Fundamentals of MIMO Wireless Communication

Tutorial

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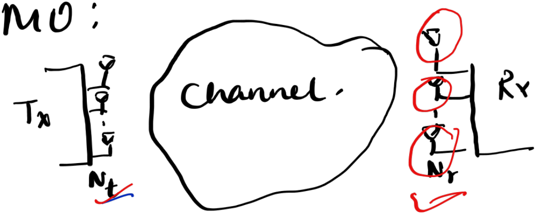
Week 1
February 7, 2023

MIMO:



$$Y = HX + N$$

$$y_1 = h_{11}x_1 + h_{12}x_2 + \cdots + h_{1,N_t}x_{N_t} + n_1$$

$$y_2 = h_{21}x_1 + h_{22}x_2 + \cdots + h_{2,N_t}x_{N_t} + n_2$$

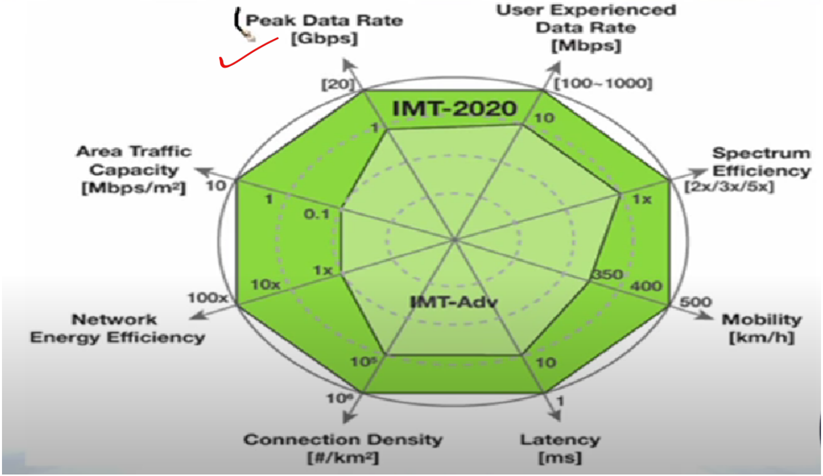
$$\vdots$$

$$y_{N_r} = h_{N_r,1}x_1 + h_{N_r,2}x_2 + \cdots + h_{N_r,N_t}x_{N_t} + n_{N_r}$$

$$\underbrace{\begin{bmatrix} y_1 \\ \vdots \\ y_{N_r} \end{bmatrix}}_{N_r \times 1} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1,N_t} \\ \vdots & & & \\ h_{N_r,1} & \cdots & \cdots & h_{N_r,N_t} \end{bmatrix} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N_t} \end{bmatrix}}_{N_t \times 1} + \underbrace{\begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_{N_r} \end{bmatrix}}_{N_r \times 1}$$

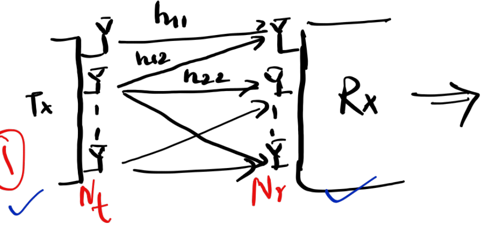
(1)

$$\boxed{H \rightarrow N_r \times N_t}$$

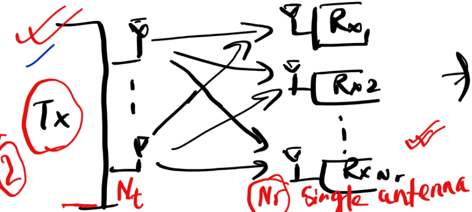


Peak Data Rate
[Gbps]
User Experienced
Data Rate
[Mbps]
[20]
[100~1000]
IMT-2020
1
10
Area Traffic
Capacity
[Mbps/m²]
10
1
0.1
Spectrum
Efficiency
[2x/3x/5x]
1x
1x
10x
100x
350
400
500
Network
Energy Efficiency
IMT-Adv
Mobility
[km/h]
10⁵
10
10⁶
1
Connection Density
[#/km²]
Latency
[ms]

Types of MIMO:

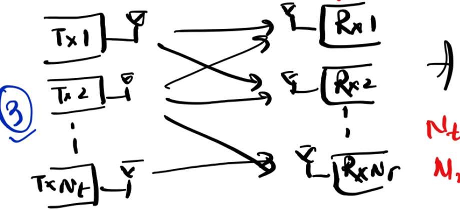


Single User MIMO



Multi-User MIMO.

