Fundamentals of MIMO Wireless Communication

Tutorial

Rubeena Aafreen

Prime Minister Research fellow
IIT Hyderabad

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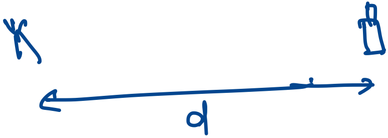
Small Scale propagation model:-

→ large scale → Path loss
→ large scale → Shadowing.

→ Small scale → multipath
→ Small scale → freq. flat fading
→ Small scale → Envelope distribution.



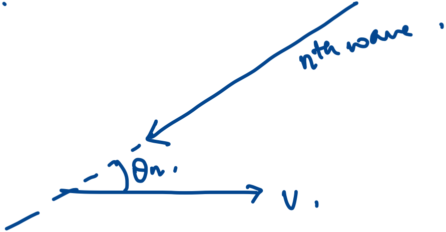
d → large → large scale model

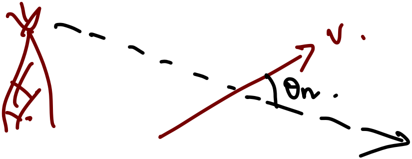
d → small → small scale model

Small scale model:-

→ Sig. will fluctuate over order of λ.

MS is moving.





$f_{D,n} = f_m \cos\theta_n$ Hz.

$f_m \rightarrow$ max. doppler shift $= \frac{v}{c}$

Tx. bandpass signal,

$r(t) = \mathrm{Re}\left[\tilde{s}(t) e^{j2\pi f_c t}\right]$

From Week 2 lecs:

Tx band pass signal:

$$s(t) = \mathrm{Re}\left\{ \tilde{s}(t)\, e^{j2\pi f_c t} \right\} \rightarrow \text{txed sig}$$

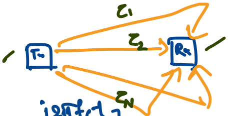
$\tilde{s}(t)$: complex envelope of tx signal.

noiseless rxed sig:

$$r(t) = \mathrm{Re}\left[\sum_{n=1}^{N} c_n\, e^{j2\pi (f_c + f_{D,n})(t-\tau_n)}\, \tilde{s}(t-\tau_n) \right]$$

c_n: amplitude scaling

Phase diff from n^{th} path: $\phi_n = 2\pi \left[f_{D,n} t - (f_c + f_{D,n}) \tau_n \right]$



$$r(t) = \mathrm{Re}\left[\sum_{n=1}^{N} c_n\, e^{j\phi_n(t)}\, \tilde{s}(t-\tau_n)\, e^{j2\pi f_c t} \right]$$

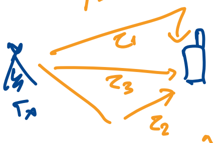
$$= \mathrm{Re}\left[\tilde{r}(t)\, e^{j2\pi f_c t} \right] \rightarrow \text{Rxed sig.}$$

$$\tilde{r}(t) = \sum_{n=1}^{N} c_n e^{j\phi_n(t)} \tilde{s}(t-\tau_n)$$

$$h(t,\tau_n) = c_n e^{j\phi_n(t)}$$

$$\boxed{\tilde{r}(t) = \sum_{n=1}^{N} h_n(t)\, \tilde{s}(t-\tau_n)}$$

$y = hx$



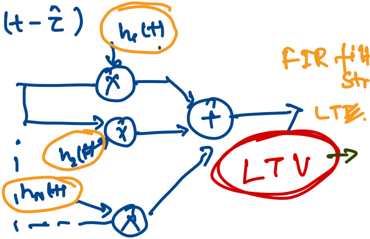
$\Rightarrow \tau_1 = \tau_2 = \cdots = \tau_N = \hat{\tau}$

$$\tilde{r}(t) = h_1(t)\tilde{s}(t-\tau_1) + h_2(t)\tilde{s}(t-\tau_2) + \cdots + h_N(t)\tilde{s}(t-\tau_N)$$

