Fundamentals of MIMO Wireless Communication

Tutorial

Rubeena Aafreen

Prime Minister Research fellow
IIT Hyderabad

Week 4
March 3, 2023

Prev. week:

Rx Signal correlation:-

$$\phi_{rr}(\Delta t) = E[r(t)\, r(t+\Delta t)]$$

$$= \phi_{h_I h_I}(\Delta t)\cos\omega_c t - \phi_{h_I h_Q}(\Delta t)\sin\omega_c t$$

$$r(t) = \mathrm{Re}\{h(t)e^{j2\pi f_c t}\}$$

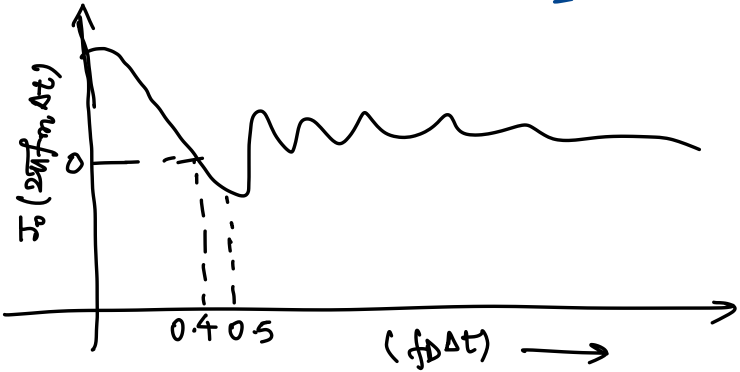
$$\phi_{rr}(\Delta t) = \mathrm{Re}\{\phi_{hh}(\Delta t)\, e^{j2\pi f_c t}\}$$

$$\phi_{hh}(\Delta t) = \phi_{h_I h_I}(\Delta t) + \phi_{h_I h_Q}(\Delta t)$$

$$\phi_{h_I h_Q}(\Delta t) = 0$$

$$\phi_{h_I h_I}(\Delta t) = \frac{\Omega_p}{2} J_0(2\pi f_{max} \Delta t)$$

$$\phi_{hh}(\Delta t) = \phi_{h_I h_I}(\Delta t) = \frac{\Omega_p}{2} J_0(2\pi f_{max} \Delta t).$$



We had considered 'isotropic scattering,

$$p(\theta) = \text{Uniform} = \frac{1}{2\pi}$$

$$\phi_{h_I h_I}(\Delta t) = \frac{\Omega_p}{2} J_0(2\pi f_m \Delta t).$$

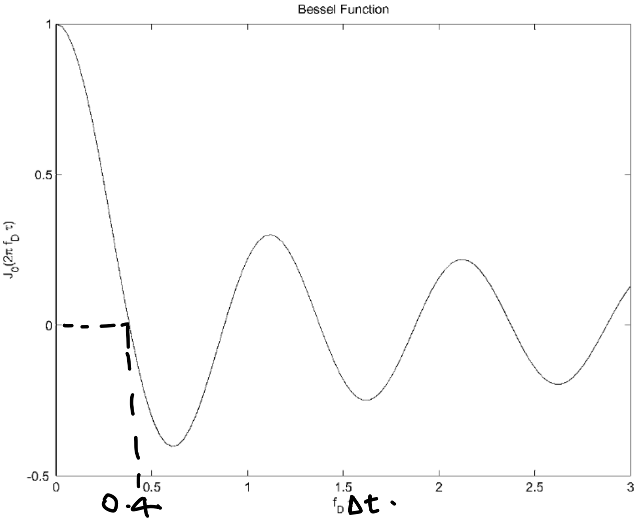


Figure 3.5: Bessel function versus $f_D \tau$.

Suppose moving mobile.