N_r single antenna users.

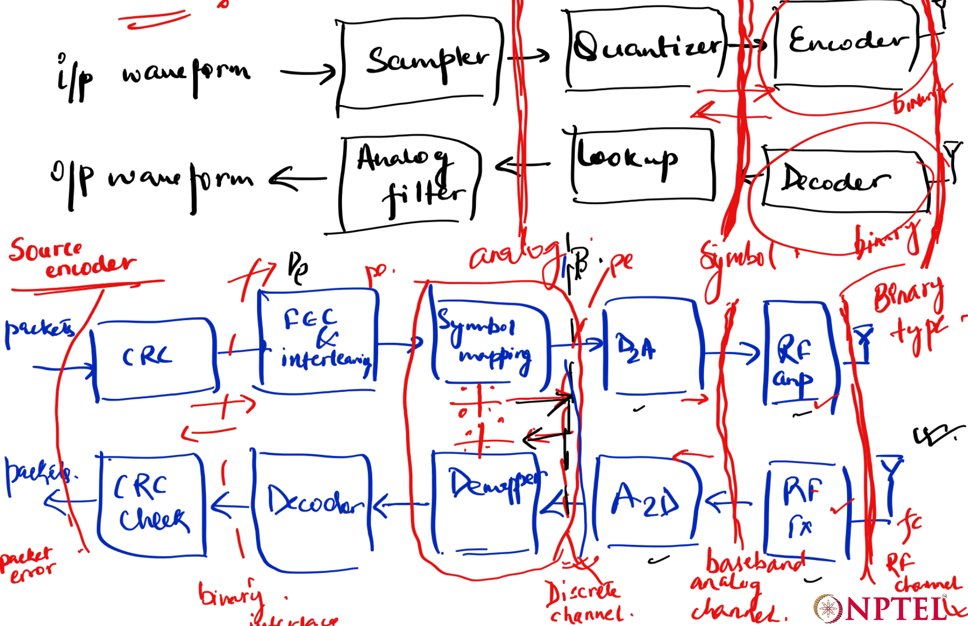


Multi-point transmission to multiple Users.

$N_t \to tx$ of single antenna.

$N_r \to rx$ of single antennas.

Channel:



i/p waveform
Sampler
Quantizer
Encoder
binary
o/p waveform
Analog filter
Lookup
Decoder
binary
Source encoder
packets
CRC
FEC & interleaving
Pe
Symbol mapping
analog I/P
Symbol
D2A
RF Amp
Binary type
packets.
CRC Check
Decoder
Demapper
A2D
RF rx
fc
packet error
binary interface
Discrete channel.
baseband analog channel.
RF channel

Probability based channel
- → Binary channel
  - → Binary Symmetric channel
  - → Binary erasure channel
- → Symbol (m-ary channel)

Waveform based channel
- → AWGN → DC

Non-ideal channel.
- → LTI
- → LTV → Wireless Commn.
- → non-linear channel.

→ If carrier freq. is getting added (RF amplifier stage)

→ at rx, the freq. component is removed at RF reciever.

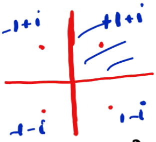
$$\left(h\, x(t)\, e^{-2\pi f_c t} + n.\right)$$

↓ at RF receiver

it does the opposite.

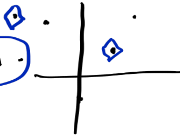
$$e^{2\pi f_c t}\left(\underline{h\, x(t)}\, e^{-2\pi f_c t} + n\right)$$

$$h\, x(t)$$



Channel →

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \rightarrow \begin{bmatrix} 1+i \\ -1+i \end{bmatrix} \xrightarrow[+n]{\text{Channel}} \begin{bmatrix} 0.7+0.3i \\ -2+3i \end{bmatrix}$$



⇒ We always assume signal as baseband.

BPSK:

$$\left.\begin{matrix} 0 \rightarrow -1 \\ 1 \rightarrow 1 \end{matrix}\right] \xrightarrow{\text{Channel}} \begin{bmatrix} -0.957 \\ 0.8 \end{bmatrix}$$

The entropy of a 2 symbol source with symbol probabilities p and $1-p$ is maximum when p is

a. 0.2.

b. 1.

c. 0.5. ✓

d. 0.

