Fundamentals of MIMO Wireless Communication

Tutorial

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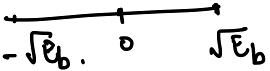
$$P_e = Q\left(\sqrt{\frac{d_{min}^2}{2N_0}}\right)$$



$d_{min} = 2\sqrt{E_b}$.

$$= Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$$= Q\left(\sqrt{2\gamma_b}\right) \approx \underline{c_1} e^{-c_2 \gamma}$$

(Chernoff bound)

$$P_s = \alpha_M Q\left(\sqrt{\beta_M \gamma_s}\right) \leq \alpha_M \underline{e}^{-\beta_M \gamma_s / 2}$$

$$c_1 = \alpha_M, \quad c_2 = \frac{-\beta_M \gamma_s}{2}$$

$$\Gamma_\Sigma = \sum_{i=1}^{M} \Gamma_i \qquad \Gamma_i \rightarrow \text{SNR experience by each branch.}$$

$$\Rightarrow P_s = \alpha_M \, e^{-\frac{\beta_M}{2}\left(\sum_{i=1}^{M}\Gamma_i\right)}$$

$$\Rightarrow \boxed{\bar{P}_s = \alpha_M \left(\frac{1}{1 + \bar{\Gamma}\frac{\beta_M}{2}}\right)^M}$$

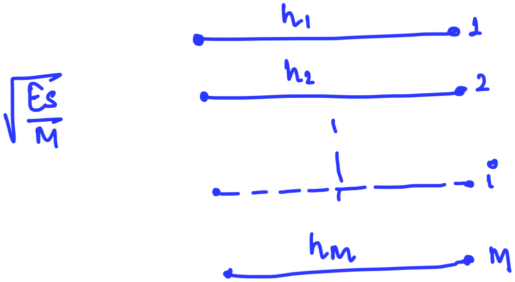
Avg. Pe for MRC.

$$\boxed{\bar{P}_s = \alpha_M \left(\frac{\bar{\Gamma}\beta_M}{2}\right)^{-M}}$$

→ ↑ branches, P_e ↓.

Diversity order → M

General order diversity :-



$$y_i = \sqrt{\frac{E_s}{M}} \cdot h_i s + n_i$$

ZMCS CG

$$\sigma_{n_i}^2 = \sigma_n^2 = N_0$$

$$E[n_i n_j^*] = 0$$

rx signal

$$z = \sum_{i=1}^{M} h_i^* y_i$$

$$= \sqrt{\frac{E_s}{M}} \sum_{i=1}^{M} |h_i|^2 s + \underbrace{\sum_{i=1}^{M} h_i^* n_i}_{\tilde{n}}$$

$$E[\tilde{n} \tilde{n}^*] = \sum_{i=1}^{M} |h_i|^2 N_0$$

$$SNR = \frac{\frac{E_s}{M} \left(\sum_{i=1}^{M} |h_i|^2 \right)^2}{\sum_{i=1}^{M} |h_i|^2 N_0}$$

NPTEL

$$= \frac{1}{M} \underbrace{\sum_{i=1}^{M} |h_i|^2}_{} \frac{E_s}{N_0}$$

$$\boxed{\eta = \frac{1}{M} \|h\|_F^2 \rho} \quad \text{— (1)}$$

Chernoff bound: $Q(x) \leq e^{-x^2/2}$

$$P_e \approx N_e Q\left(\sqrt{\frac{d_{min}^2}{2} \eta}\right) \quad \text{— (2)}$$

$$\boxed{P_e \leq N_e \, e^{-\frac{\rho}{4M} d_{min}^2 \|h\|_F^2}}$$

Error prob.

