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Exercise 3.1

(a) A possible derivation of the word "abaabaaba" is:

$$1\ (aSa) \to 2\ (abSba) \to 1\ (abaSaba) \to 1\ (abaaSaaba) \to 4\ (abaabaaba)$$

(b) $\mathcal{L}(G)$ is the definition of a Palindrome. The word can always be read either forward or backward and you get the same result.

(c) The Grammar that represents a representation of binary trees is:

$$G = \langle \{\, S \,\},\ \{\, [, \circ, \square,] \,\},\ R,\ S \rangle$$

with R:

1) $S \to \square$
2) $S \to [S \circ S]$

Exercise 3.2

(a) The grammar $G_1 = \langle \{\, S, X, Y \,\}, \{\, a, b \,\}, R_1, S \rangle$ is of type-0 because there is rule $S \to \epsilon$ and rule $S \to aS$ which violates the special case, where S never occurs on the right-hand side if S also maps to the empty word.

(b) The grammar $G_2 = \langle \{\, S, X, Y \,\}, \{\, a, b \,\}, R_2, S \rangle$ is of type-1, and so also of type-0, because $baX \to baaX$ is context-sensitive. The other rules are of type-2 or type-3.

Exercise 3.3

The regular grammar for the given DFA is:

$$G = \langle \{\, S, U, V \,\},\ \{\, a, b, c \,\},\ R,\ S \rangle$$

with rules R as follows:

$$\begin{array}{lll} S \to aU & S \to bV & S \to c \\ U \to aU & U \to cS & U \to c \\ V \to bV & V \to cS & V \to c \end{array}$$

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Exercise 3.4

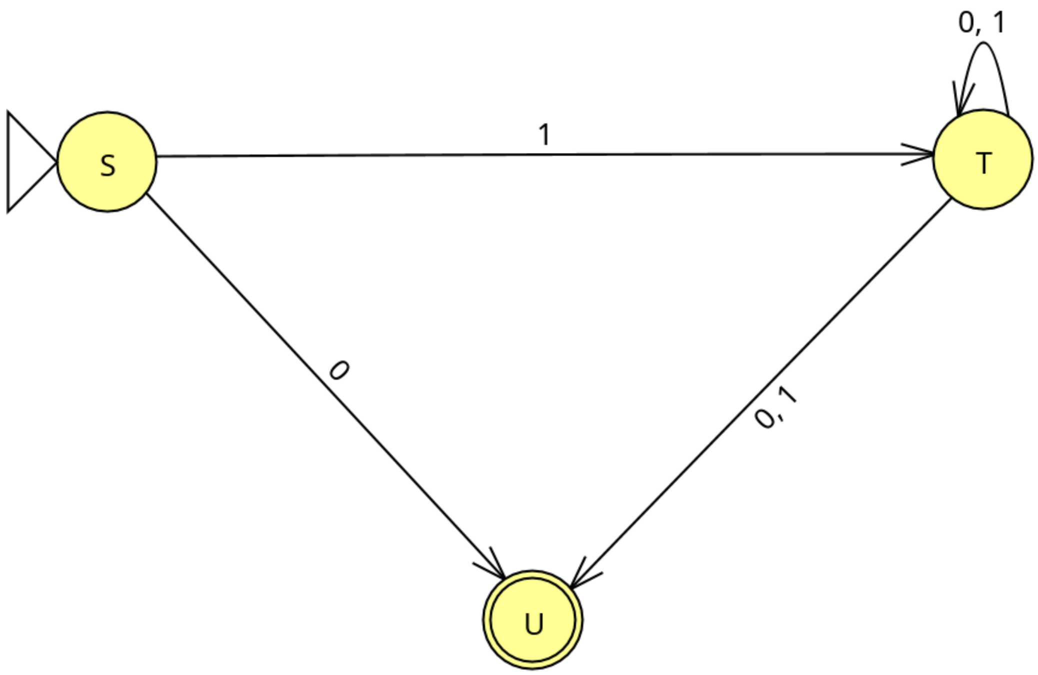

Figure 1: NFA for G

Exercise 3.5

Fist step

We find the complement of G_2 by inverting the corresponding DFA.

Second step

Given the equivalence problem: $A = B \quad \text{iff} \quad (A \cap B^c) \cup (A^c \cap B) = \emptyset$ which applies in both directions, we can extrapolate only the $(A \cap B^c)$ part in order to obtain $A \subseteq B \quad \text{iff} \quad (A \cap B^c) = \emptyset$. Because regular languages are closed under intersection and complement, and we know algorithms for these operations.

Third step

Decide if $(A \cap B^c)$ is $\emptyset$ by using the algorithm for the emptiness problem $P_\emptyset$.