

## 1. Equações de Maxwell no domínio do tempo

$$\oint_c \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int_s \vec{B} \cdot d\vec{s} \quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint_c \vec{H} \cdot d\vec{l} = \frac{d}{dt} \int_s \vec{D} \cdot d\vec{s} + \int_s \vec{J} \cdot d\vec{s} \quad \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}$$

$$\oint_s \vec{D} \cdot d\vec{s} = \int_v \rho_v dv \quad \nabla \cdot \vec{D} = \rho_v$$

$$\oint_s \vec{B} \cdot d\vec{s} = 0 \quad \nabla \cdot \vec{B} = 0$$

## 2. Vetor de Poynting

$$\oint_s (\vec{E} \times \vec{H}) \cdot d\vec{s} = -\int_v [\vec{E} \cdot \vec{J}] dv - \frac{\partial}{\partial t} \int_v \left[ \frac{\mu}{2} |\vec{H}|^2 + \frac{\epsilon}{2} |\vec{E}|^2 \right] dv$$

$$P_e = \oint_s (\vec{E} \times \vec{H}) \cdot d\vec{s} = \oint_s \vec{P} \cdot d\vec{s} \quad \vec{P} = \vec{E} \times \vec{H}$$

$$P_f = -\int_v [\vec{E} \cdot \vec{J}_i] dv \quad P_d = -\int_v [\vec{E} \cdot \vec{J}_c] dv$$

$$W_e = \int_v \frac{\epsilon}{2} |\vec{E}|^2 dv \quad W_m = \int_v \frac{\mu}{2} |\vec{H}|^2 dv$$

$$P_f = P_e + P_d + \frac{\partial(W_e + W_m)}{\partial t}$$

## 3. Campos Harmônicos

$$\mathbf{E}(\vec{r}, \omega) = \int_{-\infty}^{\infty} \vec{E}(\vec{r}, t) e^{-j\omega t} dt \quad \mathbf{H}(\vec{r}, \omega) = \int_{-\infty}^{\infty} \vec{H}(\vec{r}, t) e^{-j\omega t} dt$$

$$\vec{E}(\vec{r}, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathbf{E}(\vec{r}, \omega) e^{j\omega t} d\omega \quad \vec{H}(\vec{r}, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathbf{H}(\vec{r}, \omega) e^{j\omega t} d\omega$$

$$\vec{E}(\vec{r}, t) = \Re \{ \mathbf{E}(\vec{r}) e^{j\omega t} \} \quad \vec{H}(\vec{r}, t) = \Re \{ \mathbf{H}(\vec{r}) e^{j\omega t} \}$$

$$\vec{D}(\vec{r}, t) = \Re \{ \mathbf{D}(\vec{r}) e^{j\omega t} \} \quad \vec{B}(\vec{r}, t) = \Re \{ \mathbf{B}(\vec{r}) e^{j\omega t} \}$$

$$\vec{J}(\vec{r}, t) = \Re \{ \mathbf{J}(\vec{r}) e^{j\omega t} \} \quad \rho(\vec{r}, t) = \Re \{ q_e(\vec{r}) e^{j\omega t} \}$$

## 4. Equações de Maxwell no domínio da frequência

$$\oint_c \mathbf{E} \cdot d\vec{l} = -j\omega \int_s \mathbf{B} \cdot d\vec{s} \quad \nabla \times \mathbf{E} = -j\omega \mathbf{B}$$

$$\oint_c \mathbf{H} \cdot d\vec{l} = j\omega \int_s \mathbf{D} \cdot d\vec{s} + \int_s \mathbf{J} \cdot d\vec{s} \quad \nabla \times \mathbf{H} = j\omega \mathbf{D} + \mathbf{J}$$

$$\oint_s \mathbf{D} \cdot d\vec{s} = \int_v q_e dv \quad \nabla \cdot \mathbf{D} = q_e$$

$$\oint_s \mathbf{B} \cdot d\vec{s} = 0 \quad \nabla \cdot \mathbf{B} = 0$$

## 5. Condições de Fronteira

$$\vec{n} \times (\vec{E}_2 - \vec{E}_1) = 0 \quad \vec{n} \times (\mathbf{E}_2 - \mathbf{E}_1) = 0$$

$$\vec{n} \times (\vec{H}_2 - \vec{H}_1) = \vec{J}_s \quad \vec{n} \times (\mathbf{H}_2 - \mathbf{H}_1) = \mathbf{J}_s$$

$$\vec{n} \cdot (\vec{D}_2 - \vec{D}_1) = \rho_s \quad \vec{n} \cdot (\mathbf{D}_2 - \mathbf{D}_1) = q_e$$

$$\vec{n} \cdot (\vec{B}_2 - \vec{B}_1) = 0 \quad \vec{n} \cdot (\mathbf{B}_2 - \mathbf{B}_1) = 0$$