NPTEL

$$E\left[e^{-\nu\|h\|_F^2}\right] = \underbrace{\prod_{i=1}^{MR} \frac{1}{1+\nu\lambda_i(R)}}$$

$$\bar{P}_e = E[P_e]$$

$$= \bar{N}_e \prod_{i=1}^{M} \frac{1}{\left(1+\frac{\rho}{4M} d_{min}^2\right)}$$

$$\boxed{\bar{P}_e = \bar{N}_e \frac{1}{\left(1+\frac{\rho}{4M} d_{min}^2\right)^M}}$$

Avg probability

At high SNR

$$\boxed{\bar{P}_e = \bar{N}_e \left(\frac{\rho}{4M} d_{min}^2\right)^{-M}}$$

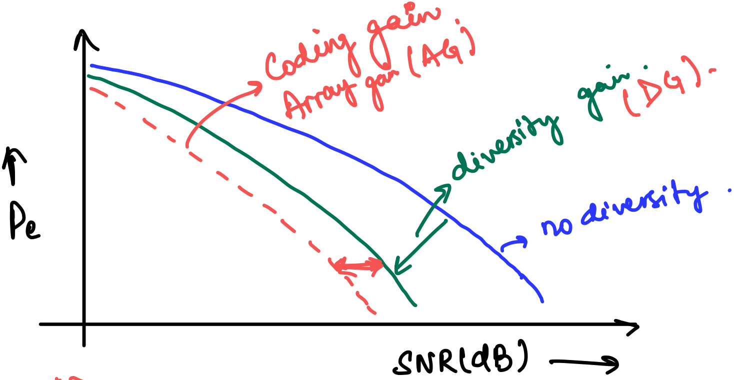
Avg. prob. at high SNR.

→ Role of diversity in reducing P_e.

$$\Rightarrow \boxed{\bar{P}_s = \alpha_M \left(\frac{1}{1 + \bar{\Gamma}\frac{\beta_M}{2}} \right)^M}$$

$\left.\begin{matrix}\alpha_M \\ \beta_M\end{matrix}\right\}$ M → modulation order.

$$\boxed{\alpha_M = N_e \quad ; \quad \beta_M = \frac{d_{min}^2}{2}}$$



$$\boxed{P_e = c\,(\Gamma_c \underset{(\beta)}{\bar{\Gamma}})^{-M}}$$

$$\log P_e = \log c - \underbrace{M}_{\text{Slope}}\left[\underbrace{\log \Gamma_c}_{\text{Shift}} + \log \bar{\Gamma}\right]$$

What is the diversity gain in **dB** for a MIMO system having the following relationship between SNR (ρ) and BER?

$$BER \approx \frac{5}{(4\rho)^2}$$

a. 3 dB ✓

b. 4 dB

c. 6 dB

d. 8 dB

$$P_e = \frac{5}{(4\rho)^2}.$$

Diversity gain = 2

$10\log_{10} 2 \approx 3$ dB.

What is the coding gain in **dB** in the above question ?

a. 3 dB

b. 4 dB

c. 6 dB ✓

d. 8 dB

$r_c = 4$

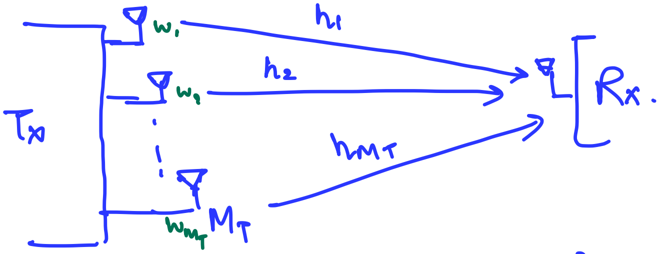
$10 \log_{10} 4 \simeq 6 \text{ dB}.$

Spatial diversity

- Rx
  - SC
  - MRC.
- Tx
  - with CSI at tx
  - without CSI at tx.

Tx Diversity:

1. CSI known at tx:



MISO: $\quad h = \begin{bmatrix} h_1 & h_2 & \cdots & h_{M_T} \end{bmatrix}_{1\times M_T}.$

$$\underset{1\times1}{y} = \sqrt{\frac{E_s}{M_T}} \cdot \underset{1\times M_T}{h} \cdot \underset{M_T\times1}{W} \cdot \underset{1\times1}{S} + n_{1\times1}$$

To maintain Tx power of Es.