Δx in time Δt with speed v.

$$\Delta x = v\,\Delta t \quad \text{—} \ (1)$$

$$\frac{v}{c} f_c = f_m$$

$$\Rightarrow v = \frac{f_m}{f_c} \cdot c \quad \text{—} \ (2)$$

$$\Delta x = \frac{f_m}{f_c} \cdot c \cdot \Delta t = \frac{f_m \Delta t}{f_c} \cdot c \,.$$

$$f_m \Delta t = \frac{\Delta x \cdot f_c}{c}.$$

$$c = f_c \cdot \lambda$$

$$\rightarrow \boxed{f_m \Delta t = \frac{\Delta x}{\lambda}}$$

$$\phi_{h_I h_I}(\Delta t) = \frac{\Omega_p}{2} J_0(2\pi f_m \Delta t)$$

$$\phi_{h_I h_I}(\Delta x) = \frac{\Omega_p}{2} J_0\left(2\pi \frac{\Delta x}{\lambda}\right).$$

$$\alpha = |h(t)|$$

$$\mu_{\alpha\alpha}(\Delta x) = \frac{\pi}{16} \Omega_p J_0^2\left(\frac{2\pi}{\lambda} \Delta x\right).$$

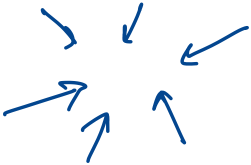
$f_D \Delta t = 0.4$ for uncorrelated sig.

$$\frac{\Delta x}{\lambda} \approx 0.4$$

$$\rightarrow \boxed{\Delta x \approx 0.4\lambda \approx \frac{\lambda}{2}}$$

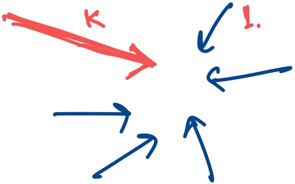
→ For uncorrelated sig, antennas should be spaced at min $\frac{\lambda}{2}$ distance.

Note: if $p(\theta)$ is different, results will change.



$$p(\theta) = \frac{1}{2\pi}$$

LOS component present:-

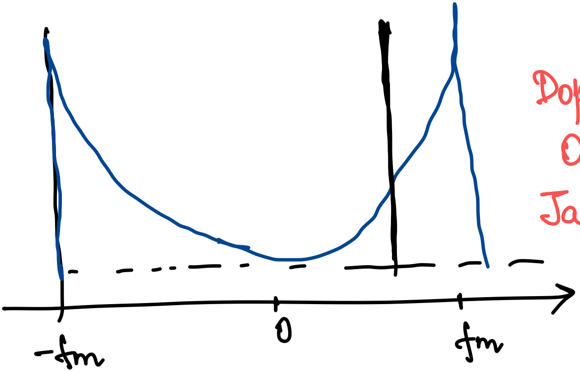


$$p(\theta) = \underbrace{\left(\frac{1}{K+1}\right)}_{\text{scattering}}\underbrace{\hat{p}(\theta)}_{\text{Uniform}} + \underbrace{\left(\frac{K}{K+1}\right)\delta(\theta-\theta_0)}_{\text{LOS.}}$$

$\theta_0 \rightarrow$ AoA of specular component.

$$S_{h_I h_I}(\Delta t) = \frac{\Omega_p}{2\pi f_m} \cdot \frac{1}{\sqrt{1-\left|\frac{f}{f_m}\right|^2}}, \quad \left|\frac{f}{f_m}\right| \leq 1$$