## 6. Vetor de Poynting Complexo

$$\vec{P} = \frac{1}{2} [\Re(\mathbf{E} \times \bar{\mathbf{H}}) + \Re(\mathbf{E} \times \mathbf{H} e^{j2\omega t})]$$

$$\vec{P}_m = \frac{1}{2} \Re(\mathbf{E} \times \bar{\mathbf{H}}) \quad \vec{P}_{rea} = \frac{1}{2} [\Re(\mathbf{E} \times \mathbf{H} e^{j2\omega t})]$$

## 7. Equação de Onda no domínio do tempo

$$\nabla^2 \vec{E} = \frac{\nabla \rho}{\epsilon} + \mu \frac{\partial \vec{J}_i}{\partial t} + \mu \sigma \frac{\partial \vec{E}}{\partial t} + \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{H} = -\nabla \times \vec{J}_i + \mu \sigma \frac{\partial \vec{H}}{\partial t} + \mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2}$$

$$\nabla^2 \vec{E} = \mu \sigma \frac{\partial \vec{E}}{\partial t} + \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{H} = \mu \sigma \frac{\partial \vec{H}}{\partial t} + \mu \epsilon \frac{\partial^2 \vec{H}}{\partial t^2}$$

## 8. Equação de Onda no domínio da frequência

$$\nabla^2 \mathbf{E} = \frac{\nabla q_e}{\epsilon} + j\omega \mu \mathbf{J}_i + j\omega \mu \sigma \mathbf{E} - \omega^2 \mu \epsilon \mathbf{E}$$

$$\nabla^2 \mathbf{H} = -\nabla \times \mathbf{J}_i + j\omega \mu \sigma \mathbf{H} - \omega^2 \mu \epsilon \mathbf{H}$$

$$\nabla^2 \mathbf{E} = \gamma^2 \mathbf{E} \quad \nabla^2 \mathbf{H} = \gamma^2 \mathbf{H}$$

$$\nabla^2 \mathbf{E} = -k^2 \mathbf{E} \quad \nabla^2 \mathbf{H} = -k^2 \mathbf{H}$$

$$\gamma^2 = j\omega \mu \sigma - \omega^2 \mu \epsilon \quad k^2 = \omega^2 \mu \epsilon$$

## 9. Equações de Maxwell para Ondas Planas

$$\mathbf{E}(\vec{r}) = \mathbf{E} e^{\pm j \vec{k} \cdot \vec{r}} \quad \mathbf{H}(\vec{r}) = \mathbf{H} e^{\pm j \vec{k} \cdot \vec{r}}$$

$$\vec{k} \times \mathbf{E} = \omega \mu \mathbf{H} \quad \vec{k} \times \mathbf{H} = -\omega \epsilon \mathbf{E}$$

$$\vec{k} \cdot \mathbf{D} = 0 \quad \vec{k} \cdot \mathbf{B} = 0$$

## 10. Parâmetros de Ondas Eletromagnéticas

$$Z = \frac{E}{H} \quad v_f = \frac{\omega}{k} \quad v_g = v_f + k \frac{dv_f}{dk} \quad v_g = v_f - \lambda \frac{dv_f}{d\lambda}$$

$$\mathbf{E} = (\vec{a}_x + \vec{a}_y A e^{j\phi}) e^{-jk_z z} \quad A = \frac{E_{y0}}{E_{x0}} \quad \phi = \phi_y - \phi_x$$

polarização linear -  $A = 0$  ou  $A = \infty$ ;  $\phi = n\pi$

polarização circular -  $A = 1$  e  $\phi = \pm (2n+1)\pi/2$

$$\text{polarização elíptica} - \begin{cases} A \neq 1 \text{ e } \phi = \pm (2n+1)\pi/2 \\ \phi \neq \pm \frac{n}{2}\pi \end{cases}$$

