Assignment 1

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Download all python codes from

https://github.com/rubeenaafreen20/EE5609/tree/master/Codes

and latex codes from

https://github.com/rubeenaafreen20/EE5609

1 Problem

A ray of light passing through the point $\begin{pmatrix}1\\2\end{pmatrix}$ reflects on the x-axis at point **A** and the reflected ray passes through the point $\begin{pmatrix}5\\3\end{pmatrix}$. Find the coordinates of **A**.

2 Explanation

Let point **P** be $\begin{pmatrix}1\\2\end{pmatrix}$ and point **Q** be $\begin{pmatrix}5\\3\end{pmatrix}$ Since, point **A** is on x-axis, ts y-coordinate is zero. Assume

$$A = \begin{pmatrix}k\\0\end{pmatrix} \tag{2.0.1}$$

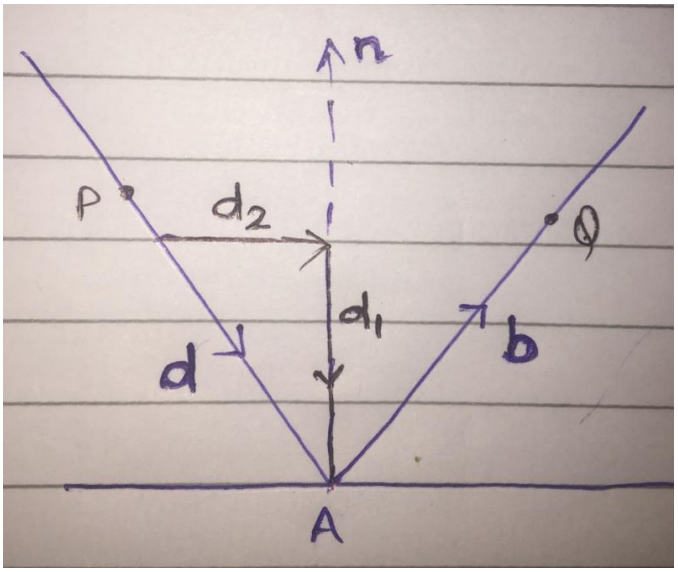

Fig. 0: Incident and reflected ray vectors

$$Incidentvector = \mathbf{P\text{-}A} \tag{2.0.2}$$

$$Reflectedvector = \mathbf{Q\text{-}A} \tag{2.0.3}$$

$$Vectoralongx-axis = \boldsymbol{a_x} = \begin{pmatrix}0\\1\end{pmatrix} \tag{2.0.4}$$

From Fig. 0,
Angle between AP and the x axis = 180°- angle between AQ and the x axis,

$$\frac{(\mathbf{P}-\mathbf{A})^{\mathbf{T}}\mathbf{a_x}}{\|\mathbf{P}-\mathbf{A}\|} = \frac{(\mathbf{Q}-\mathbf{A})^{\mathbf{T}}\mathbf{a_x}}{\|\mathbf{Q}-\mathbf{A}\|} \tag{2.0.5}$$

$$\frac{\mathbf{P^T a_x} - \mathbf{A^T a_x}}{\|\mathbf{P}-\mathbf{A}\|} = \frac{\mathbf{Q^T a_x} - \mathbf{A^T a_x}}{\|\mathbf{Q}-\mathbf{A}\|} \tag{2.0.6}$$

3 Solution

$$\frac{\begin{pmatrix}1 & 2\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} - \begin{pmatrix}k & 0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}}{\left\|\begin{pmatrix}1-k\\2\end{pmatrix}\right\|} = \frac{\begin{pmatrix}5 & 3\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} - \begin{pmatrix}k & 0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}}{\left\|\begin{pmatrix}5-k\\3\end{pmatrix}\right\|} \tag{3.0.1}$$

$$\implies \frac{2}{\sqrt{(1-k)^2+(2)^2}} = \frac{3}{\sqrt{(5-k)^2+(3)^2}} \tag{3.0.2}$$

$$\implies 5k^2 + 22k - 91 = 0 \tag{3.0.3}$$

Solving the equation (3.0.3) we get: k=2.6, -7

Since, incident ray passes through $\begin{pmatrix}1\\2\end{pmatrix}$ and reflected ray passes through $\begin{pmatrix}5\\3\end{pmatrix}$,

k cannot be negative as reflection takes place in first quadrant.

$$k = 2.6 \tag{3.0.4}$$