Simulated Power Calculations for Mediation Effects with Covariates

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Motivation

- Power Analysis [?] [?] is crucial for study design. With the increasing emphasis on understanding mechanisms, models for mediation are commonly used.
- Our work is motivated by studies in psychiatry: researchers aim to understand neural mechanisms underlying the risk for bipolar disorder in young adults by studying impulsive sensation seeking and emotional dysregulation to help identify targets to guide treatments.

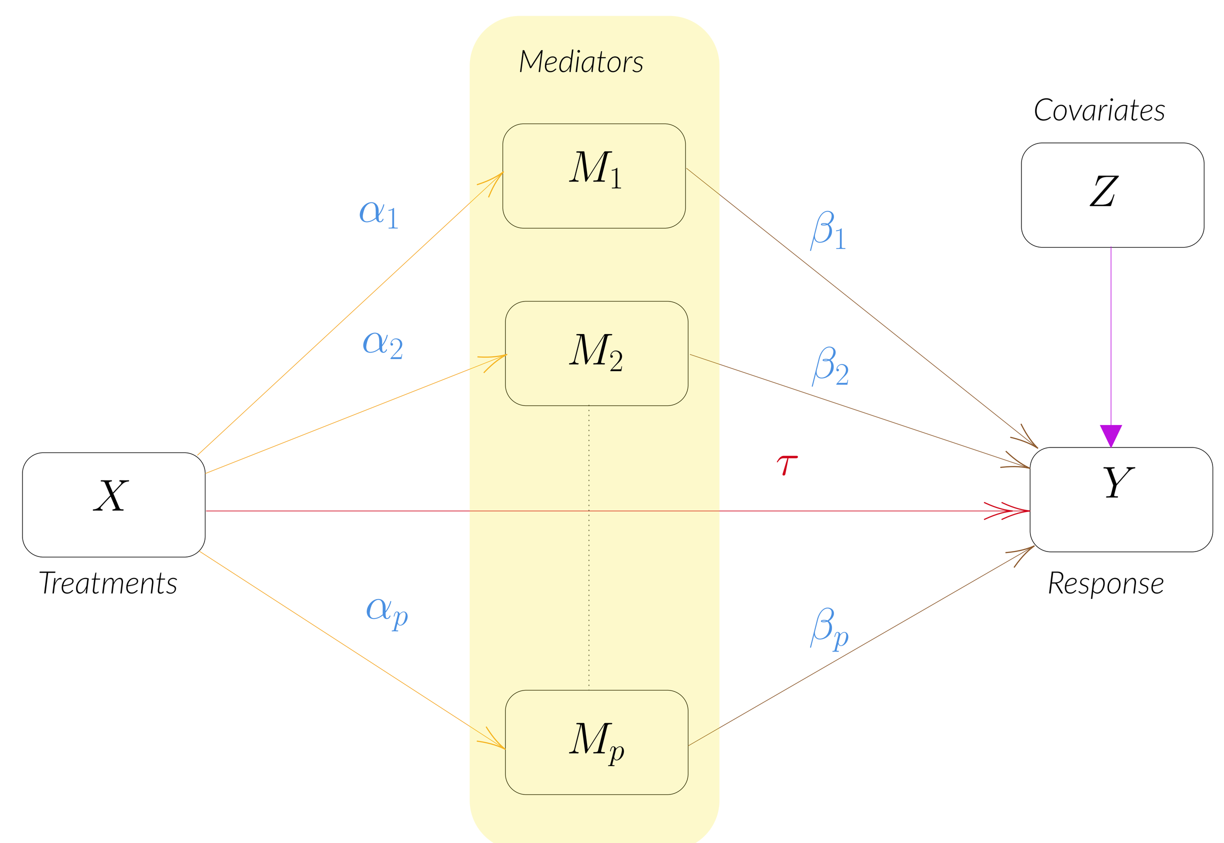

Figure 1. The Mediation structure with covariates

The Mathematical Model

M mediators with dimension p_M, Y responses with dimension p_Y, g is the link function of the generalized linear model. Identity link for linear model.

$$M_{ij} = \alpha_j X_i + \epsilon_{M,ij}$$

$$Y_{ik} = g^{-1}\left(\tau_{ik} + \sum_{k=1}^{p_M} M_{ik}\beta_{kl} + \epsilon_{Y,kl}\right)$$

$\epsilon_{M,ij} \sim N(0,\sigma_M^2),\ \tau_{ik} \sim N(\mu_\tau,\sigma_\tau^2),\ \epsilon_{Y,kl} \sim N(0,\sigma_Y^2);\ i = 1\ldots n;\ j,k = 1\ldots p_M;\ l = 1\ldots p_Y.$