$$\|w\|_F^2 = M_T.$$

$$\underline{w} = \sqrt{M_T}\,\frac{\underline{h}^H}{\sqrt{\|h\|_F^2}}$$

$$\Rightarrow\ y = \sqrt{\frac{E_s}{\cancel{M_T}}}\cdot\underline{h}\,\cancel{\sqrt{M_T}}\,\frac{h^H}{\sqrt{\|h\|_F^2}}\,s + n.$$

$$= \sqrt{E_s}\,\|h\|_F^2\,s + n.$$

$$\boxed{\Gamma_\Sigma = \frac{E_s}{N_0}\|h\|_F^2 = \bar{\Gamma}\|h\|_F^2}$$

$$P_e = \bar{N}_e\, Q\left(\sqrt{\frac{\Gamma_\Sigma d_{min}^2}{2}}\right)$$

$$\boxed{P_e \leq \bar{N}_e\, e^{-\frac{\bar{\Gamma} d_{min}^2}{4}\|h\|_F^2}}$$

error prob. for Tx MRC with CSI.

$$\boxed{\bar{P}_e = \bar{N}_e \frac{1}{\left(1+\frac{\bar{\Gamma} d_{min}^2}{4}\right)^M}}$$

Avg. Pe for Tx MRC with CSI

at high SNR,

$$\boxed{\bar{P}_e = \bar{N}_e \left(\frac{\bar{\Gamma} d_{min}^2}{4} \right)^{-M_T} = \alpha_M \left(\frac{\bar{\Gamma} \beta_M}{2} \right)^{-M_T}}$$

$$\boxed{\Gamma_\Sigma = \frac{E_s}{N_0} \|h\|_F^2 = \bar{\Gamma} \|h\|_F^2}$$

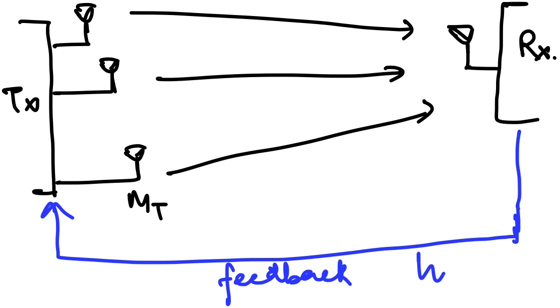
$$\bar{\Gamma}_\Sigma = \bar{\Gamma} E\left[\|h\|_F^2\right] = \bar{\Gamma} E\left[\sum_{i=1}^{M_T} |h_i|^2\right] = M_T \bar{\Gamma}$$

$$\boxed{\bar{\Gamma}_\Sigma = M_T \bar{\Gamma}}$$

Tx diversity when CSI is known:-

→ Avg SNR ↑

→ Diversity ↑



Total interval (T)

t_1 t_2 t_3

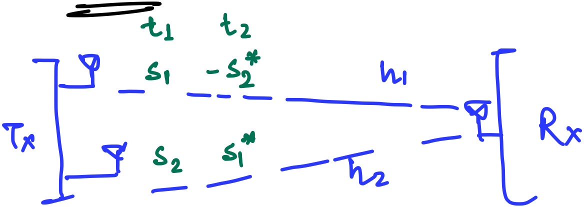
estimation feedback Tx MRC.

$h_1(t_1)$ $h_1(t_2)$ $h_1(t_3)$

Disadvantage of known CSI:-

- $T < T_c$. $h_1(t_1) \neq h_1(t_3)$
- feedback error.
- Imperfect CSI.

Alamouti Scheme: $[2\ Tx\ ,\ 1\ Rx]$



$$y_1 = \sqrt{\frac{E_s}{2}}\, h_1 s_1 + \sqrt{\frac{E_s}{2}}\, h_2 s_2 + n_1 \quad \text{--- (1)}$$

$$y_2 = -\sqrt{\frac{E_s}{2}}\, h_1 s_2^* + \sqrt{\frac{E_s}{2}}\, h_2 s_1^* + n_2$$

$$y_2^* = \sqrt{\frac{E_s}{2}}\, h_2^* s_1 - \sqrt{\frac{E_s}{2}}\, h_1^* s_1 + n_2^* \quad \text{--- (2)}$$

$$Y = \begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \sqrt{\frac{E_s}{2}} \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix}$$

