Assignment 8

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***Abstract*—This document explains solution of a problem based on nullspace.**

Download all solutions from

https://github.com/rubeenaafreen20/EE5609/tree/master/Assignment8

1 Problem

Let $s<n$ and A an $s \times n$ matrix with entries in the field $\mathbb{F}$. Use Theorem 4 to show that there is a non-zero $\mathbf{x}$ in $\mathbb{F}^{n\times 1}$ such that $\mathbf{Ax} = 0$.

2 Explanation

Theorem 4:*Let $\mathbb{V}$ be a vector space which is spanned by a finite set of vectors $\beta_1, \beta_2, ..., \beta_m$. Then any independent set of vectors in $\mathbb{V}$ is finite and contains no more than m elements.*

3 Solution

Let,$\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n$ be columns of matrix A.
$\because$ A is $s \times n$ matrix

$$\implies \alpha_i \in \mathbb{F}^{s\times 1} \tag{3.0.1}$$

$$Given\ \mathbf{x} \in \mathbb{F}^{n\times 1} \tag{3.0.2}$$

$$\implies \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{pmatrix} \tag{3.0.3}$$

Assume $(\alpha_1, \alpha_2, \dots, \alpha_s)$ form a basis for column space of A, i.e space spanned by $(\alpha_1, \alpha_2, \dots, \alpha_n)$.
$\because$ Given that n>s,
Therefore, from **Theorem 4**, $(\alpha_1, \alpha_2, \dots, \alpha_n)$ cannot be linearly independent
Therefore,

$$\mathbf{Ax} = \alpha_1 x_1 + \alpha_2 x_2 \dots \alpha_n x_n \tag{3.0.4}$$

$$= \sum_{i=1}^{s} \alpha_i x_i + \sum_{j=s+1}^{n} \alpha_j x_j \tag{3.0.5}$$

Therefore, nullspace N(A) is non-trivial as

$$\exists x_1, x_2, \dots, x_n \in \mathbb{F}^{n\times 1} \tag{3.0.6}$$

such that $\mathbf{Ax} = 0$