

Fundamentals of MIMO Wireless Communication

Tutorial

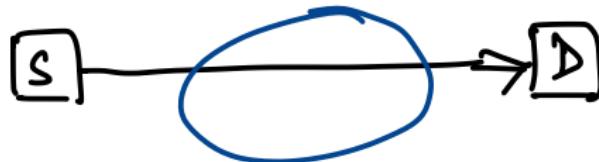
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Protocol layers.



- hosts
 - router
 - communication links
 - Application running on end user Systems
 - Protocols
 - how info on router devices
- ④ Need for proper structure. → how data goes from source to destination.

Air travel: (source)

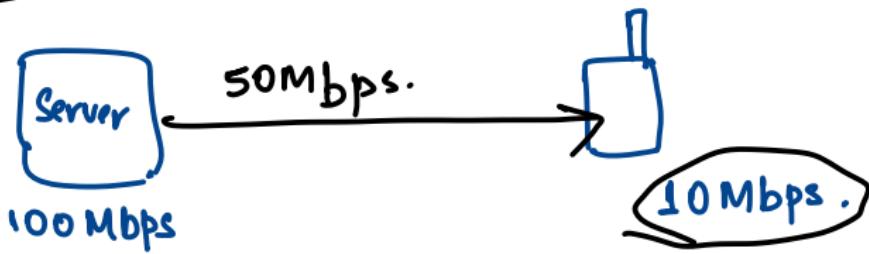


- ① Application
 - ② **Transport**
 - ③ Network layer
 - ④ Data Link layer
 - ⑤ Physical layer (Physical transmission of data).
- Presentation & Session layer.

Application layer: eg: Browser on client side
Webserver on server side.

→ HTTP, HTTPS, etc.

Transport layer: → Reliability of commⁿ. (QoS).



→ decrease data rate
→ ↑_{se} data rate.

- ④ Send Automatic Repeat Request & get the data retransmitted.

→ TCP, UDP

✓ TCP: Provides feedback (slow, reliable).

✓ UDP: faster, no feedback (movie streaming).

Network layer: Determines correct path to reach destination.

→ Routing from source to destination.

→ IP protocol.

Data link layer :- transmitting between neighbouring data elements.

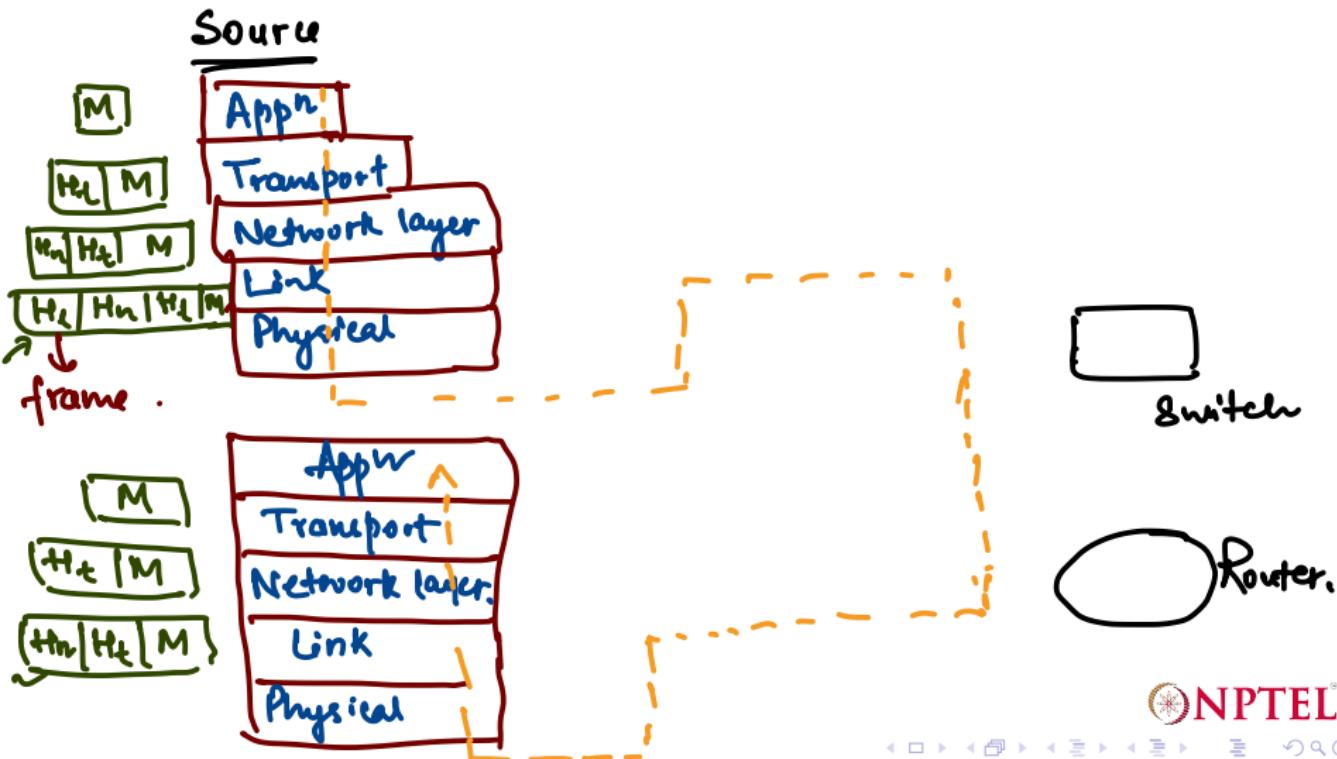
① Doesn't know source or destination.