Many effect sizes [?] have been in use, eg. Sobel (Normal) Test, Joint Testing. We use Z-adjusted R^2 effect size to evaluate mediation effect. Pseudo-R^2 for glm.

$$\begin{aligned} R^2_{\cdot|\boldsymbol{Z}} &= R^2_{\{Y\sim M\}|\boldsymbol{Z}} + R^2_{\{Y\sim X\}|\boldsymbol{Z}} - R^2_{\{Y\sim[M;X]\}|\boldsymbol{Z}} \\ &= \tilde{R}^2_{\tilde{Y}\sim M} + \tilde{R}^2_{\tilde{Y}\sim X} - \tilde{R}^2_{\tilde{Y}\sim[M;X]} \end{aligned}$$

Where $\tilde{Y}$ is the residual of Y on Z.

Power Analysis

Linear Model	n = 100	n = 200	n = 500	GLM	n = 100	n = 200	n = 500
Zero α, Small β	0.0015	0.0005	0	Zero α, Small β	0	0	0.025
Zero α, Medium β	0.014	0.037	0.046	Zero α, Medium β	0.005	0.0235	0.012

Linear Model	n = 100	n = 200	n = 500	GLM	n = 100	n = 200	n = 500
Small α, β	0.620	0.742	0.868	Small α, β	0.642	0.838	0.986
Medium α, β	0.770	0.858	0.996	Medium α, β	0.924	0.965	0.998
Large α, β	0.964	0.998	1	Large α, β	0.978	1	0.996

Table 1. Analysis of Type-I error for no mediation and Power for full-mediation

We demonstrated the efficient control of type-I error for no mediation case and the achievement of achieving the desired power in the full mediation case.

Simulation Results

For covariate-adjusted partial mediation ($\alpha \neq 0, \beta \neq 0, \tau \neq 0$), in general the power would be proportional to the increase of the sample-size as well as the effect-size one has specified.

