Assignment 3

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Optimization

***Abstract*—This document contains the solution to find the maximum value of given function, subject to given constraints by linear programming.**

Download all python codes from

https://github.com/rubeenaafreen20/EE5600AI−ML/tree/master/Assignment3/Code

Download latex-tikz codes from

https://github.com/rubeenaafreen20/EE5600AI−ML/tree/master/Assignment3

1 Problem

Solve:

$$\max_{\{x\}} Z = \begin{pmatrix} 4 & 1 \end{pmatrix} \mathbf{x} \tag{1.0.1}$$

$$s.t \quad \begin{pmatrix} 1 & 1 \\ 3 & 1 \end{pmatrix} \leq \begin{pmatrix} 50 \\ 90 \end{pmatrix} \tag{1.0.2}$$

$$\mathbf{x} \geq 0 \tag{1.0.3}$$

2 Solution

Adding slack variables to the left side of (1.0.2) and (1.0.3),we get

$$Z - 4x - y = 0 \tag{2.0.1}$$

$$x + y + s_1 = 50 \tag{2.0.2}$$

$$3x + y + s_2 = 90 \tag{2.0.3}$$

Forming simplex tableau,

$$\begin{array}{c} \begin{array}{ccccc} x & y & s_1 & s_2 & b \end{array} \\ \left(\begin{array}{ccccc} 1 & 1 & 1 & 0 & 50 \\ 3 & 1 & 0 & 1 & 90 \\ \hline -4 & -1 & 0 & 0 & 0 \end{array}\right) \end{array} \tag{2.0.4}$$

-4 is the smallest entry in the bottom row. Therefore, we determine that x is the starting variable.
Also, the smallest positive ratio is 30, therefore, we chose s_2 as the departing variable.

Hence, keeping the pivot element as 3, we perform Gauss Jordan elimination,

$$\begin{array}{c} \begin{array}{ccccc} x & y & s_1 & s_2 & b \end{array} \\ \left(\begin{array}{ccccc} 1 & 1 & 1 & 0 & 50 \\ 1 & \frac{1}{3} & 0 & \frac{1}{3} & 30 \\ \hline -4 & -1 & 0 & 0 & 0 \end{array}\right) \end{array} \tag{2.0.5}$$

$$\begin{array}{c} \begin{array}{ccccc} x & y & s_1 & s_2 & b \end{array} \\ \left(\begin{array}{ccccc} 0 & \frac{2}{3} & 1 & \frac{-1}{3} & 20 \\ 1 & \frac{1}{3} & 0 & \frac{1}{3} & 30 \\ \hline 0 & \frac{1}{3} & 0 & \frac{4}{3} & 120 \end{array}\right) \end{array} \tag{2.0.6}$$

Note that x has replaced in the basis column s_2 and the improved solution

$$(x, y, s_1, s_2) = (30, 0, 20, 0) \tag{2.0.7}$$

maximizes Z to value

$$Z = 4(30) + 3(0) \tag{2.0.8}$$

$$Z = 120 \tag{2.0.9}$$

The given problem can be expressed in the form of matrix inequality as:

$$\max_{\{x\}} \mathbf{c}^T \mathbf{x} \tag{2.0.10}$$

$$s.t \quad \mathbf{A}\mathbf{x} \leq \mathbf{b} \tag{2.0.11}$$

$$\mathbf{x} \geq 0 \tag{2.0.12}$$

$$\tag{2.0.13}$$

where

$$\mathbf{c} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \tag{2.0.14}$$

$$\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 3 & 1 \end{pmatrix} \tag{2.0.15}$$

$$\mathbf{b} = \begin{pmatrix} 50 \\ 90 \end{pmatrix} \tag{2.0.16}$$

can be solved using Python. The plot obtained from python is attached below:

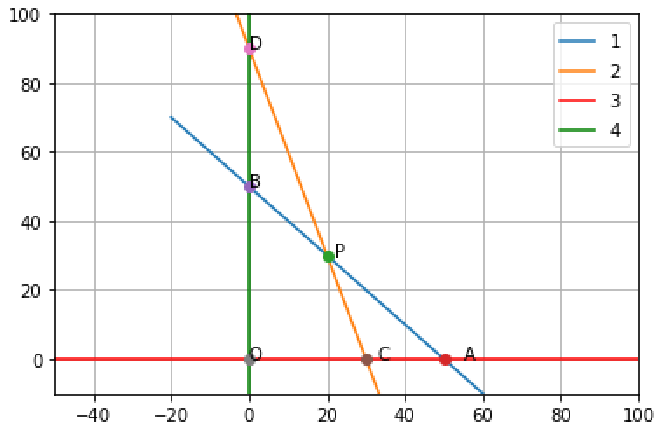

Fig. 1: Plot obtained from python code