CS1MA20 Autumn Assignment 2021

Module Code: CS1MA20

Assignment report Title: Autumn Assignment 2021

Student Number: 30021591

Date: 12/11/2021

Hrs spent for the assignment: 35

Assignment evaluation:

Question 1. Matrix Calculation

A

$$\begin{bmatrix} 1 & 9 & 7 \\ 9 & 5 & 9 \end{bmatrix}$$

B

$$\begin{bmatrix} 9 & 5 & 10 \\ 1 & 9 & 8 \end{bmatrix}$$

C

$$\begin{bmatrix} 8 & 7 \\ 10 & 10 \\ 1 & 9 \end{bmatrix}$$

1. Matrix Calculation : 30021591

This is to be done by hand, though you can use MatLab to check
Calculate 1A + 10B {2 marks}
Calculate 9A - 5C' {2}
Calculate A x C {3}
Calculate C x B {3}

$$A = \begin{bmatrix} 1 & 9 & 7 \\ 9 & 5 & 9 \end{bmatrix} B = \begin{bmatrix} 9 & 5 & 10 \\ 1 & 9 & 8 \end{bmatrix} C = \begin{bmatrix} 8 & 7 \\ 10 & 10 \\ 1 & 9 \end{bmatrix}$$

Calculate 1A + 10B

$$1A = \begin{bmatrix} 1 & 9 & 7 \\ 9 & 5 & 9 \end{bmatrix} 10B = \begin{bmatrix} 90 & 50 & 100 \\ 10 & 90 & 80 \end{bmatrix}$$

$$1A + 10B = \begin{bmatrix} 1+90 & 9+50 & 7+100 \\ 9+10 & 5+90 & 9+80 \end{bmatrix} = \begin{bmatrix} 91 & 59 & 107 \\ 19 & 95 & 89 \end{bmatrix}$$

$$\therefore\ 1A\ +\ 10B\ =\ \begin{bmatrix} 91 & 59 & 107 \\ 19 & 95 & 89 \end{bmatrix}$$

Calculate 9A - 5C`

$$9A = \begin{bmatrix} 9 & 81 & 63 \\ 81 & 45 & 81 \end{bmatrix} 5C' = \begin{bmatrix} 40 & 50 & 5 \\ 35 & 50 & 45 \end{bmatrix}$$

$$9A - 5C' = \begin{bmatrix} 9-40 & 81-50 & 63-5 \\ 81-35 & 45-50 & 81-45 \end{bmatrix} = \begin{bmatrix} -31 & 31 & 58 \\ 46 & -5 & 36 \end{bmatrix}$$

$$\therefore\ 9A\ -\ 5C'\ =\ \begin{bmatrix} -31 & 31 & 58 \\ 46 & -5 & 36 \end{bmatrix}$$

Calculate A · C

$$AC = \begin{bmatrix} (1\cdot 8)+(9\cdot 10)+(7\cdot 1) & (1\cdot 7)+(9\cdot 10)+(7\cdot 9) \\ (5\cdot 10)+(9\cdot 1)+(9\cdot 8) & (9\cdot 7)+(5\cdot 10)+(9\cdot 9) \end{bmatrix} = \begin{bmatrix} 8+90+7 & 7+90+63 \\ 50+9+72 & 63+50+81 \end{bmatrix}$$

$$\therefore\ AC\ =\ \begin{bmatrix} 105 & 160 \\ 131 & 194 \end{bmatrix}$$

Calculate C · B

$$\begin{bmatrix} (8\cdot 9)+(7\cdot 1) & (8\cdot 5)+(7\cdot 9) & (8\cdot 10)+(7\cdot 8) \\ (10\cdot 9)+(10\cdot 1) & (10\cdot 5)+(10\cdot 9) & (10\cdot 10)+(10\cdot 8) \\ (1\cdot 9)+(9\cdot 1) & (1\cdot 5)+(9\cdot 9) & (1\cdot 10)+(9\cdot 8) \end{bmatrix} = \begin{bmatrix} 72+7 & 40+63 & 80+56 \\ 90+10 & 50+90 & 100+80 \\ 9+9 & 5+81 & 10+72 \end{bmatrix}$$

$$\therefore\ CB\ =\ \begin{bmatrix} 79 & 103 & 136 \\ 100 & 140 & 180 \\ 18 & 86 & 82 \end{bmatrix}$$

Question 2. Magic Matrix Equation Solving

A

$$\begin{bmatrix} 5 & 9 \\ 5 & 10 \end{bmatrix}$$

x

$$\begin{bmatrix} x0 \\ x1 \end{bmatrix}$$

b

$$\begin{bmatrix} 117 \\ 125 \end{bmatrix}$$

2. Magic Matrix Equation Solving : 30021591

This is to be done by hand
Find matrix M, such that M A is a diagonal matrix {2}
Use M to find x in the equation Ax = b {2}
Show that Ax does equal b {2}

$$A = \begin{bmatrix} 5 & 9 \\ 5 & 10 \end{bmatrix} x = \begin{bmatrix} x0 \\ x1 \end{bmatrix} b = \begin{bmatrix} 117 \\ 125 \end{bmatrix}$$

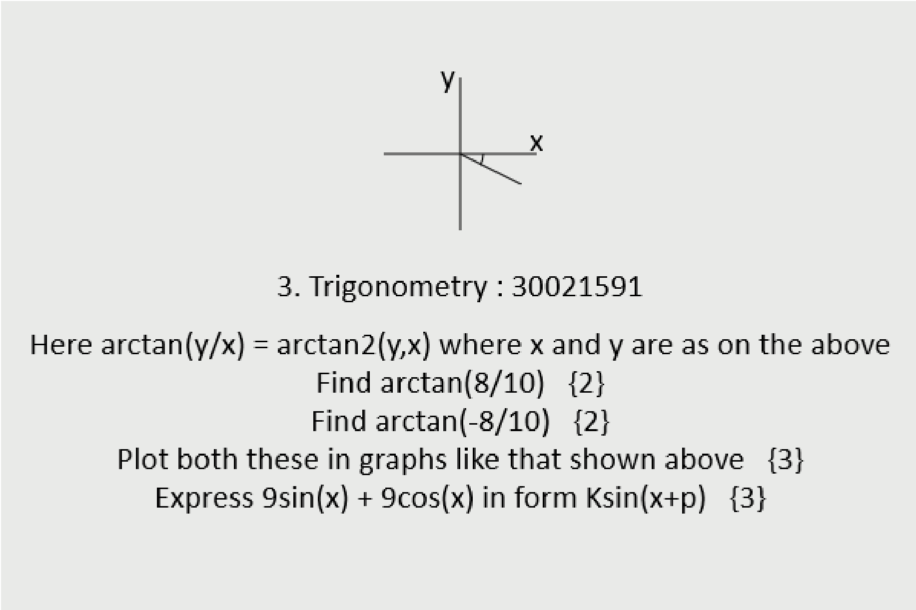
Find matrix M, such that M A is a diagonal matrix

