Fundamentals of MIMO Wireless Communication

Tutorial

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* Classical iid channel assumption:-

- Delay spread is negligible.
- $H \rightarrow \underline{Z}\underline{M}\,\underline{C}\,\underline{S}\,\underline{C}\,\underline{G}$ with unit variance.

ZM :- $h_{ij} \rightarrow 0$ mean $\rightarrow$ Scattering is uniform.

$\underline{CS}$: Circular Symmetry

$$E[e^{j\phi} z] = e^{j\phi} E[z]$$

$$h = \underbrace{h_I}_{} + j \underbrace{h_Q}_{}$$

Gaussian.

* Spatially white channel :-

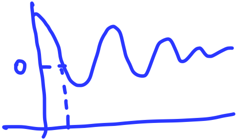
Rays arriving from all dir^n with equal probability.

$E[(H_w)_{i,j}] = 0$, $E[|(H_w)_{ij}|^2] = 1$.

$E[\underbrace{\{H_w\}_{ij}\{H_w\}^*_{mn}}_{\text{Uncorrelated.}}] = 0$ if $i \neq m$, $j \neq n$

* Spatial fading correlation :-

$\Delta x = 0.38\lambda \approx \underline{\lambda/2}$



If correlation is there,
H is modeled as,

$$\boxed{vec(H) = R^{1/2}\, vec(H_w)}$$

white channel

$$R = E\left[vec(H_w)\, vec(H_w)^H\right]$$

$H_w : M_R \times M_T$

$vec(H_w) : M_R M_T \times 1$

$R : M_R M_T \times M_R M_T$

$$A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}_{3\times3}$$

$$vec(A) = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{bmatrix}_{9\times1}$$

NPTEL

$R^H = R$

R : +ve semidefinite matrix

all eigenvalues ≥ 0

$$\boxed{vec(H) = R^{1/2}\, vec(H_w).}$$

$R = I_{M_R M_T}$ → Cross corr. is zero ⇒ $\boxed{H = H_w}$.

* R_r → rx correlation, R_t → tx correlation

$R_r \to M_r \times M_r$

$R_t \to M_t \times M_t$

$$\boxed{H = R_r^{1/2} H_w R_t^{1/2}}$$

(when tx & rx covariance matrix is given).

* Overall Covariance matrix:-

$$\boxed{R = R_t^T \otimes R_r}$$

$M_r M_t \times M_r M_t$

Kronecker product:-

$A: n \times p$, $B: m \times q$.

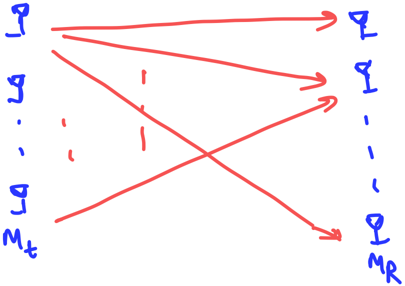
$$A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B & \cdots & a_{1p}B \\ \vdots & & & \\ a_{n1}B & - & - & a_{np}B \end{bmatrix}_{mn \times qp}.$$

* Properties of spatially wide channel H_w

- full rank matrix
- $M_r M_t$ element, each element is ZMCS CG
- $\underset{\sim}{H} = \underset{\sim}{R_r}^{1/2} \underset{\sim}{H_w} \underset{\sim}{R_t}^{1/2}$.

$\text{rank}(H) \leq \text{rank}(H_w)$

* Statistical properties of channel matrix H :-



$H \rightarrow M_R \times M_t$

- $H = UDV^H$ (SVD).

$UU^H = V^H V = I_r$ $r = \text{rank}(H)$.

