Theory of Computer Science

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Exercise Sheet 3

Due: Wednesday, March 22, 2023

Exercise 3.1 (Grammars; 1+1+1 points)

Consider the grammar $G = \langle\{\mathrm{S}\}, \{\mathtt{a}, \mathtt{b}\}, R, \mathrm{S}\rangle$ with the following production rules R:

$$\mathrm{S} \to \mathtt{a}\mathrm{S}\mathtt{a} \tag{1}$$
$$\mathrm{S} \to \mathtt{b}\mathrm{S}\mathtt{b} \tag{2}$$
$$\mathrm{S} \to \mathtt{a} \tag{3}$$
$$\mathrm{S} \to \mathtt{b} \tag{4}$$
$$\mathrm{S} \to \varepsilon \tag{5}$$

(a) Specify a derivation of the word `abaabaaba`. In each step specify the number of the used rule.

(b) What is $\mathcal{L}(G)$? Describe the language in natural language, and as simple as possible.

(c) Consider a representation of binary trees where a leaf is denoted by $\square$ and an inner node by $[L \circ R]$, where L and R are binary trees. Specify a grammar that recognizes the language of all binary trees represented this way.

Exercise 3.2 (Chomsky Hierarchy, 0.5+0.5 points)

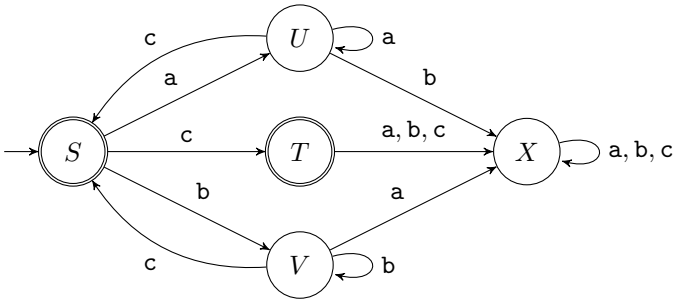
Consider the following grammars $G_i = \langle\{S, X, Y\}, \{\mathtt{a}, \mathtt{b}\}, R_i, S\rangle$ for $i \in \{1, 2\}$. In each case specify all types of the Chomsky hierarchy in which the grammar lies.

(a) $R_1 = \{S \to \mathtt{a}X, S \to \mathtt{a}S, X \to \mathtt{b}X, S \to \varepsilon, X \to \mathtt{b}Y, Y \to \mathtt{a}\}$

(b) $R_2 = \{S \to Y\mathtt{aba}X, \mathtt{ba}X \to \mathtt{baa}X, X \to \mathtt{b}X, X \to \mathtt{b}Y, Y \to \mathtt{a}\}$

Exercise 3.3 (DFA to Regular Grammar, 2 points)

Specify a regular grammar that recognizes the same language as the following DFA.



Exercise 3.4 (Regular Grammar to NFA, 1 points)

Specify an NFA that recognizes the same language as grammar $G = \langle\{S, T\}, \{0, 1\}, R, \mathrm{S}\rangle$ with the following rules in R:

$$\mathrm{S} \to 0 \qquad \mathrm{S} \to 1\mathrm{T} \qquad \mathrm{T} \to 0\mathrm{T} \qquad \mathrm{T} \to 1\mathrm{T} \qquad \mathrm{T} \to 0 \qquad \mathrm{T} \to 1$$

Exercise 3.5 (Regular Languages, 3 points)

Consider the following decision problem:

Given: regular grammars G_1 and G_2
Question: Is $\mathcal{L}(G_1) \subseteq \mathcal{L}(G_2)$?

Show that the problem is decidable by describing a decision procedure. You may use all transformations and results from the lecture without explaining them.