## 11. Ondas Estacionárias

$$\mathbf{E} = [E_0^+ e^{-jkz} + E_0^- e^{+jkz}] \vec{a}_x$$

$$E_x(z, t) = \sqrt{(E_0^+)^2 + (E_0^-)^2 + 2E_0^+ E_0^- \cos 2kz} \times \cos \left[ \omega t - \tan^{-1} \left\{ \frac{E_0^+ - E_0^-}{E_0^+ + E_0^-} \tan kz \right\} \right]$$

$$TOE = \frac{|E|_{max}}{|E|_{min}} = \frac{|E_0^+| + |E_0^-|}{|E_0^+| - |E_0^-|} \quad \Gamma = \frac{E_0^-}{E_0^+}$$

$$TOE = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

## 12. Meio com perdas

$$\nabla^2 \mathbf{E} - \gamma^2 \mathbf{E} = 0 \quad \mathbf{E} = \mathbf{E}_0 e^{\pm \vec{\gamma} \cdot \vec{r}} \quad \gamma = \alpha + j\beta$$

$$\alpha = \omega \sqrt{\frac{\mu \epsilon}{2} \left( \sqrt{1 + \left( \frac{\sigma}{\omega \epsilon} \right)^2} - 1 \right)}, \quad (Np/m)$$

$$\beta = \omega \sqrt{\frac{\mu \epsilon}{2} \left( \sqrt{1 + \left( \frac{\sigma}{\omega \epsilon} \right)^2} + 1 \right)}, \quad (rad/m)$$

$$\mathbf{E} = \mathbf{E}_0 e^{\pm \alpha \vec{a}_\gamma \cdot \vec{r}} e^{\pm j \beta \vec{a}_\gamma \cdot \vec{r}}$$

## 13. Classificação dos Meios

1. Dielétricos:  $\frac{\sigma}{\omega \varepsilon} < \frac{1}{100}$

2. Quase condutores:  $\frac{1}{100} < \frac{\sigma}{\omega \varepsilon} < 100$

3. Condutores:  $\frac{\sigma}{\omega \varepsilon} > 100$

tangente de perdas -  $\tan \theta = \frac{\sigma}{\omega \varepsilon}$

profundidade de penetração -  $\delta = \frac{1}{\alpha}$

impedância dos meios condutores -  $Z_c = \frac{E}{H} = \frac{j \omega \mu}{\gamma} \Omega$ .

$$Z_c = \sqrt{\frac{j \omega \mu}{\sigma + j \omega \varepsilon}} \quad Z_c = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\left[1 + \left(\frac{\sigma}{\omega \varepsilon}\right)^2\right]^{1/4}} e^{j(1/2) \tan^{-1}(\sigma/\omega \varepsilon)}$$

## 14. Onda Plana - Incidência normal

coeficiente de transmissão -  $T = \frac{\mathbf{E}_t}{\mathbf{E}_i} = \frac{2 Z_2}{Z_2 + Z_1}$

coeficiente de reflexão -  $\Gamma = \frac{\mathbf{E}_r}{\mathbf{E}_i} = \frac{Z_2 - Z_1}{Z_2 + Z_1}$

$$\vec{P}_m^i = \Re e \left[ \frac{1}{Z_1} \right] \frac{|\mathbf{E}_0|^2}{2} \vec{a}_z e^{-2 \alpha_1 z}$$

$$\begin{aligned} \vec{P}_m^r &= \Re e \left[ \frac{1}{Z_1} \right] \frac{-|\Gamma|^2 |\mathbf{E}_0|^2}{2} \vec{a}_z e^{+2 \alpha_1 z} \\ &= -|\Gamma|^2 P_m^i \vec{a}_z e^{+2 \alpha_1 z} \end{aligned}$$

$$\begin{aligned} \vec{P}_m^t &= \Re e \left[ \frac{1}{Z_2} \right] \frac{|T|^2 |\mathbf{E}_0|^2}{2} \vec{a}_z e^{-2 \alpha_2 z} \\ &= \frac{\Re e \left[ \frac{1}{Z_2} \right]}{\Re e \left[ \frac{1}{Z_1} \right]} |T|^2 P_m^i \vec{a}_z e^{-2 \alpha_2 z} \end{aligned}$$