$D = \text{diag}(\sigma_1, \sigma_2, \ldots \sigma_r)$, $\sigma_i \geq 0$ & $\sigma_i \geq \sigma_{i+1}$

$HH^H \rightarrow M_R \times M_R$

$$(HH^H) = Q \Lambda Q^H$$

$$\Lambda \rightarrow \text{diag}(\underbrace{\lambda_1, \lambda_2, \ldots \lambda_{M_R}}_{\text{eigen values of } HH^H})$$

$$\lambda_i = \begin{cases} \sigma_i^2, & \text{if } i = 1, 2, \ldots r \\ 0, & \text{if } i = r+1, \ldots, M_R \end{cases}$$

- $H \rightarrow$ random
  $\Rightarrow \lambda_1, \lambda_2, \ldots \lambda_{M_R}$ are random.
- $H = H_w \Rightarrow$ elements of $H_w H_w^H$ are wishart distributed.

$H_w H_w^H$

λ contains strength info of channel.

$\lambda_M \rightarrow$ exponentially distributed.

$\sigma_M \rightarrow$ Rayleigh distributed.

* Square Frobenius norm: $\|H\|_F^2$.

$$\|H\|_F^2 = \text{trace}(HH^H) = \underbrace{\sum_{i=1}^{M_R}\sum_{j=1}^{M_T} |h_{ij}|^2}_{\text{Total power gain of channel.}}$$

$(2M_tM_R$ d.o.f $)$

$$\underbrace{\|H\|_F^2}_{\text{random.}} = \sum_{i=1}^{M_R} \underbrace{\lambda_i}_{\text{random.}}$$

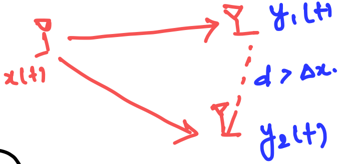
pdf of $\|H\|_F^2$ when $H = H_w$.

$$f(x) = \frac{x^{M_TM_R - 1}}{(M_TM_R - 1)!} e^{-x} \underbrace{u(x)}_{\geq 0}$$

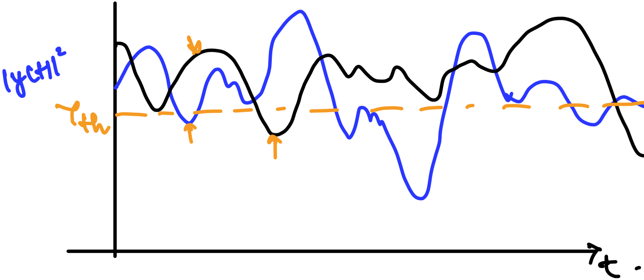
Spatial Correlation:

$$J_0\left(2\pi \frac{\Delta x}{\lambda}\right)$$

$\Delta x \approx \frac{\lambda}{2} \rightarrow$ Correlation $= 0$.



→ flat fading, slow fading

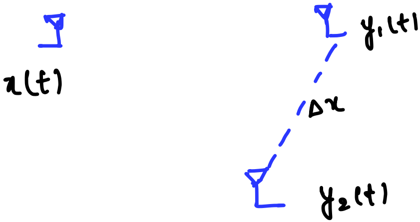


* Squared Frobenius Norm $\|H\|_F^2$

pdf of $\|H\|_F^2$ when $H = H_w$.

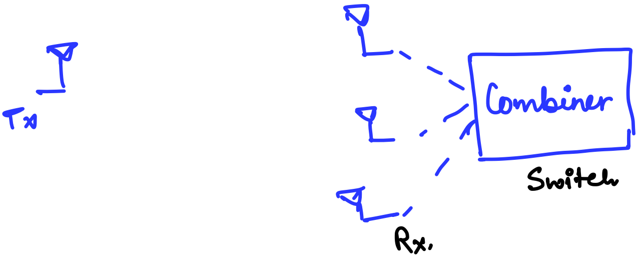
$$f(x) = \frac{x^{M_T M_R - 1}}{(M_T M_R - 1)!} e^{-x} u(x)$$