② Ensures the data reaches to next hop to reach destination.

Neighbouring hop.

Physical layer: bits are actually sent through
the medium.

wireless, wired, OF -- .



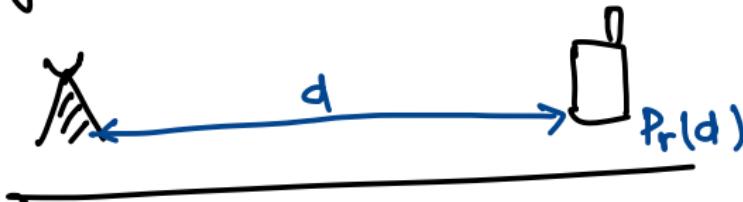
Presentation layer:

- Data is translated into bits
- Compression → lossy, lossless. (Speed).
- Encryption → data security

Session layer: ② Authorization (User has permissions ^{or} _{not}). ① Authentication (User is valid or not)

Web browser:- Works as application, presentation & session layer.

Propagation models :-

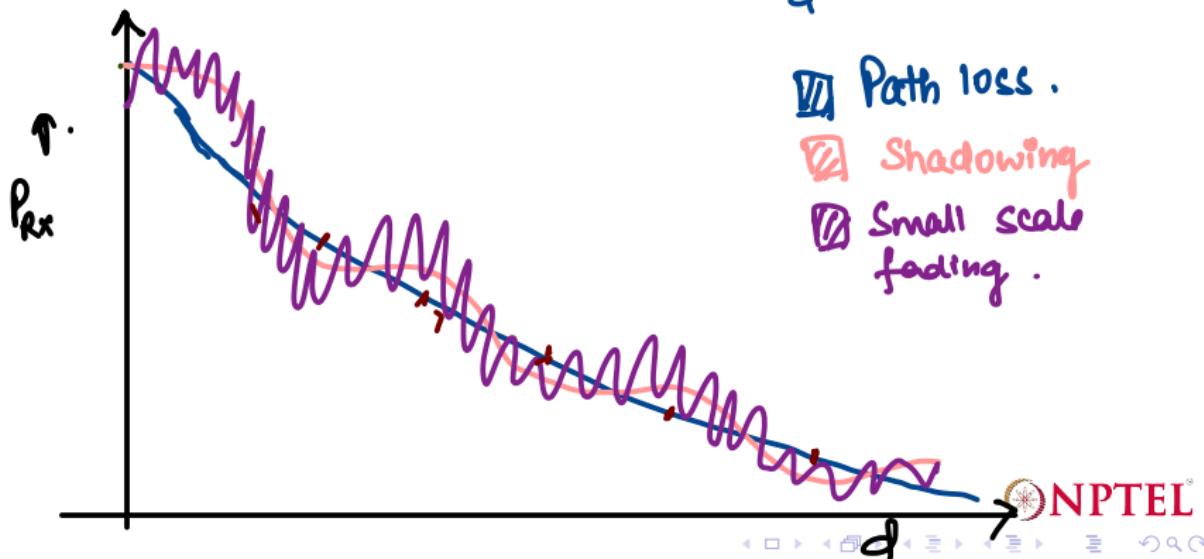
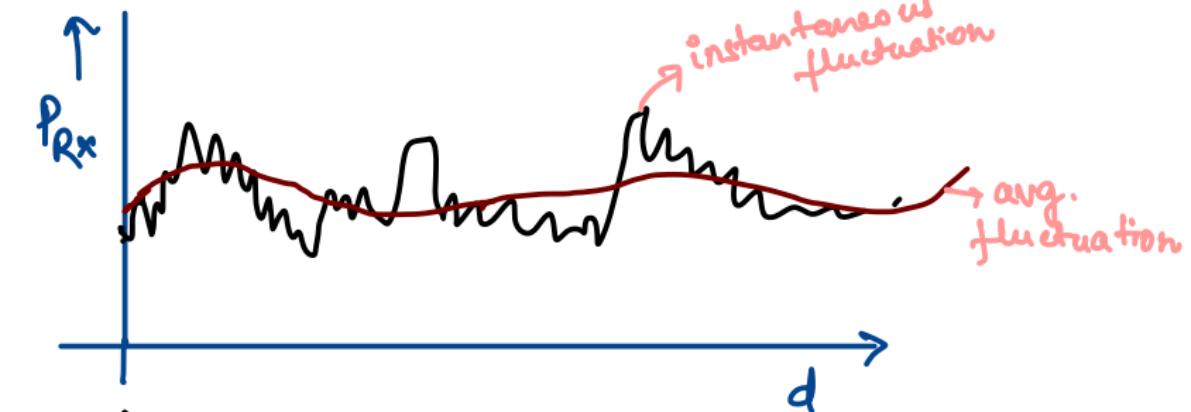


