5. Uma rede de crença (ou rede bayesiana), modela a relação entre as variáveis: oil (price of oil), inf (inflation) , eh (economy health), bp (British Petroleum Stock price), rt (retailer stock price). Cada variável tem dois estados (l:low) e (h:high), exceto a variável bp que tem adicionalmente o estado (n: normal). A rede de crença modela as variáveis de acordo com a tabela abaixo.

P(eh=l)=0.2	
P (bp=l|oil=l)=0.9	P(bp=n|oil=l)=0.1
P(bp=l|oil=h)=0.1	P(bp=n|oil=h)=0.4
P(oil=l|eh=l)=0.9	P(oil=l|eh=h)=0.05
P(rt=l|inf=l,eh=l)=0.9	P(rt=l|inf=l,eh=h)=0.1
P(rt=l|inf=h,eh=l)=0.1	P(rt=l|inf=h,eh=h)=0.01
P(inf=l|oil=l, eh=l)=0.9	P(inf=l|oil=l, eh=h)=0.1
P(inf=l|oil=h, eh=l)=0.1	P(inf=l|oil=h, eh=h)=0.01

a) Apresente a rede de crença para este problema

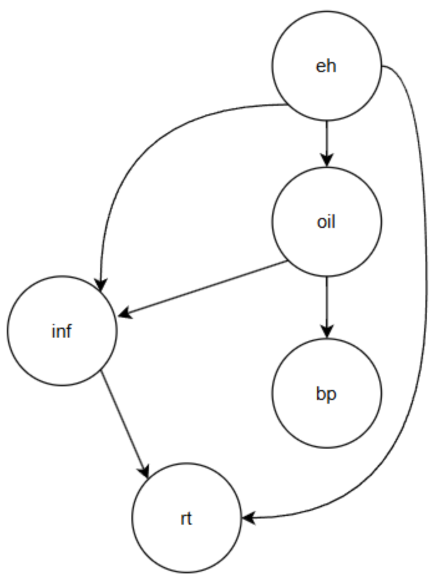
b) Dado que a bp=n e rt=h, qual é a probabilidade de que a inflação seja alta?

Respostas

a) A rede de crença é construída analisando as variáveis e como elas influenciam as outras. Dessa forma, podemos perceber verificando na tabela de probabilidades, temos as seguintes análises:

- A variável "oil" influencia diretamente "bp" e "inf";
- A variável "eh" influencia diretamente "oil", "inf" e "rt";
- A variável "inf" influencia diretamente "rt";

Com isso, podemos construir a representação gráfica da rede bayesiana, denotada abaixo:



b) Para o desenvolvimento dessa questão, precisamos encontrar as probabilidades para casos em que as variáveis dependem de outras variáveis. Portanto, a probabilidade da inflação ser alta é:

$$P(\text{inf}=h|\text{bp}=n,\text{rt}=h)=\frac{P(\text{inf}=h|\text{eh},\text{oil})\cdot P(\text{bp}=n|\text{oil})\cdot P(\text{rt}=h|\text{eh},\text{inf}=h)}{P(\text{bp}=n,\text{rt}=h)}$$

Calculando $P(\text{inf}=h|\text{eh},\text{oil})$:

$$P(\text{inf}=h|\text{eh},\text{oil})=P(\text{inf}=h|\text{eh}=l,\text{oil}=l)\cdot P(\text{oil}=l|\text{eh}=l)\cdot P(\text{eh}=l)$$

$$+P(\text{inf}=h|\text{eh}=h,\text{oil}=h)\cdot P(\text{oil}=h|\text{eh}=h)\cdot P(\text{eh}=h)$$

$$+P(\text{inf}=h|\text{eh}=h,\text{oil}=l)\cdot P(\text{oil}=l|\text{eh}=h)\cdot P(\text{eh}=h)$$

$$+P(\text{inf}=h|\text{eh}=l,\text{oil}=h)\cdot P(\text{oil}=h|\text{eh}=l)\cdot P(\text{eh}=l)$$

Resposta obtida: $P(\text{inf}=h|\text{eh},\text{oil})=0,8244$
Calculando $P(\text{bp}=n|\text{oil})$:

$$\begin{aligned}P(\text{bp}=n|\text{oil})=\ &P(\text{bp}=n|\text{oil}=l)\cdot[P(\text{oil}=l|\text{eh}=l)\cdot P(\text{eh}=l)+P(\text{oil}=l|\text{eh}=h)\cdot P(\text{eh}=h)]\\&+P(\text{bp}=n|\text{oil}=h)\cdot[P(\text{oil}=h|\text{eh}=l)\cdot P(\text{eh}=l)+P(\text{oil}=h|\text{eh}=h)\cdot P(\text{eh}=h)]\end{aligned}$$

Resposta obtida: $P(\text{bp}=n|\text{oil})=0,444$
Calculando $P(\text{rt}=h|\text{eh},\text{inf}=h)$:

$$\begin{aligned}P(\text{rt}=h|\text{eh},\text{inf}=h)=&[P(\text{rt}=h|\text{eh}=l,\text{inf}=h)\cdot(P(\text{inf}=h|\text{eh}=l,\text{oil}=l)\cdot P(\text{oil}=l|\text{eh}=l)\cdot P(\text{e}\\&+[P(\text{rt}=h|\text{eh}=h,\text{inf}=h)\cdot(P(\text{inf}=h|\text{eh}=h,\text{oil}=l)\cdot P(\text{oil}=l|\text{eh}=h)\cdot P(\end{aligned}$$

Resposta obtida: $P(\text{rt}=h|\text{eh},\text{inf}=h)=0,8129$
Calculando $P(\text{bp}=n,\text{rt}=h)$:

$$\begin{aligned}P(\text{bp}=n,\text{rt}=h)=&P(\text{bp}=n|\text{oil})\cdot P(\text{rt}=h|\text{inf},\text{eh})=0,444\cdot(P(\text{rt}=h|\text{inf}=l,\text{eh}=l)(\text{inf}=l|\text{eh}=\\&+(P(\text{rt}=h|\text{inf}=l,\text{eh}=h)\cdot P(\text{inf}=l|\text{eh}=h,\text{oil})\cdot P(\text{eh}=h))\\&+(P(\text{rt}=h|\text{inf}=h,\text{eh}=l)\cdot P(\text{inf}=h|\text{eh}=l,\text{oil})\cdot P(\text{eh}=l))\\&+(P(\text{rt}=h|\text{inf}=h,\text{eh}=h)\cdot P(\text{inf}=h|\text{eh}=h,\text{oil})\cdot P(\text{eh}=h))\end{aligned}$$

Resposta obtida: $P(\text{bp}=n,\text{rt}=h)=0,3563$
Calculando agora na fórmula final:

$$P(\text{inf}=h|\text{bp}=n,\text{rt}=h)=\frac{0,8244\cdot 0,444\cdot 0,8129}{0,3563}$$

Resposta obtida: $P(\text{inf}=h|\text{bp}=n,\text{rt}=h)=0,8351$