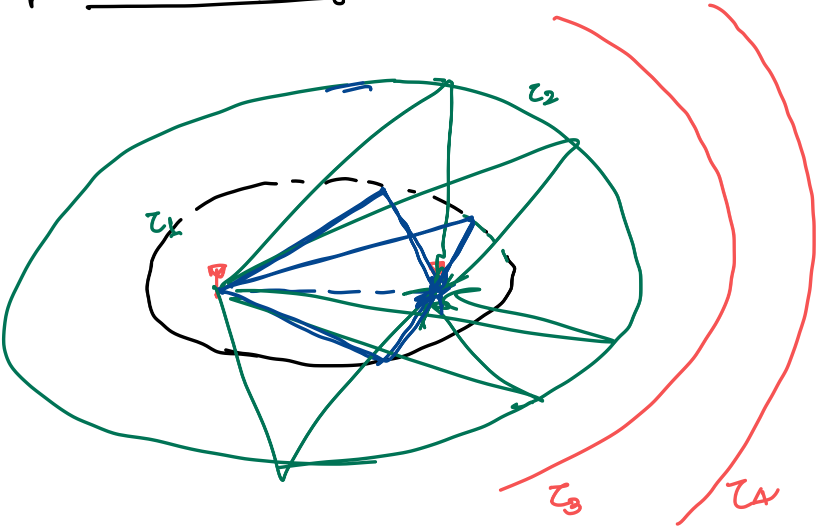
$$f \leq f_m.$$

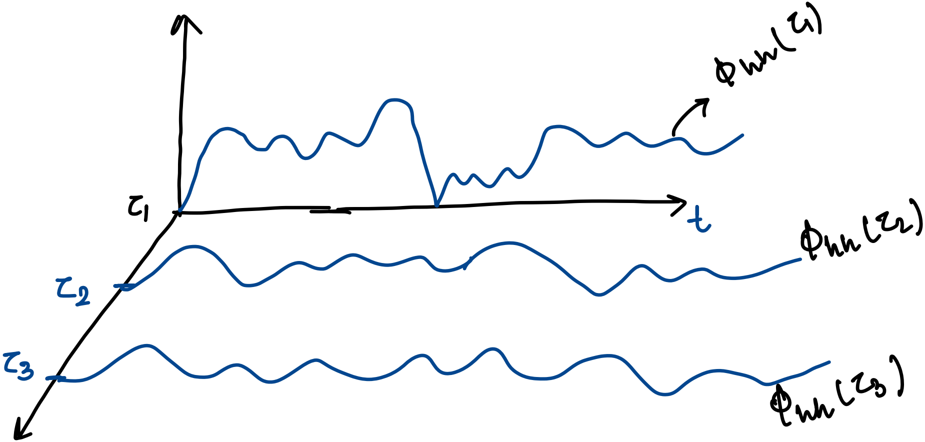


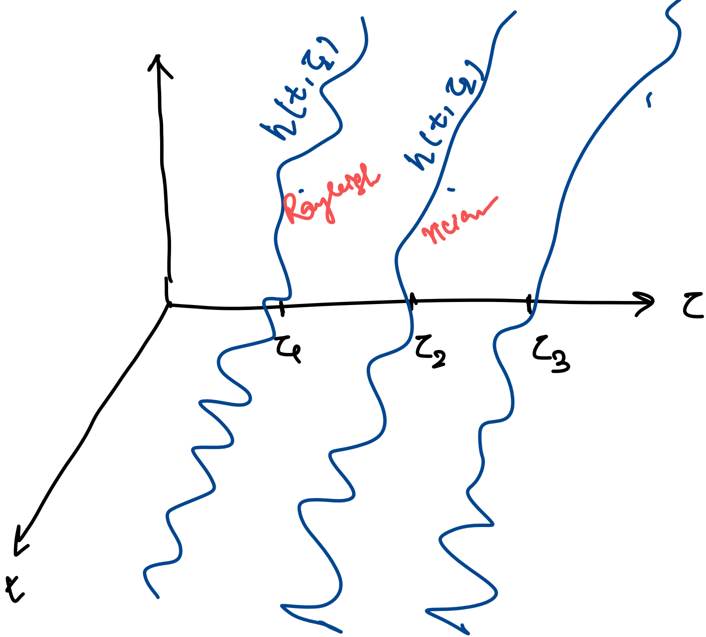
Doppler Spectrum
Or,
Jakes Spectrum

$$\boxed{p(\theta) = \frac{1}{2\pi}}$$

freq. Selective fading:




$\phi_{hh}(\tau_1)$
τ_1
t
$\phi_{hh}(\tau_2)$
τ_2
τ_3
$\phi_{hh}(\tau_3)$



h(t, τ1)
h(t, τ2)
Rayleigh
rician
τ
τ1
τ2
τ3
t

Freq. Selective fading:-

$$h(t,\tau) = \sum_{n=1}^{N} C_n \, e^{-j\phi_n(t)} \, \delta(\tau - \tau_n).$$

$$\phi_n(t) = 2\pi \{ (f_c + f_{D,n}) \tau_n - f_{D,n} t \}$$

$$\phi_{n_\theta}(t) = 2\pi \left(\left(f_c + f_{max} \cos(\theta_{n_\theta}) \right) \tau_{n_\tau} - \left(f_{max} \cos(\theta_{n_\theta}) t \right) \right)$$

$$h(t,\tau) = \sum_{n_\theta=1}^{N} C_{n_\theta,1} \, e^{-j\phi_n(t,\tau_1)} \, \delta(\tau - \tau_1) + \sum_{n_\theta=1}^{N} C_{n_\theta,2} \, e^{-j\phi_n(t,\tau_2)} \, \delta(\tau - \tau_2) +$$

$$- \; - \; —— \; -$$

f.T :