- large scale propagation model.
- depicts rx power at a dist. d .
- $d \sim 100m$

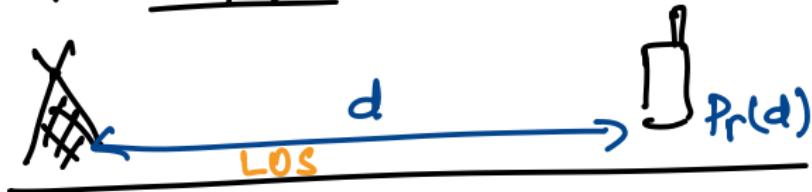
- Small scale propagation model :-
- fluctuation of signal over short dist (order of wavelength), over short duration (few seconds).



when power is below a threshold → Outage happens.



free space Propagation :-



frisis free space propagation loss :-

$$Pr(d) = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2 L^2}$$

Usually, $L = 1$.

↑
coupling loss,
filter loss,
etc.

LOS :

$\boxed{Pr(d) \propto \frac{1}{d^2}}$ $\Rightarrow 20 \text{ dB / decade.}$

$$G = \frac{4\pi A_e}{\lambda^2}$$

$A_e \rightarrow$ Antenna aperture .

$$\boxed{PL_{dB} = 10 \log_{10} \frac{P_t}{Pr} = -10 \log_{10} \left(\frac{G_t G_r \lambda^2}{(4\pi)^2 d^2} \right) *}$$

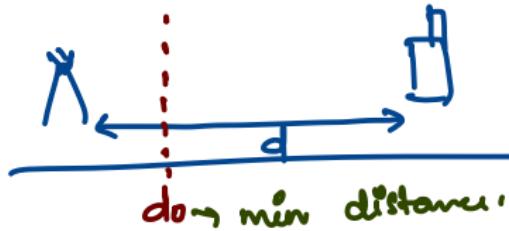
Antenna gain $G_t = G_r = 1$.

$$P_L(\text{dB}) = -10 \log \left(\frac{\lambda^2}{(4\pi)^2 d^2} \right)$$

$$\boxed{P_r(d) \propto \frac{1}{d^2}}$$

$d=0$

$$P_r(d_0) \propto \frac{1}{d_0^2} \rightarrow \text{Known.}$$



$d_0 \rightarrow \text{min distance.}$

* $\boxed{P_r(d) = P_r(d_0) \cdot \frac{d_0^2}{d^2}}$

$$\boxed{d > d_0}$$

$P_r(d_0)$ \rightarrow calculated empirically.

