EE 5837: Principles of Digital Communications

Assignment 8 - Linear Block Codes, Prof. Mohammed Zafar Ali Khan, Posted on: 17th November 2020, Due on: 24th November 2020.

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1. Calculate the improvement in probability of message error relative to an uncoded transmission for a $(24, 12)$ double-error-correcting linear block code. Assume that coherent BPSK modulation is used and that the received $\frac{E_b}{N_0} = 10\text{dB}$. (i.e here we will compare the two systems at the same $\frac{E_b}{N_0}$.

2. The telephone company uses a "best-of-three" encoder for some of its digital data channels. In this system every data bit is repeated three times, and at the receiver, a majority vote decides the value of each data bit. If the uncoded probability of bit error is .001, calculate the decoded bit-error probability when using such a best-of-three code. Here the uncoded system has excessive bit rate capacity but the error performance is to poor. We are looking for a simple technical solution to simply increase the bit error probability. Hence the $\frac{E_c}{N_0}$ in the coded case will be the same as $\frac{E_b}{N_0}$ in the uncoded case.

3. Consider a (7,4) code whose generator matrix is given by .

$$\mathbf{G} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

   (1) Find all the codewords of the code.
   (2) What is the error-correcting capability of the code?
   (3) What is the error-detecting capability of the code?
   (4) Find $\mathbf{H}$, the parity-check matrix of the code.
   (5) Construct the syndrome table for the code.
   (6) Compute the syndrome for the received vector 1101101. Is this a valid vector? If not, what was the most probable sent message?

4. BPSK is used to transmit data in an AWGN channel. To obtain error free transmission we use a (15,7) block code in combination with ARQ (automatic repeat request), which means retransmission until a code word is received correctly. The block code is only used for error detection and you can assume that it detects all errors. The data rate is 10[kbps] and the average $\frac{E_b}{N_0} = 5[\text{dB}]$

   (1) What is the word error probability without block code and ARQ (block length = 7 bits)?
   (2) What is the throughput of information bits if we use the block code and ARQ?

5. A (15,5) cyclic code has a generator polynomial as follows: $g(x) = 1 + x + x^2 + x^5 + x^8 + x^{10}$.

   (1) Find the code polynomial (in systematic form) for the message $m(x) = 1 + x^2 + x^4$.

(2) Is $V(x) = 1 + x^4 + x^6 + x^8 + x^{14}$ a code polynomial in this system? Justify your answer.

6. For an (n, k) LBC over GF(p), show that the number of rows in a standard array is p^{n-k}.

7. (Sphere packing). Consider a $Q-$ ary alphabet and consider a (n, k) LBC. We shall compute the Hamming bound, which is a bound on the code rate $R = \dfrac{k}{n}$. This bound is computed by considering the ratio of the total volume of a sphere to the volume of a smaller sphere of a given radius. Volume of the sphere is defined as the number of possible $Q-$ary elements inside the sphere. The question we ask is how many smaller spheres can be packed inside the larger sphere without the smaller spheres overlapping each other. Assume that code corrects t errors.

   A code corrects t errors if and only if spheres of radius t around codewords do not overlap. Therefore
   $$Q^k = \text{number of codewords} \leq \frac{\text{volume of space}}{\text{volume of sphere of radius } t}$$
   . Compute the the volume of the sphere of radius t. Use this to show that
   $$R \leq 1 - \frac{1}{n} \log_Q \left(1 + \binom{n}{1}(Q-1) + \binom{n}{2}(Q-1)^2 + \cdots + \binom{n}{t}(Q-1)^t \right)$$
   What is the result for binary alphabet?

8. In this question, you will do a Matlab excersie. Take an image of your choice. For simplicity convert it to a black and white image. Pass it through a channel. Encode it using a $(7, 4)$ Hamming code. The channel is such that the probability of bit error is p. Take $p = \{0.001, 0.01, 0.1, 0, 5, 0.9, 0.99, 0.999\}$. Add noise to the image according to p. Decode the noisy image after correcting errors wherever possible. Reconstruct the image. Compute the error in reconstruction. Consider the uncoded case. To the raw image data addd noise and just reconstruct the noisy image. Display all three, the original image, the reconstructed image after coding and the noisy uncoded image.. Observe and conclude.