Technical Documentation for CAAL Version 1.0

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Introduction

Modeling the growth of fish is a key component of most fish stock assessments and can strongly influence the estimated spawning biomass and exploitation levels. Growth is most often estimated by fitting a von Bertalanffy growth model (VBGM) to data consisting of age-length pairs collected from fisheries, with the assumption that each length observation is a random sample for a given age. This assumption can be approximated by randomly sampling the population, but most growth studies do not randomly sample the population and instead sample by size class due to logistical and cost reasons, which violates the assumption. A recent study proposed an alternative method (approximate length-conditional), which instead assumes each age observation is a random sample at length (Piner et al., 2015). The "conditional age at length" (CAAL) or "length-conditional" method is typically used within an assessment model because the age structure of the sampled population is required for the method. However, Piner et al. (2015) instead approximates the length conditional method by assuming an equilibrium population age structure with a constant total mortality rate and estimating the VBGM parameters outside an assessment model. Simulations by Piner et al. (2015) showed that using the approximate length conditional method results in unbiased VBGM parameter estimates when the samples are length-stratified while the traditional method results in biased estimates. In addition, the estimates were robust to small errors in the assumed mortality rate. The CAAL approach was subsequently applied to North Pacific albacore tuna, albeit with some modifications in the code to allow for more flexible binning structure and robust parameterization, which resulted in less biased estimates of length at age, especially for the youngest and oldest ages (Xu et al. 2015).

In CAAL package, we developed a user-friendly tool for biologists supporting stock assessments to estimate the parameters of growth models using a CAAL method. We also extended VBGM to several alternative common used growth models (Gompertz, inverse logistic, and Richards models).

Basic Equation

Growth models

The traditional VBGM model (option 1) is $L_t = L_\infty[1 - e^{-K(t-t_0)}]$,

where L_t is body fork length (FL cm), L_∞ (cm FL) is the mean asymptotic length, K (year^{-1}) is the Brody growth parameter, and t_0 (year^{-1}) is the length at age-0. L_∞, K and t_0 are estimable parameters.

The Gompertz model (option 2) is $L_t = L_\infty e^{-\frac{1}{K}e^{-K(t-t_0)}}$, where all parameters are the same as VBGM.

The inverse logistic model (option 3) is $L_t = \frac{\alpha}{1+\beta e^{-Kt}}$, where $\alpha, \beta,$ and K are parameters.

The Richard model (option 4) is $L_t = L_\infty[1 + \frac{1}{p}e^{-K(t-t_0)}]^{-p}$, where L_∞, K, t_0 and p are estimable parameters are parameters.

Objective function

The age-length pairs are structured as a data matrix $n_{l,a}$ in the length-conditional approach, which is the number of samples at length l and age a. Assuming that the distribution of ages at a given length is random and should therefore follow a multinomial distribution, the sub-total log-likelihood for $n_{l,a}$ given the growth parameters ($\hat{\theta}$) is calculated as $\ln L\left(\hat{\emptyset}|n\right) \sim \sum_l \sum_a n_{l,a} \ln\left(\hat{P}(a|l)\right)$, where $\left(\hat{P}(a|l)\right)$ is the expected proportion of age at a given length.

In many cases, observations are conducted in different years by different fleets. We further the objective function to account for these year and fleet effects. The total log-likelihood function is $L = \sum_{year} \sum_{fleet} \ln L\left(\hat{\emptyset}|n\right)$

Reference

Piner, K.R., H.-H. Lee, M.N. Maunder, 2015. Evaluation of using random-at-length observations and an equilibrium approximation of the population age structure in fitting the von Bertalanffy growth function. Fisheries Research, in press.

Xu, Y., S.L.H. Teo, K.R. Piner, K.-S. Chen, R.J.D. Wells, 2015. Using an approximate length-conditional approach to estimate von Bertalanffy growth parameters of North Pacific albacore (*Thunnus alalunga*). Fisheries Research, in press.

Appendix: Source Code for CAAL

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