads to smaller expected purchasing costs (note that the producer is not charged with ordered and undelivered parts), and hence to smaller total expected costs, since the purchasing costs dominate the penalty costs of unfulfilled customer orders.

The solution results for all unreliable suppliers with disruption probabilities $\pi_i,\ i \in I$ drawn from U[0.05;0.15], and global disruption probability $\pi^* = 0.01$, are additionally illustrated in Figures 1–4. The optimal risk-neutral and risk-averse (for $\alpha = 0.9$) supply portfolios are presented in Figure 1. In addition, Figure 1 shows average price per part, $\sum_{k\in K} p_{ik}/q$,

Table 4. Solutions results for the risk-averse model **CVm** (multiple sourcing).

Confidence level α	0.50	0.75	0.90	0.95	0.99
	Var. = 145,435, Bin. = 143,910, Cons. = 371,865, Nonz. = 25,679,558[a]				
	$\pi^* = 0.005, \pi_i \in U[0; 0.05]$				
CVaR	778.601	778.611	778.630	778.658	778.788
VaR	778.576	778.598	778.596	778.602	778.788
Expected cost	773.973	773.954	773.970	773.975	774.173
No. of suppliers selected	9	9	9	9	9
Expected percent of fulfilled orders[b]	83.64%	83.76%	83.70%	83.88%	83.83%
CPU[c]	2029	2376	2304	6376	3481
	$\pi^* = 0.01, \pi_i \in U[0.05; 0.15]$				
CVaR	777.716	778.789	778.794	778.795	778.795
VaR	767.600	778.788	778.794	778.795	778.795
Expected cost	748.791	752.943	752.973	752.776	752.890
No. of suppliers selected	10	9	9	9	9
Expected percent of fulfilled orders[b]	52.62%	48.35%	47.53%	47.29%	48.07%
CPU[c]	5262	8536	5421	6819	5387

[a]Var. = number of variables, Bin. = number of binary variables, Cons. = number of constraints, Nonz. = number of nonzero coefficients.
[b]$(\sum_{s\in S} P_s \sum_{j\in J} \sum_{t\in T} x^s_{jt}/n)100\%$.
[c]CPU seconds for proving optimality on a MacBookPro6.2, Intel Core i7, 2.66 GHz, RAM 8 GB/Gurobi 5.0.0.

and disruption probability, $\pi^* + (1-\pi^*)\pi_i$, for each supplier $i \in I$. The results shown in Figure 1 indicate that the number of selected suppliers for a single sourcing is smaller than that for a multiple sourcing, for both the risk-neutral and the risk-averse portfolio. The suppliers with the highest disruption probability (e.g. supplier 5) or with the highest average price (e.g. supplier 8) are allotted the lowest fraction of total demand for parts or are not selected at all. Furthermore, a single sourcing leads to a better leveled supply portfolio, with the total demand for parts more equally allocated among the selected suppliers than for a multiple sourcing.

Figure 2 presents aggregated demand pattern for products, $\sum_{j\in J:d_j=t} b_j, t \in T$, and expected aggregated production schedule, $\sum_{s\in S} P_s \sum_{j\in J} b_j x^s_{jt}, t \in T$ for the optimal risk-neutral and risk-averse supply portfolios and customer orders schedules. The aggregated schedules are similarly shaped for both single and multiple sourcing and for both risk-neutral and risk-averse portfolio. However, the expected production scheduled in each period is smaller for the risk-averse portfolio, while the expected unfulfilled demand is greater. The above comparison clearly indicates that risk-neutral solutions virtually neglect low-probability and high-cost outcomes, which leads to better average performance of a supply chain. However for the risk-neutral solutions, worst-case performance may be extremely bad. In contrast, the risk-averse solutions that account for low-probability, high-cost outcomes may lead to worse average performance (cf. the optimal expected cost in Table 2 vs. expected cost associated with the corresponding risk-averse solutions in Tables 3 and 4).