$$Y = \underset{\sim}{H_{eff}} \cdot s + n .$$

Equalization at Rx:-

$$\tilde{Y} = H_{eff}^H Y = \underbrace{H_{eff}^H H_{eff}}\, s + H_{eff}^H n .$$

$$\begin{bmatrix} h_1^* & h_2 \\ h_2^* & -h_1 \end{bmatrix} \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} = \begin{bmatrix} |h_1|^2 + |h_2|^2 & 0 \\ 0 & |h_1|^2 + |h_2|^2 \end{bmatrix}$$

$$= \begin{bmatrix} \|h\|_F^2 & 0 \\ 0 & \|h\|_F^2 \end{bmatrix} = \|h\|_F^2 I_2$$

$$y = \sqrt{\frac{E_s}{2}} \underbrace{\|h\|_F^2} I s + \underbrace{H_{eff}^H n}_{\tilde{n}}$$

$$E[\tilde{n}^H \tilde{n}] = E[n^H H_{eff} \cdot H_{eff}^H \cdot n]$$

$$= \|h\|_F^2 \, E[n^H n]$$

$$= \|h\|_F^2 N_0 I_2 .$$

SNR, $$\Gamma_\Sigma = \frac{\frac{E_s}{2} \left(\|h\|_F^2\right)^2}{\cancel{\|h\|_F^2} \cdot N_0} = \frac{1}{2} \overbrace{\frac{E_s}{N_0}}^{\bar{\Gamma}} \|h\|_F^2$$

$$\boxed{\Gamma_\Sigma = \frac{\bar{\Gamma}}{2} \|h\|_F^2}$$ SNR for Alamouti Code 2X1

$$P_e \simeq \bar{N}_e \, Q\left(\sqrt{\frac{\Gamma_{\xi}\, d_{min}^2}{2}}\right) \leq \bar{N}_e \, e^{-\frac{\Gamma_{\xi}\, d_{min}^2}{4}}.$$

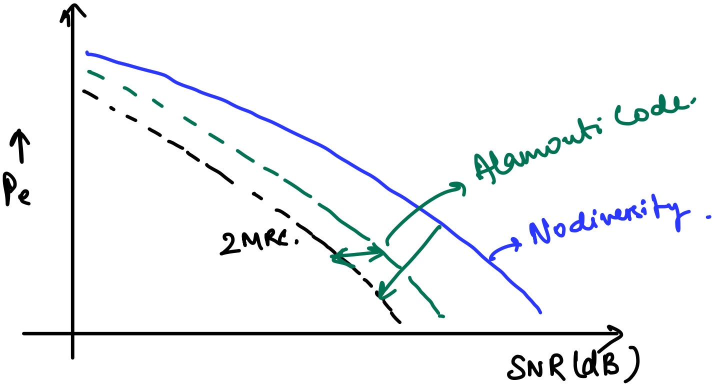
Avg. P_e

$$\bar{P}_e \simeq \bar{N}_e \prod_{i=1}^{2} \frac{1}{1+\frac{\bar{\Gamma}\, d_{min}^2}{8}}$$

At high SNR

$$\boxed{\bar{P}_e \simeq \bar{N}_e \left(\frac{\bar{\Gamma}\, d_{min}^2}{8}\right)^{-2}}$$

Avg. P_e for Alamouti code.


Pe
SNR (dB)
2MRC.
Alamouti Code.
No diversity.

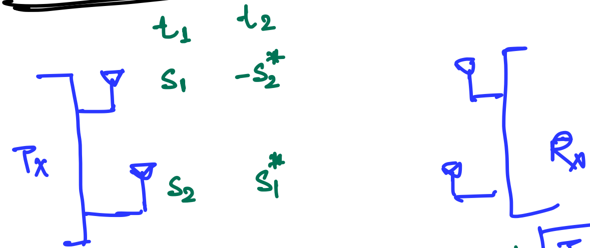
If $M \to \infty$

$$P_e \le \bar{N}_e \left(\frac{\bar{\Gamma} d_{min}^2}{4M} \right)^{-M}$$

*

$$\boxed{\bar{P}_{e_{M \to \infty}} \le \bar{N}_e \, e^{-\frac{\bar{\Gamma} d_{min}^2}{4}}} \Rightarrow \text{AWGN.}$$