$X \rightarrow x_1, x_2.$

$x_1 \downarrow p_1$, $x_2 \downarrow p_2$

n-symbol source:

$x_1, x_2, \ldots x_n.$

$p_1 \quad p_2 \quad \ldots \quad p_n.$

$$H(x) = -\sum_{i=1}^{n} p_i \log_2 p_i$$

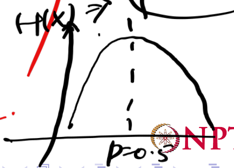
$$H = -p\log_2 p - (1-p)\log_2(1-p).$$

$$\frac{dH}{dp} = -\log_2 p - p \times \frac{1}{p}\log_2 e + \log_2(1-p) + (1-p) \times \frac{1}{1-p} \times \log_2 e.$$

$\log_2 p = \log_2(1-p)$

$p = 1-p$

$p = 0.5$



If the symbol error rate of a k-bit constellation is S then the bit error rate is approximately equal to

a. $log_2 k\mathrm{S}$.

b. $\frac{\mathrm{S}}{log_2 k}$.

c. $k\mathrm{S}$.

d. $\frac{\mathrm{S}}{k}$ ✓

$x \rightarrow b_1\ b_2 \ldots b_n$

0011

0100

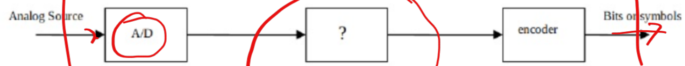
K bits.

P_b.

$$\boxed{P_S = K P_b}$$

$$\Rightarrow P_b = \frac{P_S}{K} = S/K.$$

What is the missing block in the following block diagram

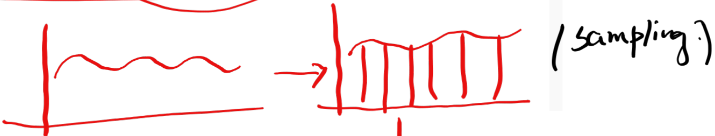


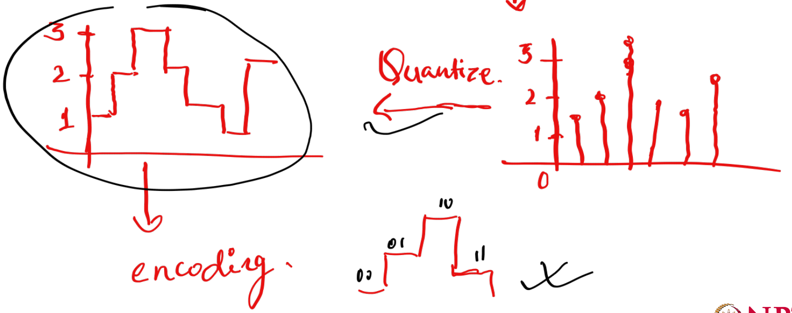
a. Decoder.

b. Amplifier.

c. RAM.

d. Quantizer. ✓





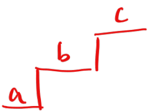
The job of a source coder is

a. To convert source symbols into bits.

b. To represent an input source message in the most efficient way possible.

c. To add parity check bits..

d. To pulse shape the data.



a	b	c
000	100	110
000	1010	1110
00	01	11

bits/symbol

bits/symbol → least method of coding is better.

In channel coding theorem, channel capacity decides the ‒ ‒ ‒ ‒ ‒ ‒ ‒ ‒ ‒ permissible rate at which error free transmission is possible.

a. Maximum.

b. Minimum.

c. Constant.

d. None of the above.

Source → introduce redundancy.

011011100

101011100

0 → 000

1 → 111

000 → 001 → 100 → 0

At the input, power is constant

In Wi-Fi the mode of transmission is

a. continuous.

b. burst.

c. analog.

d. None of these above.

- a.
- b.
- c.
- d.

→ data is sliced into smaller chunks.

→ If a packet in error → it can be retransmitted.

→ Higher data.

Wifi → bursty mode of transmission.

The rank of the matrix given below is,

$$\begin{bmatrix} 3 & 4 & 1 & 6 \\ 9 & 12 & 3 & 18 \\ 8 & 2 & 6 & 4 \\ 1 & 5 & 7 & 4 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$ $\quad A = \begin{bmatrix} 3 & 4 & 1 & 6 \\ 0 & 0 & 0 & 0 \\ 8 & 2 & 6 & 4 \\ 1 & 5 & 7 & 4 \end{bmatrix} \to$ Rank $= 3$

a. 4.

b. 3.

c. 2.

d. 1.

Row echelon form:-

1. All zero rows should be at bottom of matrix
2. Leading entry of each nonzero row occurs to the right of leading entry of previous.
3. Leading entry of any non zero row $\to 1$.
4. All column entries below 1 are zeros.