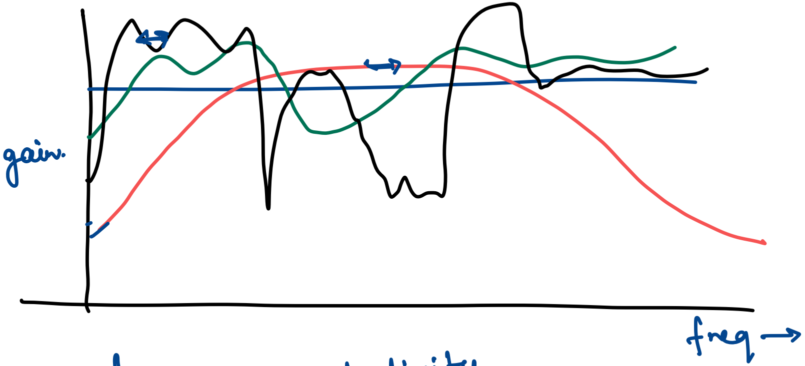
$$H(t,k) = \sum_{n_\tau=1}^{N_{\tau max}} h(t,\tau)\, e^{-j 2\pi k/N_f \cdot n_\tau}$$

$$H(t,k) = h(t,\tau_1)\, e^{-j 2\pi k/N_f \cdot 1} + h(t,\tau_2)\, e^{-j 2\pi k/N_f \cdot 2}$$

$$+ \; - - -$$

$$H(t,k) = \mathcal{F}(h(t,\tau_1)) + \mathcal{F}(h(t,\tau_2)) + - - - - + \mathcal{F}(h(t,\tau_{max}))$$

$$\boxed{H(t,k) = H(t,k_{\tau_1}) + H(t,k\tau_2) + - - - + H(t,k_{\tau max}).}$$



→ frequency selectivity.

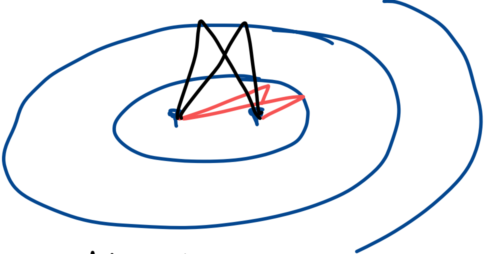
Coherence BW:

$$B_c \propto \frac{1}{\tau_{Rms}}$$

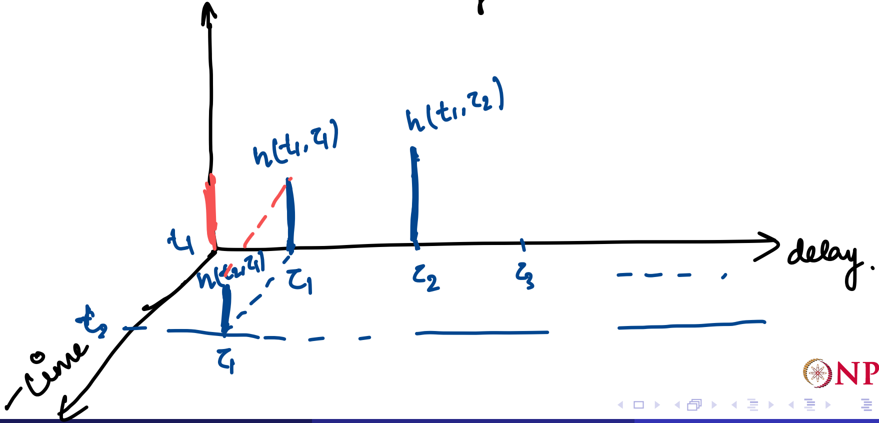
τ_{rms} → Rms delay spread.

$$B_c(0.5) = \frac{1}{5\tau_{rms}}$$ → 50% coherence BW

$$B_c(0.9) = \frac{1}{50\tau_{rms}}$$ → 90% coherence BW.



→ all resolvable delay.

