Case study: Spanish Flu in London, Birmingham, and Liverpool

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Objectives

1. To explore the use of POMP models in the context of meta-populations and multiple waves.
2. To illustrate the use of POMP models in a more complex setting.
3. To demonstrate the use of diagnostic probes for model criticism.
4. To provide an example that can be modified to apply similar approaches to other outbreaks of emerging infectious diseases.

Emerging infectious disease pandemic I

- The 1918 Spanish flu pandemic was one of the deadliest pandemics in human history.
- The pandemic lasted from January 1918 to December 1920, spreading to nearly every part of the world.
- The pandemic was caused by the H1N1 influenza A virus.
- The pandemic was responsible for the deaths of an estimated 50 million people worldwide.

Emerging infectious disease pandemic II

Key questions included:

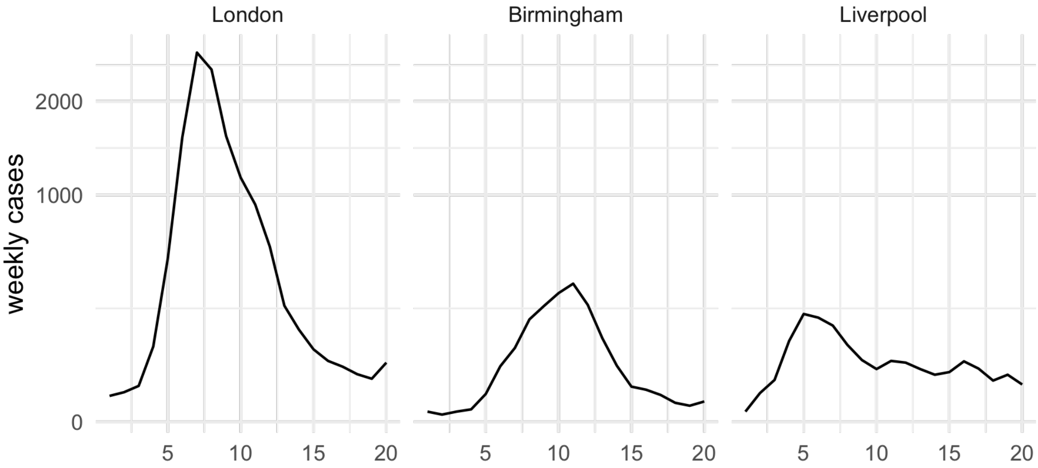
1. How fast will the pandemic unfold?
2. What are the common and differentiated characteristics of the pandemic in different cities?
3. How people will correspond to the pandemic?

Weekly reported data I

```r
dat <- read_csv("1918flu_3cities_1wave.csv")
head(dat)
```

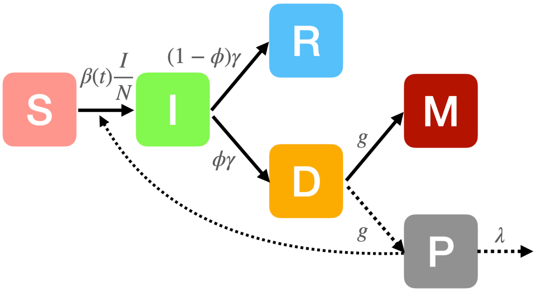
```
# A tibble: 6 x 5
  London Birmingham Liverpool  week date
   <dbl>      <dbl>     <dbl> <dbl> <date>
1     13          2         2     1 1918-09-21
2     17          1        16     2 1918-09-28
3     25          2        34     3 1918-10-05
4    110          3       127     4 1918-10-12
5    519         15       226     5 1918-10-19
6   1579         60       211     6 1918-10-26
```

Weekly reported data II



SIR model with behavioral response I

- Social distancing is a common public health intervention.
- We model the effect of social distancing as a reduction in the transmission rate.
- We incorporate demographic and climatic features to model the spread of the disease.
- The SIR model with behavioral response is given by:



SIR model with behavioral response II

$$\frac{\mathrm{d}S}{\mathrm{d}t} = -\beta(t)\, S\, \frac{I}{N}$$
$$\frac{\mathrm{d}I}{\mathrm{d}t} = \beta(t)\, S\, \frac{I}{N} - \gamma\, I$$
$$\frac{\mathrm{d}R}{\mathrm{d}t} = (1-\phi)\,\gamma\, I$$
$$\frac{\mathrm{d}D}{\mathrm{d}t} = \phi\,\gamma\, I$$
$$\frac{\mathrm{d}M}{\mathrm{d}t} = g\, D$$
$$\frac{\mathrm{d}P}{\mathrm{d}t} = g\, D - \lambda\, P$$