$$MA = \begin{bmatrix} 10 & -9 \\ -5 & 5 \end{bmatrix} \cdot \begin{bmatrix} 5 & 9 \\ 5 & 10 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

Use M to find x in the equation Ax = b-

$$\begin{bmatrix} 10 & -9 \\ -5 & 5 \end{bmatrix} \cdot \begin{bmatrix} x0 \\ x1 \end{bmatrix} x = \begin{bmatrix} 5x0 \\ 5x1 \end{bmatrix} = \begin{bmatrix} 45 \\ 40 \end{bmatrix}$$

Question 3. Trigonometry



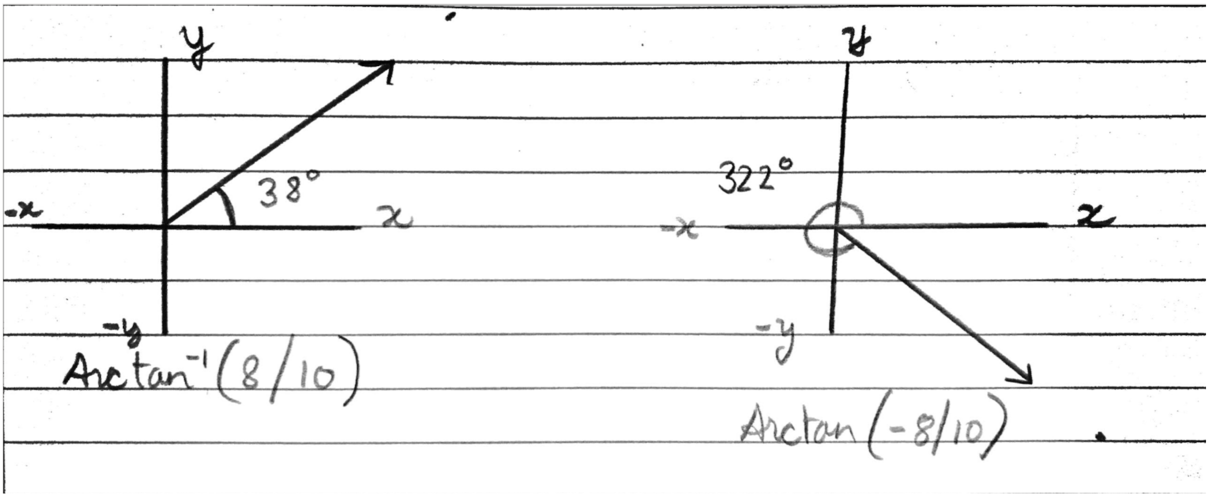
Find arctan(8/10) {2}

$$arctan = tan^{-1}(8/10) = 38.659 = 38^\circ$$

Find arctan(-8/10) {2}

$$arctan = tan^{-1}(-8/10) = 360^\circ - 38^\circ = 322^\circ$$

Plot both these in graphs like that shown above {3}



Express 9sin(x) + 9cos(x) in form Ksin(x+p) {3}

$kcos(p) = 9$ $\quad$ $ksin(p) = 9$

$tanp = 9/9$ $\quad$ $p = tan^{-1}(9/9) = 45^\circ$

$9sin(x + 45^\circ)$ $\quad$ $k = 9^2 + 9^2 = 162^\circ$

Question 4. Exp Log Hyperbolic

4. Exp Log Hyperbolic : 30021591

In MatLab create 2 by 2 subplots as follows: {4}
5 exp(-x/7) for x from 0 to 7
On log log scales from x = 1 to 10 a plot of x^5
sinh(3x/10) for x from -10 to +10
cosh(4x/9) for x from -9 to +9

In MATLAB create 2 by 2 subplots as follows: {4}

(DONE IN SUBPLOT BELLOW)

5 exp(-x/7) for x from 0 to 7

(DONE IN SUBPLOT BELLOW)

On log log scales from x = 1 to 10 a plot of x^5

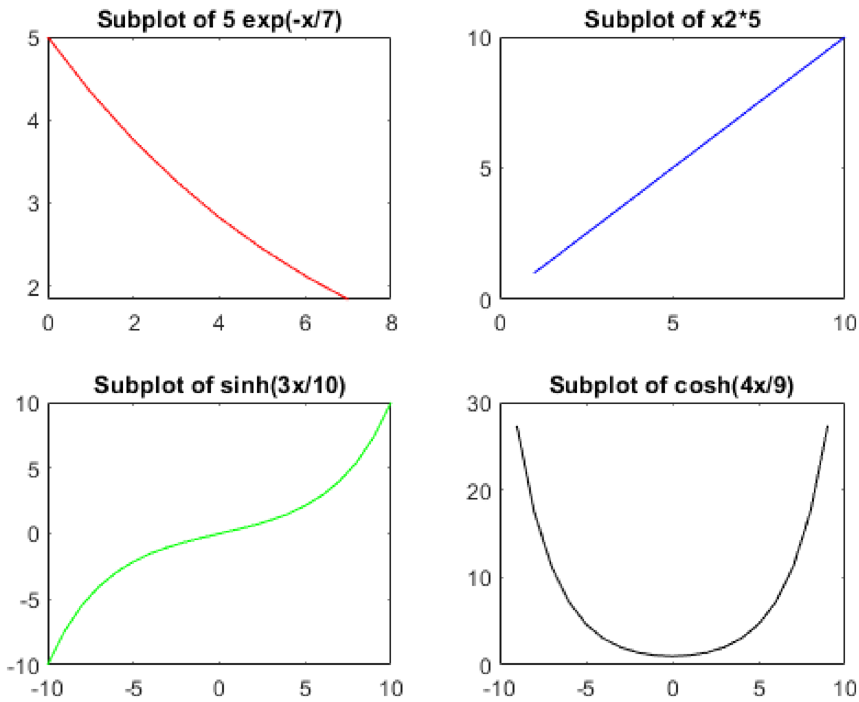
(DONE IN SUBPLOT BELLOW)

