Assignment 15

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Download the latex-tikz codes from

https://github.com/rubeenaafreen20/EE5609/tree/master/Assignment15

1 Problem

Let

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 \\ 2 & -2 & 2 \\ 2 & -3 & 2 \end{pmatrix} \tag{1.0.1}$$

Is $\mathbf{A}$ similar over the field of real numbers to a triangular matrix? If so, find such a triangular matrix.

2 Definitions

Characteristic Polynomial	For an $n \times n$ matrix $\mathbf{A}$, characteristic polynomial is defined by, $$p(x) = \left\lvert x\mathbf{I} - \mathbf{A} \right\rvert$$
Theory	$\mathbf{A}$ is similar to triangular matrix $\mathbf{J}$ if $\exists$ an invertible matrix $\mathbf{P}$ such that $$\mathbf{A} = \mathbf{P}\mathbf{J}\mathbf{P}^{-1}$$

TABLE 1: Definitions

3 Solution

Characteristic polynomial	$p(x) = \lvert x\mathbf{I} - \mathbf{A}\rvert$ $= \begin{vmatrix} x & -1 & 0 \\ -2 & x+2 & -2 \\ -2 & 3 & x-2 \end{vmatrix}$ $= x((x+2)(x-2)+6) + (-2(x-2)+4) + 0$ $= x\left(x^2+2\right) - 2x$ $= x^3$ $\implies \lambda = 0$
$dim(Ker(\mathbf{A} - \lambda\mathbf{I}))$	$(\mathbf{A} - 0\mathbf{I})\,\mathbf{X} = \mathbf{0}$ $\implies \begin{pmatrix} 0 & 1 & 0 \\ 2 & -2 & 2 \\ 2 & -3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ $\implies x = -z, y = 0$ So, $\mathbf{v_1} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$
Find $\mathbf{v_2}$ such that $\mathbf{A}\mathbf{v_2} = \mathbf{v_1}$	$\begin{pmatrix} 0 & 1 & 0 \\ 2 & -2 & 2 \\ 2 & -3 & 2 \end{pmatrix}\mathbf{v_2} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ $\implies \mathbf{v_2} = \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$
Find $\mathbf{v_3}$ such that $\mathbf{A}\mathbf{v_3} = \mathbf{v_2}$	$\begin{pmatrix} 0 & 1 & 0 \\ 2 & -2 & 2 \\ 2 & -3 & 2 \end{pmatrix}\mathbf{v_3} = \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$ $\implies \mathbf{v_3} = \begin{pmatrix} -\frac{3}{2} \\ -1 \\ 0 \end{pmatrix}$
	Let $\mathbf{P} = \left(\mathbf{v_1} \quad \mathbf{v_2} \quad \mathbf{v_3}\right)$ Then,

	$\mathbf{AP} = \begin{pmatrix} \mathbf{Av_1} & \mathbf{Av_2} & \mathbf{Av_3} \end{pmatrix}$ $\implies \mathbf{AP} = \begin{pmatrix} 0 & \mathbf{v_1} & \mathbf{v_2} \end{pmatrix}$ $= \begin{pmatrix} \mathbf{v_1} & \mathbf{v_2} & \mathbf{v_3} \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ $= \mathbf{PJ}$ $\implies \mathbf{AP} = \mathbf{PJ}$ $\implies \mathbf{A} = \mathbf{PJP}^{-1}$
Conclusion	Therefore, **A** is similar to triangular matrix and the triangular matrix is, $\mathbf{J} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

TABLE 2: Checking triangularizability of **A**