For the optimal risk-averse portfolio with $\alpha = 0.9$, Figure 3 shows the distribution of customer orders fulfillment. For both single and multiple sourcing the highest probability is concentrated at 0% (no customer order fulfilled) and at 100% (all customer orders fulfilled). For the single sourcing, however, all customer orders are either fulfilled or rejected, with a similar probability for both the events, whereas for the multiple sourcing the distribution is spread over the entire range of orders fulfillment percentage. The last results are supported by the cost distribution for the optimal risk-averse supply portfolio, presented in Figure 4, which indicates that for the multiple sourcing, cost per product is more distributed over a range of cost outcomes. For the multiple sourcing, a large probability atom, 0.79, is concentrated at the highest cost, 778.788. Similarly, for the single sourcing, large probability atoms 0.30 and 0.55, are concentrated at the highest costs, 780.856 and 781.121, respectively. In addition, Figure 4 shows a large probability atom, 0.01, at the lowest cost. For both single and multiple sourcing, the lowest cost per product, (respectively 113.061 and 113.303), is achieved under global disruption scenario, with probability 0.01, and is the sum of fixed part ordering cost and penalty cost of unfulfilled customer orders, $\sum_{i\in I} g_i u_i/B + \sum_{j\in J} f_j b_j/B$, with no purchasing cost, as all supplies are disrupted. The above results are mainly due to the parameter settings. (Note that the producer is not charged with ordered and undelivered parts and the models

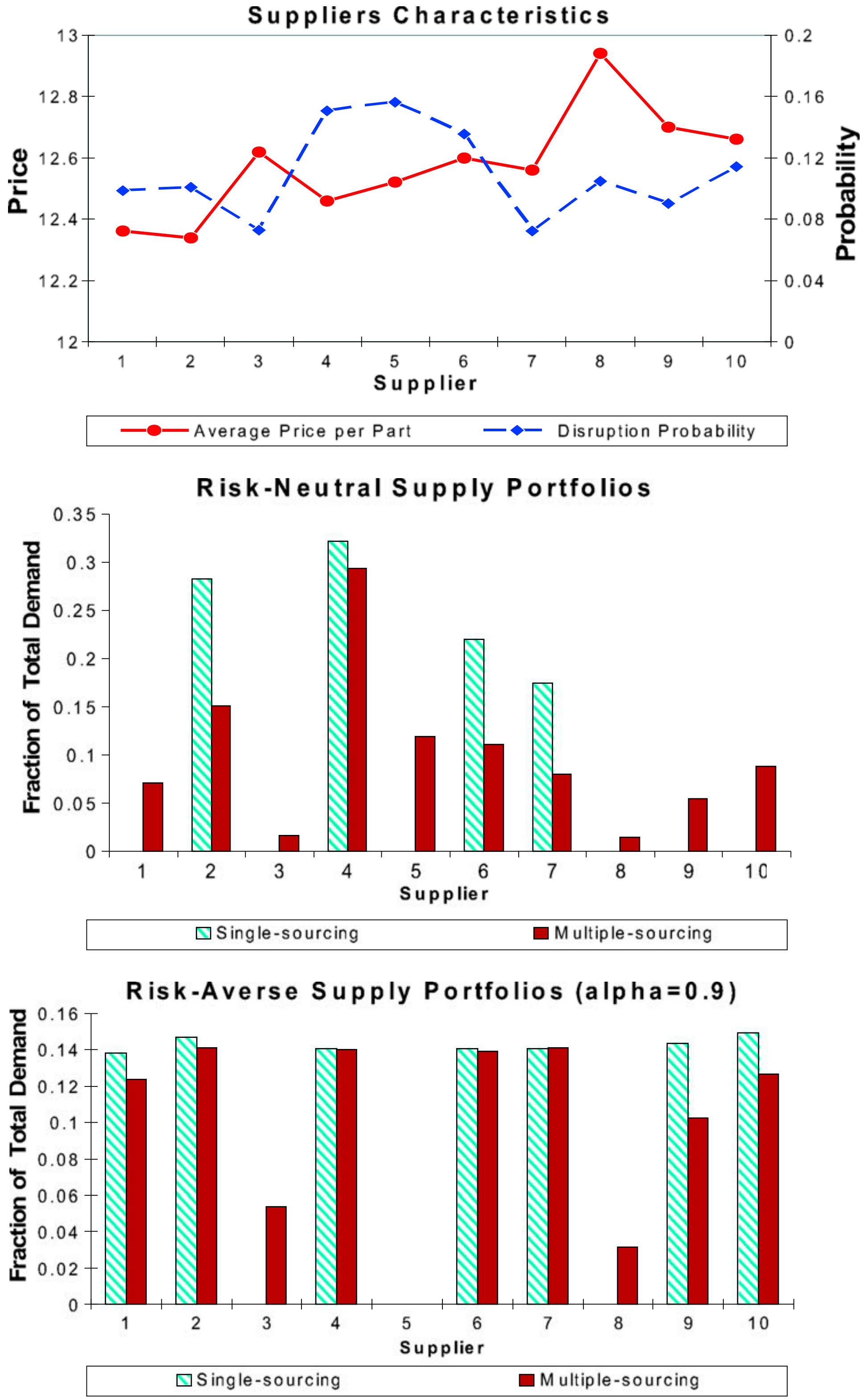

Figure 1. Supply portfolios for disruption probability $\pi^* = 0.01$, $\pi_i \in [0.05, 0.15]$, $i \in I$.

