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Assignment 8

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Abstract—This document explains solution of a problem based on nullspace.

Download all solutions from

https://github.com/rubeenaafreen20/EE5609/tree/master/Assignment8

Therefore, nullspace N(A) is non-trivial as

$$\exists x_1, x_2, \dots, x_n \in \mathbb{F}^{nx1} \tag{3.0.6}$$

such that Ax = 0

1 Problem

Let s<n and A an sxn matrix with entries in the field \mathbb{F} . Use Theorem 4 to show that there is a non-zero \mathbf{x} in \mathbb{F}^{nx1} such that $\mathbf{A}\mathbf{x} = 0$.

2 EXPLANATION

Theorem 4:Let \mathbb{V} be a vector space which is spanned by a finite set of vectors $\beta_1, \beta_2, ..., \beta_m$. Then any independent set of vectors in \mathbb{V} is finite and contains no more than m elements.

3 Solution

Let, $\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n$ be columns of matrix A. \therefore A is sxn matrix

$$\implies \alpha_i \in \mathbb{F}^{sx1}$$
 (3.0.1)

Given
$$\mathbf{x} \in \mathbb{F}^{nx1}$$
 (3.0.2)

$$\implies \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{pmatrix} \tag{3.0.3}$$

Assume $(\alpha_1, \alpha_2, ..., \alpha_s)$ form a basis for column space of A, i.e space spanned by $(\alpha_1, \alpha_2, ..., \alpha_n)$. \therefore Given that n>s,

Therefore, from **Theorem 4**, $(\alpha_1, \alpha_2, ..., \alpha_n)$ cannot be linearly independent Therefore,

$$\mathbf{A}\mathbf{x} = \alpha_1 x_1 + \alpha_2 x_2 \dots \alpha_n x_n \tag{3.0.4}$$

$$= \sum_{i=1}^{s} \alpha_i x_i + \sum_{j=s+1}^{n} \alpha_j x_j$$
 (3.0.5)