

Assignment 11

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Abstract—This assignment deals with linear functional on a vector space.

Download all solutions from

<https://github.com/rubeenaafreen20/EE5609/tree/master/Assignment11>

1 PROBLEM

Let \mathbf{V} be the vector space of $n \times n$ matrices over the field \mathbb{F} . If \mathbf{B} is a fixed $n \times n$ matrix, define a function f_B on \mathbf{V} by $f_B(\mathbf{A}) = \text{Tr}(\mathbf{B}^T \mathbf{A})$. Show that every linear functional on \mathbf{V} is of the form f_B for some \mathbf{B} .

2 SOLUTION

For $\mathbf{A} = (a_{ij}) \in \mathbf{V}$, a linear functional $f_B : \mathbf{V} \rightarrow \mathbb{F}$ is defined as:

$$f_B(\mathbf{A}) = \mathbf{A}^T \mathbf{c} \quad (2.0.1)$$

equation (2.0.1) can be written as,

$$f_B(\mathbf{A}) = \sum_{i=1}^n \sum_{j=1}^n c_{ij} a_{ij} \quad (2.0.2)$$

Now,

$$\text{Tr}(\mathbf{B}^T \mathbf{A}) = \sum_{i=1}^n (\mathbf{B}^T \mathbf{A})_{ii} \quad (2.0.3)$$

$$= \sum_{j=1}^n \sum_{i=1}^n (b_{ji})^T a_{ij} \quad (2.0.4)$$

$$= \sum_{j=1}^n \sum_{i=1}^n b_{ij} a_{ij} \quad (2.0.5)$$

Let $c_{ij} = b_{ij}$, from equation (2.0.2) and equation (2.0.5),

$$\text{Tr}(\mathbf{B}^T \mathbf{A}) = f_B \quad (2.0.6)$$

Hence, Proved.