## 15. Onda Plana - Incidência Oblíqua

$$\mathbf{E} = \mathbf{E}_\perp + \mathbf{E}_\parallel \quad \mathbf{H} = \mathbf{H}_\perp + \mathbf{H}_\parallel$$

lei de Snell da reflexão -  $\theta_i = \theta_r$

lei de Snell da refração -  $\frac{\sin \theta_i}{\sin \theta_t} = \frac{\gamma_2}{\gamma_1}$

lei de Snell da refração -  $\frac{\sin \theta_i}{\sin \theta_t} = \frac{k_2}{k_1} = \sqrt{\frac{\mu_2 \varepsilon_2}{\mu_1 \varepsilon_1}}$

$$\Gamma_\perp = \frac{\mathbf{E}_\perp^r}{\mathbf{E}_\perp^i} = \frac{Z_2 \cos \theta_i - Z_1 \cos \theta_t}{Z_2 \cos \theta_i + Z_1 \cos \theta_t}$$

$$T_\perp = \frac{\mathbf{E}_\perp^t}{\mathbf{E}_\perp^i} = \frac{2 Z_2 \cos \theta_i}{Z_2 \cos \theta_i + Z_1 \cos \theta_t}$$

$$\Gamma_\parallel = \frac{\sqrt{\frac{\mu_2}{\varepsilon_2}} \cos \theta_i - \sqrt{\frac{\mu_1}{\varepsilon_1}} \cos \theta_t}{\sqrt{\frac{\mu_2}{\varepsilon_2}} \cos \theta_i + \sqrt{\frac{\mu_1}{\varepsilon_1}} \cos \theta_t}$$

$$T_\parallel = \frac{2 \sqrt{\frac{\mu_2}{\varepsilon_2}} \cos \theta_i}{\sqrt{\frac{\mu_2}{\varepsilon_2}} \cos \theta_i + \sqrt{\frac{\mu_1}{\varepsilon_1}} \cos \theta_t}$$

$$\Gamma_\perp = \frac{\cos \theta_i - \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \sqrt{1 - \left(\frac{\varepsilon_1}{\varepsilon_2}\right) \sin^2 \theta_i}}{\cos \theta_i + \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \sqrt{1 - \left(\frac{\varepsilon_1}{\varepsilon_2}\right) \sin^2 \theta_i}}$$

$$T_\perp = \frac{2 \cos \theta_i}{\cos \theta_i + \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \sqrt{1 - \left(\frac{\varepsilon_1}{\varepsilon_2}\right) \sin^2 \theta_i}}$$

$$\Gamma_\parallel = \frac{\mathbf{E}_\parallel^r}{\mathbf{E}_\parallel^i} = \frac{-Z_1 \cos \theta_i + Z_2 \cos \theta_t}{Z_1 \cos \theta_i + Z_2 \cos \theta_t}$$

$$T_\parallel = \frac{\mathbf{E}_\parallel^t}{\mathbf{E}_\parallel^i} = \frac{2 Z_2 \cos \theta_i}{Z_1 \cos \theta_i + Z_2 \cos \theta_t}$$

$$\Gamma_\parallel = \frac{-\sqrt{\frac{\mu_1}{\varepsilon_1}} \cos \theta_i + \sqrt{\frac{\mu_2}{\varepsilon_2}} \cos \theta_t}{\sqrt{\frac{\mu_1}{\varepsilon_1}} \cos \theta_i + \sqrt{\frac{\mu_2}{\varepsilon_2}} \cos \theta_t}$$

$$T_\parallel = \frac{2 \sqrt{\frac{\mu_2}{\varepsilon_2}} \cos \theta_i}{\sqrt{\frac{\mu_1}{\varepsilon_1}} \cos \theta_i + \sqrt{\frac{\mu_2}{\varepsilon_2}} \cos \theta_t}$$

$$\Gamma_\parallel = \frac{-\cos \theta_i + \sqrt{\frac{\varepsilon_1}{\varepsilon_2}} \sqrt{1 - \left(\frac{\varepsilon_1}{\varepsilon_2}\right) \sin^2 \theta_i}}{\cos \theta_i + \sqrt{\frac{\varepsilon_1}{\varepsilon_2}} \sqrt{1 - \left(\frac{\varepsilon_1}{\varepsilon_2}\right) \sin^2 \theta_i}}$$

$$T_\parallel = \frac{2 \sqrt{\frac{\varepsilon_1}{\varepsilon_2}} \cos \theta_i}{\cos \theta_i + \sqrt{\frac{\varepsilon_1}{\varepsilon_2}} \sqrt{1 - \left(\frac{\varepsilon_1}{\varepsilon_2}\right) \sin^2 \theta_i}}$$

ângulo de Brewster -  $\theta_\beta = \tan^{-1} \left( \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \right)$

reflexão total -  $\theta_c = \sin^{-1} \left( \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \right)$