frankhafer distance:

$$d_f = \frac{2D^2}{\lambda}$$

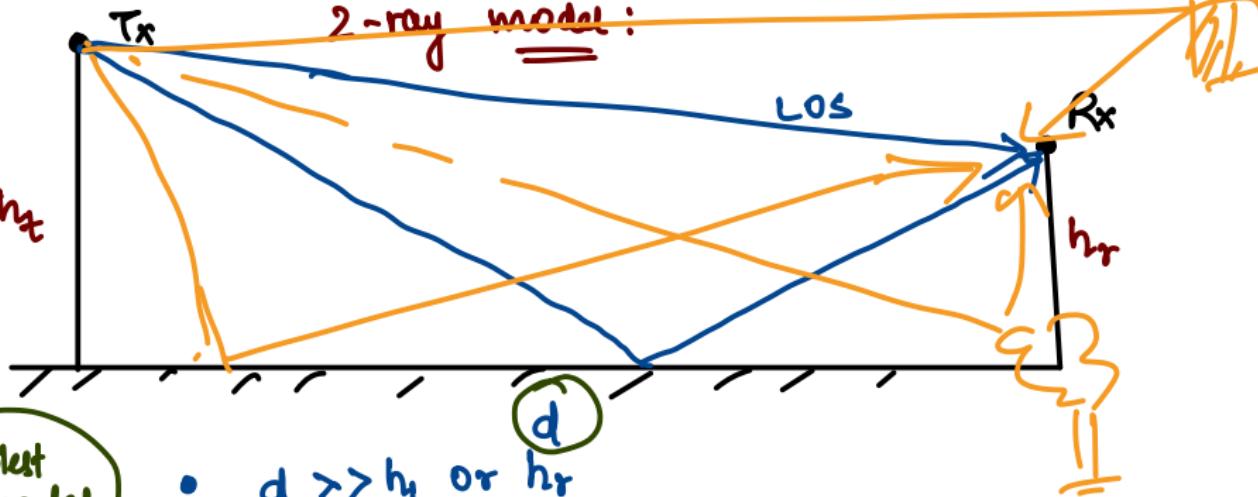
, $D \rightarrow$ Largest dimension of antenna,
 $\lambda \rightarrow$ wavelength of radio wave.

$$d_o > d_f$$

$d_o \ll$ BS-MS distance

$$\boxed{P_r(d) = P_r(d_o) \left(\frac{d_o}{d} \right)^2 \quad | \quad d \geq d_o \geq d_f}$$

$$\boxed{P_r(d)_{dB} = P_r(d_o)_{dB} + \textcircled{20} \log \left(\frac{d_o}{d} \right)}$$



Simplest model

- $d \gg h_t$ or h_r

$$P_r = P_t G_t G_r \frac{h_t^2 h_r^2}{d^4}$$

- for LOS, $P_r \propto 1/d^2$
- for two-ray, $P_r \propto \frac{1}{d^4}$.

$$P_r = P_t G_t G_r \frac{h_t^2 h_r^2}{d^4}$$

$$PL(dB) = 10 \log \frac{P_t}{P_r} = 40 \log d - (10 \log G_t + 10 \log G_r + 20 \log h_t + 20 \log h_r)$$

in real environment, $10 \times n_p$ → coeff.

n_p → Path loss exponent

$$1.4 \leq n_p \leq 6$$

Calculated empirically.

Tx band pass signal:

$$s(t) = \operatorname{Re} \left\{ \tilde{s}(t) e^{j2\pi f_{ct} t} \right\} \rightarrow \text{txed sig}$$

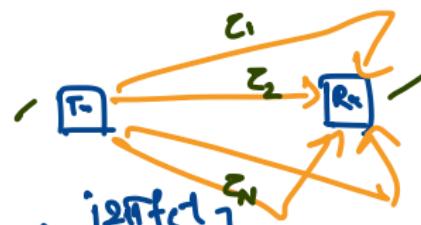
complex envelope of tx signal.

