Decimation: Low Pass filter followed by downsampler is called as Decimator.

The job of LPF is to prevent aliasing. Hence, it is also called as anti-aliasing filter.

Cutoff frequency of LPF is (π/M).

When $M=2$, then the LPF is also known as Half Band filter (HBF) with cutoff frequency $\pi/2$.

The operation of extracting every M^{th} sample is called as Down Sampling.

$$y[n] = x[Mn] \quad \text{is downsampling.}$$

To prevent aliasing in downsampling, signal bandwidth must be less than $\frac{2\pi}{M}$, where M is down sampling factor.

This is why we use low pass filter of cutoff frequency π/M before downsampling, to bandlimit the signal such that no aliasing occurs.

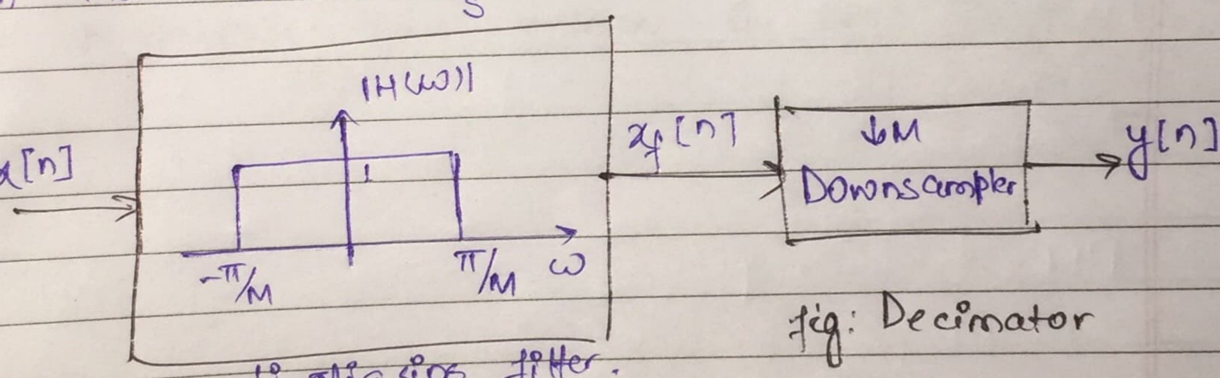

fig: Decimator

Interpolation:- Upsampler followed by Low Pass filter is known as interpolator.

The job of LPF is to remove unwanted image of $X(e^{j\omega})$. Hence, it is known as anti-imaging filter.

Cutoff frequency of LPF is π/L.

When $L=2$, the LPF is also known as Half Band filter (HBF) with cutoff frequency $\pi/2$.

Date __/__/__

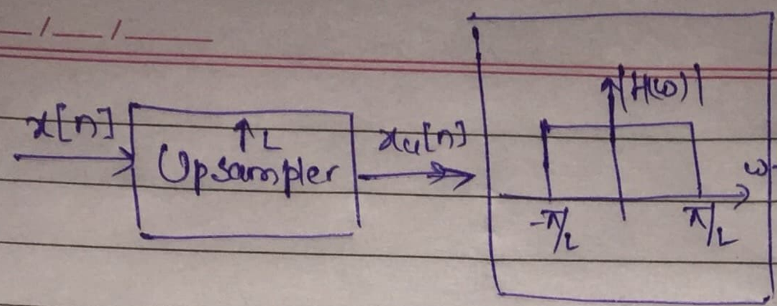

fig: Interpolator.

Anti-imaging filter

Upsampler expands a signal by a factor L, by inserting (L-1) equidistant zero valued samples between two consecutive samples of x[n].

$$x_u(n) = \begin{cases} x(n/L) & , n = 0, \pm L, \pm 2L, \ldots \\ 0 & , \text{otherwise.} \end{cases}$$

Practical Implementation of Decimation & Interpolation.

Decimator: $x_f[n] = x[n] * h[n]$, $y[n] = x_f[2n]$.

let M=2, Suppose, $l_x = 36$, $l_h = 51$.

then $L_{xf} = 36 + 51 - 1 = 86$ (LPF output)

and, $l_y = \frac{86}{2} = 43$. (downsampled)

But we want $l_y = \frac{L_x}{2} = \frac{36}{2} = 18$.

So, we discard first & last $(h_n - 1)/2$ samples from $x_f[n]$, and do downsampling after taking only middle l_x samples of $x_f[n]$.

∴ Then, $l_y = \frac{1}{2}(86 - (51-1)) = 18$.

Interpolator: – $y[n] = x_u[n] * h[n]$, $x_u[n] = x(n/L)$, $n = 0, \pm L, \ldots$
$= 0$, otherwise

Suppose, $L_x = 18$, $L_h = 51$, $L = 2$.

⇒ $l_u = 36$, & $L_y = 36 + 51 - 1 = 86$.

But l_y should be equal to $2 \times l_x$

⇒ Use similar operation and discard first & last $(h_n - 1)/2$ samples from y[n]

⇒ $l_y = 86 - (51 - 1) = 36$.