$R_1 \to R_1/3$

$$\begin{bmatrix} 1 & 4/3 & 1/3 & 2 \\ 9 & 12 & 3 & 18 \\ 8 & 2 & 6 & 4 \\ 1 & 5 & 7 & 4 \end{bmatrix} \xrightarrow[R_4 \to R_4 - R_1]{R_2 \to R_2 - 9R_1,\ R_3 \to R_3 - 8R_1} \begin{bmatrix} 1 & 4/3 & 1/3 & 2 \\ 0 & -26/3 & 10/3 & -12 \\ 0 & 0 & 0 & 0 \\ 0 & 1/3 & \frac{20}{3} & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 11/13 & 2/13 \\ 0 & 1 & -5/13 & 18/13 \\ 0 & 0 & 1 & -81/21 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

rank = No. of independent rows = No. of independent columns = 3.

The maximum eigen-value of the matrix given below is,

$$\begin{bmatrix} -1 & 3 \\ 5 & 6 \end{bmatrix}$$

a. 7.72.

b. 8.45.

c. 12.34.

d. 3.89.

Characteristic equation of matrix A

$= |A - \lambda I| = 0$, $\lambda \rightarrow$ eigen values.

$$\begin{vmatrix} -1-\lambda & 3 \\ 5 & 6-\lambda \end{vmatrix} = 0 \Rightarrow (\lambda-6)(\lambda+1) - 15 = 0$$

$$\Rightarrow \boxed{\lambda^2 - 5\lambda - 21 = 0}$$

$\lambda \rightarrow 2.27$

7.72

The Nyquist sampling frequency for a signal with frequencies between -2 Hz to 2 Hz is,

a. 2 Hz.

b. 2.5 Hz.

c. 4 Hz.

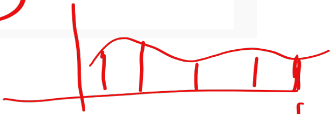
d. 6 Hz.

$-2\,Hz$ to $2\,Hz$

$$\boxed{f_s \geq 2 f_{max}}$$

$f_{max} = 2\,Hz$

$$\boxed{f_s = 2 \times 2 = 4\,HZ}$$



f_s.

$\geq 2 f_s \rightarrow$

The number of bits required for representing the symbols belonging to a 64 QAM constellation is,

a. 2.
b. 8.
c. 4.
d. 6.

64 QAM.

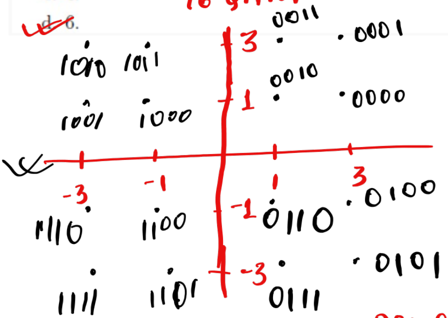
bits required to encode

4 bits

$\rightarrow 2^4$ symbols

$\rightarrow$ 16 symbols

16 QAM.



64 Symbol

64 QAM.

$\log_2 64 = 6$

M - QAM.

no. of bits required to represent the Symbols $\rightarrow \lceil \log_2 M \rceil$

The expression for covariance between two random variables X and Y is,

a. $E[XY] - E[X]E[Y]$.

b. $E[XY]$.

c. $E[XY] + E[X]E[Y]$.

d. $\frac{E[XY]}{E[X]E[Y]}$.

$E(\text{const}) = \text{const.}$

$$\text{cov}(X,Y) = E\left(\left(X-\mu_X\right)\left(Y-\mu_Y\right)\right)$$

$$\mu_X = E(X), \quad \mu_Y = E(Y)$$

$$E\left(\left(X-E(X)\right)\left(Y-E(Y)\right)\right) = E\left[XY - YE(X) - XE(Y) + E(X)E(Y)\right]$$

$$= E(XY) - E(X)E(Y) - \cancel{E(X)E(Y)} + \cancel{E(X)E(Y)}$$

$$\boxed{\text{cov}(X,Y) = E(XY) - E(X)E(Y)}$$

The null space basis vector for the following matrix is, $\begin{bmatrix} 3 & 6 \\ -9 & -18 \end{bmatrix}$

a. $\begin{bmatrix} -2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}$.

b. $\begin{bmatrix} 3/\sqrt{11} \\ -2/\sqrt{11} \end{bmatrix}$.

c. $\begin{bmatrix} 1/\sqrt{10} \\ 3/\sqrt{10} \end{bmatrix}$.

d. $\begin{bmatrix} 6/\sqrt{61} \\ -5/\sqrt{61} \end{bmatrix}$.

