Assignment 1

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Download all python codes from

https://github.com/rubeenaafreen20/EE5609/tree/master/Codes

and latex codes from

https://github.com/rubeenaafreen20/EE5609

1 Problem

A ray of light passing through the point $\begin{pmatrix}1\\2\end{pmatrix}$ reflects on the x-axis at point **A** and the reflected ray passes through the point $\begin{pmatrix}5\\3\end{pmatrix}$. Find the coordinates of **A**.

2 Explanation

Let point **P** be $\begin{pmatrix}1\\2\end{pmatrix}$ and point **Q** be $\begin{pmatrix}5\\3\end{pmatrix}$ Since, point **A** is on x-axis, ts y-coordinate is zero. Assume

$$A = \begin{pmatrix}k\\0\end{pmatrix} \tag{2.0.1}$$

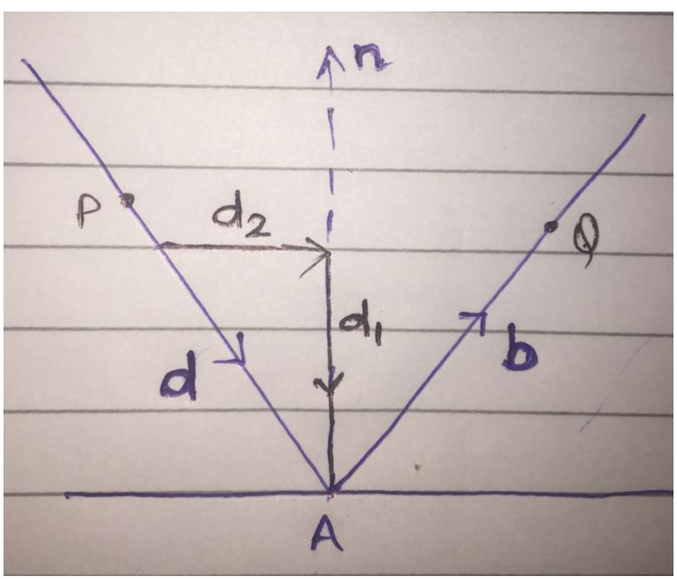

Fig. 0: Incident and reflected ray vectors

Incident vector = $\mathbf{d}$ = P-A

$$\mathbf{d} = \begin{pmatrix}1-k\\2\end{pmatrix} \tag{2.0.2}$$

Reflected vector = $\mathbf{b}$ = Q-A

$$\mathbf{b} = \begin{pmatrix}5-k\\3\end{pmatrix} \tag{2.0.3}$$

Vector along x-axis = $\boldsymbol{a_x}$

$$\boldsymbol{a_x} = \begin{pmatrix}0\\1\end{pmatrix} \tag{2.0.4}$$

From Fig. 0,
Angle between AP and the x axis = 180°- angle between AQ and the x axis,

$$\frac{\mathbf{d}^T\mathbf{a_x}}{\|\mathbf{d}\|} = \frac{\mathbf{b}^T\mathbf{a_x}}{\|\mathbf{a_x}\|} \tag{2.0.5}$$

$$\implies \frac{\begin{pmatrix}1-k & 2\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}}{\sqrt{(1-k)^2+(2)^2}} = \frac{\begin{pmatrix}5-k & 3\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}}{\sqrt{(5-k)^2+(3)^2}}$$

3 Solution

$$\implies \frac{2}{\sqrt{(1-k)^2+(2)^2}} = \frac{3}{\sqrt{(5-k)^2+(3)^2}}$$

$$\implies 5k^2+22k-91 = 0 \tag{3.0.1}$$

Solving the equation (3.0.1) we get: k=2.6, -7

Since, incident ray passes through $\begin{pmatrix}1\\2\end{pmatrix}$ and reflected ray passes through $\begin{pmatrix}5\\3\end{pmatrix}$,
k cannot be negative as reflection takes place in first quadrant.

$$k = 2.6 \tag{3.0.2}$$

4 Verification

Putting k=2.6 in equations (2.0.3) and (3.0.2), the value of incident vector **d** and reflected vector **d** are,

$$\mathbf{d} = \begin{pmatrix} 5-2.6 \\ 3 \end{pmatrix} \tag{4.0.1}$$

$$\implies \mathbf{d} = \begin{pmatrix} 2.4 \\ 3 \end{pmatrix} \tag{4.0.2}$$

and

$$\mathbf{b} = \begin{pmatrix} 1-2.6 \\ 2 \end{pmatrix} \tag{4.0.3}$$

$$\implies \mathbf{b} = \begin{pmatrix} -1.6 \\ 2 \end{pmatrix} \tag{4.0.4}$$

Unit vectors of **d** and **b** are,

$$\hat{\mathbf{d}} = \frac{\boldsymbol{d}}{\|\boldsymbol{d}\|} = \frac{3\begin{pmatrix} 0.8 \\ 1 \end{pmatrix}}{\sqrt{(2.4)^2 + (3)^2}} \tag{4.0.5}$$

$$\implies \hat{\mathbf{d}} = 0.78\begin{pmatrix} 0.8 \\ 1 \end{pmatrix} \tag{4.0.6}$$

$$\hat{\mathbf{b}} = \frac{\boldsymbol{b}}{\|\boldsymbol{b}\|} = \frac{2\begin{pmatrix} -0.8 \\ 1 \end{pmatrix}}{\sqrt{(1.6)^2 + (2)^2}} \tag{4.0.7}$$

$$\implies \hat{\mathbf{b}} = 0.78\begin{pmatrix} -0.8 \\ 1 \end{pmatrix} \tag{4.0.8}$$

From equations (4.0.6) and (4.0.8), we observe that the solution is verified.