$$\tilde{r}(t) = \left(\sum_{n=1}^{N} h_n(t)\right)\tilde{s}(t-\hat{\tau})$$

$$\tilde{s}(t) \longrightarrow \tilde{s}(t-\hat{\tau})$$

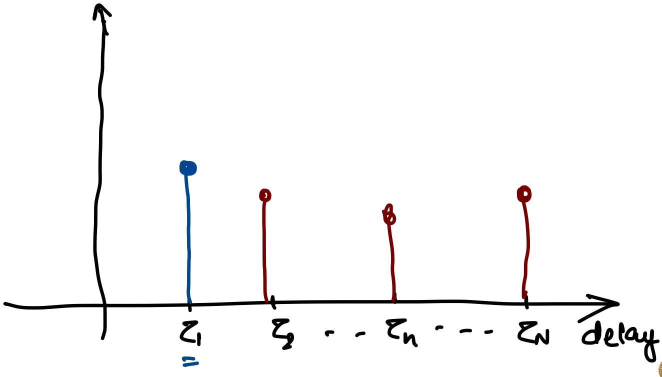
FIR filter Structure



$$\tilde{r}(t) = \sum_{n=1}^{N} h_n(t)\, \tilde{s}(t - \tau_n)$$

$$= h(t,\tau) * \tilde{s}(t).$$

$$h(t,\tau) = \sum_{n=1}^{N} \underbrace{h(t,\tau_n)}_{h_n(t)}\, \delta(\tau - \tau_n)$$

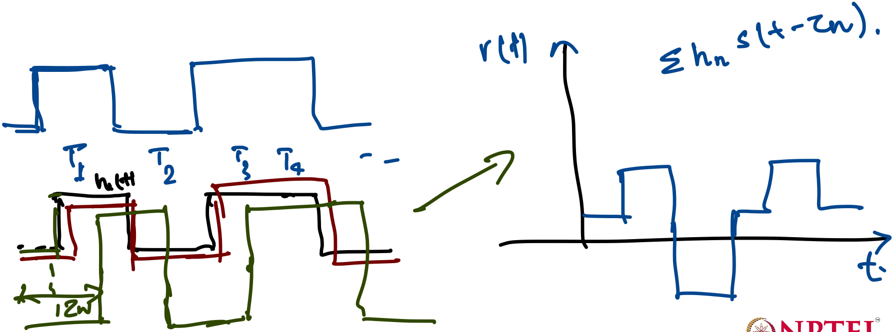


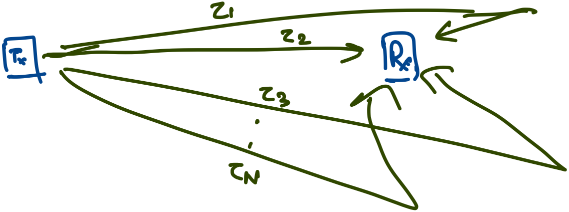
- $\vec{s}(t) \longrightarrow \underbrace{(t-\tau_1)} \longrightarrow$ multiplied by $\underbrace{h_1(t)}$ $+$

$\longrightarrow (t-\tau_2) \longrightarrow$ " " $h_2(t)$ $+$

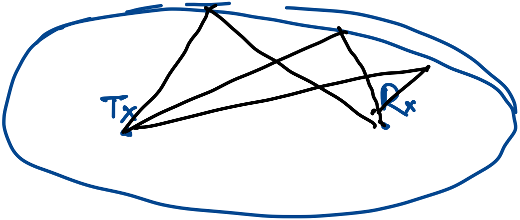
⋮

$\longrightarrow (t-\tau_N) \longrightarrow$ " " $h_N(t)$

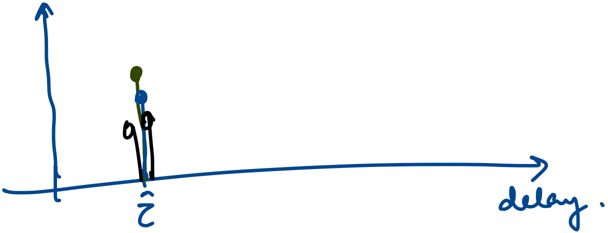




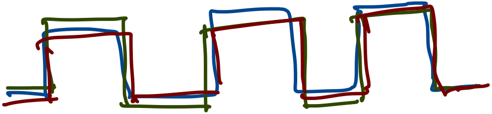
$$\tau_1 \approx \tau_2 \approx \cdots \approx \tau_N = \hat{\tau}.$$



$\tau_1 - \tau_2 \approx$ negligible .



Rx.



$\rightarrow$ decoded at rx,

$$h(t,\tau) = \sum_{n=1}^{N} h_n(t)\,\delta(\tau-\tau_n)$$

$$\approx \underbrace{\left(\sum_{n=1}^{N} h_n(t)\right)}_{h(t)}\delta(\tau-\hat{\tau}).$$

$$s(t) \longrightarrow s(t-\hat{\tau}) \longrightarrow \otimes \longrightarrow \tilde{r}(t)$$