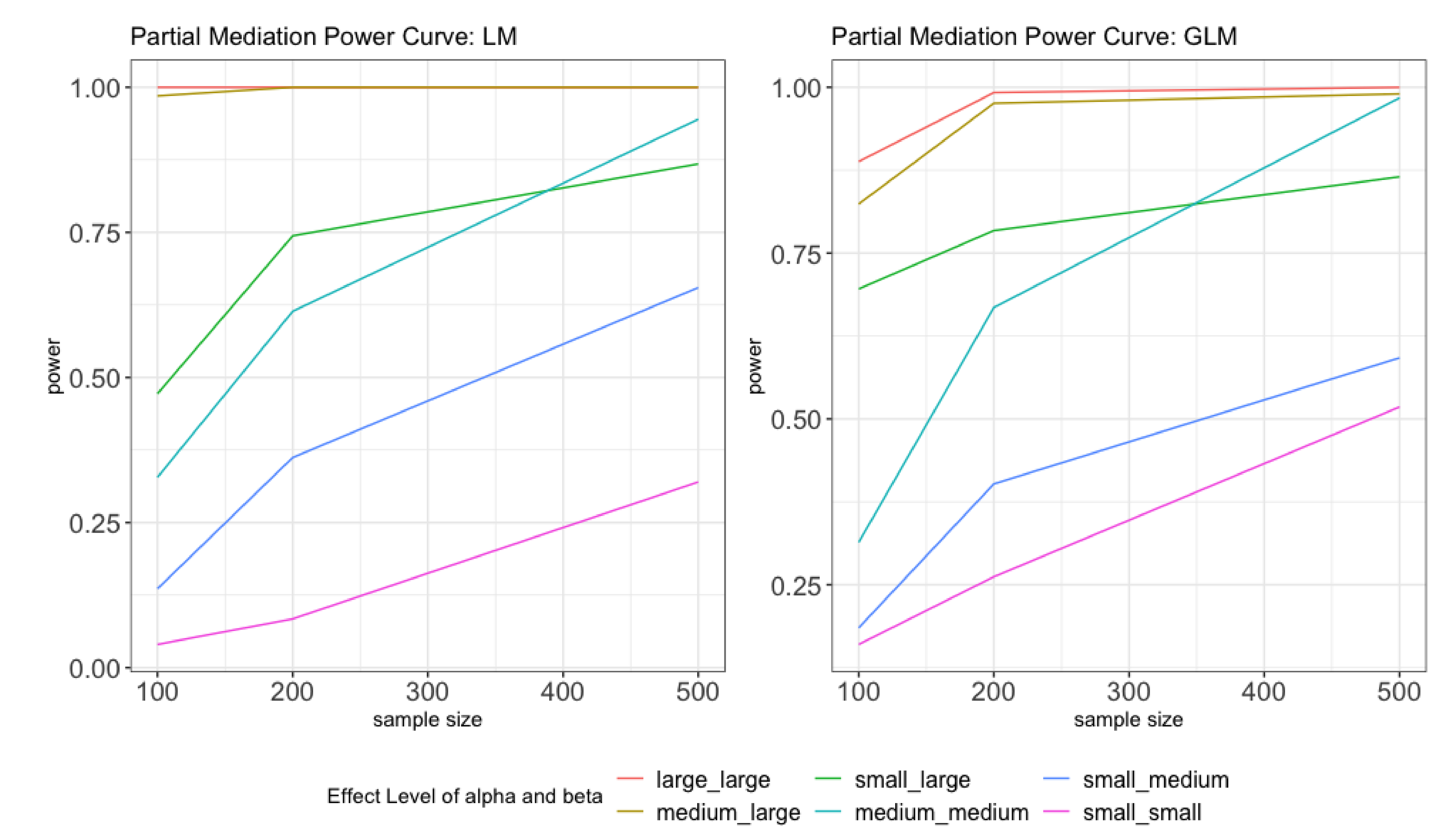

Figure 2. Power curve for simulated Linear and Generalized Linear Model

Sensitivity Analysis

We simulate Z by corrupting Y with progressively increasing Gaussian noise, thereby reducing the dependence of Y on Z.

