

Convolution:- for two sequence $x[n]$ and $h[n]$, Convolution is defined as,

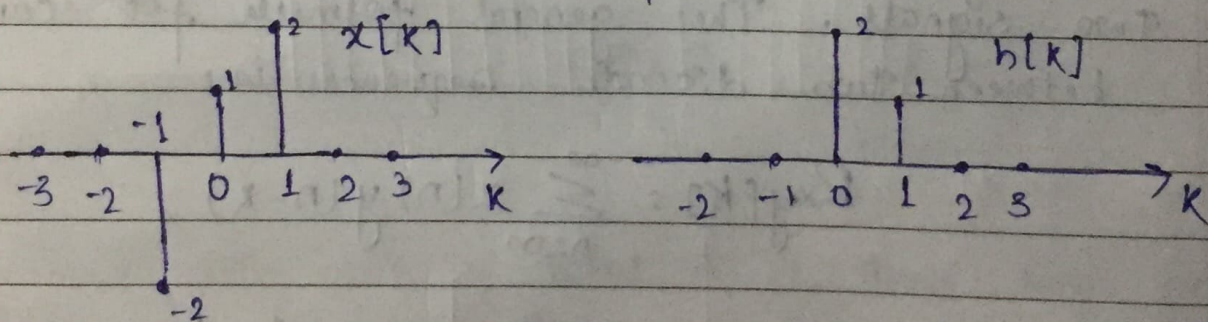
$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

In any LTI system, with the knowledge of input and impulse response, we can determine system output with convolution.

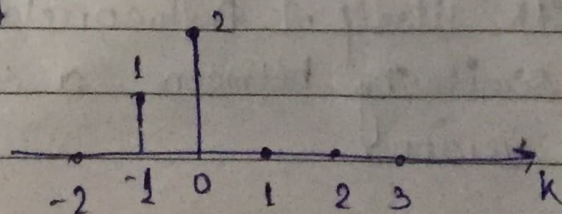
If impulse response of an LTI system is known as $h[n]$, the output $y[n]$ is convolution of input $x[n]$ and impulse response $h[n]$.

Steps for finding convolution:-

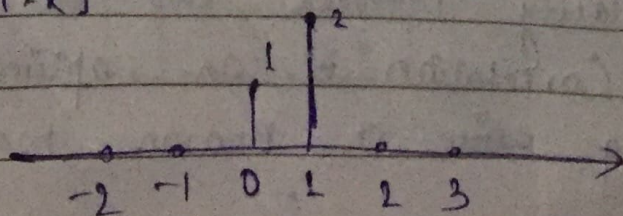
Consider two sequences $x[n]$ and $h[n]$. Plot these functions as functions of k , i.e. $x[k]$ and $h[k]$.



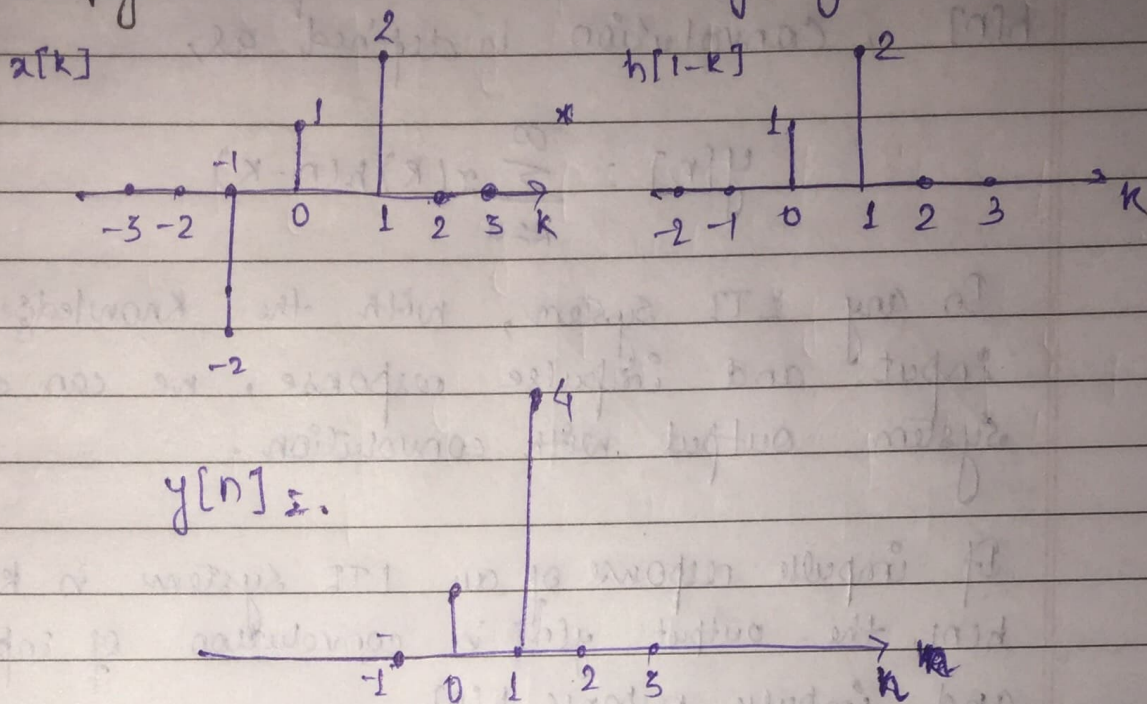
1. Invert $h[k]$ about $k=0$ to obtain $h[-k]$