$$h(t) \nearrow \otimes$$

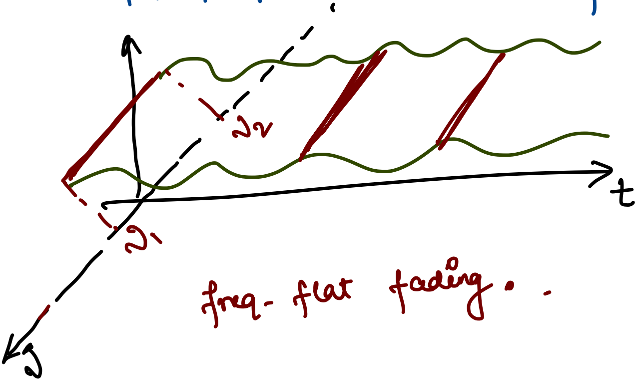
$$h(t,\tau) = h(t)\,\delta(\tau-\hat{\tau}).$$

$\downarrow$ F.T. along delay.

$$H(t,\nu) = \int_{0}^{\tau_{max}} h(t)\,\delta(\tau-\hat{\tau})\,e^{-j2\pi\nu\tau}\,d\tau$$

$$= h(t)\,\underline{\underline{e^{-j2\pi\nu\hat{\tau}}}}$$

$$|H(t,\nu)| = |h(t)| \rightarrow \text{Independent of } \nu.$$



freq-flat fading. .

Envelope distribution:

$$\tilde{r}(t) = \sum_{n=1}^{N} c_n e^{j\phi_n(t)} \tilde{s}(\tau-\tau_n)$$

$$\tilde{s}(t) = 1.$$

$$h(t) = \sum_{n=1}^{N} c_n e^{j\phi_n(t)}$$

$$= \sum_{n=1}^{N} c_n \cos\phi_n(t) + \sum_{n=1}^{N} c_n \sin\phi_n(t)$$

$$= h_I(t) + j h_Q(t)$$

$h_I(t) \rightarrow$

$\phi_n(t) \rightarrow 2\pi f_m \tau \rightarrow$

ϕ_n = Uniform random variable

$\Delta\tau_n \rightarrow$ large $\Delta\phi_n$

$\Delta\phi_n \rightarrow 2\pi$

$\phi_n \in [0, 2\pi]$

$$h_I(t) = \sum_{n=1}^{N} c_n \cos\phi_n(t) \quad , \quad h_Q(t) = \sum_{n=1}^{N} c_n \sin\phi_n(t)$$

$c_n \downarrow 1$.

if N is fairly large.
Central limit theorem,
$h_I(t)$ & $h_Q(t)$ are gaussian.

$h(t) = h_I(t) + j h_Q(t) \rightarrow$ Complex gaussian.

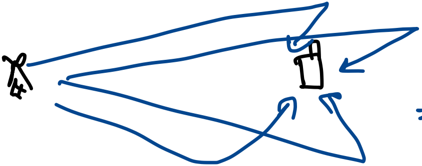
$h \rightarrow$ r.v.

o/p : $y = hx + n$.

$\alpha(t) = |h(t)| \rightarrow$ Rayleigh distribution.

$\alpha^2(t) = |h(t)|^2 \rightarrow$ Signal strength $\rightarrow$ Exponential

Rayleigh $\rightarrow$



$h(t) = \underset{\sim}{h_I(t)} + j h_Q(t)$

GRV with 0 mean.

Rician $\rightarrow$ Specular LOS component.

$\rightarrow h_I(t)$ & $h_Q(t)$ have mean $m_I(t)$ & $m_Q(t)$.

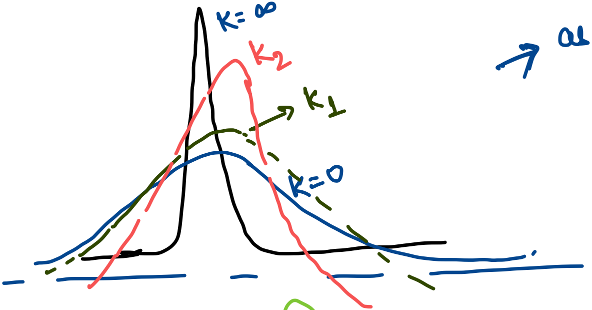
$\alpha(t) = |h(t)|$

$$\Rightarrow p_\alpha(x) = \frac{x}{b_0} \exp\left\{ -\frac{x^2+s^2}{2b_0} \right\} I_0\left(\frac{xs}{b_0}\right), \quad x \geq 0.$$