noiseless rxed
sig:

$$r(t) = \operatorname{Re} \left[\sum_{n=1}^N c_n e^{j2\pi(f_c + f_{dn})(t - z_n)} \tilde{s}(t - z_n) \right]$$

amplitude scaling

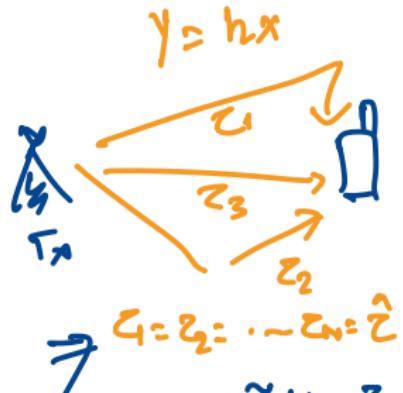
Phase diff from nth path. $\phi_n = 2\pi [f_{dn}t - (f_c + f_{dn})z_n]$



$$\begin{aligned} r(t) &= \operatorname{Re} \left[\sum_{n=1}^N c_n e^{j\phi_n(t)} \tilde{s}(t - z_n) e^{j2\pi f_{ct} t} \right] \\ &= \operatorname{Re} \left[\tilde{r}(t) e^{j2\pi f_{ct} t} \right] \rightarrow \text{Rxed sig.} \end{aligned}$$

$$\tilde{r}(t) = \sum_{n=1}^N c_n e^{j\phi_n(t)} \tilde{s}(t-z_n)$$

$\underbrace{\phantom{\sum_{n=1}^N c_n e^{j\phi_n(t)}}}_{h(t, z_n)} = c_n e^{j\phi_n(t)}.$



$$\tilde{r}(t) = \sum_{n=1}^N h_n(t) \tilde{s}(t-z_n)$$

$$\tilde{r}(t) = h_1(t) \tilde{s}(t-z_1) + h_2(t) \tilde{s}(t-z_2) + \dots + h_N(t) \tilde{s}(t-z_N)$$

$$\tilde{f}(t) = \left(\sum_{n=1}^N h_n(t) \right) \tilde{s}(t-\hat{z})$$

$h_r(t)$

$$\tilde{s}(t) \rightarrow \tilde{s}(t-\hat{z})$$



1) Among the below mentioned Open Systems Interconnection (OSI) layers, which one directly interacts with the medium

a. Data-link layer

b. Physical Layer

c. Transport Layer

d. Application Layer



- 14) The quality of service (QoS) between end to end devices is maintained by the
- a. Presentation Layer
 - b. Session Layer
 - c. Transport Layer
 - d. Application Layer

QoS → performance .

Transport layer :- Reliability of transmission .

Segmentation :-

flow Control :- Slow / speed rate .

Error Control :- ARQ .

TCP

UDP

2) The Diversity combining helps to :

- a. Increase average SNR
- b. Combat large scale fading
- c. Reduce Inter Symbol Interference (ISI)
- d. None of these



Same information from diff. paths.

Diversity: → Combat shadowing effect.

- 3) In general, the wireless channels are
- a. Linear Time Invariant (LTI)
 - b. Linear Time Varying (LTV)
 - c. Additive White Gaussian Noise (AWGN)
 - d. None of these

4) Among the listed type of fading, which is/are considered to be small scale fading?

- a. Path loss.

→ large scale.

- b. Shadow fading



- c. Rayleigh fading



- d. Rician fading

→ small scale fading.

- 5) At the transmitter the baseband signal "x(t)" is up converted to "RF" frequency of 2.4 GHz using a local oscillator. At the receiver the local oscillator used for down converting has frequency of 2.399999 GHz and phase difference between oscillator at TX and RX is $\frac{\pi}{6}$ radian. The down converted signal can be written as, $y(t) = x(t)z(t) + n(t)$, where $n(t)$ is the noise term. $z(t)$ can be given by:

a. 1

b. $e^{j(100\pi t + \frac{\pi}{6})}$

c. $e^{j(1000\pi t - \frac{\pi}{4})}$

$$tx: \checkmark d. e^{j(200\pi t - \frac{\pi}{6})} x(t) e^{j(2\pi f_{ct}t + \phi_t)}$$