* Spatial Diversity :-

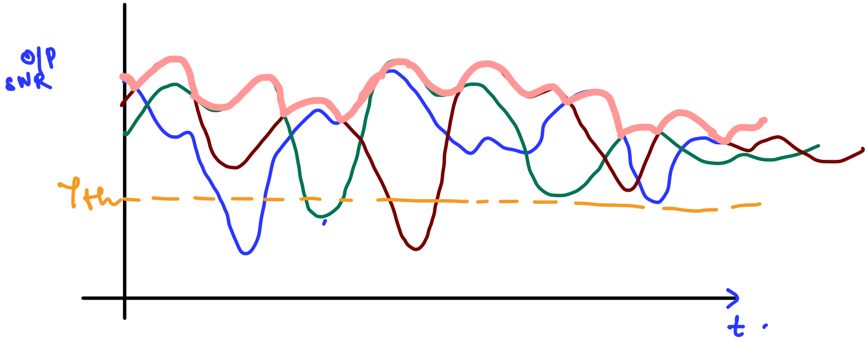


* Selection Combining (SC) :

SIMO s/s :



→ Select o/p with max. power .

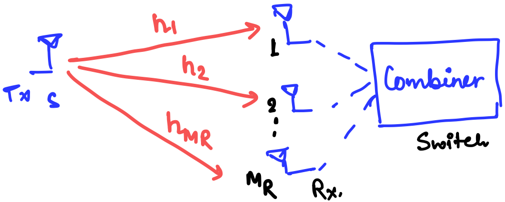


→ LCR changes

→ Avg. duration of fade changes.

→ Dynamic range of fluctuation is decreased.

→ Variability of combining signals is less.



$$y_i = h_i s + w_i \qquad i = 1, 2, \ldots M_R.$$

after selection combining,

$$y_{i^*} = h_{i^*} s + w_{i^*}$$

$$i^* = \arg\max \{ |h_i|^2 \}$$

SNR at each branch :-

$$\Gamma_i = \frac{|h_i|^2 \, E|s_i|^2}{E|w_i|^2}$$

assumption: $E\{|s_i|^2\} = P_s$

$E\{|w_i|^2\} = \sigma_w^2$.

$$\frac{E|s|^2}{E|w|^2} = \frac{P_s}{\sigma_w^2} = \Gamma$$

pdf of SNR (Γ_i) $= \frac{1}{\Gamma} e^{-\Gamma_i/\Gamma}$

$$= \frac{1}{\bar{\Gamma}} e^{-\Gamma_i/\bar{\Gamma}}$$

x

$\bar{\Gamma} \rightarrow$ Avg SNR on i^{th} branch

$\bar{\Gamma} = E[\Gamma_i] = E[\,|h_i|^2 \cdot \Gamma\,] = \underbrace{E|h_i|^2}\, E[\Gamma]$ -

Outage probability:

$$\text{prob}(\Gamma_i \leq \Gamma_{th})$$

$$= \int_0^{\Gamma_{th}} p(\Gamma_i)\, d\Gamma_i$$

$$P_o = \left(1 - e^{-\Gamma_{th}/\bar{\Gamma}}\right)$$

$\downarrow$

prob. that sig. falls below threshold for any link.

* Outage probability for SC:

→ Outage happens when all M branches

$\gamma_m < \gamma_{th}$

$$\text{Prob}(\gamma_1, \gamma_2, \gamma_3, \ldots \gamma_{M_R} < \gamma_{th})$$

$$= \left(1 - e^{-\gamma_{th}/\bar{\gamma}}\right)^M$$

Coverage probability: $1 - \left(1 - e^{-\gamma_{th}/\bar{\gamma}}\right)^M$

* pdf of γ:

$$\frac{d}{d\gamma}\left(\text{Prob}(\gamma_i > \gamma)\right) = M\left(1 - e^{-\gamma_{th}/\bar{\gamma}}\right)^{M-1} \cdot \frac{1}{\bar{\gamma}} \cdot e^{-\gamma/\bar{\gamma}}$$

Post processing SNR!

$$\bar{\Gamma}_{\Sigma} = \int_0^{\infty} \Gamma p_M(\Gamma)\, d\Gamma$$

$$\boxed{\bar{\Gamma}_{\Sigma} = \bar{\Gamma} \sum_{i=1}^{M_R} \frac{1}{i}}$$

Avg. SNR post combining for SC.