$$s^2 = m_I^2(t) + m_Q^2(t).$$

Rice factor, $K = s^2/2b_0$, $K \to 0 \rightarrow$ Rayleigh $\rightarrow$

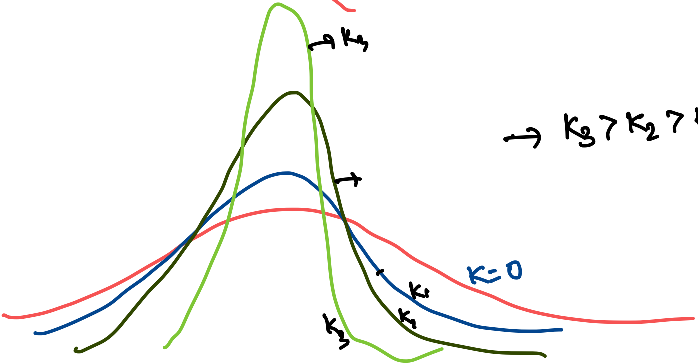
$K \to \infty \rightarrow$ No fading. $\rightarrow$



→ as $K \uparrow$ → Spread decrease.

a) $K_1 > K_2$

b) $K_2 > K_1$



→ $K_3 > K_2 > K_1$

Nakagami: Both rayleigh & rician fading explained.

$$p_\alpha(x) = 2m^m x^{2m-1} \exp\left\{\frac{-mx^2}{\Omega_p}\right\}, \quad m \geq 1/2$$

$$\Omega_p = E[\alpha^2]$$

$m = 1 \Rightarrow$ Rayleigh
$m = 1/2 \Rightarrow$ Gaussian (one side)
$m = \infty \Rightarrow$ no fading

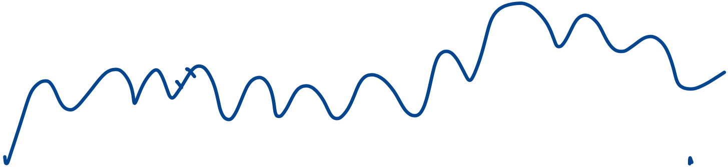
$m \uparrow \Rightarrow$ Spread decrease.

Relation between Rice factor 'K' & nakagami factor 'm'.

$$\boxed{K = \frac{\sqrt{m^2 - m}}{m - \sqrt{m^2 - m}}}, \quad m > 1.$$

$$\boxed{m = \frac{(K+1)^2}{2K+1}}$$

Receiver bandpass Signal Correlation :



$$r(t) = \mathrm{Re}\left[h(t)\, e^{j2\pi f_c t}\right] = \mathrm{Re}\left[\left[h_I(t) + j\,h_Q(t)\right] e^{j2\pi f_c t}\right]$$

$$= h_I(t) \cos \underbrace{2\pi f_c t}_{\omega_c t} - h_Q(t) \sin \underbrace{2\pi f_c t}_{\omega_c t}$$

$$\Phi_{rr}(\Delta t) = E\left[r(t)\, r(t+\Delta t)\right]$$

$$= E\Big[\left(h_I(t) \cos \omega_c t - h_Q(t) \sin \omega_c t\right)\left(h_I(t+\Delta t) \cos \omega_c (t+\Delta t) - h_Q(t+\Delta t) \sin \omega_c (t+\Delta t)\right)\Big]$$

$$= E\Big[h_I(t)\, h_I(t+\Delta t) \cos\omega_c t \cos\omega_c (t+\Delta t)$$
$$+ h_Q(t)\, h_Q(t+\Delta t) \sin\omega_c t \sin\omega_c (t+\Delta t)$$
$$- h_I(t)\, h_Q(t+\Delta t) \cos\omega_c t \sin\omega_c (t+\Delta t)$$
$$- h_Q(t)\, h_I(t+\Delta t) \sin\omega_c t \cos\omega_c (t+\Delta t) \Big]$$

$$= E\Big[\big(h_I(t)\, h_I(t+\Delta t)\big)\Big(\frac{1}{2}\cos\omega_c(2t+\Delta t) + \frac{1}{2}\cos\omega_c \Delta t\Big)\Big]$$
$$+ E\Big[h_Q(t)\, h_Q(t+\Delta t)\Big(\frac{1}{2}\cos\omega_c \Delta t - \frac{1}{2}\cos\omega_c(2t+\Delta t)\Big)\Big]$$
$$- E\Big[h_I(t)\, h_Q(t+\Delta t)\Big(\frac{1}{2}\sin\omega_c(2t+\Delta t) + \frac{1}{2}\sin\omega_c \Delta t\Big)\Big]$$
$$- E\Big[h_I(t+\Delta t)\, h_Q(t)\Big(\frac{1}{2}\sin\omega_c(2t+\Delta t) - \frac{1}{2}\sin\omega_c \Delta t\Big)\Big]$$