sinh(3x/10) for x from -10 to +10

(DONE IN SUBPLOT BELLOW)

cosh(4x/9) for x from -9 to +9

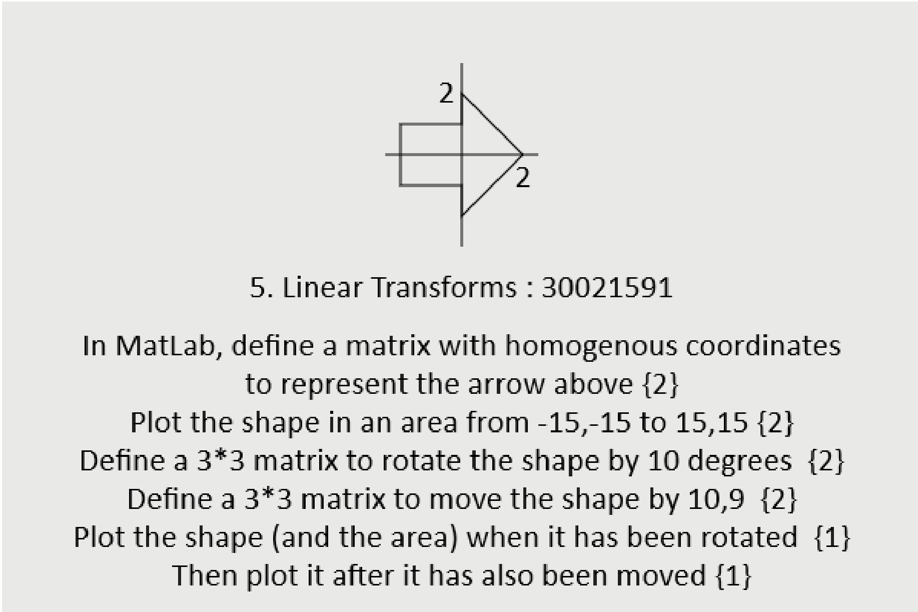
(DONE IN SUBPLOT BELLOW)




```
clc

x1 = 0:7;
x2 = 1:10;
x3 = -10:10;
x4 = -9:9;

y1 = 5 * exp(-x1/7);
y2 = x2*5;
y3 = sinh(3 * x3/10);
y4 = cosh(4 * x4 / 9);

subplot(2,2,1);
plot(x1, y1, "r");
title("Subplot of 5 exp(-x/7)")

subplot(2,2,2);
plot(x2, x2, "b");
title("Subplot of x2*5")

subplot(2,2,3);
plot(x3, y3, "g");
title("Subplot of sinh(3x/10)")

subplot(2,2,4);
```

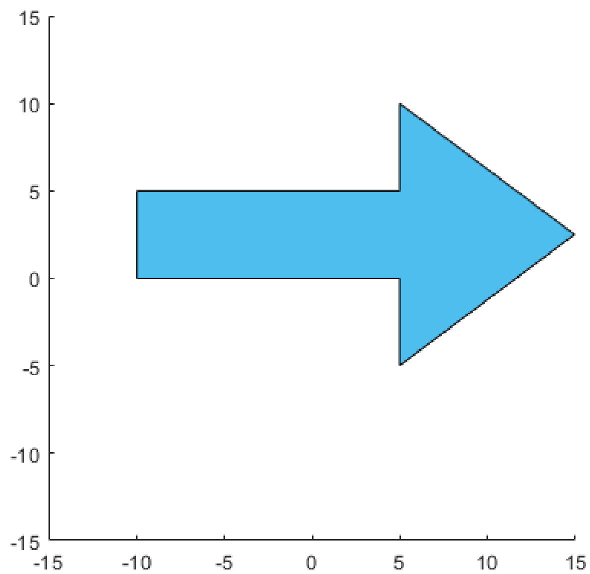
```
plot(x4, y4, "k");
title("Subplot of cosh(4x/9)")
```

Question 5. Linear Transforms



In MatLab, define a matrix with homogenous coordinates to represent the arrow above {2}

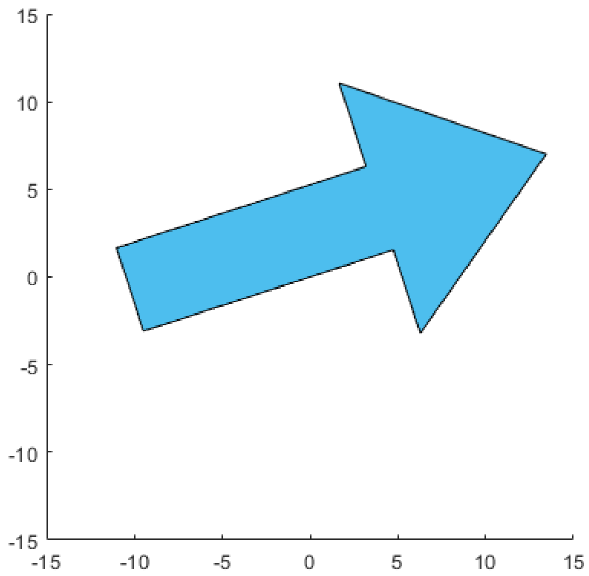
Plot the shape in an area from -15,-15 to 15,15 {2}




```
clc
clf
pts = [ 5 -10 -10 5 5 5 15 5; ...
```