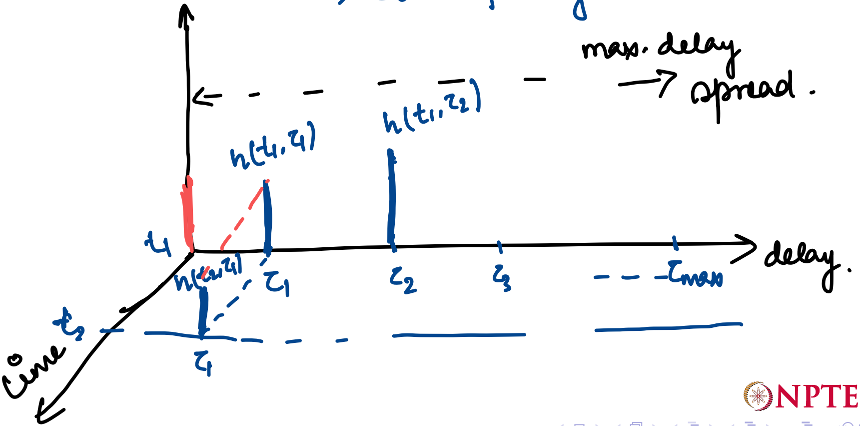


$h(t_1, \tau_1)$ & $h(t_2, t_1) \rightarrow$

if $(t_2 - t_1) << T_c$

$\rightarrow h(t_1, \tau_n) \approx h(t_2, \tau_n)$

$\rightarrow$ Slow fading



delay spread: difference b/w time of arrival of earliest multipath component & last component.

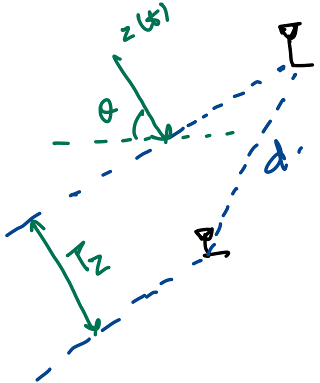
Average delay / mean delay: $\tau_m = \dfrac{\int_0^\infty \tau E|h(\tau)|^2 \, d\tau}{\int_0^\infty E\{|h(\tau)|^2\} \, d\tau}$

rms delay: $\tau_{rms} = \sqrt{\overline{\tau}^2 - \tau_m^2}$

$$\bar{\tau}^2 = \frac{\int_0^\infty \tau^2 E\left[|h(\tau)|^2\right] d\tau}{\int_0^\infty E\left[|h(\tau)|^2\right] d\tau}$$

$$\tau_{rms} = \sqrt{\bar{\tau}^2 - \tau_m^2}$$

Narrowband antenna array :-



$T_z \rightarrow$ time diff b/w reaching 1^{st} & 2^{nd} antenna.

Narrowband antenna assumption:

$B \rightarrow$ BW of signal.

$$B << \frac{1}{T_z}$$

$T_z = 5 ns$.

a. 100 MHz

b. 200 MHz

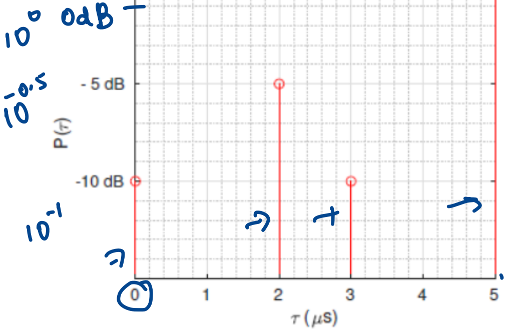
c. 10 MHz

d. 1 GHz

$$\frac{1}{T_z} = \frac{1}{5 \times 10^{-9}} = 200 MHz.$$

For narrowband assumption,

$$B << \frac{1}{T_z}.$$



$\tau_{max} = 5\,\mu s$

$$\tau_m = \frac{5\times10^{0} + 3\times10^{-1} + 2\times10^{-0.5} + 0\times10^{-1}}{10^{0} + 10^{-0.5} + 10^{-1} + 10^{-1}}$$

$\approx 3.9\,\mu s.$

The maximum delay spread and average delay spread are respectively.

a. $5\mu s$ and $3.9\mu s$.

b. $5\mu s$ and $2.9\mu s$.

c. ∞, and $0.9\mu s$.

d. None of the above.