WSS process

$$\phi_{rr}(\Delta t) \rightarrow f(\Delta t)$$

$$\phi_{rr}(\Delta t) =$$

$$\frac{1}{2}\cos\omega_c(2t+\Delta t)\Big[E[h_I(t)h_I(t+\Delta t)] - E[h_Q(t)h_Q(t+\Delta t)]\Big]$$

$$-\frac{1}{2}\sin\omega_c(2t+\Delta t)\Big[E[h_I(t)h_Q(t+\Delta t)] + E[h_Q(t)h_I(t+\Delta t)]\Big]$$

$$+\frac{1}{2}\cos\omega_c\Delta t\ (\qquad) \quad -③$$

$$-\frac{1}{2}\sin\omega_c\Delta t\ (\qquad) \quad -④$$

$$E[h_I(t)h_I(t+\Delta t)] = E[h_Q(t)h_Q(t+\Delta t)] \quad -①$$

$$E[h_I(t)h_Q(t+\Delta t)] = -E[h_Q(t)h_I(t+\Delta t)] \quad -②$$

$$\phi_{rr}(\Delta t) = \frac{1}{\cancel{2}} \cos \omega_c \Delta t \cdot \cancel{2} \cdot E\left[\underbrace{h_I(t)\, h_I(t+\Delta t)}\right] - \frac{1}{\cancel{2}} \sin \omega_c \Delta t \cdot \cancel{2} \cdot E\left[h_I(t)\, h_Q\underbrace{(t+\Delta t)}\right]$$

$$= \Delta\phi_{h_I h_I}(\Delta t) \cos \omega_c \Delta t - \Delta\phi_{h_Q h_I}(\Delta t) \sin \omega_c \Delta t.$$

$$\boxed{\phi_{hh}(\Delta t) = \phi_{h_I h_I}(\Delta t) + \phi_{h_I h_Q}(\Delta t)} \rightarrow *$$

$$\phi_{rr}(\Delta t) = \mathrm{Re}\left[\phi_{hh}(\Delta t)\, e^{j2\pi f_c t}\right]$$

$$h_I = \sum_n C_n \cos \phi_n(t) \qquad \phi_n \rightarrow [0, 2\pi]$$

$$\phi_{h_I h_I}(\Delta t) = E\left[\sum_{n=1}^{N}\sum_{m=1}^{N} C_n C_m \cos\phi_n(t)\cos\phi_m(t+\Delta t)\right]$$

$$= E\left[\sum_{n=1}^{N} C_n^2 \left(\frac{1}{2}\cos\left(\phi_n(t)+\phi_n(t+\Delta t)\right) + \cos\phi_n(\Delta t)\right)\right]$$

$$\phi_n(\Delta t) = 2\pi f_{D,n}\Delta t.$$

$$\phi_{h_I h_I}(\Delta t) = \underbrace{\sum_{n=1}^{N} E(C_n^2)}_{\Omega_p} \cdot \frac{1}{2} E\left[\cos 2\pi f_{max}\cos\theta_n \Delta t\right]$$

$$\sum_{n=1}^{N} C_n^2 = \Omega_p \Rightarrow E[C_n^2] = \frac{\Omega_p}{N}, \quad p(\theta_n) = \frac{1}{2\pi}$$

NPTEL

$$\phi_{h_I h_I}(\Delta t) = \frac{\Omega_p}{2} E_\theta\left[\cos(2\pi f_{max} \cos\theta \, \Delta t)\right]$$

$$\phi_{h_Q h_I}(\Delta t) = \frac{\Omega_p}{2} E_\theta\left[\sin(2\pi f_{max} \cos\theta \, \Delta t)\right]$$

$$\phi_{h_I h_I}(\Delta t) = \frac{\Omega_p}{2} \int_{-\pi}^{\pi} \cos(\underbrace{2\pi f_{max} \cos\theta \, \Delta t)}_{\text{even}} \, p(\theta) \, d\theta$$

$$= \frac{\Omega_p}{\cancel{2}} \cdot \frac{1}{2\pi} \cdot \cancel{2} \int_{0}^{\pi} \cos(2\pi f_{max} \Delta t \cos\theta) \, d\theta \, .$$