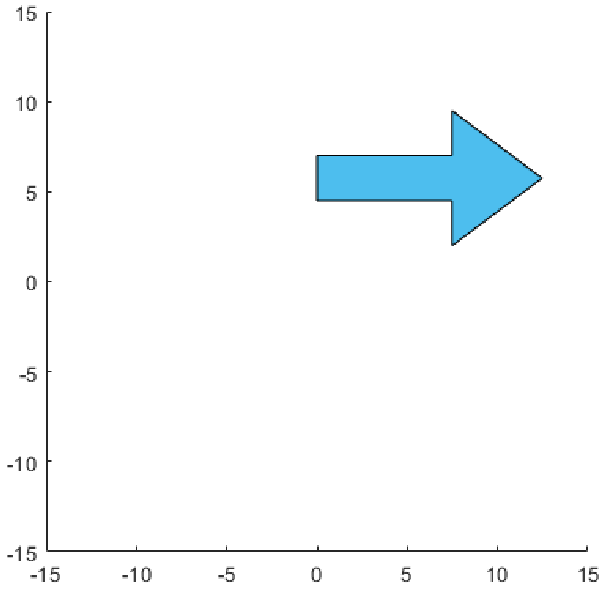
```
0  0  5  5 5 10 2.5 -5;...
1 1 1 1 1 1 1]

axis equal
xlim([-15 15])
ylim([-15 15])
h1 = patch('FaceColor',[0.3010 0.7450 0.9330]);
h1.XData = pts(1,:) ./ pts(3,:);
h1.YData = pts(2,:) ./ pts(3,:);
```

Define a 3·3 matrix to rotate the shape by 10 degrees {2}




```
mrot = [cos(pi/10), -sin(pi/10), 0; ...
sin(pi/10),  cos(pi/10), 0; ...
0,          0, 1];
```

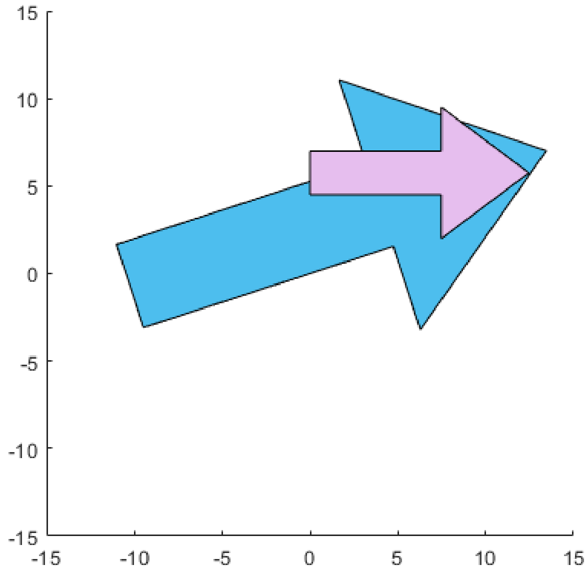
Define a 3·3 matrix to move the shape by 10,9 {2}




```
mmove = [10;9;1]
```

Plot the shape (and the area) when it has been rotated {1}

Then plot it after it has also been moved {1}



Question 6. Ax=b - 2*2 by inverse

A $\begin{bmatrix} 10 & 1 \\ 9 & 8 \end{bmatrix}$ x $\begin{bmatrix} x0 \\ x1 \end{bmatrix}$ b $\begin{bmatrix} 110 \\ 170 \end{bmatrix}$

6. Ax=b - 2*2 by inverse : 30021591

By hand do the following:
Find the determinant of A {1}
Then find the inverse of A {2}
Hence find x in the equation Ax = b {2}
Then, use MatLab to confirm your answer is correct {1}
Remember to include the MatLab commands in the report

Find the determinant of A {1}

$$det\,(A) = 10 \cdot 8 - 9 \cdot 1 = 71$$

$$Determinant of A = 71$$

Then find the inverse of A {2}

$$inv\,(A) = \begin{bmatrix} 8 & -1 \\ -9 & 10 \end{bmatrix} \cdot \frac{1}{71} = \begin{bmatrix} \frac{8}{71} & \frac{-1}{71} \\ \frac{-1}{71} & \frac{10}{71} \end{bmatrix} = \begin{bmatrix} 0.1127 & -0.0141 \\ -0.1269 & 0.1408 \end{bmatrix}$$

$$Inverse of A = \begin{bmatrix} \mathbf{0.1127} & \mathbf{-0.0141} \\ \mathbf{-0.1269} & \mathbf{0.1408} \end{bmatrix}$$

Hence find x in the equation Ax = b {2}

$$\begin{bmatrix} 0.1127 & -0.0141 \\ -0.1269 & 0.1408 \end{bmatrix} \cdot \begin{bmatrix} 10 & 1 \\ 9 & 8 \end{bmatrix} \cdot \begin{bmatrix} x0 \\ x1 \end{bmatrix} = \begin{bmatrix} 0.1127 & -0.0141 \\ -0.1269 & 0.1408 \end{bmatrix} \cdot \begin{bmatrix} 110 \\ 170 \end{bmatrix}$$

$$\begin{bmatrix} 0.1127 & -0.0141 \\ -0.1269 & 0.1408 \end{bmatrix} \cdot \begin{bmatrix} 110 \\ 170 \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \end{bmatrix}; \;\; \mathbf{x} = \begin{bmatrix} \mathbf{10} \\ \mathbf{10} \end{bmatrix}$$

$$Ax = b = \begin{bmatrix} 10 & 1 \\ 9 & 8 \end{bmatrix} \cdot \begin{bmatrix} 10 \\ 10 \end{bmatrix} = \begin{bmatrix} 110 \\ 170 \end{bmatrix}$$

Then, use MATLAB to confirm your answer is correct {1}

```
inv(A)
```

ans =

```
0.1127   -0.0141
-0.1268    0.1408
```

```
x = inv(A) * b
```

x =

```
10.0000
10.0000
```

A * x

ans =

```
110
170
```

b

b =

```
110
170
```

Question 7. Ax=b - 3*3 by inverse

$$A = \begin{bmatrix} 1 & 9 & 8 \\ 10 & 9 & 7 \\ 5 & 10 & 9 \end{bmatrix} \quad x = \begin{bmatrix} x0 \\ x1 \\ x2 \end{bmatrix} \quad b = \begin{bmatrix} 134 \\ 205 \\ 185 \end{bmatrix}$$

7. Ax=b - 3*3 by inverse : 30021591

By hand do the following:
Find the determinant of A {2}
Then find the inverse of A {3}
Hence find x in the equation Ax = b {2}
Then, use MatLab to confirm your answer is correct {1}
Remember to include the MatLab commands in the report

