EE 5837: Principles of Digital Communications
EE 2320: Digital Modulation Techniques

Assignment 4, Prof. Mohammed Zafar Ali Khan, Posted on: 01st October 2020, Due on: 07th October 2020.

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1. Bipolar pulse signals, $s_i(t), (i = 1, 2)$, of amplitude ± 1V and duration of 1 sec are received in the presence of AWGN that has a variance 0.1V^2. Determine the optimum (minimum probability of error) detection threshold γ_0, for matched filter detection if the a priori probabilities are

   (a) $\Pr(s_1) = 0.5$,

   (b) $\Pr(s_1) = 0.7$,

   (c) $\Pr(s_1) = 0.2$.

   Also, explain the effect of a priori probabilities on the value of γ_0.

2. In a binary antipodal signalling scheme, the signals are given by

$$s_1(t) = -s_2(t) = \begin{cases} \frac{2At}{T}, & 0 \le t \le T/2 \\ 2A\left(1 - \frac{t}{T}\right), & T/2 \le t \le T \\ 0, & \text{otherwise} \end{cases}$$

   The channel is AWGN and where the noise has the power spectral density $G_n(f) = \frac{N_0}{2}$. The two signals have prior probabilities p_1 and $p_2 = 1 - p_1$.

   (a) Determine the structure of the optimal receiver.

   (b) Determine an expression for the bit error probability

   (c) Plot the bit error probability as a function of $0 \le p_1 \le 1$

3. The $16-$QAM signal constellation shown in Fig. 1 is an international standard for telephone-line modems (called V.29). Determine the optimum decision boundaries for the detector, assuming that the SNR is sufficiently high so that errors only occur between adjacent points.

4. Determine the bit rate that can be transmitted through a 4KHz voice-band telephone (band-pass) channel if the following modulation methods are used:

   1. Binary PSK
   2. $4-$PSK(QPSK)
   3. 8-QAM
   4. Binary orthogonal FSK with non-coherent detection
   5. Orthogonal 8 - FSK with non-coherent detection.

   For parts $1-3$ assume that the transmitter pulse shape has a raised cosine spectrum with a 50% roll-off.

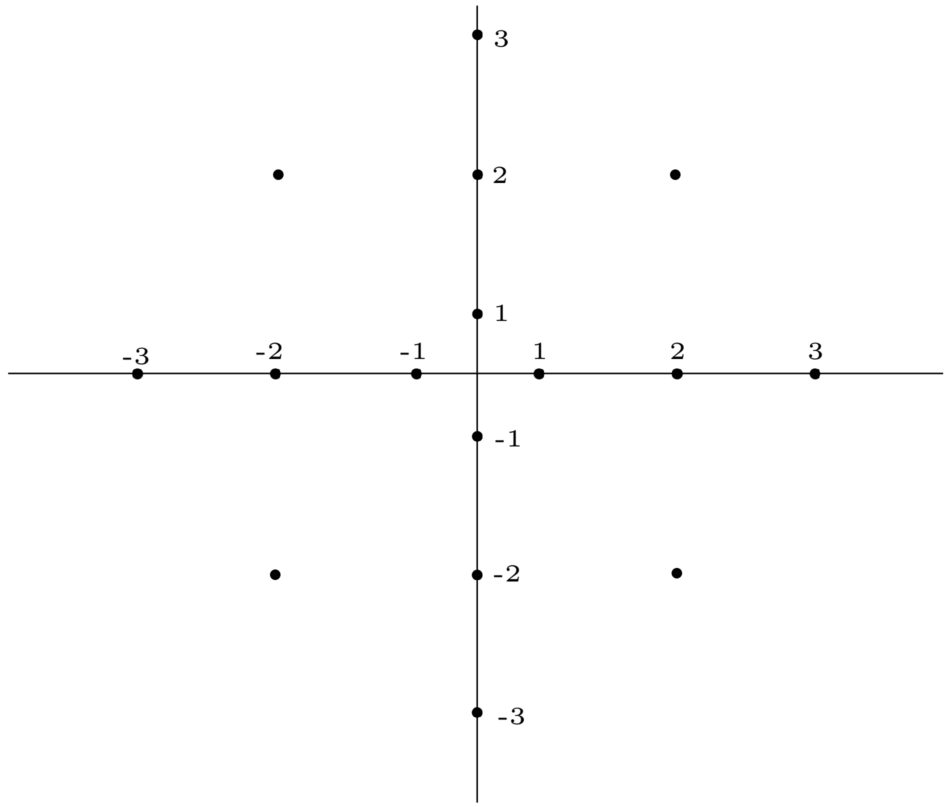

Figure 1: Signal Constellation of V.29 Modem

5. The frequency response characteristics of a lowpass channel can be approximated by

$$H(f) = \begin{cases} 1 + \alpha \cos(2\pi f t_0), & |\alpha| < 1, \quad |f| \leq W \\ 0, & \text{otherwise} \end{cases}$$

where W is the channel bandwidth. An input signal $s(t)$ whose spectrum is bandlimited to WHz is passed through the channel.

1. Show that
$$y(t) = s(t) + \frac{\alpha}{2}\left[s(t - t_0) + s(t + t_0)\right]$$
The channel produces a pair of echoes.
2. Suppose that the received signal $y(t)$ is passed through a filter matched to $s(t)$. Determine the output of the matched filter at $t = kT, k = 0, \pm 1, \pm 2, \cdots$, where T is the symbol duration.
3. What is the ISI pattern resulting from the channel if $t_0 = T$?

6. Consider a BPSK system with equally likely waveforms $s_1(t) = \cos(\omega_0 t)$ and $s_2(t) = -\cos(\omega_0 t)$. Assume that the received $E_b/N_0 = 9.6$dB, giving rise to a bit error probability of 10^{-5}, when the synchronization is perfect. Consider that carrier recovery with the PLL suffers some fixed error ϕ associated with the phase estimate, so that the reference signals are expressed as $\cos(\omega_0 t + \phi)$ and $-\cos(\omega_0 t + \phi)$.

(a) How badly does the bit-error probability degrade when $\phi = 25$ degrees?

(b) How large a phase error would cause the bit-error probability to degrade to 10^{-3}?

7.
   - An M-ary PSK, ISI-free system is to operate with 2^k PSK symbols over a 120[kHz] channel. The minimum required bit rate is 900[kbps]. What minimum SNR is required to maintain reception without a P_b no worse than 10^{-6}?
   - Repeat the above task for an M-ary QAM system, recalculating the new value for the minimum required SNR to maintain reception with a P_b no worse than 10^{-6} and comment on this new result.

8. Use Matlab for the following excercise. We will do simulaion for 3 well known modulation technniques for BER. The baseband equivalent representation is given by $y_k = a_k + v_k$, where a_k and y_k are the baseband equilvalent transmitted and received signal, with v_k being noise. Do stepwise as follows in Matlab:

   1. Generate random binary data (of length 100000).
   2. Map the data to the BPSK signal constellation.
   3. Add AWGN noise to this signal. Vary the noise variance to have SNR range between [-5dB, 10 dB].
   4. The noisy signal will then be used to detect the transmitted bits as per threshold based rule.
   5. Compare the detecded bits with the transmitted ones and plot the BER.

   Do the same for 4−FSK and 16QAM constellations.