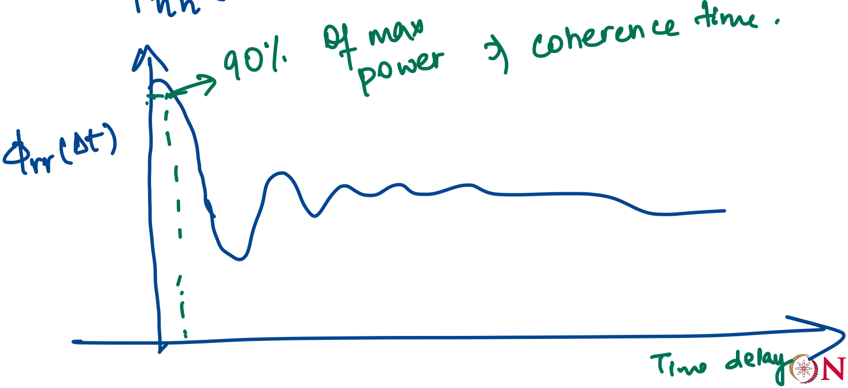
$$= \frac{\Omega_p}{2\pi} \int_{0}^{\pi} \cos(2\pi f_{max} \Delta t \sin\theta) \, d\theta$$

$$\phi_{h_I h_I}(\Delta t) = \frac{\Omega_p}{2} J_0(2\pi f_{max} \Delta t)$$

$$\phi_{h_Q h_I}(\Delta t) = \frac{\Omega_p}{2} \cdot \frac{1}{2\pi} \int_{-\pi}^{\pi} \underbrace{\sin(2\pi f_{max} \Delta t \cos\theta)}_{\text{odd}} \, d\theta$$

$$= 0$$

$$\phi_{hh}(\Delta t) = \phi_{h_I h_I}(\Delta t)$$



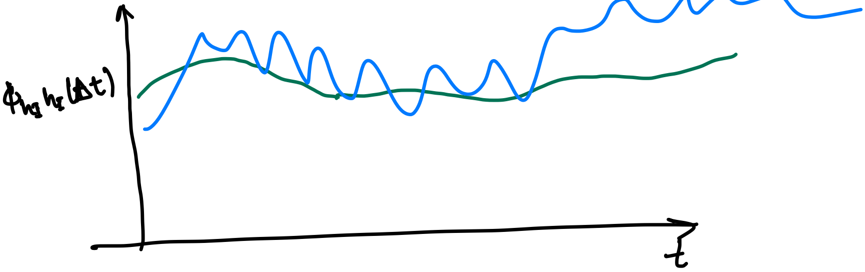
Coherence time :- Separation of time for which signals are correlated.

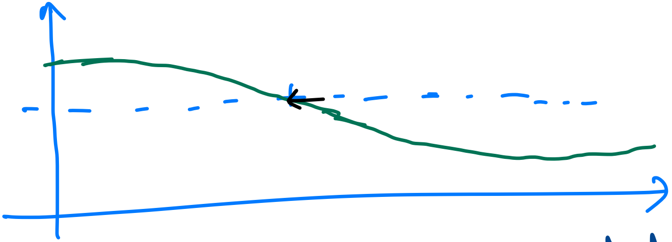
$$\boxed{T_c = \frac{0.423}{f_m} \quad ; \quad T_c = \frac{9}{16 f_m}}$$

relaxed

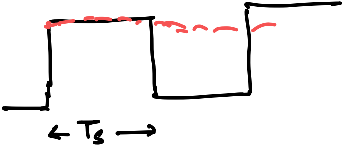
Stricter formula

$f_m \rightarrow$ max. doppler shift.





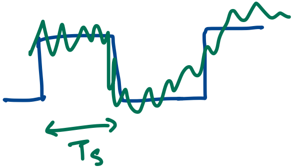
Outage $\rightarrow$ because of slow fading.



$T_c > T_s$ $\rightarrow$ slow fading channel.

$T_c \leq T_s$

If $T_c \leq T_s$.



Summary: To calculate coherence time:-

1. From correlation plot (Exact).
2. $T_c = \frac{0.423}{f_m}$ (relaxed condⁿ)
3. $T_c = \frac{9}{16 f_m}$ (stricter condⁿ).
4. $T_c > T_s \longrightarrow$ slow fading condition
5. $T_c \leq T_s \longrightarrow$ fast fading.

Slow fading condⁿ:

① T_s is very small.

② Mobility is very slow → f_m is less.

if mobility is faster ; $f_m \uparrow \Rightarrow T_c \downarrow \Rightarrow$ fast fading.

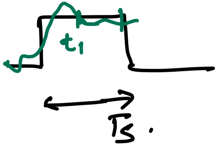
for slow fading :

$$y = hx + n$$

T_s. $h \rightarrow$ constant.

