Assignment 8

Rubeena Aafreen

Download all solutions from

https://github.com/rubeenaafreen20/EE5609/tree/master/Assignment8

1 Problem

Let $s<n$ and A an $s \times n$ matrix with entries in the field $\mathbb{F}$. Use Theorem 4 to show that there is a non-zero $\mathbf{x}$ in $\mathbb{F}^{n\times 1}$ such that $\mathbf{Ax} = 0$.

2 Explanation

Theorem 4: *Let $\mathbb{V}$ be a vector space which is spanned by a finite set of vectors $\beta_1, \beta_2, ..., \beta_m$. Then any independent set of vectors in $\mathbb{V}$ is finite and contains no more than m elements.*

3 Solution

Let

$$\mathbf{A} = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \dots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \dots & \alpha_{2n} \\ \vdots & \vdots & & \vdots \\ \alpha_{s1} & \alpha_{s2} & \dots & \alpha_{sn} \end{pmatrix} \tag{3.0.1}$$

Let $(\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n)$ be columns of matrix A, such that,

$$\alpha_i = \begin{pmatrix} \alpha_{i1} \\ \alpha_{i2} \\ \vdots \\ \alpha_{is} \end{pmatrix}, i = 1, 2, \dots, n \tag{3.0.2}$$

$$\implies \alpha_i \in \mathbb{F}^{s\times 1} \tag{3.0.3}$$

Let a vector $\mathbf{x}$ in field $\mathbb{F}^{n\times 1}$, such that

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \tag{3.0.4}$$

$$\mathbf{Ax} = 0 \tag{3.0.5}$$

$$\implies \begin{pmatrix} \alpha_{11} & \alpha_{12} & \dots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \dots & \alpha_{2n} \\ \vdots & \vdots & & \vdots \\ \alpha_{s1} & \alpha_{s2} & \dots & \alpha_{sn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = 0 \tag{3.0.6}$$

$$\implies \begin{pmatrix} \alpha_1 & \alpha_2 & \dots & \alpha_n \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = 0 \tag{3.0.7}$$

$$\implies \alpha_1 x_1 + \alpha_2 x_2 + \dots + \alpha_n x_n = 0 \tag{3.0.8}$$

$$\tag{3.0.9}$$

where $\alpha_1, \alpha_2, \dots, \alpha_n$ are columns of dimension $s\times 1$.
$\because$ It is given that n>s,
Equation (3.0.7) leads to a homogeneous system of linear equations with s *equations and* n *unknowns.*
Since, s rows can hold at most s pivots, there must be $(n - s)$ free variables.
These free variables can be assigned any value. Hence, there are more solutions to equation (3.0.8) than the trivial $x = 0$.
$\implies$ Equation (3.0.8) will have at least one special solution.
Therefore, there exists a non-zero $\mathbf{x}$ in $\mathbb{F}^{n\times 1}$, where

$$x_1, x_2, \dots, x_n \in \mathbb{F}^{n\times 1} \tag{3.0.10}$$

such that $\mathbf{Ax} = 0$.