Null space :

$NS(A) = \{ \bar{x} \,;\, A\bar{x} = 0 \}$

Set of all vector $\bar{x}$, such that $A\bar{x} = 0$

$x \to 2\times 1.$ $\quad \bar{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$A\bar{x} = 0$

$\Rightarrow \begin{bmatrix} 3 & 6 \\ -9 & -18 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \Rightarrow \left. \begin{array}{l} 3x_1 + 6x_2 = 0 \\ -9x_1 - 18x_2 = 0 \end{array} \right\} \Rightarrow x_1 + 2x_2 = 0$

$\bar{x} = \begin{bmatrix} -2 \\ 1 \end{bmatrix} \to \begin{bmatrix} -2/\sqrt{4+1} \\ 1/\sqrt{4+1} \end{bmatrix} \to \begin{bmatrix} -2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}$

$A\bar{x} = 0$ $\quad \boxed{\bar{y} = \lambda \bar{x}}$ $\quad$ NS: $A\bar{y} \Rightarrow A(\lambda \bar{x}) \Rightarrow \lambda (A\bar{x}) = 0$.

If $\bar{x}$ is nullspace of A, then $y = \lambda\bar{x}$ is also nullspace

The determinant of the following matrix is, $\begin{bmatrix} 6 & -5 & 2 \\ 7 & 1 & -3 \\ 3 & -1 & 0 \end{bmatrix}$

a. 9.

~~b.~~ 7.

c. 6.

d. 2.

$$\begin{bmatrix} 6 & -5 & 2 \\ 7 & 1 & -3 \\ 3 & -1 & 0 \end{bmatrix} \rightarrow 6(-3) + 5(9) + 2(-10)$$

$$-18 + 45 - 20$$

$$7$$

The Fourier Transform of a rectangular pulse is,

a. Rectangular.

b. Triangular.

c. Sinc pulse.

d. Impulse.

$$\frac{e^{jx} - e^{-jx}}{2}$$

$\sin x$

$$\Pi\left(t/\tau\right) = \text{rect}\left(t/\tau\right) = \begin{cases} 1, & |t| \leq (\tau/2) \\ 0, & \text{o.w.} \end{cases}$$

fourier transform :-

$$F(\Pi) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t}\, dt = \int_{-\tau/2}^{\tau/2} 1 \cdot e^{-j\omega t}\, dt = \left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-\tau/2}^{\tau/2}$$

$$X(\omega) = \frac{e^{-j\omega\tau/2} - e^{j\omega\tau/2}}{-j\omega} = \frac{e^{j\omega\tau/2} - e^{-j\omega\tau/2}}{j\omega\frac{\tau}{2}} \cdot \frac{\tau}{2} = \frac{\sin \omega\tau/2}{\omega\tau/2}.$$

$$= \tau \cdot \text{sinc}\left(\frac{\omega\tau}{2}\right)$$

Following are statements related to cellular communication:

(i) 1G used analog communication.

(ii) 2G was developed for voice and data communication both.

(iii) GPRS is also known as 2.5G.

(iv) MIMO was introduced in 3G.

(v) Massive MIMO is used in 4G.

(vi) The sole aim of 5G communication system is to increase the data rate for high mobility users.

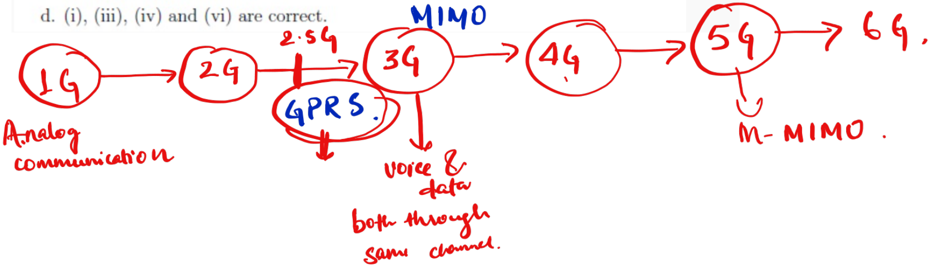
lecture 1

a. (i), (ii), (iii) and (iv) are correct.

b. (i), (iii) and (iv) are correct.

c. (i), (iii) and (vi) are correct.

d. (i), (iii), (iv) and (vi) are correct.



Youtube

→ Sharing the link.