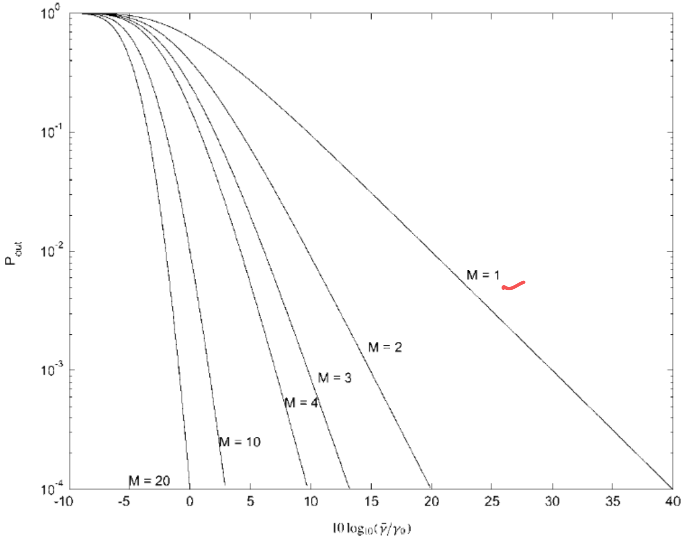


Figure 7.2: Outage probability of selection combining in Rayleigh fading.

$P_{out_1} = 0.1$

$M_R = 2$, $P_{out} = (0.1)^2 = 0.01$

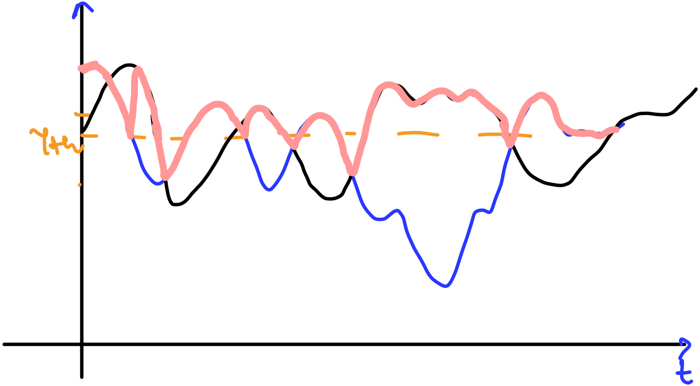
$M_R = 3$, $P_{out} = (0.1)^3 = 0.001$

Disadvantage:

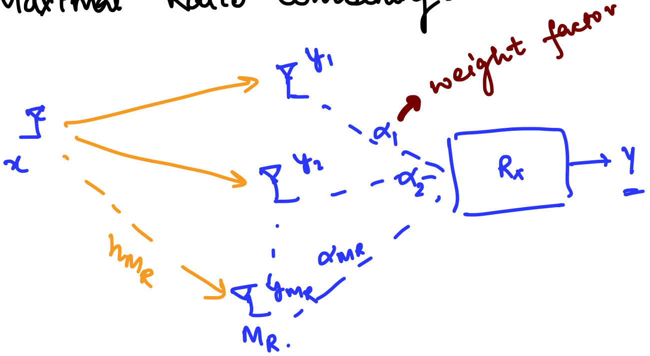
→ At every instant, you have to measure power.

Switched Beam Combining:-

→ Switches only when SNR goes below γ_{th}.



