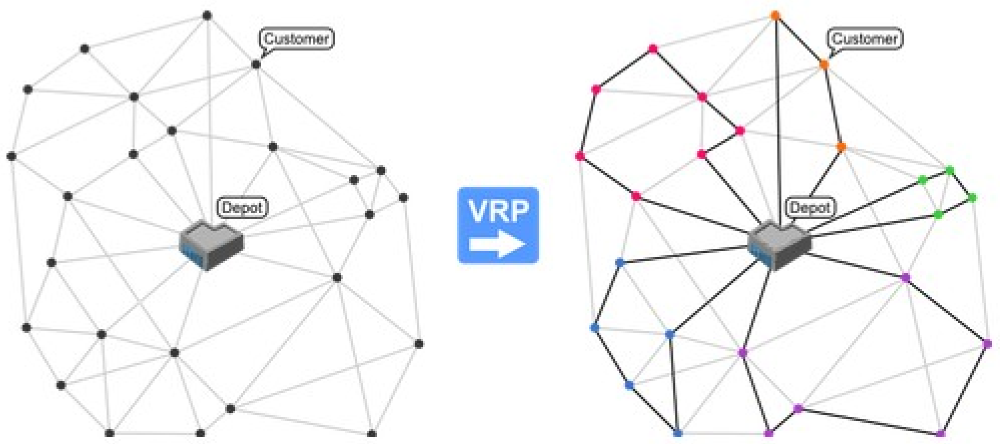
Vehicle Routing Problem

Introduction

The vehicle routing problem (VRP) is a combinatorial optimization and integer programming problem seeking to service a number of customers with a fleet of vehicles. Proposed by Dantzig and Ramser in 1959, VRP is an important problem in the fields of transportation, distribution, and logistics.

Description

The Vehicle Routing Problem (VRP) is a generic name given to a whole class of problems in which a set of routes for a fleet of vehicles based at one or several depots must be determined for a number of geographically dispersed cities or customers. The objective of the VRP is to deliver a set of customers with known demands on minimum-cost vehicle routes originating and terminating at a depot. In the two figures below we can see a picture of a typical input for a VRP problem and one of its possible outputs:



An instance of a VRP (left) and its solution (right)

Formulation

The VRP is a combinatorial problem whose ground set is the edges of a graph $G(V,E)$. The notation used for this problem is as follows:

- $V=\{v_0,v_1,\ldots,v_n\}$ is a vertex set, where:
  - Consider a depot to be located at v_0.
  - Let $V'=V\setminus\{v_0\}$ be used as the set of n cities.
- $A=\{(v_i,v_j)|v_i,v_j\in V;i\neq j\}$ is an arc set.
- C is a matrix of non-negative costs or distances c_{ij} between customers v_i and v_j.
- d is a vector of the customer demands.
- R_i is the route for vehicle i.
- m is the number of vehicles (all identical). One route is assigned to each vehicle.

When $c_{ij}=c_{ji}$ for all $(v_i,v_j)\in A$ the problem is said to be symmetric and it is then common to replace A with the edge set $E=\{(v_i,v_j)|v_i,v_j\in V;i<j\}$.
With each vertex v_i in V' is associated a quantity qi of some goods to be delivered by a vehicle. The VRP thus consists of determining a set of m vehicle routes of minimal total cost, starting and ending at a depot, such that every vertex in V' is visited exactly once by one vehicle.

Capacitated VRP

CVRP is a VRP in which a fixed fleet of delivery vehicles of uniform capacity must service known customer demands for a single commodity from a common depot at minimum transit cost. That is, CVRP is like VRP with the additional constraint that every vehicles must have uniform capacity of a single commodity.

We can find below a formal description for the CVRP:

Objective

The objective is to minimize the vehicle fleet and the sum of travel time, and the total demand of commodities for each route may not exceed the capacity of the vehicle which serves that route.

Feasibility

A solution is feasible if the total quantity assigned to each route does not exceed the capacity of the vehicle which services the route.

Formulation

Let Q denote the capacity of a vehicle. Mathematically, a solution for the CVRP is the same that VRP's one, but with the additional restriction that the total demand of all customers supplied on a route R_i does not exceed the vehicle capacity Q: $\sum m_i=1 d_i \leq Q$.

Example of file and optimal solution

- Arquivo A-n33-k5.vpr

NAME : A-n33-k5
COMMENT : (Augerat et al, Min no of trucks: 5, Optimal value: 661)
TYPE : CVRP
DIMENSION : 33
EDGE_WEIGHT_TYPE : EUC_2D
CAPACITY : 100
NODE_COORD_SECTION
1 42 68
2 77 97
3 28 64
4 77 39

5 32 33
6 32 8
7 42 92
8 8 3
9 7 14
10 82 17
11 48 13
12 53 82
13 39 27
14 7 24
15 67 98
16 54 52
17 72 43
18 73 3
19 59 77
20 58 97
21 23 43
22 68 98
23 47 62
24 52 72
25 32 88
26 39 7
27 17 8
28 38 7
29 58 74
30 82 67
31 42 7
32 68 82
33 7 48
DEMAND_SECTION
1 0
2 5
3 23
4 14
5 13
6 8
7 18
8 19
9 10
10 18
11 20
12 5
13 9
14 23
15 9
16 18
17 10
18 24
19 13
20 14
21 8
22 10

23 19
24 14
25 13
26 14
27 2
28 23
29 15
30 8
31 20
32 24
33 3
DEPOT_SECTION
 1
 -1
EOF

Optimal Solution
Route #1: 15 17 9 3 16 29
Route #2: 12 5 26 7 8 13 32 2
Route #3: 20 4 27 25 30 10
Route #4: 23 28 18 22
Route #5: 24 6 19 14 21 1 31 11
cost 661
