

# Assignment 12

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Download the latex-tikz codes from

<https://github.com/rubeenaafreen20/EE5609/tree/master/Assignment12>

## 1 PROBLEM

(UGC-june2017,74) :

For any  $n \times n$  matrix  $B$ , let  $N(B) = \{X \in \mathbb{R}^n : BX = 0\}$  be the null space of  $B$ . Let  $A$  be a  $4 \times 4$  matrix with  $\dim(N(A - 4I)) = 2$ ,  $\dim(N(A - 2I)) = 1$  and  $\text{rank}(A) = 3$  Which of the following are true?

- 1) 0,2 and 4 are eigenvalues of  $A$
- 2)  $\det(A) = 0$
- 3)  $A$  is not diagonalizable
- 4)  $\text{trace}(A) = 8$

## 2 SOLUTION

Given	$A$ is a $4 \times 4$ matrix. $\dim(N(A - 4I)) = 2$ , $\dim(N(A - 2I)) = 1$ , and $\text{rank}(A) = 3$
Eigenvalues of a matrix	<p>The number <math>\lambda</math> is an eigenvalue of a matrix <math>A</math> if and only if <math>A - \lambda I</math> is singular, i.e. <math> A - \lambda I  = 0</math></p> <p>For <math>\lambda = 2</math>          Given that <math>\dim(N(A - 2I)) = 1</math>  <math>\implies \text{rank}(A - 2I) = 4 - 1 = 3</math>  <math>\implies (A - 2I)</math> is not a full rank matrix          Therefore <math> A - 2I  = 0</math>  <math>\implies 2</math> is an eigenvalue of <math>A</math> with multiplicity 2.</p> <p>Similarly, for <math>\lambda = 4</math>          Given that <math>\dim(N(A - 4I)) = 2</math>  <math>\implies \text{rank}(A - 4I) = 4 - 2 = 2</math>  <math>\implies (A - 4I)</math> is not a full rank matrix          Therefore <math> A - 4I  = 0</math>  <math>\implies 4</math> is an eigenvalue of <math>A</math> with multiplicity 1.</p>

	<p>For <math>\lambda = 0</math>  Given that <math>\text{rank}(A) = 3</math>  <math>\implies A</math> is not a full rank matrix  Therefore <math> A  = 0</math>  <math>\implies 0</math> is an eigenvalue of <math>A</math> with multiplicity 1.</p>
Determinant	<p>Given that <math>\text{rank}(A) = 3</math>  <math>\implies A</math> is not a full rank matrix  Therefore <math> A  = 0</math></p>
Diagonalizability	<p>An <math>n \times n</math> matrix <math>A</math> is diagonalizable if and only if <math>A</math> has <math>n</math> linearly independent eigen vectors.  <math>\text{rank}(A) + \text{nullity}(A) = n</math>  <math>\implies</math> for <math>\lambda = 0</math>,  <math>\text{nullity}(A - \lambda I) = \text{nullity}(A) = 4 - 3 = 1</math>  <math>\implies</math> There exists only one linearly independent eigen vector corresponding to 0 eigen value  Thus, matrix <math>A</math> is not diagonalizable.</p>
Trace	<p><math>\text{Trace}(A) = \text{sum of eigen values}</math>  <math>\implies \text{Trace}(A) = 0 + 2 + 2 + 4 = 8</math></p>
Conclusion	<p>Option (1), (2) and (4) are correct</p>