Homework 08 Root Finding

1. Use zero-, first-, second- and third-order Taylor series expansions to predict $f(2)$

$$f(x) = 25x^3 - 6x^2 + 7x - 88$$

function_x(x) = $25\,x^3 - 6\,x^2 + 7\,x - 88$

trueValue_2 = 102

Using a base point at x = 1 and step size = 0.1, 0.5, 1 with **Backward Divided Difference.**

$$f^{(n)}\left(x\right) = \frac{\nabla_h^n[f](x)}{h^n} = \frac{1}{h^n}\sum_{k=0}^{n}(-1)^k\binom{n}{k}f(x-kh),$$

Calculate the error for each of the Taylor series expansions

$$f\left(x_{i+1}\right) = \sum_{n=0}^{\infty}\frac{f^{(n)}(x_i)}{n!}(x_{i+1}-x_i)^n,$$

$$\text{error} = \frac{\text{trueValue} - \text{approximatedValue}}{\text{trueValue}}$$

Zero-Order Taylor Series Expansion

$$f(x_{i+1}) \cong f(x_i)$$

```
Approximation Value (Taylor + Backward Zero-Order Step 0.10) : -62.00   error : 1.61
Approximation Value (Taylor + Backward Zero-Order Step 0.50) : -62.00   error : 1.61
Approximation Value (Taylor + Backward Zero-Order Step 1.00) : -62.00   error : 1.61
```

One-Order Taylor Series Expansion

$$f(x_{i+1}) \cong f(x_i) + f'(x_i)(x_{i+1} - x_i)$$

```
Approximation Value (Taylor + Backward First-Order Step 0.10) : 1.35   error : 0.99
Approximation Value (Taylor + Backward First-Order Step 0.50) : -20.25   error : 1.20
Approximation Value (Taylor + Backward First-Order Step 1.00) : -36.00   error : 1.35
```

Second-Order Taylor Series Expansion

$$f\left(x_{i+1}\right) \cong f\left(x_i\right) + f'\left(x_i\right)\left(x_{i+1} - x_i\right) + \frac{f''\left(x_i\right)(x_{i+1} - x_i)^2}{2!}$$

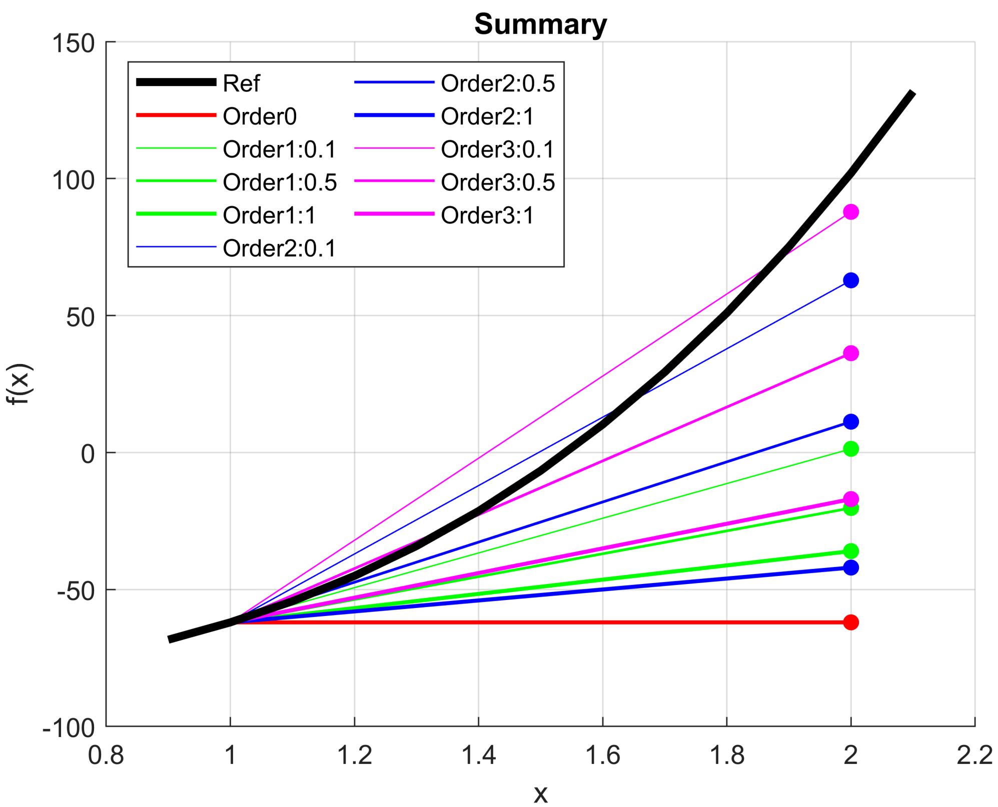
```
Approximation Value (Taylor + Backward Second-Order Step 0.10) :62.85   error : 0.38
Approximation Value (Taylor + Backward Second-Order Step 0.50) :11.25   error : 0.89
Approximation Value (Taylor + Backward Second-Order Step 1.00) :-42.00   error : 1.41
```