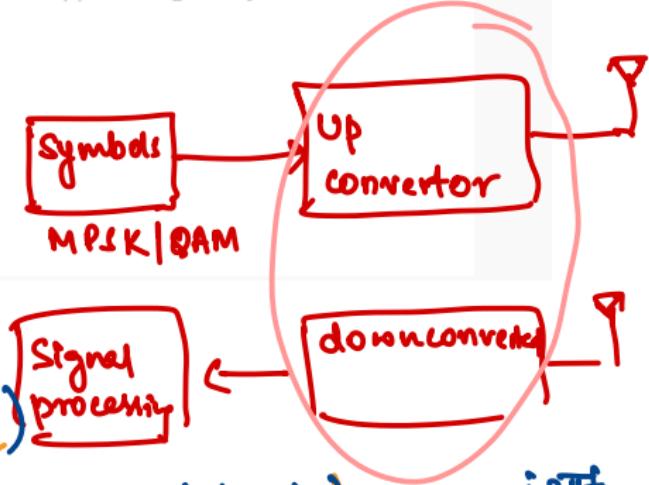
$$rx: -j(2\pi f_{cr}t + \phi_r)$$

$$y(t) = r(t) e^{-j(2\pi f_{cr}t + \phi_r)}$$

$$r(t) = x(t+n\Delta t) \Rightarrow$$

$$\begin{aligned} f_{ct} - f_{cr} &= 10^{-3} \times 10^9 \text{ Hz} \\ &= 100 \text{ Hz} \end{aligned}$$

$$\begin{aligned} &e^{-j(2\pi(f_{ct}-f_{cr})t + (\phi_x - \phi_r))} \left\{ x(t) e^{j2\pi f_{ct}t + \phi_t} \right. \\ &\quad \left. + n(t) \right\} + \tilde{n}(t) \\ &= x(t) e^{j2\pi(200\pi t - \frac{\pi}{6})} + n(t). \end{aligned}$$



6) A cellular communication system is operating at the carrier frequency of 6 GHz. What should be the range of distance over which the average received signal power should be computed to model the large scale fading?

a. 0.1m - 10 m

b. 0.25m - 10m

c. Greater than 10m

d. 0.25m - 2m

$$f_c = 6 \text{ GHz} -$$

$$\lambda = \frac{c}{f_c} = \frac{3 \times 10^8}{6 \times 10^9} = 0.05 \text{ m}.$$

for large scale fading,

$$d \rightarrow 5\lambda \text{ to } 40\lambda$$

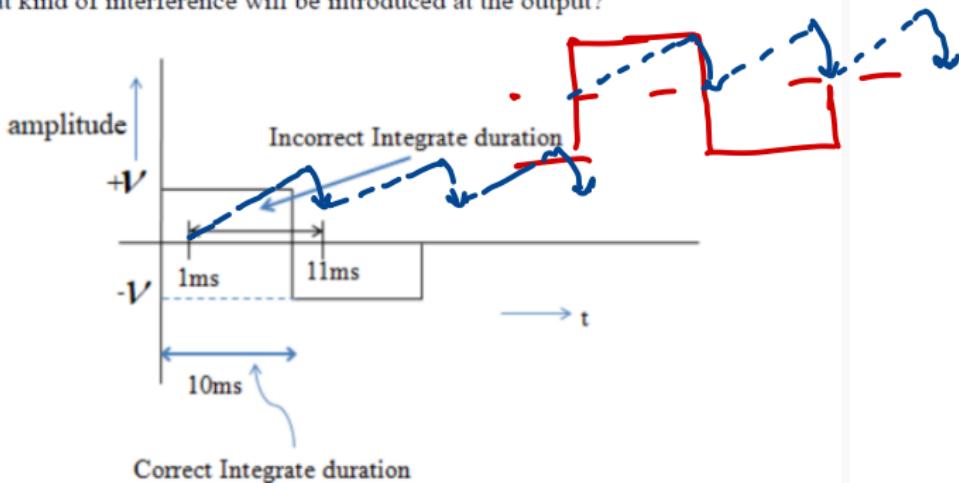
$$\Rightarrow 0.25 \text{ m} - 2 \text{ m}$$

7) MIMO systems consisting of:

- a. Multiple antennas at both the transmitter and receiver side.
- b. Single antenna at the transmitter side and multiple antennas at the receiver side.
- c. Multiple antennas at the transmitter and single antenna at the receiver side
- d. Single antenna at both the transmitter and the receiver side.

	$\boxed{\text{Tx}}$	$\boxed{\text{Rx}}$
SISO	1	1
SIMO	1	multiple
MISO	multiple	1 , multiple .
MIMO	multiple	multiple .