* Maximal Ratio Combining:-



$$y_i = h_i x + w_i$$

MRC: $$y = \sum_{i=1}^{M_R} \alpha_i (h_i x + w_i)$$

$i = 1, 2, \ldots M_R$.

$$\alpha = h_i^* \quad \text{(error prob)}$$

$$\text{Or } \frac{h_i^*}{\sqrt{\sum_k |h_k|^2}} \quad \text{(decoding).}$$

$$y = \sum_{i=1}^{M} \alpha_i (h_i x + w_i)$$

$$= \sum_{i=1}^{M} \left(|h_i|^2 x + h_i^* w_i \right)$$

$$= x \sum_{i=1}^{M} |h_i|^2 + \sum_{i=1}^{M} h_i^* w_i \, .$$

$$P_{sig}: \ E|x^2| \left(\sum_{i=1}^{M} |h_i|^2 \right)^2$$

$$P_{noise}: \ E|w^2| \sum_{i=1}^{M} |h_i|^2$$

* Post processing SNR for MRC :

$$\underset{(\Gamma_{M_{MRC}})}{SNR} = \frac{E[|x|^2]}{E[|w|^2]} \cdot \left(\sum_{i=1}^{M} |h_i|^2 \right)$$

$$= \Gamma_1 + \Gamma_2 + \cdots + \Gamma_{MR}.$$

$$\boxed{\Gamma_{M_{MRC}} = \sum_{i=1}^{M} \Gamma_i} \rightarrow \text{Chi square } (2M \text{ d.o.f})$$

Avg. post. processing SNR : $E[\Gamma_M]$

$$= \sum_{i=1}^{M} E[\Gamma_i] = \underline{\underline{M\bar{\Gamma}}}$$

Outage prob. for MRC :-

$$P_{out} = \text{Prob}\left(\Gamma_M < \Gamma_{th}\right)$$

$$= \int_0^{\Gamma_{th}} P_{MRC}(\Gamma_M)\, d\Gamma$$

$$= 1 - e^{-\Gamma_{th}/\bar{\Gamma}} \sum_{k=1}^{M} \frac{\left(\Gamma_{th}/\bar{\Gamma}\right)^{k-1}}{(k-1)!}$$

Coverage prob. & pdf of SNR.

$$P_{MRC}(\Gamma) = \frac{\Gamma^{M-1} e^{-\Gamma/\bar{\Gamma}}}{\bar{\Gamma}^{M} (M-1)!}$$

$$\bar{P}_b = \int_0^\infty Q(\sqrt{2\gamma})\, p(\gamma)\, d\gamma$$

$$= \left(\frac{1-G}{2}\right)^M \sum_{m=0}^{M-1} \binom{M-1+m}{m} \left(\frac{1+G}{2}\right)^m$$

Error prob. for MRC

$$\boxed{\bar{P}_{s_{MRC}} \propto (\Gamma)^{-M_R}.}$$

→ As M_R ↑, prob. of error ↓

M_R → diversity order.

$M_R = 1.$

$P_s \propto \Gamma^{-1}$

$M_R = 2$

$P_s \propto \Gamma^{-2}.$

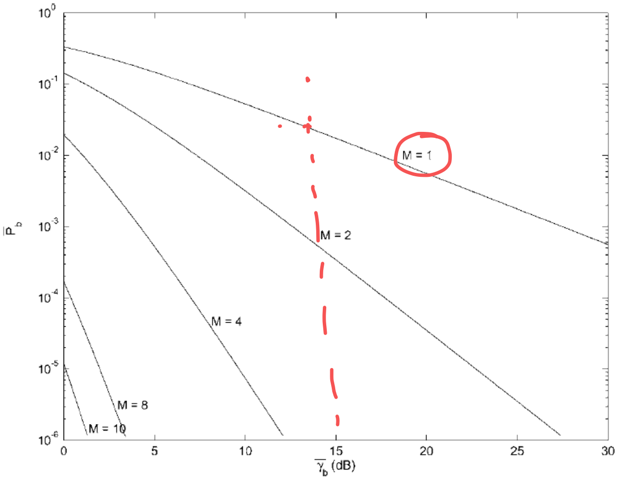


Figure 7.6: Average P_b for maximal-ratio combining with i.i.d. Rayleigh fading.

Consider a 2×3 MIMO system where spatial correlation exists both in transmitter and receiver side. The principle square root of transmit and receive correlation matrix are given respectively :-

2Rx 3Tx.