Third-Order Taylor Series Expansion

$$f\left(x_{i+1}\right) \cong f\left(x_i\right) + f'\left(x_i\right)\left(x_{i+1} - x_i\right) + \frac{f''\left(x_i\right)\left(x_{i+1} - x_i\right)^2}{2!} + \frac{f'''\left(x_i\right)\left(x_{i+1} - x_i\right)^3}{3!}$$

```
Approximation Value (Taylor + Backward Third-Order Step 0.10) : 87.85   error : 0.14
Approximation Value (Taylor + Backward Third-Order Step 0.50) : 36.25   error : 0.64
Approximation Value (Taylor + Backward Third-Order Step 1.00) : -17.00   error : 1.17
```

Summary Plot



2. Write a computer program that find the root of

$$f(x) = x^3 - 3x^2 + 1$$

function_x(x) = $x^3 - 3\,x^2 + 1$

by using both bisection and false-position method in period $[0, 2]$

and terminate the program when approximated

relative error $\varepsilon_a \leq 1\%$ **the program must generate a report file,**

report.txt, that shows in each line:

(1) the number of iterations,

(2) an estimated root in each iteration,

(3) the approximate relative error in each iteration.

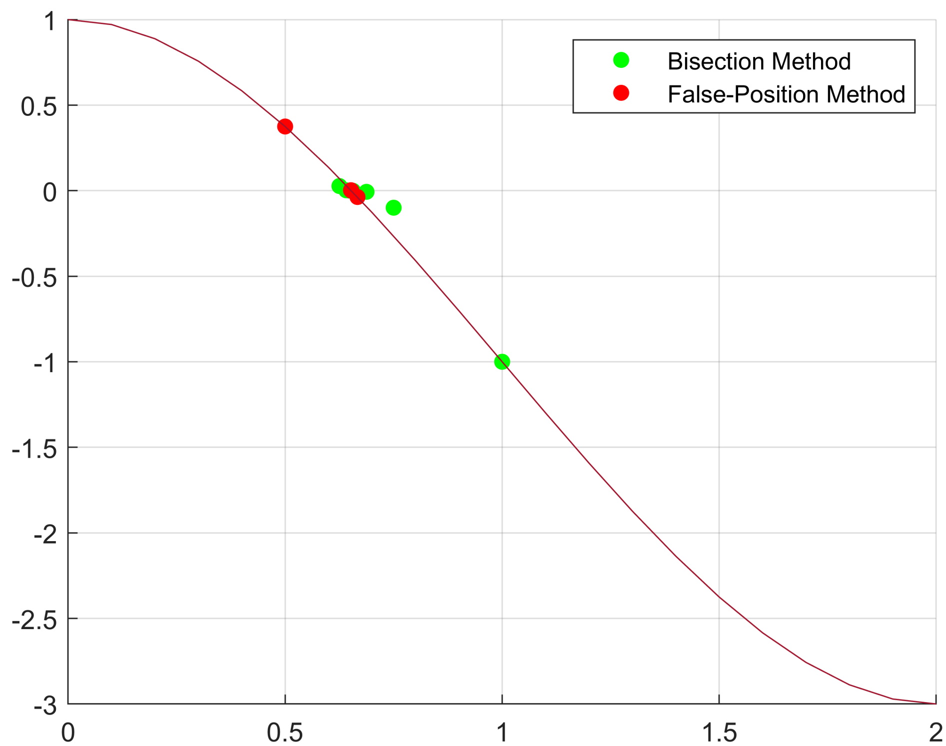
Bracketing Method: Bisection Method

```
Iteration : 1    Estimated root : 1.0000    Approximate Relative Error : 100.0000
Iteration : 2    Estimated root : 0.5000    Approximate Relative Error : 100.0000
Iteration : 3    Estimated root : 0.7500    Approximate Relative Error : 33.3333
Iteration : 4    Estimated root : 0.6250    Approximate Relative Error : 20.0000
Iteration : 5    Estimated root : 0.6875    Approximate Relative Error : 9.0909
Iteration : 6    Estimated root : 0.6563    Approximate Relative Error : 4.7619
Iteration : 7    Estimated root : 0.6406    Approximate Relative Error : 2.4390
Iteration : 8    Estimated root : 0.6484    Approximate Relative Error : 1.2048
Iteration : 9    Estimated root : 0.6523    Approximate Relative Error : 0.5988
```

Bracketing Method: False-Position Method

```
Iteration : 1    Estimated root : 0.5000    Approximate Relative Error : 100.0000
Iteration : 2    Estimated root : 0.6667    Approximate Relative Error : 25.0000
Iteration : 3    Estimated root : 0.6517    Approximate Relative Error : 2.2989
Iteration : 4    Estimated root : 0.6527    Approximate Relative Error : 0.1552
```

Summary Plot



Appendix

Binomial coefficient

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

Backward Divided Difference Taylor Series

$$f\left(x_{i+1}\right) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_i)}{n!}(x_{i+1} - x_i)^n,$$

$$f^{(n)}\left(x\right) = \frac{\nabla_h^n[f](x)}{h^n} = \frac{1}{h^n} \sum_{k=0}^{n} (-1)^k \binom{n}{k} f(x - kh),$$