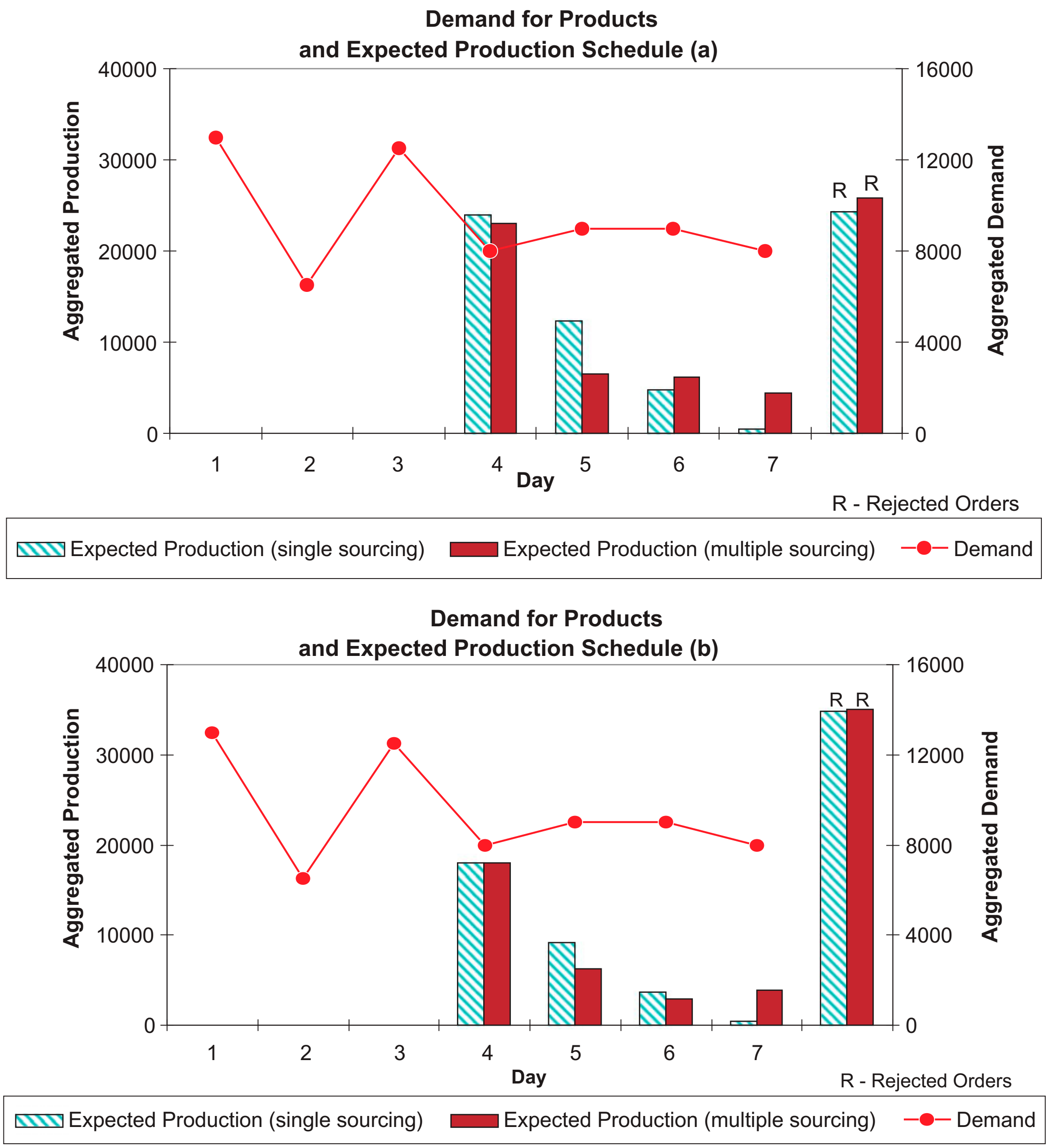

Figure 2. Expected production schedule for $\pi^* = 0.01$, $\pi_i \in [0.05, 0.15]$, $i \in I$: (a) risk-neutral portfolio, (b) risk-averse portfolio for $\alpha = 0.9$.