H_w : 2×3.

$$\mathbf{R}_T^{\frac{1}{2}} = \begin{bmatrix} 1 & 0.35 & 0.05 \\ 0.35 & 1 & 0.25 \\ 0.05 & 0.25 & 1 \end{bmatrix}$$

$R_T^{1/2}$:

$$\mathbf{R}_R^{\frac{1}{2}} = \begin{bmatrix} 1 & 0.2 \\ 0.2 & 1 \end{bmatrix}$$

$R_R^{1/2}$:

Consider an instance of spatially white channel being:-

$$\mathbf{H}_W = \begin{bmatrix} 0.25 & 0.65 & 0.15 \\ 0.85 & 0.75 & 0.35 \end{bmatrix}$$

$$\boxed{H = R_r^{1/2} H_w R_t^{1/2}.}$$

Compute the spatially correlated channel matrix :-

a. $\begin{bmatrix} 1.41 & 1.26 & 0.46 \\ 1.89 & 1.63 & 0.22 \end{bmatrix}$

b. $\begin{bmatrix} 1.9 & 0.22 & 1.12 \\ 0.68 & 0.41 & 1.31 \end{bmatrix}$

c. $\begin{bmatrix} 0.271 & 0.12 & 0.05 \\ 1.21 & 1.556 & 2.07 \end{bmatrix}$

d. $\begin{bmatrix} 0.711 & 1.002 & 0.441 \\ 1.227 & 1.29 & 0.645 \end{bmatrix}$

NPTEL

Consider a SIMO system where the receiver has four branches with instantaneous SNRs of −10 dB, 5 dB, −3 dB, and 12 dB, respectively. Find the instantaneous post SNR that can be achieved with selection combining.

a. 5 dB

b. 12 dB

c. 12.92 dB

d. 13.18 dB

$M_R = 4$

−10 dB

5 dB

−3 dB

12 dB

$$\bar{\Gamma}_{SC} = \bar{\Gamma} \sum_{i=1}^{M_R} \frac{1}{i}$$

Find the outage probability for a selection combining receiver of 4 receive branches when threshold SNR (γ_{th}) is 18 dB and average SNR ($\bar{\gamma}$) of branches is 20 dB.

a. 5.21 %

b. 4.79 %

c. 3.69 %

d. 2.45 %

$M = 4.$

$\Gamma_{th} = 18\,dB = 10^{1.8}$

$\bar{\Gamma} = 20\,dB = 10^{2}.$

$$P_{out} = \left(1 - e^{-\Gamma_{th}/\bar{\Gamma}}\right)^4$$

$$= \left(1 - e^{-\frac{10^{1.8}}{10^2}}\right)^4 = 0.0479$$

$$= 4.79\%$$

What is the average post SNR of selction combining for 4 receive branches in a SIMO system if average SNR ($\bar{\gamma}$) on each branch is 10 dB ?

a. 5 dB

b. 12 dB

c. 12.92 dB

d. 13.18 dB

$$\bar{\Gamma} = 10\,dB = 10$$

$$\bar{\Gamma}_{sc} = \bar{\Gamma} \sum_{i=1}^{M_R} \frac{1}{i}$$

$$= 10\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) = 20.83$$

$$\bar{\Gamma}_{sc}\Big|_{dB} = 10\log_{10}(20.83) = 13.18\ dB.$$

In the above question, find the post processing SNR if MRC combining is used.

a. 5 dB

b. 12 dB

c. 12.92 dB

d. 13.18 dB

$$\boxed{\bar{\Gamma}_{MRC} = M\bar{\Gamma}}$$

$$\Gamma_{MRC} = \left(\sum_{i=1}^{M_R} \Gamma_i \right)$$

$\Gamma_1 = 10\,dB = 10$

$\Gamma_2 = 5\,dB = 3.16$

$\Gamma_3 = -3\,dB = 0.5$

$\Gamma_4 = 12\,dB = 15.84$

$\Gamma_{MRC} = 29.5 = 14.69\,dB.$

Consider a MIMO system with 1 transmit and M receive antennas. If y_i is the output of the i^{th} branch at the receiver and h_i be the channel coefficient for the i^{th} branch, then which of the following represents the equalization / combining for a MRC receiver?