$$\tau_m = 3.9\ \mu s.$$

$$\bar{\tau}^2 = \frac{\tau_1^2 P(\tau_1) + \tau_2^2 P(\tau_2) + \tau_3^2 P(\tau_3) + \tau_4^2 P(\tau_4)}{P(\tau_1) + P(\tau_2) + P(\tau_3) + P(\tau_4)}$$

$$= \frac{0 \times 10^{-1} + 4 \times 10^{-0.5} + 9 \times 10^{-1} + 25 \times 10^{0}}{0.1 + 0.316 + 0.1 + 1}$$

$$= 17.918\ \mu s.$$

$$\tau_{rms} = \sqrt{\bar{\tau}^2 - \tau_m^2} = \sqrt{17.918 - (3.9)^2}$$

$$= 1.6\ \mu s.$$

The RMS delay spread in the previous problem is obtained as,

a. $4.6\mu s$

b. $3.6\mu s$

c. $2.6\mu s$

d. $1.6\mu s$

Let $h(t_1, \tau)$ and $h(t_2, \tau)$ are impulse responses of a channel at time instants t_1 and t_2. If $|t_1 - t_2| \ll T_c$, where, T_c is the channel coherence time, then the impulses received at a delay of τ_1 at time instants t_1 and t_2 would be

a. Totally unrelated

b. Almost same in magnitude and phase angle

c. Almost same in magnitude but would have totally unrelated phase angles

d. Almost same in phase angle but would have totally unrelated magnitudes

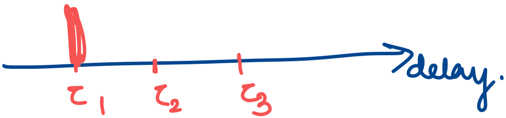
$$|t_1 - t_2| << T_c$$

$$h(t_1, \tau) \approx h(t_2, \tau)$$

rms delay spread : τ_{rms}.

$\tau_{rms} \rightarrow$

for any impulse sent, how much is the spread of envelope.



$$\boxed{\tau_{max} \approx (5-10)\,\tau_{rms}}$$

The rms delay spread of a wireless channel experiencing frequency selective fading is 1 μs. The maximum excess delay is likely to be

a. 0.5 μs

b. 1 μs

c. 5 μs

d. 1 ms

$\tau_{rms} = 1 \mu s.$

$\tau_{max} \simeq (5-10)\, \tau_{rms}$

$\approx 5 \mu s - 10 \mu s.$

Coherence distance is inversely proportional to

a. delay spread $B_c \propto \frac{1}{\text{delay spread}}$

b. doppler spread $\left(T_c \propto \frac{1}{\quad}\right)$

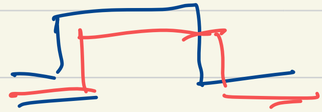
c. angle spread

d. all the above

Wireless channel

- Time selective
  - Slow fading
  - fast fading
  - doppler spread.
  - $T_c \propto \frac{1}{\text{doppler spread}}$
- freq. selective
  - flat fading
  - freq. selective fading
  - delay spread
  - $B_c \propto \frac{1}{\text{delay spread}}$
- Space selective
  - Rich
  - poor
  - Angle spread
  - $D_c \propto \frac{1}{\text{Angle spread}}$
    - Separation of distance for which sig. components are correlated.

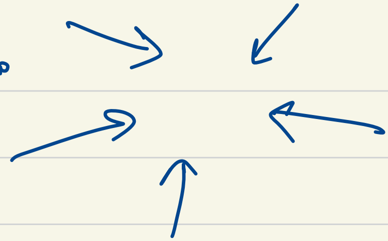
multipath delay < $T_s/10$	multipath delay > $T_s/10$.
→ delay << T_s → flat fading	- frequency selective fading
→ not time resolvable	→ Time resolvable.
→ components not distinguishable	→ Components are distinguishable.
→ No ISI	→ ISI



$T_c > T_s$ → flat

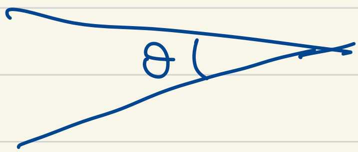
$T_c < T_s$ →

Rich Scattering:
(lots of reflection)



→ coherence distance → low
angle spread → more.

Poor Scattering



angle spread → less.
coherence distance → more.

