Assignment 5

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The link to the solution is

https://github.com/rubeenaafreen20/EE5609/tree/master/Assignment3

***Abstract*—This documents solves a problem based on circles.**

1 Problem

Find the points on the curve $\mathbf{x}^T\mathbf{x}-2\begin{pmatrix}1 & 0\end{pmatrix}\mathbf{x}-3=0$ at which the tangents are parallel to the x-axis

2 Solution

General equation of circle is

$$\mathbf{x}^T\mathbf{x}+2\mathbf{u}^T\mathbf{x}+f=0 \tag{2.0.1}$$

The centre and the radius can be obtained as,

$$\mathbf{u}=\begin{pmatrix}-1\\0\end{pmatrix} \tag{2.0.2}$$

$$f=-3 \tag{2.0.3}$$

$$\mathbf{c}=-\mathbf{u}=\begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.4}$$

$$r=\sqrt{\|\mathbf{u}\|^2-f}=2 \tag{2.0.5}$$

$\because$ The tangents are parallel to the x-axis, their direction and normal vectors, $\mathbf{m}$ and $\mathbf{n}$ are respectively,

$$\mathbf{m}=\begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.6}$$

$$\mathbf{n}=\begin{pmatrix}0\\1\end{pmatrix} \tag{2.0.7}$$

For a circle, given the normal vector $\mathbf{n}$, the tangent points of contact to circle given by equation (2.0.1) are given by

$$\mathbf{q_i}=(\kappa_i\mathbf{n}-\mathbf{u}),\ i=1,2 \tag{2.0.8}$$

where

$$\kappa_i=\pm\sqrt{\frac{\mathbf{u^T u}-f}{\mathbf{n^T n}}} \tag{2.0.9}$$

$$\kappa=\pm\sqrt{\frac{\begin{pmatrix}-1 & 0\end{pmatrix}\begin{pmatrix}-1\\0\end{pmatrix}-(-3)}{\begin{pmatrix}0 & 1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}}} \tag{2.0.10}$$

$$\implies \kappa=\pm\sqrt{\frac{4}{1}} \tag{2.0.11}$$

$$\implies \kappa=\pm2 \tag{2.0.12}$$

and from (2.0.8) , the point of contact $\mathbf{q_i}$ are,

$$\mathbf{q_1}=2\begin{pmatrix}0\\1\end{pmatrix}-\begin{pmatrix}-1\\0\end{pmatrix} \tag{2.0.13}$$

$$=\begin{pmatrix}1\\2\end{pmatrix} \tag{2.0.14}$$

$$\mathbf{q_2}=-2\begin{pmatrix}0\\1\end{pmatrix}-\begin{pmatrix}-1\\0\end{pmatrix} \tag{2.0.15}$$

$$=\begin{pmatrix}1\\-2\end{pmatrix} \tag{2.0.16}$$

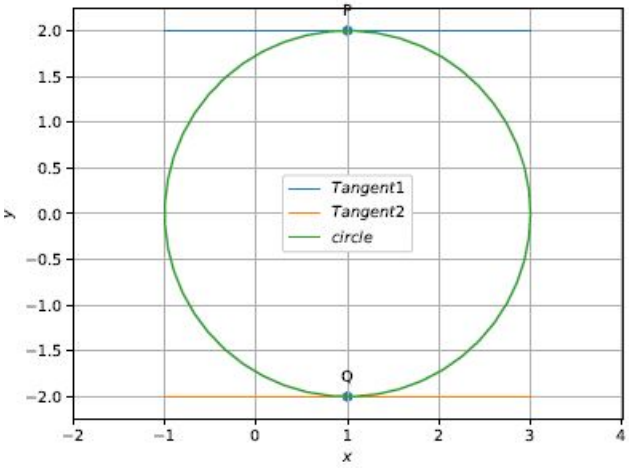

Fig. 1: Figure depicting tangents of circle parallel to x-axis