proposed do not explicitly account for the lost profits of unfulfilled customer orders due to supply disruptions.) If the unit penalty costs of unfulfilled customer orders (f_j) would dominate the corresponding maximum unit prices of required parts ($\sum_{k \in K} a_{jk} \max_{i \in I_k}(p_{ik})$), then instead of the lowest cost, the global disruption scenario would result in the highest cost outcome with probability $\pi^* + (1 - \pi^*) \sum_{i \in I} \pi_i$.

The computational experiments were performed using the AMPL programming language and the Gurobi 5.0.0 solver on a laptop MacBookPro 6.2 with Intel Core i7 processor running at 2.66 GHz and with 8GB RAM. The Gurobi solver was capable of finding proven optimal solutions for all examples with CPU time ranging from several to several thousands seconds. The longest CPU time was required to prove optimality of the risk-neutral solutions for all unreliable suppliers (for $\pi_i \in [0.05, 0.15]$, $i \in I$ and $\pi^* = 0.01$). For a higher disruption probability, the fraction of unfulfilled customer orders and the corresponding expected penalty cost are increasing, and the latter cost becomes an important part of the total expected cost to be minimized in the optimal risk-neutral solution.

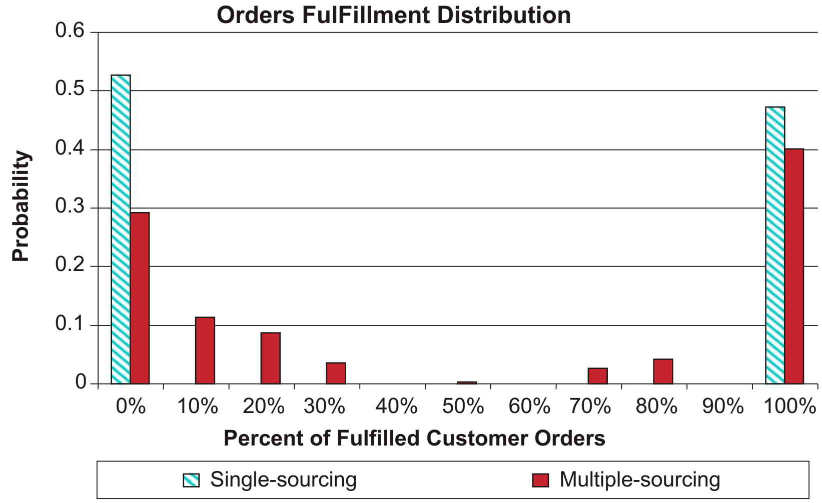

Figure 3. Distribution of customer orders fulfillment for $\alpha = 0.9$, $\pi^* = 0.01$, $\pi_i \in [0.05, 0.15]$, $i \in I$.

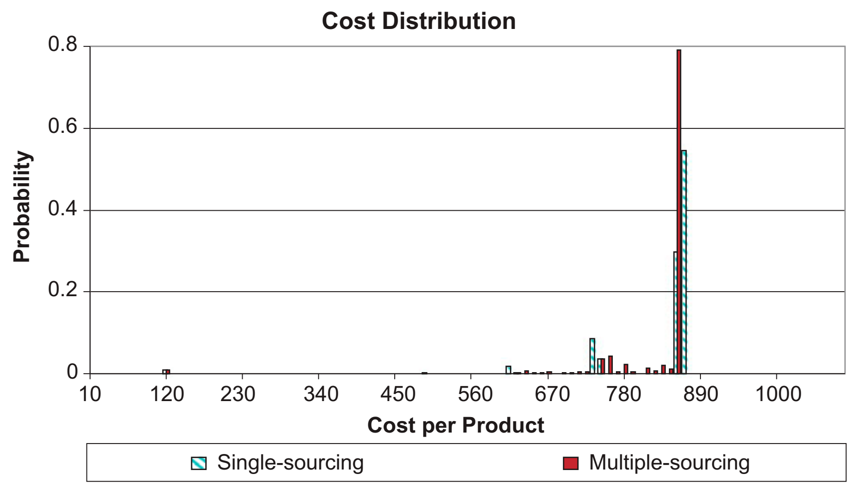

Figure 4. Cost distribution for $\alpha = 0.9$, $\pi^* = 0.01$, $\pi_i \in [0.05, 0.15]$, $i \in I$.

