Theory of Computer Science

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Exercise Sheet 7

Due: Wednesday, April 26, 2023

Exercise 7.1 (Turing-Computable Numerical Functions; 1+1 Point)

Consider function $f : \mathbb{N}_0^3 \to \mathbb{N}_0$ and a DTM M computing f^{code}. Specify x_1, x_2, x_3 and y with $f(x_1, x_2, x_3) = y$ for the following start and end configurations of M:

(a) $\langle \varepsilon, q_0, \texttt{101\#0\#11} \rangle \vdash^* \langle \varepsilon, q_{\text{accept}}, \texttt{1001} \rangle$

(b) $\langle \varepsilon, q_0, \texttt{11\#100\#1} \rangle \vdash^* \langle \varepsilon, q_{\text{accept}}, \texttt{01} \rangle$

Exercise 7.2 (Exponentiation is Turing-computable; 2 Points)

Describe the main idea of a proof showing that exponentiation of binary numbers is Turing-computable, i.e., describe the high-level idea of a Turing machine that computes exp^{code} for the function $exp : \mathbb{N}_0^2 \to_{\text{p}} \mathbb{N}_0$ with $exp(n_1, n_2) = n_1^{n_2}$.
We recommend to have the tape content represent a tuple of numbers for most of the computation, and refer to the numbers by index. For example, if the tape currently encodes 3 numbers, you can refer to the second number by n_2. It is then sufficient to describe how your TM manipulates these numbers (including introducing new numbers or deleting numbers).
You can simulate any of the following DTMs in your TM: M_{succ}, M_{pred_1}, M_{pred_2}, M_{add}, M_{sub}, M_{mul}, M_{div} (they compute the function denoted in their subscript). If you do so, specify which numbers you use as input, and in which number the output is stored.

Exercise 7.3 (Composition of Computable Functions; 2 Points)

Let $g : \Sigma_1^* \to \Sigma_2^*$ and $f : \Sigma_2^* \to \Sigma_3^*$ be Turing-computable partial functions for alphabets $\Sigma_1, \Sigma_2, \Sigma_3$. Show that the *composition* $(f \circ g) : \Sigma_1^* \to \Sigma_3^*$ is also Turing-computable.
In general the composition of two functions is defined as $(f \circ g)(x) = f(g(x))$. Specifically, the value $(f \circ g)(x)$ is undefined if $g(x)$ is undefined.

Exercise 7.4 (Transitivity of Reductions; 2 Points)

Show for any languages A, B and C: if $A \leq B$ and $B \leq C$, then $A \leq C$.

You may use the result from exercise 7.3, even if you have not solved it.

Exercise 7.5 (Reductions; 0.5+0.5+0.5+0.5 Points)

Consider a decidable language D, a language R that is Turing-recognizable but not decidable, and a language U that is not Turing-recognizable. Given the following reductions, is L decidable? Is it Turing-recognizable? To both, the answer is either “yes”, “no”, or “we cannot tell from the given information”.

(a) $R \leq L$

(b) $L \leq U$

(c) $D \leq L$

(d) $L \leq D$