

Assignment 7

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Row Reduced Echelon Form

Abstract—This document solves problem based on solution of system of linear equations using Row Reduction

Download all solutions from

<https://github.com/rubeenaafreen20/EE5609/tree/master/Assignment7>

1 PROBLEM

Find all solutions of

$$2x_1 - 3x_2 - 7x_3 + 5x_4 + 2x_5 = -2 \quad (1.0.1)$$

$$x_1 - 2x_2 - 4x_3 + 3x_4 + x_5 = -2 \quad (1.0.2)$$

$$2x_1 - 4x_3 + 2x_4 + x_5 = 3 \quad (1.0.3)$$

$$x_1 - 5x_2 - 7x_3 + 6x_4 + 2x_5 = -7 \quad (1.0.4)$$

2 SOLUTION

The given equations can be written as,

$$\begin{pmatrix} 2 & -3 & -7 & 5 & 2 \\ 1 & -2 & -4 & 3 & 1 \\ 2 & 0 & -4 & 2 & 1 \\ 1 & -5 & -7 & 6 & 2 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -2 \\ -2 \\ 3 \\ 7 \end{pmatrix} \quad (2.0.1)$$

Now, we form the augmented matrix and perform Row reduction,

$$\left(\begin{array}{ccccc|c} 2 & -3 & -7 & 5 & 2 & -2 \\ 1 & -2 & -4 & 3 & 1 & -2 \\ 2 & 0 & -4 & 2 & 1 & 3 \\ 1 & -5 & -7 & 6 & 2 & 7 \end{array} \right) \quad (2.0.2)$$

$$\xleftrightarrow{R_3=R_3-R_1} \left(\begin{array}{ccccc|c} 2 & -3 & -7 & 5 & 2 & -2 \\ 1 & -2 & -4 & 3 & 1 & -2 \\ 0 & 3 & 3 & -3 & -1 & 5 \\ 1 & -5 & -7 & 6 & 2 & 7 \end{array} \right) \quad (2.0.3)$$

$$\xleftrightarrow{R_1=\frac{1}{2}R_1} \left(\begin{array}{ccccc|c} 1 & -\frac{3}{2} & -\frac{7}{2} & \frac{5}{2} & 1 & -1 \\ 1 & -2 & -4 & 3 & 1 & -2 \\ 0 & 3 & 3 & -3 & -1 & 5 \\ 1 & -5 & -7 & 6 & 2 & 7 \end{array} \right) \quad (2.0.4)$$

$$\xleftrightarrow{R_2=R_2-R_1, R_4=R_4-R_1} \left(\begin{array}{ccccc|c} 1 & -\frac{3}{2} & -\frac{7}{2} & \frac{5}{2} & 1 & -1 \\ 0 & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & 0 & -1 \\ 0 & 3 & 3 & -3 & -1 & 5 \\ 0 & -\frac{7}{2} & -\frac{7}{2} & \frac{7}{2} & 1 & -6 \end{array} \right) \quad (2.0.5)$$

$$\xleftrightarrow{R_1=R_1-3R_2} \left(\begin{array}{ccccc|c} 1 & 0 & -2 & 1 & 1 & 2 \\ 0 & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & 0 & -1 \\ 0 & 3 & 3 & -3 & -1 & 5 \\ 0 & -\frac{7}{2} & -\frac{7}{2} & \frac{7}{2} & 1 & -6 \end{array} \right) \quad (2.0.6)$$

$$\xleftrightarrow{R_3=R_3+6R_2, R_4=R_4-7R_2} \left(\begin{array}{ccccc|c} 1 & 0 & -2 & 1 & 1 & 2 \\ 0 & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array} \right) \quad (2.0.7)$$

$$\xleftrightarrow{R_2=-2R_2} \left(\begin{array}{ccccc|c} 1 & 0 & -2 & 1 & 1 & 2 \\ 0 & 1 & 1 & -1 & 0 & 2 \\ 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array} \right) \quad (2.0.8)$$

$$\xleftrightarrow{R_1=R_1+R_3, R_4=R_4+R_3, R_3=-R_3} \left(\begin{array}{ccccc|c} 1 & 0 & -2 & 1 & 0 & 1 \\ 0 & 1 & 1 & -1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right) \quad (2.0.9)$$

So,

$$x_1 - 2x_3 + x_4 = 1 \quad (2.0.10)$$

$$x_2 + x_3 - x_4 = 2 \quad (2.0.11)$$

$$x_5 = 1 \quad (2.0.12)$$

Solving the equations we get,

$$x_1 = 1 + 2x_3 - x_4 \quad (2.0.13)$$

$$x_2 = 2 - x_3 + x_4 \quad (2.0.14)$$

$$x_5 = 1 \quad (2.0.15)$$

which can be written as,

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \quad (2.0.16)$$

$$\Rightarrow \mathbf{x} = \begin{pmatrix} 1 + 2x_3 - x_4 \\ 2 - x_3 + x_4 \\ x_3 \\ x_4 \\ 1 \end{pmatrix} \quad (2.0.17)$$

where $x_3, x_4 \in \mathbb{R}$.

Note that the above solution space is not a vector space. As $x_5 = 1$, the zero vector is not included in the solution space.

Since, \mathbf{x} is not a vector space, it cannot be expressed as a linear combination of basis vectors.