2 tx , 2 rx antennas (Modification of alamouti)



$* \; \boxed{T_c > 2T_s}$

$$H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$$

$$\begin{bmatrix} y_1(t_1) \\ y_2(t_1) \end{bmatrix} = Y_1 = \sqrt{\frac{E_s}{2}} \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$Y_2 = \sqrt{\frac{E_s}{2}} \begin{bmatrix} h_{21} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} -s_2^* \\ s_1^* \end{bmatrix} + \begin{bmatrix} n_3 \\ n_4 \end{bmatrix}$$

$$Y_2^* = \sqrt{\frac{E_s}{2}} \begin{bmatrix} h_{11}^* & h_{12}^* \\ h_{21}^* & h_{22}^* \end{bmatrix} \begin{bmatrix} -s_2 \\ s_1 \end{bmatrix} + \begin{bmatrix} n_3^* \\ n_4^* \end{bmatrix}$$

$$= \sqrt{\frac{E_s}{2}} \begin{bmatrix} h_{12}^* & -h_{11}^* \\ h_{22}^* & -h_{21}^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_3^* \\ n_4^* \end{bmatrix}$$

$$Y = \begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \sqrt{\frac{E_s}{2}} \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \\ h_{12}^* & -h_{11}^* \\ h_{22}^* & -h_{21}^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3^* \\ n_4^* \end{bmatrix}$$

$$\rightarrow \quad Y = \sqrt{E_s/2}\, \underline{H_{eff}}\, \underline{s} + \underline{n}$$

Equalization

$$\underline{z} = \underline{H_{eff}}^H \underline{Y} = \sqrt{E_s/2}\, \underline{H_{eff}}^H \underline{H_{eff}}\, \underline{s} + \underbrace{\underline{H_{eff}}^H \underline{n}}_{\tilde{n}}$$

$$H_{eff}^H H_{eff} = \begin{bmatrix} h_{11}^* & h_{21}^* & h_{12} & h_{22} \\ h_{12}^* & h_{22}^* & -h_{11} & -h_{21} \end{bmatrix} \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \\ h_{12}^* & -h_{11}^* \\ h_{22}^* & -h_{21}^* \end{bmatrix}$$

$$= \begin{bmatrix} \|H\|_F^2 & 0 \\ 0 & \|H\|_F^2 \end{bmatrix} = \|H\|_F^2 I_2$$

$$E[\tilde{n}^H \tilde{n}] = \|H\|_F^2 I_2 N_0.$$

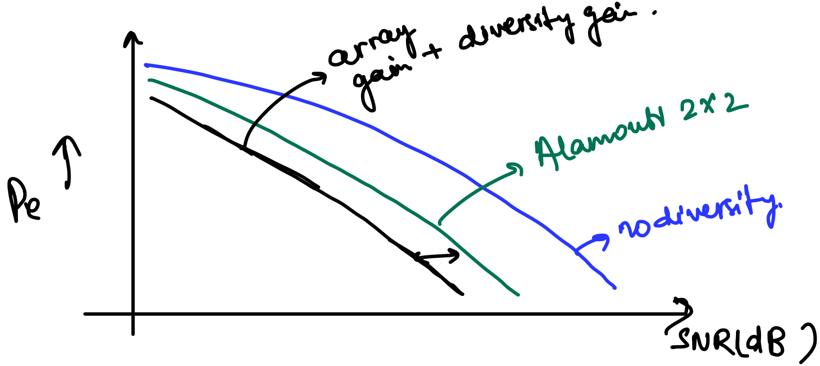
$$\Gamma_\Sigma = \frac{\frac{E_s}{2}\left(\|H\|_F^2\right)^2}{\cancel{\|H\|_F^2} N_0} = \frac{1}{2}\frac{E_s}{N_0}\|H\|_F^2$$

SNR for 2x2 $\boxed{\Gamma_\Sigma = \frac{\bar{\Gamma}}{2}\|H\|_F^2}$

Avg. SNR $\boxed{\bar{\Gamma}_\Sigma = \frac{\bar{\Gamma}}{2} E\left[\|H\|_F^2\right] = \frac{\bar{\Gamma}}{2} \times 4 = 2\bar{\Gamma}}$

additional SNR gain

→ Array gain.


array gain + diversity gain.
Alamouti 2x2
no diversity
Pe
SNR(dB)

MIMO Diversity:

$$H \to M_R \times M_T .$$

$$Y_{M_R \times 1} = \sqrt{\frac{E_s}{M_T}}\, \underline{\underline{H}}\, \underline{W}\, S + n_{M_R \times 1}$$

w must satisfy $\|w\|_F^2 = M_T$

$z = \underset{1\times M_R}{g^H} y_{M_R \times 1}$ $\quad g \rightarrow$ complex weight vector at rx.