Find the determinant of A {2}

$$det\,(A) = 1 \cdot ((9 \cdot 9) - (10 \cdot 7)) - 9 \cdot ((10 \cdot 9) - (5 \cdot 7)) + 8 \cdot ((10 \cdot 10) - (5 \cdot 9))$$

Determinant of A = -44

Then find the inverse of A {3}

$$inv\,(A) = \begin{bmatrix} \begin{bmatrix} 9 & 7 \\ 10 & 9 \end{bmatrix} & \begin{bmatrix} 10 & 7 \\ 5 & 9 \end{bmatrix} & \begin{bmatrix} 10 & 9 \\ 5 & 10 \end{bmatrix} \\ \begin{bmatrix} 9 & 8 \\ 10 & 9 \end{bmatrix} & \begin{bmatrix} 1 & 8 \\ 5 & 9 \end{bmatrix} & \begin{bmatrix} 1 & 9 \\ 5 & 10 \end{bmatrix} \\ \begin{bmatrix} 9 & 8 \\ 9 & 7 \end{bmatrix} & \begin{bmatrix} 1 & 8 \\ 10 & 7 \end{bmatrix} & \begin{bmatrix} 1 & 9 \\ 10 & 9 \end{bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 81 - 70 & 90 - 35 & 100 - 45 \\ 81 - 80 & 9 - 40 & 10 - 45 \\ 63 - 72 & 7 - 18 & 9 - 90 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 55 & 55 \\ 1 & -31 & -35 \\ -9 & -73 & -81 \end{bmatrix} \cdot \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix} = \begin{bmatrix} 11 & -55 & 55 \\ -1 & -31 & 3 \\ -9 & 73 & 81 \end{bmatrix}$$

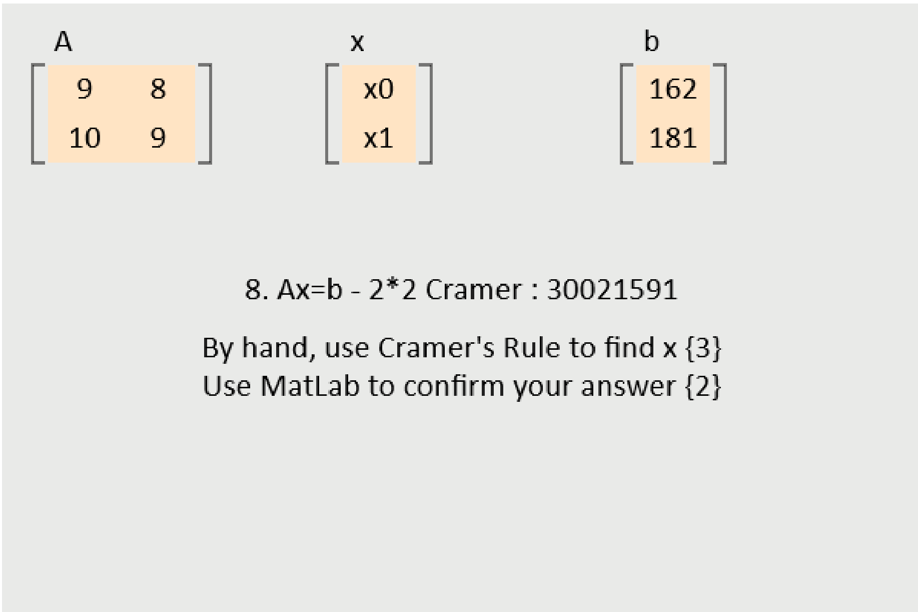
$$inv\,(A)' = \begin{bmatrix} -0.25 & 1.25 & -1.25 \\ 0.0227 & 0.7045 & -0.0681 \\ 0.2045 & -1.6590 & 1.8409 \end{bmatrix}$$

Hence find x in the equation Ax = b {2}

$$\begin{bmatrix} -0.25 & 1.25 & -1.25 \\ 0.0227 & 0.7045 & -0.0681 \\ 0.2045 & -1.6590 & 1.8409 \end{bmatrix} \cdot \begin{bmatrix} 134 \\ 205 \\ 185 \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 10 \end{bmatrix};\quad x = \begin{bmatrix} \mathbf{9} \\ \mathbf{5} \\ \mathbf{10} \end{bmatrix}$$

$$Ax = b = \begin{bmatrix} 1 & 9 & 8 \\ 10 & 9 & 7 \\ 5 & 10 & 9 \end{bmatrix} \cdot \begin{bmatrix} 9 \\ 5 \\ 10 \end{bmatrix} = \begin{bmatrix} 134 \\ 205 \\ 185 \end{bmatrix}$$

Question 8. Ax=b - 2*2 Cramer



By hand, use Cramer's Rule to find x {3}

$$det(A) = \begin{bmatrix} 9 & 8 \\ 10 & 9 \end{bmatrix} = 9 \cdot 9 - 10 \cdot 8$$
$$= 81 - 80 = 1$$

$$det(A) = \begin{bmatrix} 9 & 8 \\ 10 & 9 \end{bmatrix} = 9 \cdot 9 - 10 \cdot 8$$
$$= 81 - 80 = 1$$
$$D = 1$$

$$det(x0) = \begin{bmatrix} 162 & 8 \\ 181 & 9 \end{bmatrix} = 162 \cdot 9 - 181 \cdot 8 = 10$$

$$det(x1) = \begin{bmatrix} 9 & 162 \\ 10 & 181 \end{bmatrix} = 9 \cdot 181 - 10 \cdot 162 = 9$$

$$x1 = \frac{Dx1}{D} = \frac{9}{1} = 9$$

$$Ax = b = \begin{bmatrix} 9 & 8 \\ 10 & 9 \end{bmatrix} \cdot \begin{bmatrix} 10 \\ 9 \end{bmatrix} = \begin{bmatrix} 162 \\ 181 \end{bmatrix}$$