SIMO

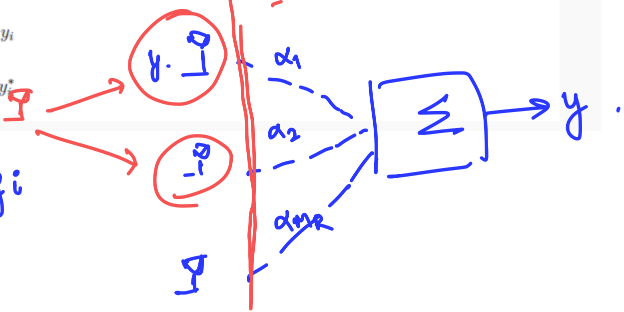
a. $z = \sum_{i=1}^{M} y_i$

b. $z = \sum_{i=1}^{M} h_i y_i$

c. $z = \sum_{i=1}^{M} h_i^* y_i$ ✓

d. $z = \sum_{i=1}^{M} h_i y_i^*$

$y_i = h_i x + w_i$



$z = \sum_{i=1}^{M} h_i^* y_i$

NPTEL

What is the Diversity order for a MRC receive diversity scheme using M_R number of receive antennas and 1 transmit antenna ?

a. M_R

b. $\frac{M_R}{2}$

c. $\frac{M_R}{3}$

d. None of the above

$$P_s \propto (\Gamma)^{-M_R}$$

diversity.

Consider a MIMO system with one transmit antenna and 2 receive antennas. Given, each branch receives an independent Rayleigh faded signal and the average SNR ($\bar{\gamma}$) for each branch 10 dB. If MRC combining is used at the receiver, what is the average output SNR?

a. 16 dB

b. 20 dB

c. 4 dB

d. 13 dB

$M_R = 2.$

$$\boxed{\bar{\Gamma}_{M_{MRC}} = M\bar{\Gamma}}$$

$$\bar{\Gamma} = 10\,dB = 10 \Rightarrow \bar{\Gamma}_{M_{MRC}} = 20$$

$$\bar{\Gamma}_{M_{MRC}}\Big|_{dB} = 10\log 20 \simeq 13.01 \approx 13\,dB.$$

What is the minimum spacing between two antennas to observe two independent fading channels if λ is wave length

a. λ

b. $\lambda/2$

c. $\lambda/4$

d. None of these

$\Delta x = \lambda/2.$

$J_0\left(2\pi \frac{\Delta x}{\lambda}\right) \rightarrow 0$ when $\Delta x \approx 0.38\lambda$

$\approx \lambda/2.$

Which statement are true for a Hermitian Matrix ?

a. Eigen values can be complex. ×

b. The eigen vectors if they come from different eigen values are orthogonal to one another.

c. Eigen vectors are columns of Fourier Matrix.

d. None of the above.

$A^H = A$. $\rightarrow$ λ_i are real.

Which is/ are the correct statement related to cauchy–Schwarz inequality, where $| < u, v > |$ signifies the inner product of u and v vector and $||v||$ denotes the length of vector v.

a. $| < u, v > | > ||u||.||v||$

b. $| < u, v > | \leq ||u||.||v||$

c. two sides of equation are equal if and only if u and v are linearly independent

d. None of the above

$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ $\mathbf{B} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

What is $\mathbf{A} \bigotimes \mathbf{B}$ and $\mathbf{B} \bigotimes \mathbf{A}$, where $\bigotimes$ is Kronecker product.

a. $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & -1 & 0 \\ 1 & 1 & -1 & -1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

c. $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$

$$A \otimes B = \begin{bmatrix} 1\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} & 1\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \\ 1\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} & -1\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & -1 & 0 \\ 1 & 1 & -1 & -1 \end{bmatrix}$$

$$B \otimes A = \begin{bmatrix} 1\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \\ 1\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} & 1\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \end{bmatrix}$$