$z = \sqrt{\frac{E_s}{M_T}}\, \underline{g}^H \underline{H}\, \underline{w}\, \underline{s} + \underbrace{g^H n}_{\tilde{n}}$

$E[\tilde{n}^H \tilde{n}] = \|g^H\|_F^2 N_0$

$\Gamma_\Sigma = \frac{\|g^H H w\|_F^2}{M_T \|g^H\|_F^2} \cdot \bar{\Gamma}$ $\qquad \bar{\Gamma} = \frac{E_s}{N_0}$

$$\max \frac{\| g^H H w \|_F^2}{\| g^H \|_F^2}$$

$$H = U \Sigma V^H \quad (SVD).$$

We will take,

$\frac{w}{\sqrt{M_T}}$ & g from U & V.

$$\Sigma = \begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_r \end{bmatrix} \qquad \sigma_1 > \sigma_2 \cdots$$

$\boxed{\sigma_i^2 = \lambda_i}$ Dominant eigen value technique

$$\Sigma = \begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_r \end{bmatrix}$$

$\lambda \rightarrow$ eigenvalue of HH^H

$$\sigma_{max} = \sqrt{\lambda_{max}}.$$

$$H = U \Sigma V^H$$

$$\underbrace{U^H U}_{I} \quad \Sigma \quad \underbrace{V^H V}_{I}$$

$$\boxed{\frac{w}{\sqrt{M_T}} \rightarrow 1^{st} \text{ column of } V}$$

$$\boxed{g \rightarrow 1^{st} \text{ row of } U}$$

$$z = \sqrt{E_s}\,\sigma_{max}\, s + n.$$

$$\boxed{\Gamma_\Sigma = \lambda_{max}\bar{\Gamma}}$$

Dominant eigen value mode of transmission

$$\boxed{\lambda_{max} = \frac{\Gamma_\Sigma}{\bar{\Gamma}}}$$

$$\|H\|_F^2 = \sum_{i=1}^{r} \lambda_i$$

$$\Rightarrow \frac{1}{r}\sum_{i=1}^{r} \lambda_i = \frac{1}{r}\|H\|_F^2$$

$$\Rightarrow \frac{1}{r}\sum_{i=1}^{r}\lambda_i \leq \lambda_{max} \leq \|H\|_F^2$$

$$\Rightarrow \frac{\|H\|_F^2}{\min(M_T, M_R)} \leq \lambda_{max} \leq \|H\|_F^2$$

$$\Rightarrow \frac{\|H\|_F^2}{\min(M_T, M_R)} \leq \frac{\Gamma_\leq}{\bar{\Gamma}} \leq \|H\|_F^2$$

$$\Rightarrow \frac{\|H\|_F^2\,\bar{\Gamma}}{\min(M_T, M_R)} \leq \Gamma_\leq \leq \bar{\Gamma}\,\|H\|_F^2$$

when H is full rank,

$$\boxed{\bar{\Gamma}\max(M_R, M_T) \leq \Gamma_\Sigma \leq M_T M_R \bar{\Gamma}}$$

SNR range for MIMO.

$$\bar{N}_e \left(\frac{\bar{\Gamma} d_{min}^2}{4\max(M_T, M_R)}\right)^{-M_T M_R} \geq P_e \geq \bar{N}_e \left(\frac{\bar{\Gamma} d_{min}^2}{4}\right)^{-M_T M_R}$$

full diversity obtained MIMO with known CSI.