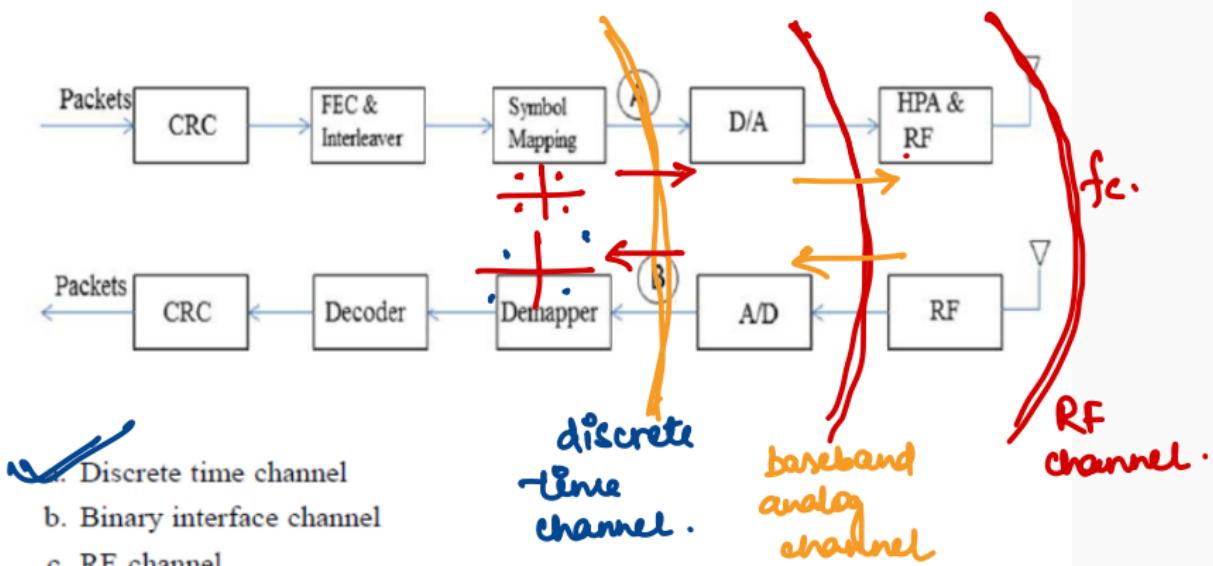
- 8) In the figure below, the received baseband pulse as shown. Symbol duration, $T=10\text{ms}$. The pulse is given at the input of integrate and dump circuit.
Due to an error in time synchronization, if the integration starts at 1 ms instead of origin, what kind of interference will be introduced at the output?



- a. Adjacent Channel Interference
- b. Inter Carrier Interference
- c. Co-channel Interference
- d. Inter Symbol Interference

9)

The interface channel between point A and B in the communication system shown in figure is:



- a. Discrete time channel
- b. Binary interface channel
- c. RF channel
- d. Binary symmetric channel

The power of a certain transmitted signal is 10 mW, the power in dBm is

- a. 10 dBm
- b. 20 dBm
- c. 30 dBm
- d. 40 dBm

$$P_t = 10 \text{ mW}.$$

$$\begin{aligned} P_t^{\text{dBm}} &= 10 \log_{10} \left(\frac{P_t}{10^{-3}} \right) \\ &= 10 \log_{10} \left(\frac{10 \times 10^{-3}}{10^{-3}} \right) = 10 \log_{10} 10 \\ &= 10 \text{ dBm.} \end{aligned}$$

Path Loss Exponent of indoor line of sight communication is

- a. 1
- b. 3
- c. < 2
- d. > 4

$$PL(\text{dB}) = \underbrace{10n_p \log d}_{\text{path loss exponent}} - (10 \log G_t + 10 \log G_r + 20 \log h_t + 20 \log h_r)$$

$$4 \leq n_p \leq 6$$

Given received power at a distance of 30 m as -20 dBm. Find the received power at 100 m by assuming free space propagation.

- a. -30.45 dBm
- b. -60.45 dBm
- c. -90.45 dBm
- d. -120.54 dBm

$$\begin{aligned} \text{Pr}(d_0) &= -20 \text{ dBm}, \quad d_0 = 30 \text{ m} \\ &= -50 \text{ dB} = 10^{-5} \text{ W} \\ d &= 100 \text{ m} \end{aligned}$$

$$n = 2.$$
$$\text{Pr}(d) = \text{Pr}(d_0) \cdot \left(\frac{d_0}{d} \right)^n = 10^{-5} \left(\frac{30}{100} \right)^2$$

$$\text{Pr}(d) \Big|_{\text{dBm}} = -30.45 \text{ dBm}.$$

The distribution followed by the received power in linear scale for a shadow fading channel is