Use MATLAB to confirm your answer {2}

```
A = [9 8; 10 9]
```

A =

```
 9     8
10     9
```

```
b = [162;181]
```

b =

```
162
181
```

```
det(A)
```

ans =

```
  1.0000
```

```
x0 = [162 8;181 9]
```

x0 =

```
162     8
181     9
```

```
x1 = [9 162;10 181]
```

x1 =

```
 9   162
10   181
```

```
det(x0)
```

ans =

```
10.0000
```

```
det(x1)
```

ans =

```
9.0000
```

```
x=[det(x0); det(x1)]
```

x =

```
10.0000
9.0000
```

```
b
```

```
A*x
```

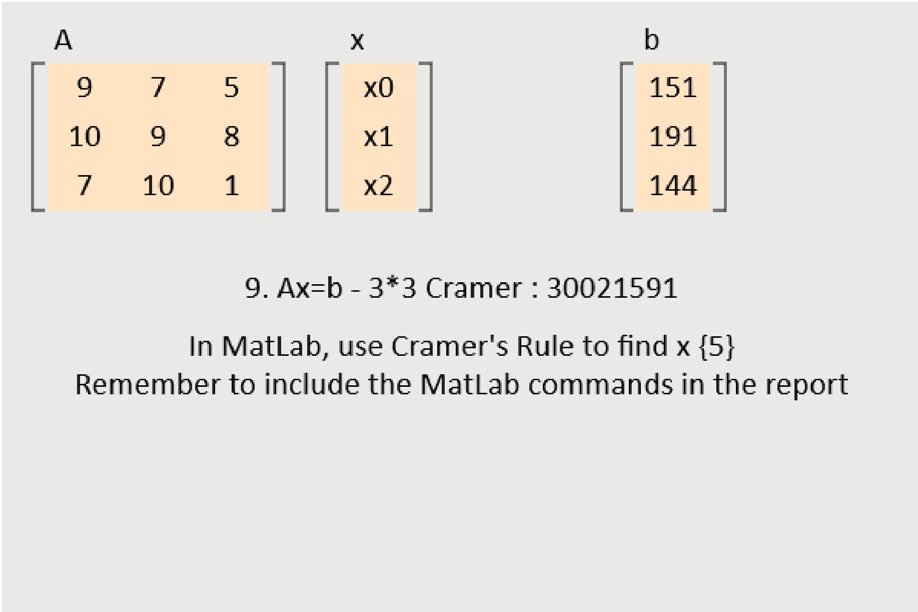
b =

```
162
181
```

ans =

```
162.0000
181.0000
```

Question 9. Ax=b - 3*3 Cramer



In MATLAB, use Cramer's Rule to find x {5}

```
A = [9, 7, 5; 10, 9, 8; 7, 10, 1];
b = [151; 191; 144];
```

```
x0 = [151, 7, 5; 191, 9, 8; 144, 10, 1];
x1 = [9, 151, 5; 10,191, 8; 7, 144, 1];
x2 = [9, 7, 151; 10, 9, 191; 7, 10, 144];
```

```
detA=det(A);
detX0 = det(x0)/detA;
detX1 = det(x1)/detA;
detX2 = det(x2)/detA;
x = [detX0; detX1; detX2];
```

A

A =

```
 9     7     5
10     9     8
 7    10     1
```

b

b =

```
151
191
144
```

x0

x0 =

```
151     7     5
191     9     8
144    10     1
```

x1

x1 =

```
  9   151     5
 10   191     8
  7   144     1
```

x2

x2 =

```
  9     7   151
 10     9   191
  7    10   144
```

detA

detA =

```
-132
```

detX0

detX0 =

```
  7
```

detX1

detX1 =

```
9.0000
```

detX2

detX2 =

```
5.0000
```

x =

```
7.0000
9.0000
5.0000
```

b

b =

```
151
191
144
```

A * x

ans =

```
151.0000
191.0000
144.0000
```

Question 10. Vectors

P1 $\begin{bmatrix} -8 \\ -9 \\ 48 \end{bmatrix}$ P2 $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ P3 $\begin{bmatrix} 6 \\ -4 \\ 5 \end{bmatrix}$

10. Vectors : 30021591

Points P1..P3 are on a plane. Find vectors P1 to P2 and P2 to P3 {2}
Hence, find the vector normal to the plane {2}
Now find the equation of the plane {2}
The plane is viewed by an eye at 4,-7,4
Find the vector from P2 to the eye, and show the plane is visible {2}
In MatLab, plot the plane, the vector P2 to the eye
and the normal from P2 of length 5 {2}

Points P1..P3 are on a plane. Find vectors P1 to P2 and P2 to P3 {2}

$$p1\vec{p}2 =< (-8-1), (-9-1), (48-0) >= -9, -10, 48$$

$$p2\vec{p}3 =< (1-6, (1-(-4)), (0-5) >= -5, 5, -5$$

Hence, find the vector normal to the plane {2}

$$\begin{vmatrix} i & j & k \\ -9 & -10 & 48 \\ -5 & 5 & -5 \end{vmatrix}$$

$$\begin{vmatrix} -10 & 48 \\ 5 & -5 \end{vmatrix} i - \begin{vmatrix} -9 & 48 \\ -5 & -5 \end{vmatrix} j + \begin{vmatrix} -9 & -10 \\ -5 & 5 \end{vmatrix} k$$

$$(-10 \times -5) - (5 \times 48)i - (-9 \times -5) - (-5 \times 48)j + (-9 \times 5) - (-5 \times -10)k$$

$$\begin{gathered} = (50 - 240)i - (45(-240))J + (-45 - 50)k \\ = (-190)i - (285)j + (-95)k \\ =< -190, -285, -95 > \end{gathered}$$

Now find the equation of the plane {2}

The plane is viewed by an eye at 4,-7,4. Find the vector from P2 to the eye, and show the plane is visible {2}

In MatLab, plot the plane, the vector P2 to the eye and the normal from P2 of length 5 {2}

Question 11. Gaussian Elimination

A

$$\begin{bmatrix} 10 & 10 & 1 \\ 9 & 8 & 10 \\ 9 & 7 & 5 \end{bmatrix}$$

x

$$\begin{bmatrix} x0 \\ x1 \\ x2 \end{bmatrix}$$

b

$$\begin{bmatrix} 171 \\ 153 \\ 138 \end{bmatrix}$$

11. Gaussian Elimination : 30021591

By hand:
Form the Augmented Matrix {1}
Use Row Operations so this is in Row Echelon form {3}
Hence find x in the equation Ax = b {2}
Go to MatLab, and first confirm your answer {2}
Then use determinants to find Augmented Matrix's Rank {2}
Show all working. Is it what you expect?

