Assignment 11

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***Abstract*—This assignment deals with linear functional on a vector space.**

Download all solutions from

https://github.com/rubeenaafreen20/EE5609/tree/master/Assignment11

1 Problem

Let $\mathbf{V}$ be the vector space of $n \times n$ matrices over the field $\mathbb{F}$. If $\mathbf{B}$ is a fixed $n \times n$ matrix, define a function f_B on $\mathbf{V}$ by $f_B(\mathbf{A}) = Tr(\mathbf{B}^T\mathbf{A})$. Show that every linear functional on $\mathbf{V}$ is of the form f_B for some $\mathbf{B}$.

2 Solution

For $\mathbf{A} = (a_{ij}) \in \mathbf{V}$, a linear functional $f : \mathbf{V} \longrightarrow \mathbb{F}$ is defined as:

$$f(\mathbf{A}) = \mathbf{A}^T\mathbf{c} \tag{2.0.1}$$

equation (2.0.1) can be written as,

$$f(\mathbf{A}) = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}a_{ij} \tag{2.0.2}$$

Now let $\mathbf{B} = (b_{ij}) \in \mathbf{V}$ be any $n \times n$ matrix. Then

$$Tr(\mathbf{B}^t\mathbf{A}) = \sum_{i=1}^{n}(\mathbf{B}^t\mathbf{A})_{ii} \tag{2.0.3}$$

$$= \sum_{i=1}^{n}\sum_{k=1}^{n}(b_{ik})^T a_{ki} \tag{2.0.4}$$

$$= \sum_{i=1}^{n}\sum_{k=1}^{n} b_{ki}a_{ki} \tag{2.0.5}$$

From equation (2.0.2) and equation (2.0.5), we get an appropriate matrix $\mathbf{B}$ such that $Tr(\mathbf{B}^T\mathbf{A}) = f$ if

$$c_{ki} = b_{ki} \tag{2.0.6}$$

Hence, Proved.