Consider a Matrix

$$\mathbf{A} = \begin{bmatrix} 6 & 4 & 5 \\ 0 & 8 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

The eigenvalues, trace and determinant of A satisfies which of the following?

a. $\lambda_1 = 6, \lambda_2 = 8, \lambda_3 = 0$, trace $=14$, and det $= 0$

b. $\lambda_1 = 6, \lambda_2 = 5, \lambda_3 = 3$, trace $= 10$, and det $= 0$

c. $\lambda_1 = 3, \lambda_2 = 1, \lambda_3 = 1$, trace $=5$, and det $= 3$

d. $\lambda_1 = 3, \lambda_2 = 2, \lambda_3 = 1$, trace $=6$, and det $= 6$

- Product of eigen values = det(A)

  = 0

  $\Rightarrow$ One eigen value must be zero.

- Sum of eigenvalues = trace(A).

A MIMO channel matrix,$\mathbf{H}$ is given as,

$$\mathbf{H} = \begin{bmatrix} 0.3 & 0.2 \\ 0.1 & 0.2 \end{bmatrix}_{2\times 3}$$

What are the eigen value of $\mathbf{HH}^H$ and a squared Frobenious norm of $\mathbf{H}$?

a. $\lambda_1, \lambda_2 = 0.699, 0.447$ *and* $\| \mathbf{H} \|_F^2 = 1.1471$

b. $\lambda_1, \lambda_2 = 0.1706, 0.0094$ *and* $\| \mathbf{H} \|_F^2 = 0.1800$

c. $\lambda_1, \lambda_2 = 0.24, 0.04$ *and* $\| \mathbf{H} \|_F^2 = 0.28$

d. $\lambda_1, \lambda_2 = 0.707, 0.0094$ *and* $\| \mathbf{H} \|_F^2 = 0.7164$

$$HH^H = \begin{bmatrix} 0.3 & 0.2 \\ 0.1 & 0.2 \end{bmatrix} \begin{bmatrix} 0.3 & 0.1 \\ 0.2 & 0.2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.13 & 0.07 \\ 0.07 & 0.05 \end{bmatrix}$$

$$tr(HH^H) = \sum_{i=1}^{2} \lambda_i = \|H\|_F^2$$

$$= 0.18$$

L -26

Find the upper bound on Probability of symbol error with MRC combining for a 5×1 MIMO system when average SNR $\bar{\gamma}$ is 30 dB and 16-QAM modulation is used. Assume Rayleigh frequency falt and spacially white channel.

a. 2×10^{-7}

b. 1.25×10^{-6}

c. 2×10^{-5}

d. 5.9×10^{-8}

$$P_e \approx \bar{N}_e \left(\frac{\rho d_{min}^2}{4 M_R} \right)^{-M_R}$$

$N_e \rightarrow$ nearest neighbour

$d_{min} \rightarrow 0.6325 \sqrt{E_s} \approx 0.6325$

$\rho \rightarrow$ Avg. SNR

$M_R \rightarrow$ No. of branches.

$$F = 30\,\text{dB} = 10^3 = 1000$$

$$N_e = 4.$$

$$P_e \leq 4\left(\frac{\overset{20}{\cancel{1000}} \times 0.\cancel{4}}{\cancel{4} \times \cancel{5} \times \cancel{10}}\right)^{-5}$$

$$= 4(20)^{-5} = 1.25 \times 10^{-6}.$$

The following matrix is, $A = \begin{bmatrix} 4 & 5 & 7 \\ 8 & 0 & 2 \\ 6 & 7 & 2 \end{bmatrix}$

$$\det(A) = 4(-14) - 5(4) + 7(56)$$
$$\neq 0.$$

a. Rank-2 matrix ✗

b. Symmetric matrix ✗

c. Full rank matrix ✓

d. None of These