Effect of correlation

$$\bar{P}_e = \max \sum_{i=1}^{4} \log\left(1 + \bar{\Gamma}\frac{d_{min}^2}{8}\lambda_i(R)\right)$$

Correlation $\uparrow$, P_e $\uparrow$.

Rician channel :-

$E[H] = \bar{H} \rightarrow$ non zero mean.

$$H = \sqrt{\frac{K}{1+K}}\,\bar{H} + \sqrt{\frac{1}{1+K}}\,H_w$$

$K \to \infty \Rightarrow$ LOS strong.

$$\boxed{\bar{P}_e \leq e^{-\bar{r}\frac{d_{min}^2}{8}\|H\|_F^2}}$$

For a 4×8 MIMO configuration, given the channel is spatially white, what is the expectation of the Frobenious norm of the channel matrix $\mathbf{H}$

a. 16

b. 64

c. 32 ✓

d. 8

$$E\left[\|H\|_F^2\right] = M_T M_R$$

$$M_T = 4, \quad M_R = 8.$$

$$H_{M_R \times M_T}$$

$$E\left[\|H\|_F^2\right] = 4 \times 8 = 32$$

The original Alamouti's scheme consists of M transmit and N receive antennas. Then 1

a. $M = N = 1$

b. $M = N = 2$

c. $M = 2$ and $N = 1$

d. $M = 1$ and $N = 2$

2 Tx , 1 Rx antenna

$M = 2$, $N = 1$.

Rx diversity

Tx diversity.

A 1×2 SIMO system employing Alamouti scheme and a 2×1 MISO system employing Maximum Ratio Combining have

a. same diversity gain (DG) and array gain (AG)

b. different DG and AG

c. same DG but different AG

d. same AG but different DG

For a MIMO system without any array gain, if the order of diversity is increased till it tends to infinity without increasing the total transmitted power then the probability of error tends to

a. zero

b. 0.5

c. same as an additive white Gaussian noise (AWGN) channel with the same signal to noise ratio

d. same as if the diversity order was unity

The diversity order of a $M_R \times 2$ MIMO system (without channel state information at transmitter) is

a. 0

b. 2

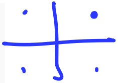
c. M_R

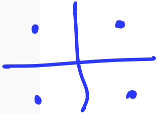
d. $2M_R$

Find the average Bit Error Rate (BER) for 2×1 Alamouti transmit diversity scheme for average symbol energy to noise spectral density ratio $\frac{E_s}{N0}$ of 10 dB and QPSK modulation.

(BER$= \frac{\overline{P_e}}{log_2 M}$, $\overline{P_e}$ is average Symbol Error Rate, M is the order of the modulation)

a. 0.16
b. 0.32
c. 0.08
d. 0.04





2×1 Alamouti

$\bar{\Gamma} = 10\text{dB} = 10.$

$$\bar{P}_e \leq \bar{N}_e \left(\frac{\bar{\Gamma} d_{min}^2}{8} \right)^{-2} \qquad d_{min} = \sqrt{2}$$

$$\leq 2 \left(\frac{10 \times 2}{8} \right)^{-2} = 0.32$$

$$\bar{P}_b = \frac{\bar{P}_s}{\log_2 M} = \frac{0.32}{2} = 0.16$$

If correlation is present in the MIMO channel the probability of error

a. increases.

b. decreases.

c. remains unchanged.

d. depends on the probability distribution of the channel coefficients.

For large values of Rician K-factor, a line-of-sight MIMO channel approaches

a. A correlated Rayleigh channel.
b. Additive White Gaussian Noise channel.
c. An uncorrelated Rayleigh channel.
d. None of the above.

