Assignment 4

Rubeena Aafreen

The link to the solution is

https://github.com/rubeenaafreen20/EE5609/tree/master/Assignment4

***Abstract*—This documents solves a problem based on conics.**

1 Problem

Find the points on the curve $\mathbf{x}^T \begin{pmatrix} \frac{1}{9} & 0 \\ 0 & \frac{1}{16} \end{pmatrix} \mathbf{x} = 1$ at which the tangents are

a) parallel to x-axis
b) parallel to y-axis

2 Solution

General equation of conics is

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \tag{2.0.1}$$

Comparing with the equation given,

$$\mathbf{V} = \begin{pmatrix} \frac{1}{9} & 0 \\ 0 & \frac{1}{16} \end{pmatrix} \tag{2.0.2}$$

$$\mathbf{u} = \mathbf{0} \tag{2.0.3}$$

$$f = -1 \tag{2.0.4}$$

$$\left|\mathbf{v}\right| = \left|\begin{pmatrix} \frac{1}{9} & 0 \\ 0 & \frac{1}{16} \end{pmatrix}\right| > 0 \tag{2.0.5}$$

$\because |\mathbf{V}| > 0$, the given equation is of ellipse.
a)The tangents are parallel to the x-axis, hence, their direction and normal vectors, $\mathbf{m_1}$ and $\mathbf{n_1}$ are respectively,

$$\mathbf{m_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.6}$$

$$\mathbf{n_1} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.7}$$

For an ellipse, given the normal vector $\mathbf{n}$, the tangent points of contact to the ellipse are given by

$$\mathbf{q} = (\kappa \mathbf{n} - \mathbf{u}) = \kappa \mathbf{n} \tag{2.0.8}$$

where

$$\kappa = \pm \sqrt{\frac{\mathbf{u^T V^{-1} u} - f}{\mathbf{n^T V^{-1} n}}} \tag{2.0.9}$$

$$= \pm \sqrt{\frac{-f}{\mathbf{n^T V^{-1} n}}} \tag{2.0.10}$$

$$\mathbf{V}^{-1} = \begin{pmatrix} 9 & 0 \\ 0 & 16 \end{pmatrix} \tag{2.0.11}$$

$$\kappa_1 = \pm \sqrt{\frac{-(-1)}{\begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 9 & 0 \\ 0 & 16 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}}} \tag{2.0.12}$$

$$\implies \kappa_1 = \pm \sqrt{\frac{1}{16}} \tag{2.0.13}$$

$$\implies \kappa_1 = \pm \frac{1}{4} \tag{2.0.14}$$

From (2.0.8) , the point of contact $\mathbf{q_i}$ are,

$$\mathbf{q_1} = \frac{1}{4} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.15}$$

$$= \begin{pmatrix} 0 \\ \frac{1}{4} \end{pmatrix} \tag{2.0.16}$$

$$\mathbf{q_2} = -\frac{1}{4} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.17}$$

$$= \begin{pmatrix} 0 \\ -\frac{1}{4} \end{pmatrix} \tag{2.0.18}$$

b) The tangents are parallel to the y-axis, hence, their direction and normal vectors, $\mathbf{m_2}$ and $\mathbf{n_2}$ are respectively,

$$\mathbf{m_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.19}$$

$$\mathbf{n_2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.20}$$

Using equation (2.0.9), the values of κ for this case are

$$\kappa_2 = \pm\sqrt{\frac{-(-1)}{\begin{pmatrix}1 & 0\end{pmatrix}\begin{pmatrix}9 & 0\\0 & 16\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}}} \tag{2.0.21}$$

$$\implies \kappa_2 = \pm\sqrt{\frac{1}{9}} \tag{2.0.22}$$

$$\implies \kappa_2 = \pm\frac{1}{3} \tag{2.0.23}$$

and from (2.0.8) , the point of contact $\mathbf{q_i}$ are,

$$\mathbf{q_3} = \frac{1}{3}\begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.24}$$

$$= \begin{pmatrix}\frac{1}{3}\\0\end{pmatrix} \tag{2.0.25}$$

$$\mathbf{q_4} = -\frac{1}{3}\begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.26}$$

$$= \begin{pmatrix}-\frac{1}{3}\\0\end{pmatrix} \tag{2.0.27}$$