Assignment 10

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***Abstract*—This assignment deals with annihilator of vector space.**

Download all solutions from

https://github.com/rubeenaafreen20/EE5609/tree/master/Assignment10

1 Problem

Let W be the subspace of R^5 which is spanned by the vectors

$$\begin{aligned}\alpha_1 &= \epsilon_1 + 2\epsilon_2 + \epsilon_3, \\ \alpha_2 &= \epsilon_2 + 3\epsilon_3 + 3\epsilon_4 + \epsilon_5, \\ \alpha_3 &= \epsilon_1 + 4\epsilon_2 + 6\epsilon_3 + 4\epsilon_4 + \epsilon_5\end{aligned} \tag{1.0.1}$$

Find a basis for W^0

2 Annihilator

2.1 Definition

If V is a vector space over the field $\mathbb{F}$ and W is a subset of V, the annihilator of W is the set W^0 of linear functionals f on V such that $f(\alpha) = 0$ for every α in W.

2.2 Properties of Annihilator

If f is a linear functional on R^n:

$$f(x_1, x_2, \dots, x_n) = \sum_{j=1}^{n} c_j x_j \tag{2.2.1}$$

Then f is in W^0 if and only if

$$\forall \alpha \in W : f(\alpha) = 0 \iff f = 0 \tag{2.2.2}$$

3 Solution

Let matrix A with row vectors $\alpha_1, \alpha_2, \alpha_3$:

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & 3 & 1 \\ 1 & 4 & 6 & 4 & 1 \end{pmatrix} \tag{3.0.1}$$

Converting into row reduced echelon form,

$$\begin{pmatrix} 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & 3 & 1 \\ 1 & 4 & 6 & 4 & 1 \end{pmatrix} \xrightarrow{rref} \begin{pmatrix} 1 & 0 & 0 & 4 & 3 \\ 0 & 1 & 0 & -3 & -2 \\ 0 & 0 & 1 & 2 & 1 \end{pmatrix} \tag{3.0.2}$$

Given that f is a linear functional on R^5, therefore,

$$f(x_1, \dots, x_5) = \sum_{j=1}^{5} c_j x_j \tag{3.0.3}$$

Then f is in W^0 if and only if,

$$f(\alpha_i) = 0, i = 1, 2, 3 \tag{3.0.4}$$

$$\implies \sum_{j=1}^{5} A_{ij} c_j = 0, 1 \le i \le 3 \tag{3.0.5}$$

Therefore, from equation (3.0.2)

$$c_1 + 4c_4 + 3c_5 = 0, \tag{3.0.6}$$
$$c_2 - 3c_4 - 2c_5 = 0, \tag{3.0.7}$$
$$c_3 + 2c_4 + c_5 = 0 \tag{3.0.8}$$
$$\implies c_1 = -(4c_4 + 3c_5) \tag{3.0.9}$$
$$c_2 = (3c_4 + 2c_5) \tag{3.0.10}$$
$$c_3 = -(2c_4 + c_5) \tag{3.0.11}$$

Let $c_4 = a, c_5 = b$, then

$$c_1 = -(4a + 3b) \tag{3.0.12}$$
$$c_2 = 3a + 2b \tag{3.0.13}$$
$$c_3 = -(2a + b) \tag{3.0.14}$$

therefore,

$$f(x_1, \dots, x_5) = -(4a + 3b)x_1 + (3a + 2b)x_2 - (2a + b)x_3 + ax_4 + bx_5 \tag{3.0.15}$$

Therefore, dimension of W^0 is 2 and a basis $\{f_1, f_2\}$ can be obtained by putting $a = 0, b = 1$ and $a = 1, b = 0$ in equation (3.0.15)

$$f_1(x_1, \dots, x_5) = -3x_1 + 2x_2 - x_3 + x_5 \tag{3.0.16}$$
$$f_2(x_1, \dots, x_5) = -4x_1 + 3x_2 - 2x_3 + x_4 \tag{3.0.17}$$