An instance of 2×2 spatially white MIMO channel is given as $\mathbf{H} = \begin{bmatrix} 0.9+0.1j & 0.2+0.1j \\ 0.1+0.3j & 0.8+0.8j \end{bmatrix}$. Find the transmit ($\mathbf{w}$) and receive weight vectors ($\mathbf{g}$) for dominant Eigenmode transmission.

a. $\mathbf{g} = \begin{bmatrix} -0.8 \\ -1.15-0.2j \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} -0.8 \\ -1.15-0.2j \end{bmatrix}$.

b. $\mathbf{g} = \begin{bmatrix} -0.5-0.13j & 0.85+0.01j \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} -0.8 \\ -1.15-0.2j \end{bmatrix}$.

c. $\mathbf{g} = \begin{bmatrix} -0.8 \\ -1.15-0.2j \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} -0.5-0.13j & 0.85+0.01j \end{bmatrix}$.

d. $\mathbf{g} = \begin{bmatrix} -0.5-0.13j & 0.85+0.01j \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} -0.5-0.13j & 0.85+0.01j \end{bmatrix}$.

$$\Sigma = \begin{bmatrix} 1.3 & 0 \\ 0 & 0.75 \end{bmatrix}$$

NPTEL

$$\frac{w}{\sqrt{M_T}} \rightarrow \begin{bmatrix} -0.56 \\ -0.8+0.14j \end{bmatrix}$$

$g \rightarrow$ 1st row of $U \rightarrow [0.5-8.2j \quad 0.85+0.0j]$

Consider a MIMO system specified with channel matrix H. If the maximum eigen value λ_{max} of the matrix HH^H is 4 and the average symbol energy to noise spectral density ratio $\frac{E_s}{N_0}$ is 10 dB, find the post processing SNR with dominant Eigen mode transmission.

a. 10 dB
b. 6 dB
c. 14 dB
d. 16 dB

$\lambda_{max} = 4.$

$\bar{\Gamma} = 10\,dB \approx 10.$

$\Gamma_{\Sigma} = \lambda_{max}\bar{\Gamma} = 40$

$10\log_{10} 40 = 16\,dB.$

An instance of a 2×2 spatially white MIMO channel is given by a channel matrix $\mathbf{H}$ which has two non zero singular values 1 and 0.5. The SNR per branch is 13 dB and Alamouti scheme is employed, the corresponding received SNR is approximately

a. 16 dB

b. 11 dB

c. 14 dB

d. 22 dB

$\sigma_1 = 1, \ \sigma_2 = 0.5.$

$$\|H\|_F^2 = \sigma_1^2 + \sigma_2^2$$

$$= 1 + 0.25 = 1.25$$

$$\Gamma = 13\,dB \simeq 20$$

$$\Gamma_\Sigma = \frac{\Gamma}{2}\|H\|_F^2 = \frac{20}{2} \times 1.25 = 12.5$$

$$\approx 11\,dB.$$

For a 2×2 MIMO system with $\mathbf{H} = \mathbf{H}_w$ and assuming Alamouti scheme is employed, if $\overline{\eta}$ is the average received SNR and ρ is the SNR per branch, then $\frac{\overline{\eta}}{\rho}$ is

a. 1

b. 2

c. $\sqrt{2}$

d. 4

$\overline{\eta} \rightarrow$ avg. rx SNR.

$\rho \rightarrow$ SNR per branch.

$$\eta = \frac{\rho}{2} \|H\|_F^2$$

$$\overline{\eta} = \frac{\rho}{2} E\left[\|H\|_F^2\right] = \frac{\rho}{2} \times 4 = 2\rho.$$

$$\frac{\overline{\eta}}{\rho} = 2$$

A 4×2 MIMO system with $\mathbf{H} = \mathbf{H}_w$ uses dominant eigenmode transmission. The SNR per branch is 10 dB. The range of received SNR is

a. $12 \text{ dB} \leq \overline{\eta} \leq 14 \text{ dB}$

b. $15 \text{ dB} \leq \overline{\eta} \leq 17 \text{ dB}$

c. $16 \text{ dB} \leq \overline{\eta} \leq 19 \text{ dB}$

d. $18 \text{ dB} \leq \overline{\eta} \leq 21 \text{ dB}$

4×2 MIMO

$M_T = 4,\ M_R = 2$

$\bar{\Gamma} \max(M_T, M_R) \leq \Gamma_\Sigma \leq M_T M_R \bar{\Gamma}$

$\bar{\Gamma} = 10$

$40 \leq \Gamma_\Sigma \leq 8 \times 10$

$\Rightarrow 40 \leq \Gamma_\Sigma \leq 80$

$\Rightarrow 16 \text{ dB} \leq \Gamma_\Sigma \leq 19 \text{ dB}$