- a. Gaussian
- ~~b. log-normal~~
- c. exponential
- d. Rayleigh

Match the channel path loss models given in column A with the corresponding n_p given in Column B

	Column A		Column B
1	$PL(d) = 42 + 25\log_{10}(d) + 14\log_{10}(f)$	1	6.5
2	$PL(d) = 27 - 35\log_{10}(d) + 17\log_{10}(f)$	2	3.5
3	$PL(d) = 36 + 48\log_{10}(d) + 13\log_{10}(f)$	3	2.5
4	$PL(d) = 52 - 65\log_{10}(d) + 24\log_{10}(f)$	4	4.8

- a. A1-B1; A2-B2; A3-B3; A4-B4
- b. A1-B2; A2-B1; A3-B4; A4-B3
- ~~c. A1-B3; A2-B2; A3-B4; A4-B1~~
- d. A1-B4; A2-B3; A3-B2; A4-B1

$$10n_p \log(d).$$

$$A1: 10n_p = 25 \Rightarrow n_p = 2.5$$

$$A2: 10n_p = 35 \Rightarrow n_p = 3.5$$

$$A3: 10n_p = 48 \Rightarrow n_p = 4.8$$

$$A4: 10n_p = 65 \Rightarrow n_p = 6.5 \Rightarrow B1$$

Find the doppler shift when the mobile user is moving away from base station with a speed of 90 kmph with angle of arrival of 30° . Consider carrier $f_c = 2$ GHz.

- a. 150 Hz ~~x~~
- b. 144 Hz ~~x~~
- c. -150 Hz
- d. 144 Hz ~~x~~

When MS is moving away from BS
→ doppler shift is -ve.

$$v = 90 \text{ kmph} = 90 \times \frac{5}{18} = 25 \text{ m/s.}$$

$$f_D = \frac{v \cos \theta}{\lambda} = \frac{v \cos \theta}{c} \cdot f_c.$$

$$25 \times \frac{\sqrt{3}}{2} \times \frac{2 \times 10^9}{3 \times 10^8} = 144 \text{ Hz.}$$

∴ MS is moving away \Rightarrow $f_D = -144 \text{ Hz.}$

Local average fluctuations of received signal power with transmitter-receiver separation distance is observed due to,

- a. small scale fading
- b. path loss
- ~~c.~~ shadowing
- d. none of the above

Given the transmit power is 0 dBm and received power at a distance d is -90 dBm ($d \gg$ Antenna dimensions). Find the corresponding path loss in dB

a. -120 dBm

b. 90 dB

c. 60 dBm

d. -60 dB

$$\begin{aligned} PL|_{dB} &= 10 \log_{10} \frac{P_T}{P_R} \\ &= 10 \log_{10} P_T - 10 \log_{10} P_R \\ &= P_T|_{dB} - P_R|_{dB}. \end{aligned}$$

$$P_T = 0 \text{ dBm} = 30 \text{ dB}$$

$$P_R = -90 \text{ dBm} = -90 + 30 = -60 \text{ dB}$$

$$\begin{aligned} PL|_{dB} &= P_T|_{dB} - P_R|_{dB} = 30 - (-60) \text{ dB} \\ &= 90 \text{ dB}. \end{aligned}$$

Match the pairs from column A and column B. (K = Rician K-factor)

	Column A		Column B
1	NLOS channel	1	$K = 0$
2	Rayleigh fading	2	$K > 0$
3	Almost no fading	3	$K = \infty$

- a. A1-B1; A2-B2; A3-B3
~~✓~~ b. A1-B2; A2-B1; A3-B3
c. A1-B1; A2-B3; A3-B1
d. A1-B3; A2-B1; A3-B2

A1: B2
A2: B1
A3: B3 .

Rician fading channel: One path is dominant.

$$P_d(x) = \frac{x}{b_0} \exp\left\{-\frac{x^2 + s^2}{2b_0}\right\} I_0\left(\frac{xs}{b_0}\right), x \geq 0$$

$$K = \frac{s^2}{2b_0}$$

, $K \rightarrow \infty \Rightarrow$ No fading \Rightarrow LOS channel
 $K = 0 \Rightarrow$ Rayleigh
 $K > 0 \Rightarrow$ NLOS.



