Exercise 4.1

(a)

$w \in L$	$w \notin L$
0011	0
1001	10

(b)

$w \in L$	$w \notin L$
110	111
001	0

(c)

$w \in L$	$w \notin L$
0001	1
0011	10

(d)

$w \in L$	$w \notin L$
$\emptyset$	1
0	10

Exercise 4.2

The language L is defined by the regular expression: $(b^*ab^*ab^*)^*|b^*$

Exercise 4.3

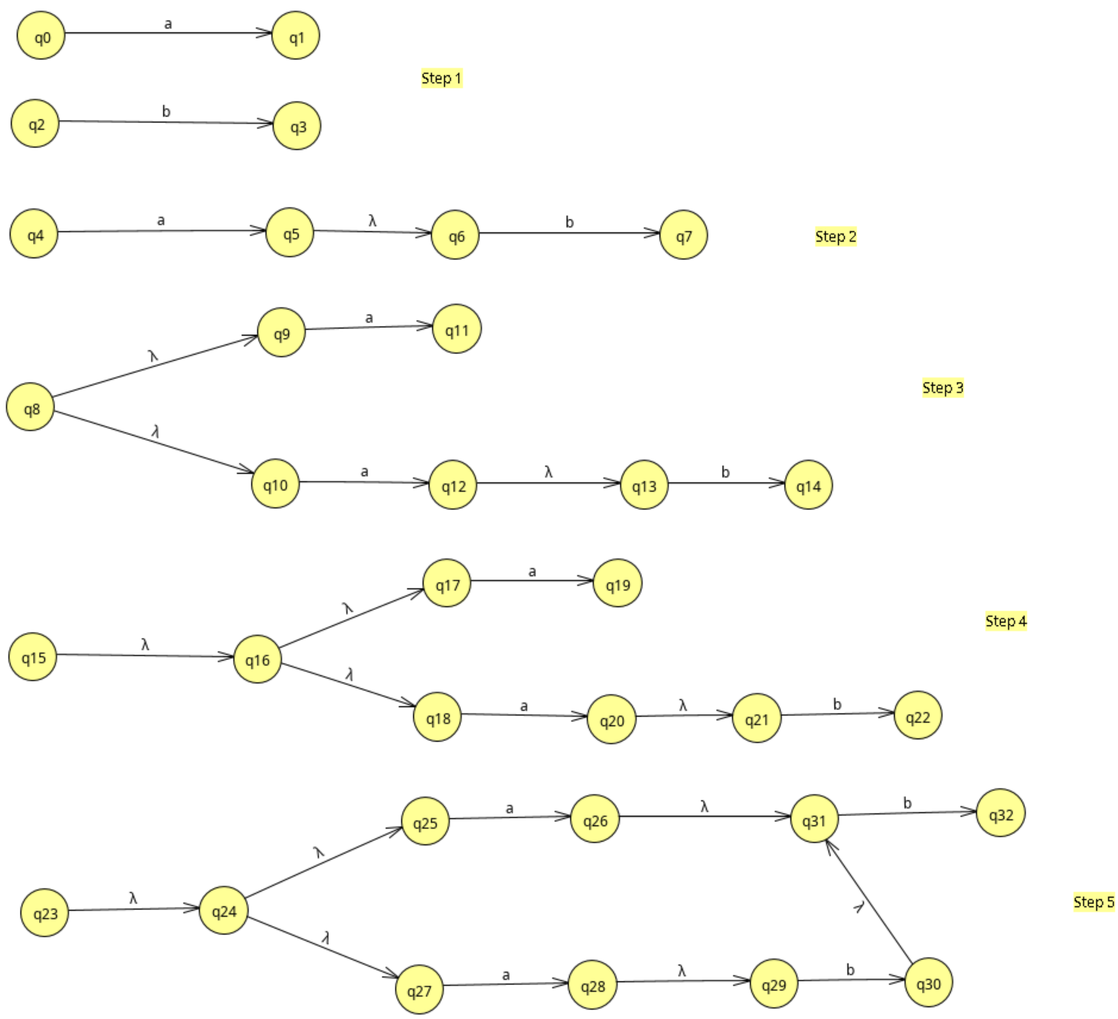

Figure 1: NFA for the regular expression $\gamma = (a|ab)^*b$

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Exercise 4.4

(a) *Proof.* The language $L = \{ a^n b^m c^{n+m} \mid n, m \in \mathbb{N}_0 \}$ is not regular.
Assume L is regular. Then let p be a pumping number for L.
The word $x = a^p b^{2p} c^{p+2p}$ is in L and has length $\geq p$.
Let $x = uvw$ be a split with the properties of the pumping lemma.
Then the word $x' = uv^2w$ is also in L. Since $|uv| \leq p$, uv consists only of symbols a and $x' = a^{p+|v|} b^{2p} c^{p+2p}$.
Since $|v| \geq 1$ it follows that $(p + |v|) + 2p \neq p + 2p$ and thus $x' \notin L$.

Example:
$u = a^i$ $\quad$ $v = a^j$ $\quad$ $w = a^{n-i-j} b^m c^{n+m}$
$v = v^2 \rightarrow v^2 = a^{2j}$
The word is then: $x = uvvw$
$uvvw = a^i a^{2j} a^{n-i-j} b^m c^{n+m} = a^{j+n} b^m c^{n+m}$
$n = 2$ $\quad$ $m = 3$ $\quad$ $j = 4$ $\quad$ $\rightarrow$ $\quad$ $x = aaaaaabbbccccc$

$\square$