- S, I, R: the susceptible, infectious, and recovered populations
- D: those who have lost of infectiousness and are progressing to death due to influenza or/and pneumonia
- M: those who have died
- P: general public's risk perception based on recent influenza deaths
- N: $N = S + I + R + D + M$, the constant total population size

SIR model with behavioral response III

The time-dependent transmission rate $\beta(t)$ consists of two components:

$$\beta(t) = \beta_0 \cdot \left(1 - \frac{P(t)}{N}\right)^{\kappa} \tag{1}$$

- β_0: the baseline transmission rate
- $\left(1 - \frac{P(t)}{N}\right)^{\kappa}$: represent the effect of reactive social distancing on transmission rate based on the public's risk perception $P(t)$

The number of reported cases C follows a Negative Binomial distribution, in London, Birmingham, and Liverpool, respectively, with ratio ρ and over-dispersion parameter k, given the cumulative number of cases H:

$$C \sim \mathrm{NegBin}(\rho H, k)$$

SIR model with behavioral response IV

- Fixed parameters:
  - The total population sizes for London, Birmingham, and Liverpool are fixed at 4484523, 919444, and 802940, respectively
  - γ^{-1}: the average infectious period, fixed at 4 days
  - g^{-1}: the mean time from loss-of-infectiousness to death, fixed at 8 days
- Parameters to be estimated, initial conditions:
  - η_L, η_B, η_L: the initial fraction of susceptible individuals for London, Birmingham, and Liverpool, respectively
  - ψ_L, ψ_B, ψ_L: the initial fraction of infectious individuals for London, Birmingham, and Liverpool, respectively
- Parameters to be estimated, common features:
  - β_0: the baseline transmission rate
  - κ: the exponent of the social distancing effect
  - λ: the rate at which the public's risk perception decays

The implementation in pomp: the state process for meta-populations

```
sir_meta <- Csnippet("
  double *S = &S1, *I = &I1, *R = &R1, *D = &D1, *M = &M1;
  double *P = &P1, *H = &H1; int N[3] = {N1, N2, N3};
  double Beta, dN_SI, dN_IRD, dN_IR, dN_ID, dN_DM, dN_P;
  for (int i = 0; i < 3; i++) {
    Beta = beta0*pow(1-P[i]/N[i],kappa);
    dN_SI = rbinom(S[i],1-exp(-Beta*I[i]/N[i]*dt));
    dN_IRD = rbinom(I[i],1-exp(-gamma*dt));
    dN_IR = nearbyint((1-phi)*dN_IRD); dN_ID = nearbyint(phi*dN_IRD);
    dN_DM = rbinom(D[i],1-exp(-g*dt));
    dN_P = rbinom(P[i],1-exp(-lambda*dt));
    S[i] -= dN_SI; I[i] += dN_SI - dN_IRD; R[i] += dN_IR;
    D[i] += dN_ID - dN_DM; M[i] += dN_DM; P[i] += dN_DM - dN_P;
    H[i] += dN_IRD;}
")
```

The implementation in pomp: the initial conditions

```
sir_meta_rinit <- Csnippet("
  double *S = &S1, *I = &I1, *R = &R1, *D = &D1, *M = &M1;
  double *P = &P1, *H = &H1; int N[3] = {N1, N2, N3};
  double eta[3] = {eta1, eta2, eta3};
  double psi[3] = {psi1, psi2, psi3};
  for (int i = 0; i < 3; i++) {
    S[i] = nearbyint(N[i]*eta[i]);
    I[i] = nearbyint(N[i]*psi[i]);
    R[i] = nearbyint(N[i]*(1-eta[i]-psi[i]));
    D[i] = M[i] = P[i] = H[i] = 0;
  }
")
```

