Assignment 7

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Row Reduced Echelon Form

***Abstract*—This document solves problem based on solution of system of linear equations using Row Reduction**

Download all solutions from

https://github.com/rubeenaafreen20/EE5609/tree/master/Assignment7

1 Problem

Find all solutions of

$$2x_1 - 3x_2 - 7x_3 + 5x_4 + 2x_5 = -2 \tag{1.0.1}$$

$$x_1 - 2x_2 - 4x_3 + 3x_4 + x_5 = -2 \tag{1.0.2}$$

$$2x_1 - 4x_3 + 2x_4 + x_5 = 3 \tag{1.0.3}$$

$$x_1 - 5x_2 - 7x_3 + 6x_4 + 2x_5 = -7 \tag{1.0.4}$$

2 Solution

The given equations can be written as,

$$\begin{pmatrix} 2 & -3 & -7 & 5 & 2 \\ 1 & -2 & -4 & 3 & 1 \\ 2 & 0 & -4 & 2 & 1 \\ 1 & -5 & -7 & 6 & 2 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -2 \\ -2 \\ 3 \\ 7 \end{pmatrix} \tag{2.0.1}$$

Now, we form the augmented matrix and perform Row reduction,

$$\left(\begin{array}{ccccc|c} 2 & -3 & -7 & 5 & 2 & -2 \\ 1 & -2 & -4 & 3 & 1 & -2 \\ 2 & 0 & -4 & 2 & 1 & 3 \\ 1 & -5 & -7 & 6 & 2 & 7 \end{array}\right) \tag{2.0.2}$$

$$\xleftrightarrow{R_3=R_3-R_1} \left(\begin{array}{ccccc|c} 2 & -3 & -7 & 5 & 2 & -2 \\ 1 & -2 & -4 & 3 & 1 & -2 \\ 0 & 3 & 3 & -3 & -1 & 5 \\ 1 & -5 & -7 & 6 & 2 & 7 \end{array}\right) \tag{2.0.3}$$

$$\xleftrightarrow{R_1=\frac{1}{2}R_1} \left(\begin{array}{ccccc|c} 1 & \frac{-3}{2} & \frac{-7}{2} & \frac{5}{2} & 1 & -1 \\ 1 & -2 & -4 & 3 & 1 & -2 \\ 0 & 3 & 3 & -3 & -1 & 5 \\ 1 & -5 & -7 & 6 & 2 & 7 \end{array}\right) \tag{2.0.4}$$

$$\xleftrightarrow{R_2=R_2-R_1,R_4=R_4-R_1} \left(\begin{array}{ccccc|c} 1 & \frac{-3}{2} & \frac{-7}{2} & \frac{5}{2} & 1 & -1 \\ 0 & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & 0 & -1 \\ 0 & 3 & 3 & -3 & -1 & 5 \\ 0 & -\frac{7}{2} & -\frac{7}{2} & \frac{7}{2} & 1 & -6 \end{array}\right) \tag{2.0.5}$$

$$\xleftrightarrow{R_1=R_1-3R_2} \left(\begin{array}{ccccc|c} 1 & 0 & -2 & 1 & 1 & 2 \\ 0 & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & 0 & -1 \\ 0 & 3 & 3 & -3 & -1 & 5 \\ 0 & -\frac{7}{2} & -\frac{7}{2} & \frac{7}{2} & 1 & -6 \end{array}\right) \tag{2.0.6}$$

$$\xleftrightarrow{R_3=R_3+6R_2,R_4=R_4-7R_2} \left(\begin{array}{ccccc|c} 1 & 0 & -2 & 1 & 1 & 2 \\ 0 & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right) \tag{2.0.7}$$

$$\xleftrightarrow{R_2=-2R_2} \left(\begin{array}{ccccc|c} 1 & 0 & -2 & 1 & 1 & 2 \\ 0 & 1 & 1 & -1 & 0 & 2 \\ 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right) \tag{2.0.8}$$

$$\xleftrightarrow{R_1=R_1+R_3,R_4=R_4+R_3,R_3=-R_3} \left(\begin{array}{ccccc|c} 1 & 0 & -2 & 1 & 0 & 1 \\ 0 & 1 & 1 & -1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \tag{2.0.9}$$

So,

$$x_1 - 2x_3 + x_4 = 1 \tag{2.0.10}$$
$$x_2 + x_3 - x_4 = 2 \tag{2.0.11}$$
$$x_5 = 1 \tag{2.0.12}$$

Solving the equations we get,

$$x_1 = 1 + 2x_3 - x_4 \tag{2.0.13}$$
$$x_2 = 2 - x_3 + x_4 \tag{2.0.14}$$
$$x_5 = 1 \tag{2.0.15}$$

which can be written as,

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \tag{2.0.16}$$

$$\implies \mathbf{x} = \begin{pmatrix} 1 + 2x_3 - x_4 \\ 2 - x_3 + x_4 \\ x_3 \\ x_4 \\ 1 \end{pmatrix} \tag{2.0.17}$$

We can express (2.0.17) as a sum of linear combination of vectors,

$$\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \mathbf{x_3} + \begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \mathbf{x_4} \tag{2.0.18}$$

where $x_3, x_4 \in \mathbb{R}$.
Note that the above solution space is not closed on vector addition and scalar multiplication.As $x_5 = 1$, the zero vector is not included in the solution space. Hence, $\mathbf{x}$ is not a vector space.
Since, $\mathbf{x}$ is not a vector space, it cannot be expressed in the form of linear combination of basis vectors.