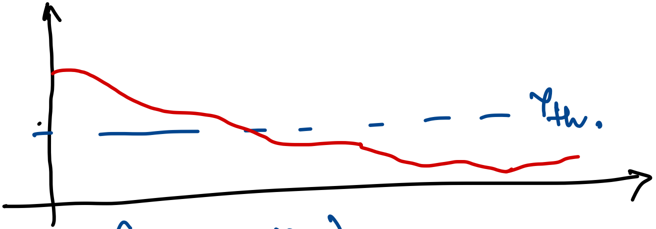
If we know h, $\boxed{\hat{x} = h^{-1} y}$ → Slow fading → Receiver design is easy fast fading scenario:-

$$y = h_1 x(0 - t_1) + h_2 x(t_1 + \delta - T_s).$$



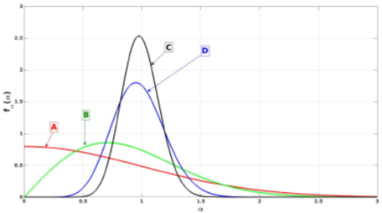
Slow fading vs -

Outage:



Coverage $\left(P_r(d) > \gamma_{th} \right)$

The probability density function (PDF) of Nakagami-m distribution is given for different values of m.



Which one of the following PDFs has a high value of m.

a. PDF labeled $\boxed{A}$

b. PDF labeled $\boxed{B}$

c. PDF labeled $\boxed{C}$

d. PDF labeled $\boxed{D}$

$m\uparrow$ → spread decreasing.

m for plot A : ½

B : 1.

m = 1 → Rayleigh fading

m = ½ → one sided gaussian

The value of Ricean K-factor for a wireless channel is 1. The parameter m for the Nakagami distribution can be obtained as,

a. 3.44

b. 2.28

c. 1.88

d. 1.33

$$m = \frac{(K+1)^2}{2K+1}$$

$$K=1 \Rightarrow m = \frac{2^2}{3} = 4/3 = 1.33$$

In the derivation of band pass signal correlation using Clarke's model, which one of the following assumptions influences the correlation to follow a Bessel function? (probability density function of angle of arrival is denoted as $p(\theta)$)

a. $p(\theta)$ is Gaussian distributed in $[-\pi \quad \pi]$

b. $p(\theta)$ is Uniformly distributed in $[-\pi \quad \pi]$

c. $p(\theta)$ is Laplacian distributed in $[-\pi \quad \pi]$

d. None of these

$p(\theta)$

Find the coherence time of a wireless channel for vehicular speeds of 80 kmph and carrier frequency of 1 GHz

a. 1.4 ms

b. 2.4 ms

c. 3.4 ms

d. 4.1 ms

$v = 80\,\text{kmph}$, $f_c = 1\,\text{GHz}$.

$v = 80\,\text{kmph} = 80 \times \frac{5}{18}\ \text{m/s}$.

$$f_m = \frac{v}{c} f_c = \frac{80 \times 5}{18 \times 3 \times 10^8} \times 1 \times 10^9 = 74\ \text{Hz}.$$

$$T_c = \frac{9}{16 f_m} = 7.6\ \text{ms}.$$

For a Rayleigh faded channel consider the Doppler frequency $(f_m) = 100$Hz. Find the approximate level crossing rate for threshold level $(\rho) = 1$

a. 103 crossings/s.

b. 55 crossings/s.

c. 92 crossings/s.

d. 79 crossings/s.

$$\boxed{LCR = \sqrt{2\pi}\, f_m \rho e^{-\rho^2}}$$

level crossing rate:

→ Avg. no. of time sig. crosses threshold level.

$f_m = 100$ Hz

$\rho = 1$

LCR = 92 crossings/sec

A wireless communication system is using QPSK modulation with the bit rate of 1 Mbps. A mobile user, moving in a bullet train measures the coherence time of the channel as one microsecond. The channel experienced by the mobile user is

a. Slow Fading.

b. Fast Fading.

c. Data is insufficient

d. None of these

$T_c = 1\,\mu s.$

$T_b = 1\,\mu s$

$T_s = nT_b = 2 \times 1\,\mu s$

$= 2\,\mu s$

$T_c < T_s$

$\Rightarrow$ fast fading.

1 Mb per second.

1 bit $\rightarrow \frac{1}{10^6}\,s \rightarrow 1\,\mu s.$

For a Rayleigh faded channel the average fade duration is observed to be 10 ms. Calculate the maximum Doppler frequency (f_m) assuming threshold level $(\rho) = 1$

a. 35 Hz.

b. 46 Hz.

c. 57 Hz.

d. 68 Hz.

Avg. fade duration

$$= \frac{e^{-\rho^2}}{\rho f_m} \cdot \frac{1}{\sqrt{2\pi}}$$

$$10^{-2} = \frac{e^{-1}}{f_m \sqrt{2\pi}}$$

$$\Rightarrow f_m = \frac{1}{\sqrt{2\pi}\, e} \times 10^2 = 68$$

A mobile station is moving at a speed of 500 kmph. It is using 2.4 GHz carrier frequency. To accomplish channel equalization, it measures the channel coefficients in a regular interval. What should be the maximum measurement interval?

