

# Assignment 1

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Download all python codes from

<https://github.com/rubeenaafreen20/EE5609/tree/master/Codes>

and latex codes from

<https://github.com/rubeenaafreen20/EE5609>

## 1 PROBLEM

A ray of light passing through the point  $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$  reflects on the x-axis at point **A** and the reflected ray passes through the point  $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$ . Find the coordinates of **A**.

## 2 EXPLANATION

Let point **P** be  $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$  and point **Q** be  $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$  Since, point **A** is on x-axis, its y-coordinate is zero. Assume

$$A = \begin{pmatrix} k \\ 0 \end{pmatrix} \quad (2.0.1)$$

$$\text{Incident vector} = \mathbf{P} - \mathbf{A} \quad (2.0.2)$$

$$\text{Reflected vector} = \mathbf{Q} - \mathbf{A} \quad (2.0.3)$$

$$\text{Vector along y-axis} = \mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.4)$$

$$\text{Vector along x-axis} = \mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.5)$$

From Fig. 0,

Angle between AP and the x axis =  $180^\circ$  - angle between AQ and the x axis,

$$\frac{(\mathbf{P} - \mathbf{A})^T \mathbf{e}_2}{\|\mathbf{P} - \mathbf{A}\|} = \frac{(\mathbf{Q} - \mathbf{A})^T \mathbf{e}_2}{\|\mathbf{Q} - \mathbf{A}\|} \quad (2.0.6)$$

$$\frac{\mathbf{P}^T \mathbf{e}_2 - \mathbf{A}^T \mathbf{e}_2}{\|\mathbf{P} - \mathbf{A}\|} = \frac{\mathbf{Q}^T \mathbf{e}_2 - \mathbf{A}^T \mathbf{e}_2}{\|\mathbf{Q} - \mathbf{A}\|} \quad (2.0.7)$$

## 3 SOLUTION

$$\frac{\begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} k & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}}{\left\| \begin{pmatrix} 1-k \\ 2 \end{pmatrix} \right\|} = \frac{\begin{pmatrix} 5 & 3 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} k & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}}{\left\| \begin{pmatrix} 5-k \\ 3 \end{pmatrix} \right\|} \quad (3.0.1)$$

$$\Rightarrow \frac{2}{\sqrt{(1-k)^2 + (2)^2}} = \frac{3}{\sqrt{(5-k)^2 + (3)^2}} \quad (3.0.2)$$

$$\Rightarrow 5k^2 + 22k - 91 = 0 \quad (3.0.3)$$

Solving the equation (3.0.3) we get:  $k=2.6, -7$

Since, incident ray passes through  $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$  and reflected ray passes through  $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$ ,

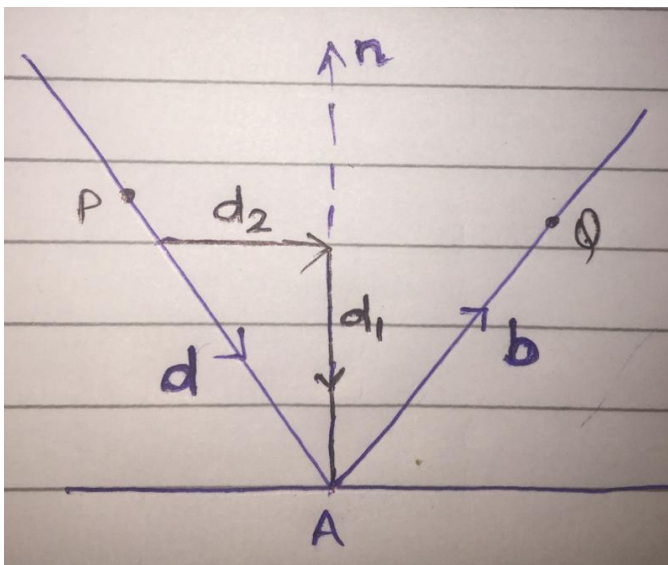


Fig. 0: Incident and reflected ray vectors

$k$  cannot be negative as reflection takes place in first quadrant.

$$k = 2.6 \quad (3.0.4)$$

#### 4 VERIFICATION

Figure plotted using python code:

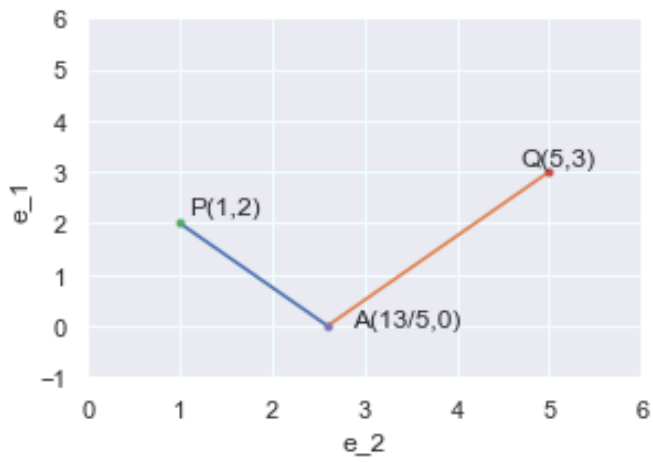


Fig. 0: Incident and reflected ray vectors plotted via Python code