Assignment 1

Rubeena Aafreen (EE20RESCH11012)

1 PROBLEM

A ray of light passing through the point $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ reflects on the x-axis at point A and the reflected ray passes through the point $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$. Find the coordinates of A.

2 EXPLANATION

Let point P be $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and point Q be $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$ Since, point A is on x-axis, its y-coordinate is zero. Assume

$$A = \begin{pmatrix} k \\ 0 \end{pmatrix} \tag{2.0.1}$$

$$Incident vector = \mathbf{P} - \mathbf{A} \tag{2.0.2}$$

$$Reflected vector = \mathbf{Q} - \mathbf{A} \tag{2.0.3}$$

$$Vector along y-axis = \mathbf{e_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.4}$$

$$Vector along x-axis = \mathbf{e_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.5}$$

Angle between AP and the x axis = 180°- angle between AQ and the x axis,

$$\frac{(\mathbf{P} - \mathbf{A})^\mathbf{T}\mathbf{e_2}}{\|\mathbf{P} - \mathbf{A}\|} = \frac{(\mathbf{Q} - \mathbf{A})^\mathbf{T}\mathbf{e_2}}{\|\mathbf{Q} - \mathbf{A}\|} \tag{2.0.6}$$

$$\frac{\mathbf{P^T}\mathbf{e_2} - \mathbf{A^T}\mathbf{e_2}}{\|\mathbf{P} - \mathbf{A}\|} = \frac{\mathbf{Q^T}\mathbf{e_2} - \mathbf{A^T}\mathbf{e_2}}{\|\mathbf{Q} - \mathbf{A}\|} \tag{2.0.7}$$

3 SOLUTION

$$\frac{\begin{pmatrix} 1 & 2 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} k & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}}{\left\|\begin{pmatrix} 1 - k \\ 2 \end{pmatrix}\right\|} = \frac{\begin{pmatrix} 5 & 3 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} k & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}}{\left\|\begin{pmatrix} 5 - k \\ 3 \end{pmatrix}\right\|} \tag{3.0.1}$$

$$\implies \frac{2}{\sqrt{(1 - k)^2 + (2)^2}} = \frac{3}{\sqrt{(5 - k)^2 + (3)^2}} \tag{3.0.2}$$

$$\implies 5k^2 + 22k - 91 = 0 \tag{3.0.3}$$

Solving (3.0.3) we get: k=2.6, -7

Since, incident ray passes through $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and reflected ray passes through $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$, k cannot be negative as reflection takes place in first quadrant.

$$k = 2.6 \tag{3.0.4}$$
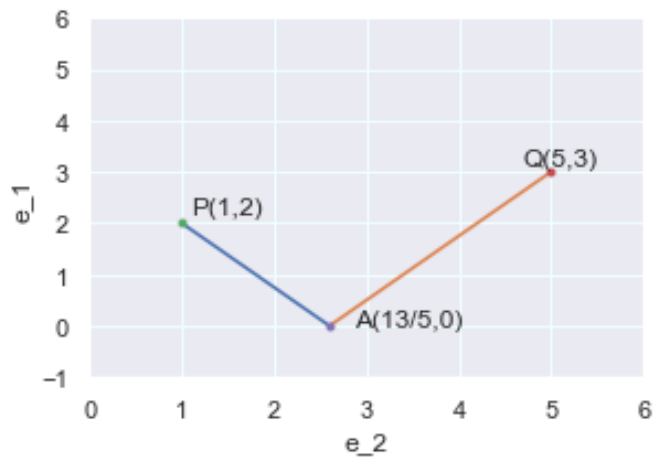
4 VERIFICATION

Figure plotted using python code:

Fig. 0: Incident and reflected ray vectors plotted via Python code