Assignment 12

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Download the latex-tikz codes from

https://github.com/rubeenaafreen20/EE5609/tree/master/Assignment12

1 Problem

(UGC-june2017,74) :
For any $n \times n$ matrix B, let $N(B) = \{X \in \mathbb{R}^n : BX = 0\}$ be the null space of B. Let A be a 4×4 matrix with $dim(N(A-2I)) = 2, dim(N(A-4I)) = 1$ and $rank(A) = 3$ Which of the following are true?

1) 0,2 and 4 are eigenvalues of A
2) determinant(A)=0
3) A is not diagonalizable
4) trace(A)=8

2 Solution

Given	A is a 4×4 matrix. $dim\left(N\left(A-2I\right)\right) = 2,$ $dim\left(N\left(A-4I\right)\right) = 1,$ and $rank\left(A\right) = 3$
Eigenvalues of a matrix	The number λ is an eigenvalue of a matrix A if and only if$A - \lambda I$ is singular, i.e. $\left\|A - \lambda I\right\| = 0$ For $\lambda = 2$ Given, $dim\left(N\left(A-2I\right)\right) = 2$ $\implies nullity(A-2I) = 2$ $rank(A) + nullity(A) = n$ $\implies rank\left(A-2I\right) = 4-2 = 2$ $\implies (A-2I)$ is not a full rank matrix Therefore $\left\|A-2I\right\| = 0$ Also, $\implies N\left(A-2I\right) = \{X \in \mathbb{R}^4 : (A-2I)X = 0\}$ $\implies (A-2I)X = 0$ gives two eigen vectors $\implies$ 2 is an eigenvalue of A with multiplicity 2. Similarly, for $\lambda = 4$ Given, $dim\left(N\left(A-4I\right)\right) = 1$ $\implies rank\left(A-4I\right) = 4-1 = 3$ $\implies (A-4I)$ is not a full rank matrix

	Therefore $\left\|A - 4I\right\| = 0$ $\implies$ 4 is an eigenvalue of A with multiplicity 1. For $\lambda = 0$ Given that $rank\,(A) = 3$ $\implies$ A is not a full rank matrix Therefore $\left\|A\right\| = 0$ $\implies$ 0 is an eigenvalue of A with multiplicity 1.
Determinant	Given that $rank\,(A) = 3$ $\implies$ A is not a full rank matrix Therefore $\left\|A\right\| = 0$
Diagonalizability	An $n \times n$ matrix A is diagonalizable if and only if A has n linearly independent eigen vectors. $rank\,(A) + nullity\,(A) = n$ $\implies$ for $\lambda = 0$, $nullity\,(A - \lambda I) = nullity\,(A) = 4 - 3 = 1$ $\implies$ There exists only one linearly independent eigen vector corresponding to 0 eigen value Thus, matrix A is not diagonalizable.
Trace	$Trace\,(A)$=sum of eigen values $\implies$ $Trace\,(A) = 0 + 2 + 2 + 4 = 8$
Conclusion	Option (1), (2) and (4) are correct