Assignment 13

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Download the latex-tikz codes from

https://github.com/rubeenaafreen20/EE5609/tree/master/Assignment13

1 Problem

Suppose that $\mathbf{A}$ is a 2×2 matrix with real entries which is symmetric ($\mathbf{A}^t = \mathbf{A}$). Prove that $\mathbf{A}$ is similar over $\mathbb{R}$ to a diagonal matrix.

2 Solution

Given	$\mathbf{A}$ is a 2×2 matrix with real entries and $\mathbf{A}$ is symmetric ($\mathbf{A}^t = \mathbf{A}$)
To Prove	$\mathbf{A}$ is similar to diagonal matrix over $\mathbb{R}$
Theory	$\mathbf{A}$ is similar to diagonal matrix $\mathbf{\Lambda}$ if $\exists$ an invertible matrix $\mathbf{P}$ such that: $\mathbf{A} = \mathbf{P}\mathbf{\Lambda}\mathbf{P}^{-1}$
Proof	Let $\mathbf{A} = \begin{pmatrix} a & c \\ c & b \end{pmatrix}, a, b, c \in \mathbb{R}$ Characteristic polynomial: $p(t) = \left\lvert \mathbf{A} - \lambda \mathbf{I} \right\rvert$ $p(t) = \begin{vmatrix} a-t & c \\ c & b-t \end{vmatrix}$ $\implies p(t) = t^2 - (a+b)t + ab - c^2 = 0$ Roots of p(t) are eigenvalues of $\mathbf{A}$ Discriminant of $p(t)$ is given by $(a+b)^2 - 4(ab - c^2) = a^2 + b^2 - 2ab + c^2$ $= (a-b)^2 + 4c^2 > 0$ We observe that the above equation has positive discriminant, hence λ has real values

	Eigen vectors are obtained by: $(\mathbf{A} - \lambda\mathbf{I})\,\mathbf{X} = 0$ Since, eigen values are real $\implies$ eigen vectors $\mathbf{v_1}$ and $\mathbf{v_2}$ are linearly independent. Let $\mathbf{P} = \begin{pmatrix}\mathbf{v_1} & \mathbf{v_2}\end{pmatrix}$ $\implies$ $\mathbf{AP} = \begin{pmatrix}\lambda_1\mathbf{v_1} & \lambda_2\mathbf{v_2}\end{pmatrix}$ $\implies$ $\mathbf{AP} = \mathbf{P\Lambda}$, where $\mathbf{\Lambda} = \begin{pmatrix}\lambda_1 & 0 \\ 0 & \lambda_2\end{pmatrix}$ $\implies$ $\mathbf{A} = \mathbf{P\Lambda P}^{-1}$ Therefore, $\mathbf{A}$ is similar to diagonal matrix $\mathbf{\Lambda}$ Hence, Proved.