

Assignment 1

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Download all python codes from

<https://github.com/rubeenaafreen20/EE5609/tree/master/Codes>

and latex codes from

<https://github.com/rubeenaafreen20/EE5609>

$$\frac{(\mathbf{P} - \mathbf{A})^T \mathbf{e}_2}{\|\mathbf{P} - \mathbf{A}\|} = \frac{(\mathbf{Q} - \mathbf{A})^T \mathbf{e}_2}{\|\mathbf{Q} - \mathbf{A}\|} \quad (2.0.7)$$

$$\frac{\mathbf{P}^T \mathbf{e}_2 - \mathbf{A}^T \mathbf{e}_2}{\|\mathbf{P} - \mathbf{A}\|} = \frac{\mathbf{Q}^T \mathbf{e}_2 - \mathbf{A}^T \mathbf{e}_2}{\|\mathbf{Q} - \mathbf{A}\|} \quad (2.0.8)$$

3 SOLUTION

1 PROBLEM

A ray of light passing through the point $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ reflects on the x-axis at point A and the reflected ray passes through the point $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$. Find the coordinates of A.

2 EXPLANATION

Let point P be $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and point Q be $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$ Since, point A is on x-axis, its y-coordinate is zero. Assume

$$A = \begin{pmatrix} k \\ 0 \end{pmatrix} \quad (2.0.1)$$

Incident vector

$$= \mathbf{P} - \mathbf{A} \quad (2.0.2)$$

Reflected vector

$$= \mathbf{Q} - \mathbf{A} \quad (2.0.3)$$

Vector along y-axis

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.4)$$

$$(2.0.5)$$

Vector along x-axis

$$\mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.6)$$

Angle between AP and the x axis = 180° - angle between AQ and the x axis,

$$\frac{\begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} k & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}}{\left\| \begin{pmatrix} 1-k \\ 2 \end{pmatrix} \right\|} = \frac{\begin{pmatrix} 5 & 3 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} k & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}}{\left\| \begin{pmatrix} 5-k \\ 3 \end{pmatrix} \right\|} \quad (3.0.1)$$

$$\Rightarrow \frac{2}{\sqrt{(1-k)^2 + (2)^2}} = \frac{3}{\sqrt{(5-k)^2 + (3)^2}} \quad (3.0.2)$$

$$\Rightarrow 5k^2 + 22k - 91 = 0 \quad (3.0.3)$$

Solving (3.0.3) we get: $k=2.6, -7$

Since, incident ray passes through $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and reflected ray passes through $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$,

k cannot be negative as reflection takes place in first quadrant.

$$k = 2.6 \quad (3.0.4)$$

4 VERIFICATION

Figure plotted using python code:

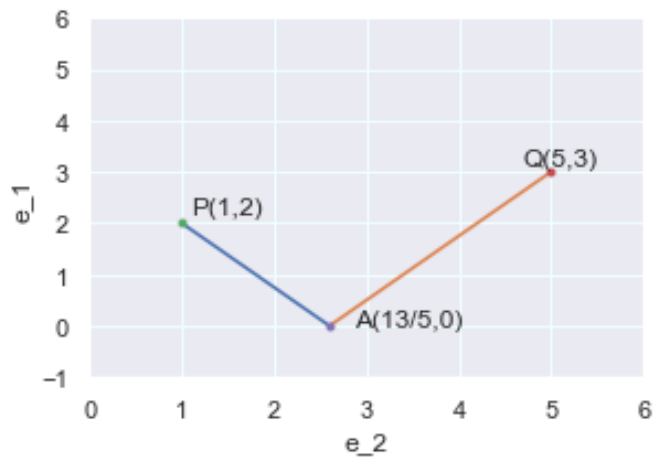


Fig. 0: Incident and reflected ray vectors plotted via Python code