Which of the following is **NOT** true for a Wide Sense Stationary Uncorrelated Scattering (WSSUS) channel?

a. Impulse response of the channel at two different delays τ_1 and τ_2 are uncorrelated, no matter how small $|\tau_1 - \tau_2|$ is.

b. Channel scattering functions at two different Doppler frequencies ν_1 and ν_2 are uncorrelated, no matter how small $|\nu_1 - \nu_2|$ is.

c. Impulse response of the channel at two different time instants t_1 and t_2 are uncorrelated, no matter how small $|t_1 - t_2|$ is.

d. The channel may be time variant.

WSSUS   HO → Δd.

WSS : $\Delta\tau$

US : Δf ,

The channel matrix of a single input multiple output (SIMO) channel can be represented as

a. scalar.
b. column vector.
c. row vector.
d. square matrix.

$$Y = Hx + n.$$

MIMO:-
$$\underbrace{\begin{bmatrix} y(t) \end{bmatrix}}_{M_R \times 1} = \underbrace{\begin{bmatrix} H \end{bmatrix}}_{M_R \times M_T} \underbrace{\begin{bmatrix} x(t) \end{bmatrix}}_{M_T \times 1} + \underbrace{\begin{bmatrix} n(t) \end{bmatrix}}_{M_R \times 1}$$

SISO: $M_R = 1$, $M_T = 1$.

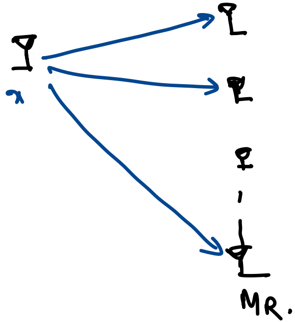
SIMO: $M_T = 1$, $M_R = M_R$.

MISO: $M_T = M_T$, $M_R = 1$.

Flat fading channel :-

Assuming flat fading

SIMO:



$$h(t,\tau) \approx h(t).$$

$$h(t) = [h_1(t) \quad h_2(t) \quad \cdots \quad h_{MR}(t)]^T.$$

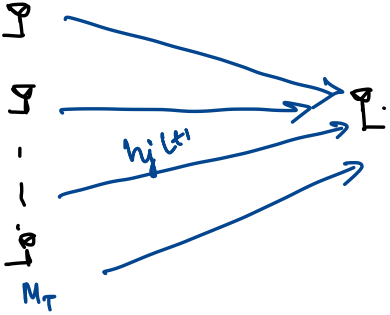
$$= \begin{bmatrix} h_1(t) \\ \vdots \\ \vdots \\ h_{MR}(t) \end{bmatrix}$$

$$\boxed{y = hx + n}$$

$$\begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_{M_R}(t) \end{bmatrix}_{M_R \times 1} = \begin{bmatrix} h_1(t) \\ h_2(t) \\ \vdots \\ h_{M_R}(t) \end{bmatrix} x + \begin{bmatrix} n_1(t) \\ n_2(t) \\ \vdots \\ n_{M_R}(t) \end{bmatrix}$$

$$y(t) = h_1(t)\,x(t) + h_2(t)\,x(t) + \dots + \tilde{n}(t)$$

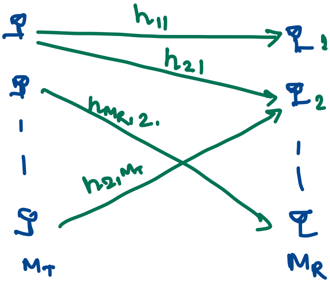
MISO:



$$y = hx + n.$$

$$y(t) = \underbrace{\begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_{MT}(t) \end{bmatrix}}_{M_T \times 1} \begin{bmatrix} h_1(t) & \cdots & h_{M_T}(t) \end{bmatrix} + n(t)$$

MIMO:



$$y = hx + n$$

$$\underbrace{\begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ \\ \end{bmatrix}}_{M_R \times 1} = \underbrace{\begin{bmatrix} h_{11}(t) & \cdots & h_{1M_T}(t) \\ \vdots & & \\ h_{M_R,1}(t) & \cdots & h_{M_R M_T}(t) \end{bmatrix}}_{M_R \times M_T} \underbrace{\begin{bmatrix} x_1(t) \\ \vdots \\ x_{M_T}(t) \end{bmatrix}}_{M_T \times 1} + \underbrace{\begin{bmatrix} \\ \\ \\ \end{bmatrix}}_{M_R \times 1}$$

$$Y = Hs + n.$$

$$y_i(t) = \sum_{j=1}^{M_T} h_{ij}(t)\, s_j(t)$$

A mobile station is moving at a speed of 500 kmph. It is using 2.4 GHz carrier frequency. To accomplish channel equalization, it measures the channel coefficients in a regular interval. What should be the maximum measurement interval?

a. 120.26 μs.

b. 161.14 μs.

c. 180.24 μs.

d. 200.68 μs.

