Assignment 1

Rubeena Aafreen (EE20RESCH11012)

Download all python codes from

https://github.com/rubeenaafreen20/EE5609/tree/master/Codes

and latex codes from

https://github.com/rubeenaafreen20/EE5609

1 Problem

A ray of light passing through the point $\begin{pmatrix}1\\2\end{pmatrix}$ reflects on the x-axis at point **A** and the reflected ray passes through the point $\begin{pmatrix}5\\3\end{pmatrix}$. Find the coordinates of **A**.

2 Explanation

Let point **P** be $\begin{pmatrix}1\\2\end{pmatrix}$ and point **Q** be $\begin{pmatrix}5\\3\end{pmatrix}$ Since, point **A** is on x-axis, ts y-coordinate is zero. Assume

$$A = \begin{pmatrix}k\\0\end{pmatrix} \tag{2.0.1}$$

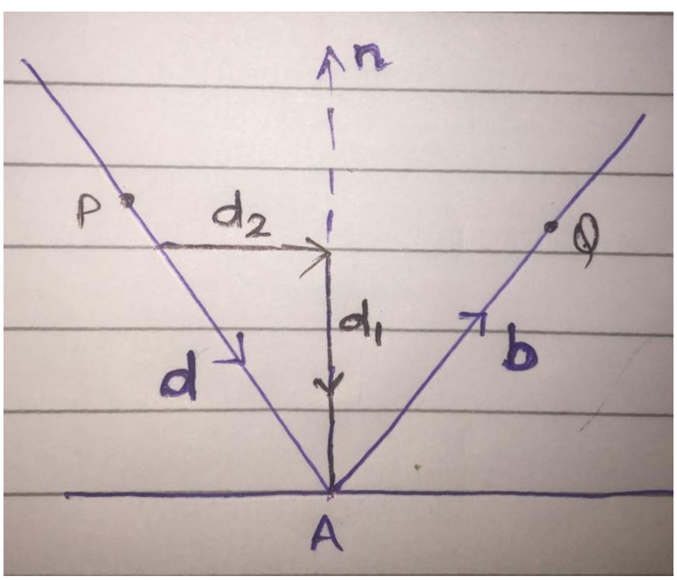

Fig. 0: Incident and reflected ray vectors

Incident vector = **d** = P-A

$$\mathbf{d} = \begin{pmatrix}1-k\\2\end{pmatrix} \tag{2.0.2}$$

Reflected vector = **r** = Q-A

$$\mathbf{r} = \begin{pmatrix}5-k\\3\end{pmatrix} \tag{2.0.3}$$

Normal vector

$$\mathbf{n} = \begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.4}$$

From Fig. 0,
Projection of d in the direction of n is given by

$$\mathbf{d_1} = (\mathbf{d}^T\hat{\mathbf{n}})\hat{\mathbf{n}} \tag{2.0.5}$$

Projection of d in the orthogonal direction is given by

$$\mathbf{d_2} = \mathbf{d} - (\mathbf{d}^T\hat{\mathbf{n}})\hat{\mathbf{n}} \tag{2.0.6}$$

Projection of **b**= -(Projection of **d**)

$$\mathbf{b} = -(\mathbf{d}^T\hat{\mathbf{n}})\hat{\mathbf{n}} + (\mathbf{d} - (\mathbf{d}^T\hat{\mathbf{n}})\hat{\mathbf{n}}) \tag{2.0.7}$$

$$\implies \mathbf{b} = \mathbf{d} - 2(\mathbf{d}^T\hat{\mathbf{n}})\hat{\mathbf{n}} \tag{2.0.8}$$

$$\implies \mathbf{b} = \mathbf{d} - 2\frac{\mathbf{d}^T\mathbf{n}}{\|\mathbf{n}\|^2}\mathbf{n} \tag{2.0.9}$$

$$\left\|\mathbf{n}^2\right\| = 1 \tag{2.0.10}$$

Hence,equation for reflected vector can be written as:

$$\mathbf{b} = \mathbf{d} - 2(\mathbf{d}^T\mathbf{n})\mathbf{n} \tag{2.0.11}$$

3 Solution

Solving the equation (2.0.11):

$$\mathbf{b} = \begin{pmatrix}1-k\\2\end{pmatrix} - 2\left(\begin{pmatrix}k-1 & 2\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}\right)\cdot\begin{pmatrix}1\\0\end{pmatrix} \tag{3.0.1}$$

$$\implies \mathbf{b} = \begin{pmatrix} k-1 \\ 2 \end{pmatrix} \tag{3.0.2}$$

From equations (2.0.3) and (3.0.2), we get

$$k = \frac{13}{5} = 2.6 \tag{3.0.3}$$

4 Verification

Putting k=2.6 in equations (2.0.3) and (3.0.2), the value of calculated reflected vector **d** and given reflected vector **d** are,

$$\mathbf{r} = \begin{pmatrix} 5-2.6 \\ 3 \end{pmatrix} \tag{4.0.1}$$

$$\implies \mathbf{r} = \begin{pmatrix} 2.4 \\ 3 \end{pmatrix} \tag{4.0.2}$$

and

$$\mathbf{b} = \begin{pmatrix} 2.6-1 \\ 2 \end{pmatrix} \tag{4.0.3}$$

$$\implies \mathbf{b} = \begin{pmatrix} 1.6 \\ 2 \end{pmatrix} \tag{4.0.4}$$

Value of k is correct if unit vectors of both **r** and **b** are same.

$$b = \frac{\boldsymbol{b}}{\|\boldsymbol{b}\|} = \frac{2\begin{pmatrix} 0.8 \\ 1 \end{pmatrix}}{\sqrt{(1.6)^2 + (2)^2}} \tag{4.0.5}$$

$$\implies b = 0.78\begin{pmatrix} 0.8 \\ 1 \end{pmatrix} \tag{4.0.6}$$

and

$$r = \frac{\boldsymbol{r}}{\|\boldsymbol{r}\|} = \frac{3\begin{pmatrix} 0.8 \\ 1 \end{pmatrix}}{\sqrt{(2.4)^2 + (3)^2}} \tag{4.0.7}$$

$$\implies b = 0.78\begin{pmatrix} 0.8 \\ 1 \end{pmatrix} \tag{4.0.8}$$

From equations (4.0.6) and (4.0.8), we observe that the solution is verified.