

Assignment 1

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Download all python codes from

<https://github.com/rubeenaafreen20/EE5609/tree/master/Codes>

and latex codes from

<https://github.com/rubeenaafreen20/EE5609>

1 PROBLEM

A ray of light passing through the point $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ reflects on the x-axis at point **A** and the reflected ray passes through the point $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$. Find the coordinates of **A**.

2 EXPLANATION

Let point **P** be $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and point **Q** be $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$. Since, point **A** is on x-axis, its y-coordinate is zero. Assume

$$A = \begin{pmatrix} k \\ 0 \end{pmatrix} \quad (2.0.1)$$

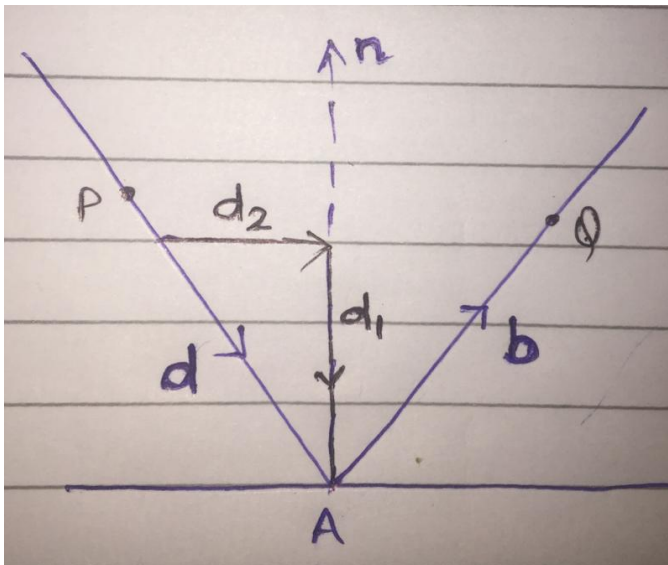


Fig. 0: Incident and reflected ray vectors

Incident vector = $\mathbf{d} = \mathbf{P} - \mathbf{A}$

$$\mathbf{d} = \begin{pmatrix} 1-k \\ 2 \end{pmatrix} \quad (2.0.2)$$

Reflected vector = $\mathbf{r} = \mathbf{Q} - \mathbf{A}$

$$\mathbf{r} = \begin{pmatrix} 5-k \\ 3 \end{pmatrix} \quad (2.0.3)$$

Normal vector

$$\mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.4)$$

From Fig. 0,

Projection of \mathbf{d} in the direction of \mathbf{n} is given by

$$\mathbf{d}_1 = (\mathbf{d}^T \mathbf{n}) \mathbf{n} \quad (2.0.5)$$

Projection of \mathbf{d} in the orthogonal direction is given by

$$\mathbf{d}_2 = \mathbf{d} - (\mathbf{d}^T \mathbf{n}) \mathbf{n} \quad (2.0.6)$$

$$\mathbf{d} + \mathbf{b} = 2\mathbf{d}_2 \quad (2.0.7)$$

$$\Rightarrow \mathbf{d} + \mathbf{b} = 2\mathbf{d} - 2(\mathbf{d}^T \mathbf{n}) \mathbf{n} \quad (2.0.8)$$

$$\Rightarrow \mathbf{b} = \mathbf{d} - 2(\mathbf{d}^T \mathbf{n}) \mathbf{n} \quad (2.0.9)$$

3 SOLUTION

Solving the equation (2.0.11):

$$\mathbf{b} = \begin{pmatrix} 1-k \\ 2 \end{pmatrix} - 2 \left((k-1 \quad 2) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right) \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (3.0.1)$$

$$\Rightarrow \mathbf{b} = \begin{pmatrix} k-1 \\ 2 \end{pmatrix} \quad (3.0.2)$$

From equations (2.0.3) and (3.0.2), we get

$$k = \frac{13}{5} = 2.6 \quad (3.0.3)$$

4 VERIFICATION

Putting $k=2.6$ in equations (2.0.3) and (3.0.2), the value of calculated reflected vector \mathbf{d} and given reflected vector \mathbf{d} are,

$$\mathbf{r} = \begin{pmatrix} 5 - 2.6 \\ 3 \end{pmatrix} \quad (4.0.1)$$

$$\Rightarrow \mathbf{r} = \begin{pmatrix} 2.4 \\ 3 \end{pmatrix} \quad (4.0.2)$$

and

$$\mathbf{b} = \begin{pmatrix} 2.6 - 1 \\ 2 \end{pmatrix} \quad (4.0.3)$$

$$\Rightarrow \mathbf{b} = \begin{pmatrix} 1.6 \\ 2 \end{pmatrix} \quad (4.0.4)$$

Value of k is correct if unit vectors of both \mathbf{r} and \mathbf{b} are same.

$$b = \frac{\mathbf{b}}{\|\mathbf{b}\|} = \frac{2 \begin{pmatrix} 0.8 \\ 1 \end{pmatrix}}{\sqrt{(1.6)^2 + (2)^2}} \quad (4.0.5)$$

$$\Rightarrow b = 0.78 \begin{pmatrix} 0.8 \\ 1 \end{pmatrix} \quad (4.0.6)$$

and

$$r = \frac{\mathbf{r}}{\|\mathbf{r}\|} = \frac{3 \begin{pmatrix} 0.8 \\ 1 \end{pmatrix}}{\sqrt{(2.4)^2 + (3)^2}} \quad (4.0.7)$$

$$\Rightarrow b = 0.78 \begin{pmatrix} 0.8 \\ 1 \end{pmatrix} \quad (4.0.8)$$

From equations (4.0.6) and (4.0.8), we observe that the solution is verified.