Rx

$$v = 500\,\text{kmph} = 138.8\ \text{m/s}.$$

$$f_c = 2.4\ \text{GHz}.$$

$$f_D = \frac{v}{c} \cdot f_c = \frac{138.8}{3 \times 10^8} \times 2.4 \times 10^9 = 1110.4$$

q9a.png

Z is an element of the channel matrix of a Zero Mean Circularly Symmetric Complex Gaussian (ZMCSCG) MIMO channel. Then the expectation of $Ze^{j\phi}$ is

a. always zero.

b. zero only if $\phi = 0$.

c. zero only if $Z = 0$.

d. zero if either $\phi = 0$ or $Z = 0$.

$$E[Ze^{j\phi}] = e^{j\phi} \cancel{E[Z]} \to 0.$$
$$= 0.$$

Classic iid channel:

$T_{rms} \approx 0.$

$\rightarrow$ H can be modelled as $\underbrace{ZM}CSC\underbrace{G}$

$Z = X + jY$

$X \rightarrow$ Gaussian, with
$Y \rightarrow$ zero mean.

Circular symmetry.

$$E[e^{j\phi} Z] = e^{j\phi} E[Z].$$

$\mathbf{A}$ is a matrix of the order $m \times n$. Then the order of $\underline{\text{vec}(\mathbf{A})}$ would be

a. $m \times n$

b. $n \times m$

c. $mn \times 1$ ✓

d. $1 \times mn$

$$A = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}_{m \times n}$$

$$\text{vec } A = \begin{bmatrix} \\ \\ \\ \\ \\ \end{bmatrix}_{mn \times 1}.$$

The rms delay spread in a channel is found to be 1.5 μs. The 50% and 90% coherence bandwidth of the channel are respectively:-

a. 55 kHz, 5 kHz

b. 25 kHz, 2.5 kHz

c. 155 kHz, 15.5 kHz

d. 133 kHz, 13.3 kHz

$\tau_{rms} = 1.5\,\mu s.$

$B_c(0.5) = \frac{1}{5\tau_{rms}} = \frac{1}{7.5\times10^{-6}}$

$= 133\,kHz.$

$B_c(0.9) = \frac{1}{50\tau_{rms}} = 13.3\,kHz.$

What is the minimum physical separation required between adjacent antennas in a rich scattering environment to have independently faded received signals? (λ denotes wavelength)

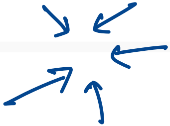
a. λ

b. $\frac{\lambda}{2}$ ✓

c. $\frac{\lambda}{3}$

d. $\frac{\lambda}{4}$

$$p(\theta) = \frac{1}{2\pi}$$



→ Rich Scattering environment.

$\lambda/2$

NPTEL

The channel covariance matrix $\mathbf{R}$ is identified as a positive semidefinite hermitian matrix. What are the properties of $\mathbf{R}$?

a. Eigen values are real

b. Eigen values are distinct

c. Eigen values are non negative

d. All the above

$$R = E\left[\mathrm{vec}(H)\,\mathrm{vec}(H)^H\right]$$

MIMO:

$$H \rightarrow M_R \times M_T \rightarrow \mathrm{vec}(H) \rightarrow M_R M_T \times 1.$$

$$A = \underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 9 & 8 \end{bmatrix}}_{3\times 3} \rightarrow \mathrm{vec}(A) \rightarrow 9\times 1 \rightarrow \begin{bmatrix} 1 \\ 4 \\ 7 \\ 2 \\ 5 \\ 9 \\ 3 \\ 6 \\ 8 \end{bmatrix}$$

$$R = E\left[vec(H)\, vec(H)^H \right]$$

$$(M_R M_T \times 1)(1 \times M_R M_T)$$

$\rightarrow$ Square matrix.

$R \rightarrow$ +ve semidefinite matrix.

All eigen values are real, distinct and non-negative ≥ 0.