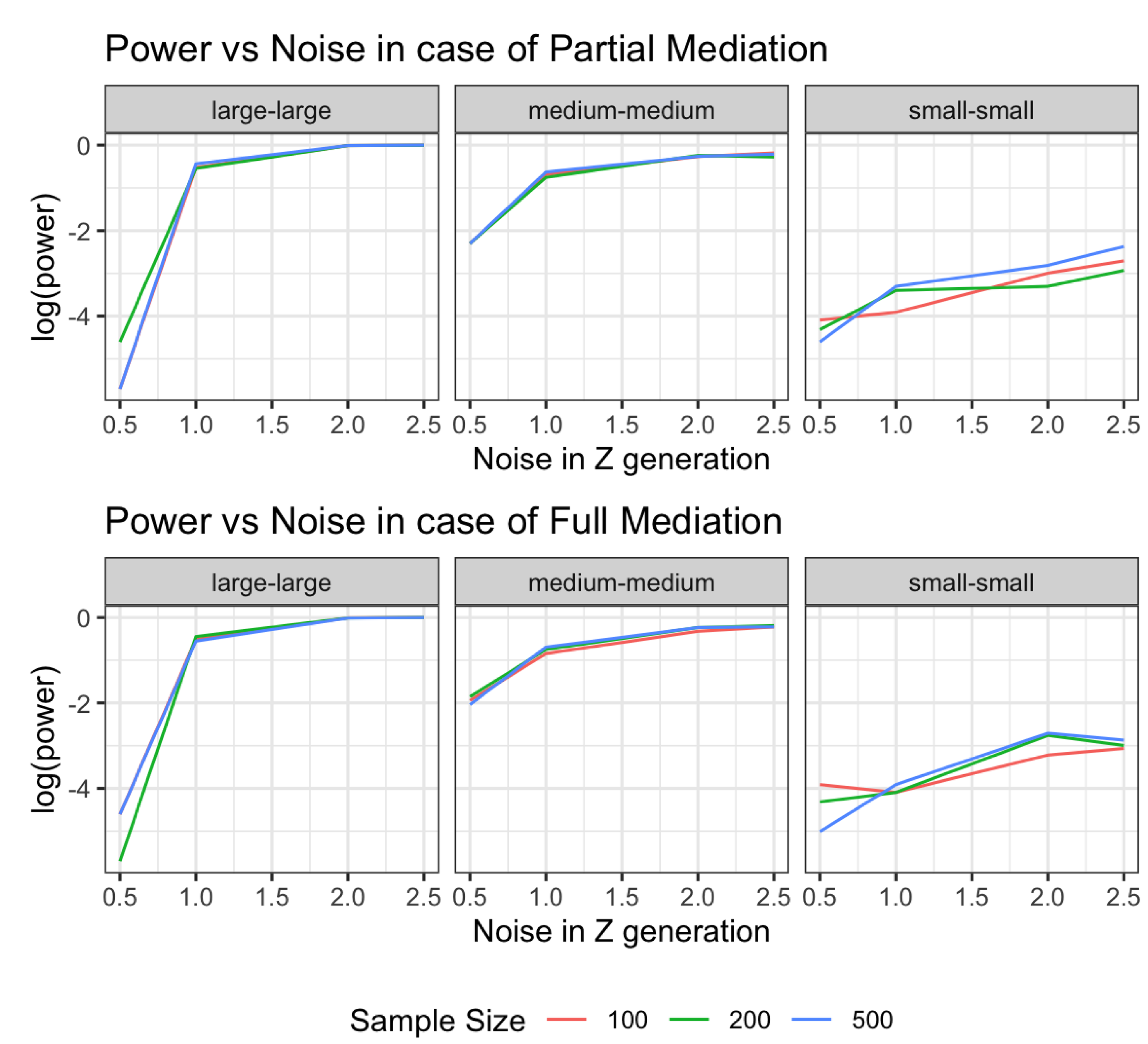

Figure 3. Power vs Noise for two effect-level frameworks.

Adjusting for the effect of Z, we can still recover most of the information about the dependence structure of the underlying mediation model.

Application to Psychiatric Studies

Two real-world case study demonstrate the usefulness of covariate-adjusted power calculation for both linear model and glm.

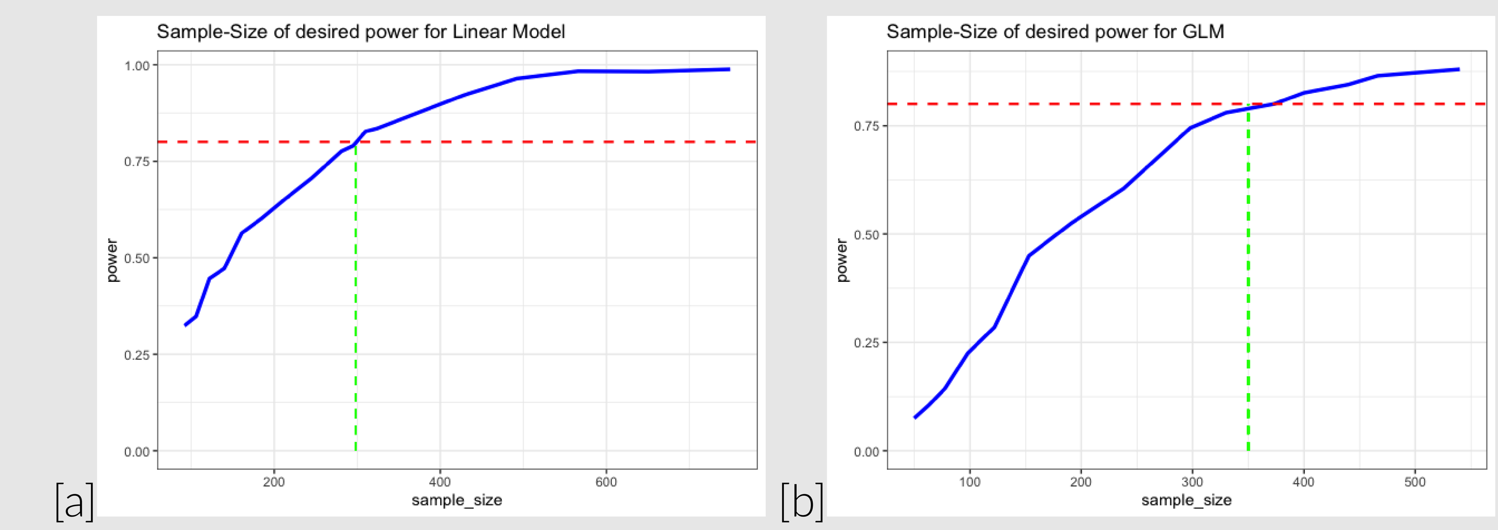

Figure 4. (a) Small (α)-Medium (β) effect-size for mediation analysis with linear model (b) Small (α) - Small (β) effect-size for mediation analysis with logistic regression. Blue line: power curve. Red line: desired power(0.8) and green line: required sample-size.

The curve demonstrates that in estimating the required sample size for a planned study, one can start from a small sample size and increment by a certain step iteratively until reaching a desirable power level.

The results above are based on an ongoing study at the Department of Psychiatry, University of Pittsburgh.

Future work

What's new?

- We developed a way to do power calculation for covariate-adjusted mediation analysis and demonstrated the efficiency and robustness.
- We extended the R^2 effect-size [?] for testing mediation effects to generalized linear model (glm) and non-normal responses.

Future Work

- Adapt the covariate-adjusted R^2 into causal framework for non-normal response and continuous treatments.
- Extend the R^2 to deal with the mediation analysis including longitudinal observations with non-normal response.
- Establish the large-sample properties for covariate-adjusted R^2 effect-size for more complex mediation designs.

References

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