Assignment 8

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https://github.com/rubeenaafreen20/EE5609/tree/master/Assignment8

1 Problem

Let $s<n$ and A an $s \times n$ matrix with entries in the field $\mathbb{F}$. Use Theorem 4 to show that there is a non-zero $\mathbf{x}$ in $\mathbb{F}^{n\times 1}$ such that $\mathbf{Ax} = \mathbf{0}$.

2 Explanation

Theorem 4: *Let $\mathbb{V}$ be a vector space which is spanned by a finite set of vectors $\beta_1, \beta_2, ..., \beta_m$. Then any independent set of vectors in $\mathbb{V}$ is finite and contains no more than m elements.*

3 Solution

Let $\mathbb{V}$ be a vector space spanned by $a_1, a_2, \ldots, a_n$, where a_i, i=1,2,…,n are columns of matrix $\mathbf{A}_{s\times n}$.

$$\mathbf{A} = \begin{pmatrix} a_1 & a_2 & \ldots & a_n \end{pmatrix} \tag{3.0.1}$$

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & & & \vdots \\ a_{s1} & a_{s2} & \ldots & a_{sn} \end{pmatrix} \tag{3.0.2}$$

Let us take a_i,i=1,2,…,n as standard $s \times 1$ bases.

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & \ldots & 0 & a_{1,s+1} & \ldots & a_{1n} \\ 0 & 1 & \ldots & 0 & a_{2,s+1} & \ldots & a_{2n} \\ \vdots & & \vdots & & \vdots & & \vdots \\ 0 & 0 & \ldots & 1 & a_{s,s+1} & \ldots & a_{sn} \end{pmatrix} \tag{3.0.3}$$

From (3.0.3), it is clear that

$$dim(col(A)) \leq s \tag{3.0.4}$$
$$\implies rank(A) \leq s \tag{3.0.5}$$

Now, from rank-nullity theorem,

$$rank(A) + nullity(A) = n \tag{3.0.6}$$
$$nullity(A) = n - rank(A) \tag{3.0.7}$$
$$\implies nullity(A) > 0 \tag{3.0.8}$$

From equation (3.0.8) it is clear that there will be a non zero $\mathbf{x}$ such that $\mathbf{Ax} = \mathbf{0}$