The implementation in pomp: the measurements

```
sir_meta_dmeas <- Csnippet("
  double lik1, lik2, lik3;
  lik1 = (ISNA(London)) ? dnbinom_mu(London,rho*H1,k,1) : 0;
  lik2 = (ISNA(Birmingham)) ? dnbinom_mu(Birmingham,rho*H2,k,1) : 0;
  lik3 = (ISNA(Liverpool)) ? dnbinom_mu(Liverpool,rho*H3,k,1) : 0;
  lik = lik1 + lik2 + lik3;
  lik = (give_log) ? lik : exp(lik);
")

sir_meta_rmeas <- Csnippet("
  London = rnbinom_mu(k,rho*H1);
  Birmingham = rnbinom_mu(k,rho*H2);
  Liverpool = rnbinom_mu(k,rho*H3);
")
```

The implementation in pomp: build the model I

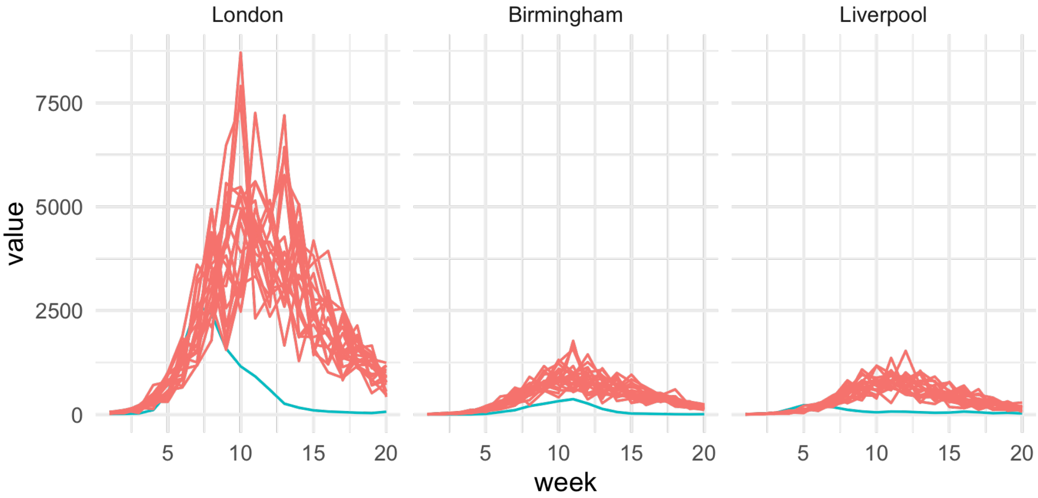
```
dat |> select(-date) |>
  pomp(
    times = "week", t0 = 0,
    rprocess=euler(sir_meta,delta.t=1/7),
    rinit=sir_meta_rinit, rmeasure=sir_meta_rmeas,
    dmeasure=sir_meta_dmeas, accumvars = sprintf("H%d",1:3),
    statenames=c(sprintf("S%d",1:3),sprintf("I%d",1:3),
      sprintf("R%d",1:3), sprintf("D%d",1:3),sprintf("M%d",1:3),
      sprintf("P%d",1:3),sprintf("H%d",1:3)),
    paramnames=c(
      "beta0","kappa","gamma","phi","g","lambda","rho","k",
      sprintf("N%d",1:3),sprintf("eta%d",1:3),sprintf("psi%d",1:3))
  ) -> pomp_meta
```

The implementation in pomp: build the model II

```r
params <- c(beta0 = 5, kappa = 5, gamma = 1/4, phi = 0.0119, g = 1/8,
  lambda = 1/2, rho = 0.05, k = 10, N1 = 4484523, N2 = 919444,
  N3 = 802940, eta1 = 0.2, eta2 = 0.2, eta3 = 0.2,
  psi1 = 0.0005, psi2 = 0.0005, psi3 = 0.0005)

pomp_meta |>
  simulate(
    params = params, nsim=20,format="data.frame",include.data=TRUE
  ) |>
  select(week,.id,London,Birmingham,Liverpool) |>
  reshape2::melt(id.vars = c("week",".id")) |>
  ggplot(aes(x=week, y=value, group=.id, color=.id=="data")) +
  geom_line() + facet_wrap(variable ~ .) +
  theme_minimal() + guides(color="none")
```

The implementation in `pomp`: build the model III



License, acknowledgments, and links

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- The materials build on previous versions of this course and related courses.
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- Produced with R version 4.3.2 and pomp version 5.10.
- Compiled on 2024-07-24.

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R code for this lesson