Assignment 8

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https://github.com/rubeenaafreen20/EE5609/tree/master/Assignment8

1 Problem

Let $s<n$ and A an $s \times n$ matrix with entries in the field $\mathbb{F}$. Use Theorem 4 to show that there is a non-zero $\mathbf{x}$ in $\mathbb{F}^{n\times 1}$ such that $\mathbf{Ax} = \mathbf{0}$.

2 Explanation

Theorem 4: *Let $\mathbb{V}$ be a vector space which is spanned by a finite set of vectors $\beta_1,\beta_2,...,\beta_m$. Then any independent set of vectors in $\mathbb{V}$ is finite and contains no more than m elements.*

3 Solution

Let $\mathbf{A_1}, \mathbf{A_2}, \mathbf{A_3}, \dots, \mathbf{A_n}$ be s-tuples in vector space $\mathbb{V}$.

$$\mathbf{A} = \begin{pmatrix} \mathbf{A_1} & \mathbf{A_2} & \dots & \mathbf{A_n} \end{pmatrix} \tag{3.0.1}$$

$$\mathbf{A_i} = \begin{pmatrix} \alpha_{1i} \\ \alpha_{2i} \\ \vdots \\ \alpha_{si} \end{pmatrix}, i = 1, 2, \dots, n \tag{3.0.2}$$

$$\mathbf{A} = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \dots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \dots & \alpha_{2n} \\ \vdots & \vdots & & \vdots \\ \alpha_{s1} & \alpha_{s2} & \dots & \alpha_{sn} \end{pmatrix} \tag{3.0.3}$$

Let,

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \tag{3.0.4}$$

$$\mathbf{Ax} = \mathbf{0_{s\times 1}} \tag{3.0.5}$$

$$\implies \begin{pmatrix} \mathbf{A_1} & \mathbf{A_2} & \dots & \mathbf{A_n} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \mathbf{0} \tag{3.0.6}$$

$$\implies \mathbf{A_1}x_1 + \mathbf{A_2}x_2 + \dots + \mathbf{A_n}x_n = \mathbf{0} \tag{3.0.7}$$

$$\implies \begin{pmatrix} \mathbf{A_1} & \mathbf{A_2} & \dots & \mathbf{A_n} \end{pmatrix} \mathbf{x} = \mathbf{0} \tag{3.0.8}$$

$\because$ It is given that n>s,
Equation (3.0.7) leads to a homogeneous system of linear equations with *s equations and n unknowns*.
Since, s rows can hold at most s pivots, there must be $(n - s)$ free variables.
These free variables can be assigned any value. Hence, there are more solutions to equation (3.0.7) than the trivial $x = 0$.
$\implies$ Equation (3.0.7) will have at least one special solution.
Therefore, at least one $x_j \neq 0, j = 1, 2, \dots, n$ esixts such that $\mathbf{Ax} = 0$.