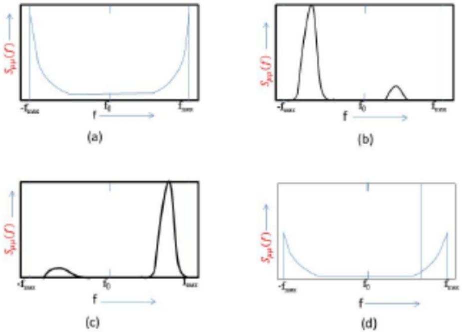
a. 120.26 μs.

b. 161.14 μs.

c. 180.24 μs.

d. 200.68 μs.

Which of the following figures represents Doppler power spectral density of Rayleigh channel where LOS is absent?



a. Figure (a).

b. Figure (b).

c. Figure (c).

d. Figure (d).

With increase in velocity of the receiver, (the correct statements are),

i Coherence time increases.

ii Level crossing rate increases.

iii Average envelope fade duration increases.

iv Coherence bandwidth decreases.

a. ii only.

b. i only.

c. iii and iv.

d. ii and iii.

AFD and LCR stand for

i Average fade duration and logical crossing rate, respectively.

ii Average fade duration and level crossing rate, respectively.

iii Actual fade duration and level crossing rate, respectively.

iv Adaptive fade duration and logical crossing rate, respectively.

The power angular spectrum in a LOS channel is given by, $p(\theta) = \frac{1}{K+1}\hat{p(\theta)} + \frac{K}{K+1}\delta(\theta - \pi/3)$ where $\hat{p(\theta)} = \frac{1}{2\pi}\ [-\pi, \pi]$. The correlation of the real part of the small scale channel $\phi_{h_I h_I}(\Delta t)$ is given as (assume $G(\theta) = 1$),

a. $\frac{\Omega_p}{2}\frac{1}{K+1}J_0(2\pi f_{max}\Delta t) + \frac{\Omega_p}{2}\frac{K}{K+1}cos(\sqrt{2}\pi f_{max}\Delta t)$.

b. $\frac{\Omega_p}{2}\frac{1}{K+1}J_0(2\pi f_{max}\Delta t)$.

c. $\frac{\Omega_p}{2}\frac{1}{K+1}cos(2\pi f_{max}\Delta t) + \frac{\Omega_p}{2}\frac{K}{K+1}cos(\pi f_{max}\Delta t)$.

d. $\frac{\Omega_p}{2}\frac{K}{K+1}cos(\sqrt{2}\pi f_{max}\Delta t)$.

$p(\theta) \in [-\pi, \pi]$

$$p(\theta) = \frac{1}{K+1}\hat{p}(\theta) + \frac{K}{K+1}\delta(\theta - \pi/3)$$

$$\hat{p}(\theta) = \frac{1}{2\pi}[-\pi, \pi]$$

$$\phi_{h_I h_I}(\Delta t) =$$

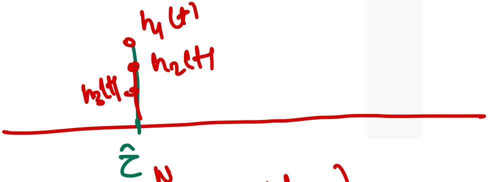
Number of resolvable multipath(s) observed for a flat fading channel is

a. zero

b. one ✓

c. 2-5

d. greater than 5



$$h(t) = \left(\sum_{n=1}^{N} h_n(t) \right)$$

Four transmitters with same transmit power of 100W are operating in a region with carrier frequencies of 5 GHz, 1800 MHz, 2.4 GHz and 900 MHz. The average received power measured at a distance 10 km from the transmitters are $P1, P2, P3$ and $P4$ respectively. Given the Path loss expression as,

$$P.L = K + 10log_{10}(f_c(Hz)) + 10n_p log_{10}(d(km))$$

Then,

a. $P1 > P2 > P3 > P4$

b. $P4 > P2 > P3 > P1$ ✓

c. $P4 > P3 > P2 > P1$

d. $P1 > P3 > P2 > P4$

$P_1, P_2, P_3, P_4.$

$PL \uparrow \rightarrow P_{rxed} \downarrow.$

$PL \propto f_c$

$f_{c_1} > f_{c_3} > f_{c_2} > f_{c_4}.$

$PL_1 > PL_3 > PL_2 > PL_4.$

$P_4 > P_2 > P_3 > P_1$