Assignment 4

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Gradient Descent

***Abstract*—This document contains the solution to find the maximum / minimum value of given function by gradient descent method**

Download all python codes from

```
https://github.com/rubeenaafreen20/EE5600AI−ML
    /tree/master/Assignment4/Code
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Download latex-tikz codes from

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https://github.com/rubeenaafreen20/EE5600AI−ML
    /tree/master/Assignment4
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1 Problem

Find the maximum and minimum values, if any, of the follwing function

$$f(x) = 9x^2 + 12x + 2 \tag{1.0.1}$$

2 Solution

A function is said to be convex if following inequality is true:

$$\lambda f(x_1) + (1-\lambda)f(x_2) \geq f(\lambda x_1 + (1-\lambda)x_2) \tag{2.0.1}$$

and for $\lambda \in [0,1]$

$$\lambda(9x_1^2 + 12x_1 + 2) + (1-\lambda)((9x_2^2 + 12x_2 + 2) \geq 9(\lambda x_1 + (1-\lambda)x_2)^2 + 12(\lambda x_1 + (1-\lambda)x_2) + 2 \tag{2.0.2}$$

$$x_1^2(9\lambda - 9\lambda^2) + x_2^2(9\lambda - 9\lambda^2) - 2x_1x_2(9\lambda - 9\lambda^2) \geq 0 \tag{2.0.3}$$

$$(9\lambda - 9\lambda^2)(x_1^2 + x_2^2 - 2x_1x_2) \tag{2.0.4}$$

$$9\lambda(1-\lambda)(x_1 - x_2)^2 \geq 0 \tag{2.0.5}$$

Equation (2.0.7) holds true for all $\lambda \in (0,1)$. Hence the given function f(x) is convex.
Fora general quadratic equation

$$f(x) = ax^2 + bx + c \tag{2.0.6}$$

The update equation for gradient descent to find minimum of a function is given by:

$$\lambda_{n+1} = \lambda_n - \mu f'(\lambda_n) \tag{2.0.7}$$

$$= \lambda_n - \mu(2a\lambda_n + b) \tag{2.0.8}$$

In equation (2.0.7) λ_0 is an initial guess and μ is a variable parameter,known as step size λ_{n+1} is the next position. The minus sign refers to the minimization part of gradient descent.
Assume,

$$\lambda_0 = 1 \tag{2.0.9}$$

$$\mu = 0.001 \tag{2.0.10}$$

$$precision = 0.00000001 \tag{2.0.11}$$

$$\implies \lambda_1 = 1 - 0.001(2 \times 9 \times 1 + 12) \tag{2.0.12}$$

$$\implies \lambda_1 = 1 - 0.03 \tag{2.0.13}$$

$$= 0.97 \tag{2.0.14}$$

following the above method, we keep doing iterations until $\lambda_{n+1} - \lambda_n$ becomes less than the value of precision we have chosen.

3 Results

Using python, the results are:

1) The local minimum occurs at -0.666666130125316.
2) The value of f(x) at minima is -1.9999999999974087

Figure 1 shows plot of parabola obtained from python code:

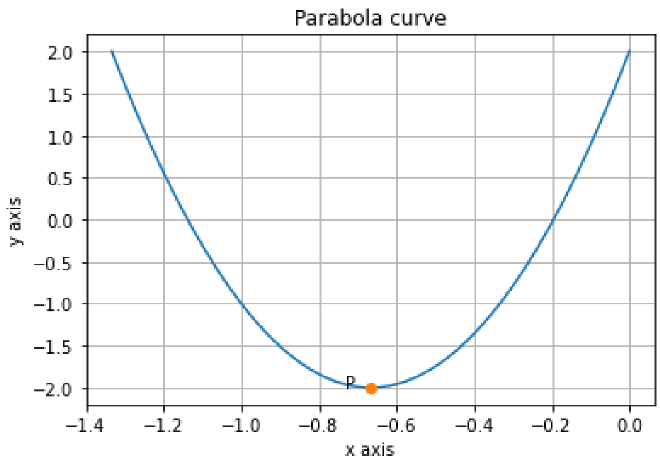

Fig. 1: Plot obtained from python code