Form the Augmented Matrix {1}

$$aug(A) = \begin{bmatrix} 10 & 10 & 1 & 171 \\ 9 & 8 & 10 & 153 \\ 9 & 7 & 5 & 138 \end{bmatrix}$$

Use Row Operations so this is in Row Echelon form {3}

$$\frac{1}{10} \cdot R1 \to R1 \qquad \begin{bmatrix} 1 & 1 & \frac{1}{10} & \frac{171}{10} \\ 9 & 8 & 10 & 153 \\ 9 & 7 & 5 & 138 \end{bmatrix}$$

$$1/9 \cdot R2 \to R2 \qquad \begin{bmatrix} 1 & 1 & \frac{1}{10} & \frac{171}{10} \\ 1 & \frac{8}{9} & \frac{10}{9} & 17 \\ 9 & 7 & 5 & 138 \end{bmatrix}$$

$$\frac{1}{9} \cdot R3 \to R3 \qquad \begin{bmatrix} 1 & 1 & \frac{1}{10} & \frac{171}{10} \\ 1 & \frac{8}{9} & \frac{10}{9} & 17 \\ 1 & \frac{7}{9} & \frac{5}{9} & \frac{46}{3} \end{bmatrix}$$

$$R2 - 1 \cdot R1 \to R2 \qquad \begin{bmatrix} 1 & 1 & \frac{1}{10} & \frac{171}{10} \\ 1 & \frac{8}{9} & \frac{10}{9} & 17 \\ 1 & \frac{7}{9} & \frac{5}{9} & \frac{46}{3} \end{bmatrix}$$

$$R3 - 1 \cdot R1 \to R3 \qquad \begin{bmatrix} 1 & 1 & \frac{1}{10} & \frac{171}{10} \\ 0 & -\frac{1}{9} & \frac{91}{90} & -\frac{1}{10} \\ 0 & -\frac{2}{9} & \frac{41}{90} & -\frac{53}{30} \end{bmatrix}$$

$$-9 \cdot R2 \to R2$$

$$\begin{bmatrix} 1 & 1 & \frac{1}{10} & \frac{171}{10} \\ 0 & 1 & -\frac{91}{10} & \frac{9}{10} \\ 0 & -\frac{2}{9} & \frac{41}{90} & -\frac{53}{30} \end{bmatrix}$$

$$-\frac{9}{2} \cdot R3 \to R3 \qquad \begin{bmatrix} 1 & 1 & \frac{1}{10} & \frac{171}{10} \\ 0 & 1 & -\frac{91}{10} & \frac{9}{10} \\ 0 & 1 & \frac{41}{90} & \frac{159}{20} \end{bmatrix}$$

$$R3 - 1 \cdot R2 \to R3 \qquad \begin{bmatrix} 1 & 1 & \frac{1}{10} & \frac{171}{10} \\ 0 & 1 & -\frac{91}{10} & \frac{9}{10} \\ 0 & 0 & \frac{141}{20} & \frac{141}{20} \end{bmatrix}$$

$$\frac{20}{141} \cdot R3 \to R3 \qquad \begin{bmatrix} 1 & 1 & \frac{1}{10} & \frac{171}{10} \\ 0 & 1 & -\frac{91}{10} & \frac{9}{10} \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$R2 + 91/10 \cdot R3 \to R2 \qquad \begin{bmatrix} 1 & 1 & \frac{1}{10} & \frac{171}{10} \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$R1 - 1/10 \cdot R3 \to R1 \qquad \begin{bmatrix} 1 & 1 & 0 & 17 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$R1 - 1 \cdot R2 \to R1 \qquad \begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Hence find x in the equation Ax = b {2}

$$Ax = b = \begin{bmatrix} 10 & 10 & 1 \\ 9 & 8 & 10 \\ 9 & 7 & 5 \end{bmatrix} \cdot \begin{bmatrix} 7 \\ 10 \\ 1 \end{bmatrix} = \begin{bmatrix} 171 \\ 153 \\ 138 \end{bmatrix}$$

Go to MATLAB, and first confirm your answer {2}

> A = [10 10 1; 9 8 10; 9 7 5]

A =

```
10    10     1
 9     8    10
 9     7     5
```

> b = [171 ; 153 ; 138]

b =

```
171
153
138
```

> x = mldivide(A,b)

x =

```
7.0000
10.0000
1.0000
```

b

b =

171
153
138

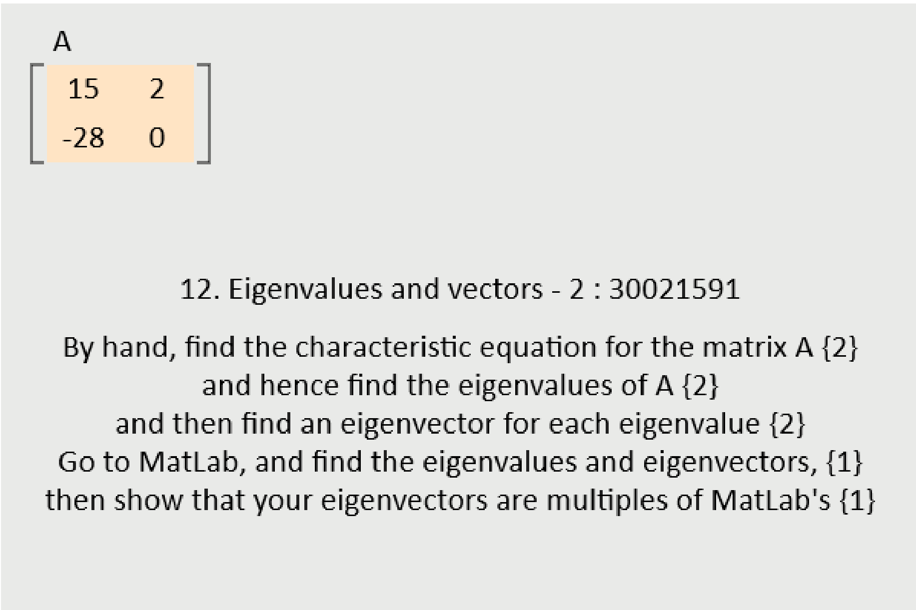
A * x

ans =

171
153
138

Then use determinants to find Augmented Matrix's Rank {2}

Question 12. Eigenvalues and vectors - 2


A

$$\begin{bmatrix} 15 & 2 \\ -28 & 0 \end{bmatrix}$$

12. Eigenvalues and vectors - 2 : 30021591

By hand, find the characteristic equation for the matrix A {2}
and hence find the eigenvalues of A {2}
and then find an eigenvector for each eigenvalue {2}
Go to MatLab, and find the eigenvalues and eigenvectors, {1}
then show that your eigenvectors are multiples of MatLab's {1}