5. Conclusion

The integrated selection of supply portfolio and assignment of customer orders to planning periods under disruption risks is a computationally difficult stochastic combinatorial optimization problem. The proposed mixed integer programming approach with conditional value-at-risk as a risk measure allows the decision maker for coordinating the flows of parts from suppliers to producer and the flows of finished products to customers. Moreover, the selection ahead of time of the optimal risk-averse supply portfolio and the optimal schedule of customer orders for each disruption scenario, reshapes the cost distribution in such a way that the worst-case cost probability is significantly reduced.

The computational experiments indicate that the suppliers with high disruption probability or with high prices are allotted the lowest fractions of the total demand for parts or are not selected at all. The computational results also demonstrate that when the purchasing cost dominates the shortage cost of parts, the optimal number of selected suppliers remains constant over a range of confidence level α and the worst-case costs that are mainly based on the purchasing costs, are nearly independent on α. In contrast, when the shortage cost of parts dominates the purchasing cost, the worst-case cost outcomes occur under global disruption scenario and the optimal number of selected suppliers usually increases with the confidence level α (e.g. Sawik 2011a) to mitigate the impact of disruption risks by diversification of supply portfolio.

Comparison of single and multiple sourcing strategy indicates that for both the risk-neutral and the risk-averse portfolio the number of selected suppliers for single sourcing is smaller than the corresponding number for multiple sourcing. In addition, single sourcing leads to a better leveled supply portfolio, with the total demand for parts more equally allocated among the selected suppliers than for multiple sourcing. Furthermore, the expected optimal production schedules are similarly shaped for both single and multiple sourcing and for both risk-neutral and risk-averse portfolio. However, the expected production scheduled in each period is smaller for the risk-averse portfolio, while the expected unfulfilled demand is greater, which clearly indicates that risk-neutral solutions virtually neglect low-probability and high-cost outcomes.

For the limited number of scenarios considered, the proven optimal solution can be found, using the Gurobi solver for mixed integer programming. However, in the proposed models the number of scheduling variables x_{jt}^{s} is $O(hnr)$ and the number of constraints is $O(hqr)$ ((6), (16)) or $O(nqr)$ (cutting constraints (23) for a single sourcing), i.e. they grow linearly in the number r of disruption scenarios and hence exponentially in the number m of suppliers, if all $r = 2^m$ potential scenarios are considered.

In the proposed models the suppliers have different limited capacity and differ in price of offered parts and transportation time, while the quality of supplied parts is not considered. However, the models can be easily enhanced to account for the suppliers defect rates for each part type, e.g. Sawik (2011a). In addition, quantity discounts offered by the suppliers for the purchased parts may be considered, e.g. Kang and Lee (2013).

The supply chain management focuses on a variety of different optimality criteria. In view of the global competition and flow disruption risks, how to best schedule the flows of supply and demand, with reduced cost, improved responsiveness, and higher customer service level, becomes a crucial issue. Cost is the most commonly used criterion for a global supply chain performance. Responsiveness is regarded as an important performance metric of a supply chain in a rapid changing market environment. A responsive supply chain, however, usually has a higher cost, while a cost-efficient supply chain often operates at the expense of market responsiveness. Another fundamental characteristic determining the performance of a global supply chain is customer service level (e.g. Sawik forthcoming), which measures the percentage of customer demand satisfied on time. A low customer service level may cause lost sales or lost customers, which result in profit loss for the whole supply chain. The above three performance metrics are basic to the control and optimization of flows in global supply chains and the decision makers often do not have preference to any objective, i.e. all the objectives are equally important. The future research should concentrate on a multiple objective coordinating the flows of supply and demand over a supply chain network, with different types of supply chain risks and mitigation measures considered (e.g. Micheli, Mogre, and Perego forthcoming) and the inventory holding and transportation costs introduced. The future research should also focus on finding valid inequalities to strengthen the proposed mixed integer programs and on the enhancement of the proposed disruption scenarios for a semi-global disruption risks associated with different geographical regions that may result in disruption of all suppliers in the same region, simultaneously.

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