2. Obtain $h[n-k]$ by shifting $h[-k]$ by n .
eg: $n=1 \Rightarrow h[1-k]$



3. Multiply $x[k]$ with $h[1-k]$ to get $y[1]$



Correlation:-

Correlation is a measure of similarity between two signals. The general formula for correlation between two discrete sequences is,

$$R_{xy}[k] = \sum_{n=-\infty}^{\infty} x(n) y(n-k)$$

There are two types of correlation:-

1. AutoCorrelation: It is defined as correlation of a signal with itself. Autocorrelation is a measure of similarity between a signal and its time delayed version.

2. Cross Correlation: It is defined as a measure of similarity between two different signals. Correlation is an optimal technique for detecting ~~noise~~ a known waveform in random

noise. Using correlation to detect a waveform is frequently called matched filtering.

Mathematically, steps for correlation is identical, except for signal inside convolution machine is flipped left-to-right.

Hence, ~~convolution~~
Correlation can also be written as,

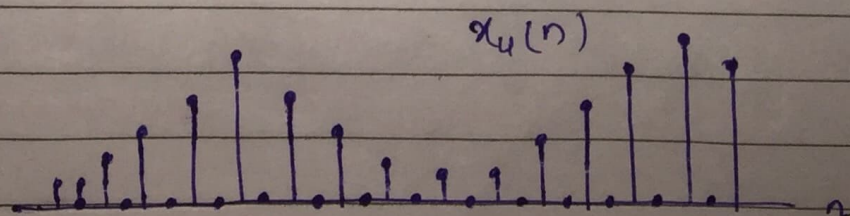
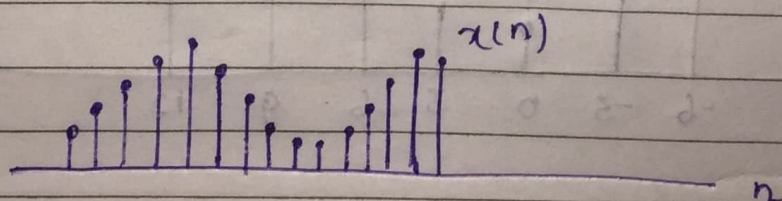
$$R_{xy}[k] = x[k] * h[-k]$$

Upsampling:- The process of converting a sequence into a higher equivalent sampling rate is referred to as Upsampling or Interpolation.

In Upsampling by an integer factor $L > 1$, $L-1$ equidistant zero-valued samples are inserted between each two consecutive samples of the input signal $x(n)$ to generate an output signal $x_u(n)$, such that,

$$x_u(n) = \begin{cases} x(n/L) & , n = 0, \pm L, \pm 2L, \dots \\ 0 & , \text{otherwise} \end{cases}$$

The sampling rate of $x_u(n)$ is L times larger than that of original signal $x(n)$.

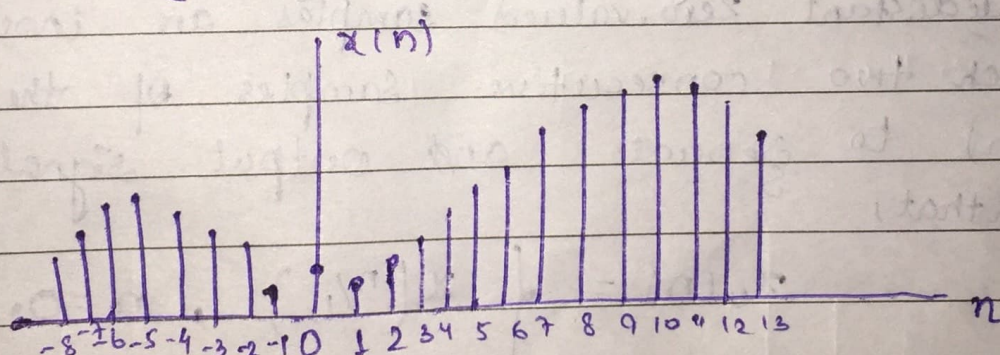


Down-Sampling:- The operation of extracting every N_s^{th} sample is commonly referred to as decimation.

Thus, the decimation operation by an integer factor $N_s > 1$ on a signal $x(n)$ consists of keeping every N_s^{th} sample of $x(n)$ and removing $N_s - 1$ in between samples, such that,

$$x_d(n) = x(N_s \cdot n)$$

This results in a sequence $x_d(n)$ whose sampling rate is $\frac{1}{N_s}$ of that of $x(n)$.



for $N_s = 3$.