By hand, find the characteristic equation for the matrix A {2}

$$A = \begin{bmatrix} 15 & 2 \\ -28 & 0 \end{bmatrix}$$

$$det(A - \lambda I)\vec{x} = 0$$

$$|A - \lambda 1| = 0$$

$$\begin{vmatrix} 15 - \lambda & 2 \\ -28 & 0 - \lambda \end{vmatrix} = 0$$

and hence find the eigenvalues of A {2}

$$\begin{vmatrix} 15-\lambda & 2 \\ -28 & 0-\lambda \end{vmatrix} = 0$$

$$\begin{gathered} (15-\lambda)(0-\lambda)-(28)(2)=0 \\ 15\lambda-\lambda^2-(-56)=0 \\ \lambda^2-7\lambda-8\lambda+56=0 \\ (\lambda-7)(\lambda-8)=0 \\ \lambda_1=7 \\ \lambda_2=8 \end{gathered}$$

$$\vec{x}=\begin{bmatrix} 7 \\ 8 \end{bmatrix}$$

and then find an eigenvector for each eigenvalue {2}

$$(A-\lambda I)\vec{x}=\vec{0}$$

$$A\vec{x}=\lambda\vec{x}$$

$$A(c\vec{x})=\lambda(c\vec{x})$$

$$A-7I=\begin{bmatrix} 15 & 2 \\ -28 & 0 \end{bmatrix}-\begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}=\begin{bmatrix} 8 & 2 \\ -28 & -7 \end{bmatrix}$$
$$R_2+2R_1$$

$$\frac{1}{8}R_1 \to R_1 \qquad \begin{bmatrix} 1 & \frac{1}{4} \\ -28 & -7 \end{bmatrix}$$

$$-\frac{1}{28}R_2 \to R_2 \qquad \begin{bmatrix} 1 & \frac{1}{4} \\ 1 & \frac{1}{4} \end{bmatrix}$$

$$R_2-1\cdot R_1 \to R_2 \qquad \begin{bmatrix} 1 & \frac{1}{4} \\ 0 & 0 \end{bmatrix}$$

$$1x_1+1/4x_2=0$$

$$A-8I=\begin{bmatrix} 15 & 2 \\ -28 & 0 \end{bmatrix}-\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}=\begin{bmatrix} 120 & 16 \\ -224 & 0 \end{bmatrix}$$
$$R_2+2R_1$$

$$-1224\cdot R2 \to R2 \qquad \begin{bmatrix} 1 & \frac{2}{15} \\ 1 & 0 \end{bmatrix}$$

$$R1{-}2/15\cdot R2 \to R1 \qquad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$1x_1 + 1x_2 = 0$$

Go to MATLAB, and find the eigenvalues and eigenvectors, {1}

> eig(A)

ans =

```
8
7
```

then show that your eigenvectors are multiples of MATLAB{1}

> A * x

ans =

```
134
-224
```

> A * B

ans =

```
134
-224
```

Question 13. Eigenvalues and vectors - 2

A

$$\begin{bmatrix} 2 & 0 & 1 \\ -36 & 5 & -4 \\ 2 & 1 & 11 \end{bmatrix}$$

13. Eigenvalues and vectors - 3 : 30021591

In MatLab, find the eigenvalues and eigenvectors of A {2}
By hand, find the characteristic equation of A {2}
Show that the 3 eigenvalues are roots of this {2}
In MatLab, form matrix X from the three eigenvectors and
show Inv(X) * A * X is a diagonal matrix {2}

In MATLAB, find the eigenvalues and eigenvectors of A {2}

> [X,D] = eig(A)

X =

```
-0.1231 -0.0478 0.0572
-0.9847 0.9567 -0.9147
```

D =

```
1.0000         0         0
     0    8.0000         0
```

```
0.1231 -0.2870 0.4002
```

```
     0         0    9.0000
```

> A*X(:,2)

ans =

```
-0.3827
7.6538
-2.2962
```

> D(2,2)*X(:,2)

ans =

```
-0.3827
7.6538
-2.2962
```

> norm(AX - XD)

ans =

```
5.9247e-15
```

By hand, find the characteristic equation of A {2}

$$\begin{vmatrix} 2 & 0 & 1 \\ -36 & 5 & -4 \\ 2 & 1 & 11 \end{vmatrix} = \det \begin{vmatrix} 2-\lambda & 0 & 1 \\ -36 & 5-\lambda & -4 \\ 2 & 1 & 11-\lambda \end{vmatrix}$$

$$(2-\lambda)(5-\lambda)(11-\lambda) - 36 - (5-\lambda)2 + 4(2-\lambda)$$

Show that the 3 eigenvalues are roots of this {2}

$$\begin{matrix} -0.1231 & -0.0478 & 0.0572 \\ -0.9847 & 0.9567 & -0.9147 \\ 0.1231 & -0.2870 & 0.4002 \end{matrix} = \begin{matrix} 1 \\ 8 \\ 9 \end{matrix}$$

In MATLAB, form matrix X from the three eigenvectors and show Inv(X) · A · X is a diagonal matrix {2}

> A = [2 0 1; -36 5 -4; 2 1 11]

A =

```
  2     0     1
-36     5    -4
  2     1    11
```

> [X,D] = eig(A)

X =

```
0.1231 -0.0478 0.0572
-0.9847 0.9567 -0.9147
0.1231 -0.2870 0.4002
```

D =

```
1.0000         0         0
     0    8.0000         0
     0         0    9.0000
```

> inv(A) * A * X

ans =

```
0.1231 -0.0478 0.0572
-0.9847 0.9567 -0.9147
0.1231 -0.2870 0.4002
```