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Exercise 5.1

Step 1: ϵ Rules
$S \to \epsilon$ is the only occurrence of ϵ and S never occurs on the right-hand side. We don't have to do anything here.

Step 2: Chain Rules
Since the CMS can only have a terminal or **two** Variables on the right-hand side, we have to eliminate every Rule that has only a single Variable on the right.

Chains in G:
$S \to Z \to bb$ $S \to Z \to Za$ $X \to Y \to bY$ $Y \to X \to aZb$

We eliminate the chains by linking the end result of the chain directly with the first Variable. Since $X \to Y$ and $Y \to X$, $X = Y$ so we can just write them together as X that points to the results of X and Y.

$S \to bb \mid Za \mid XX$ $Z \to bb \mid Za$ $X \to aZb \mid bX$

Step 3: Order
Now we have to eliminate all the rules that have terminals **and** Variables on the right-hand side. We do so by adding a new Variable for every terminal that points to only that terminal.

So we create: $A \to a$ and $B \to b$ and can now rewrite the Rules above to:
$S \to BB \mid ZA \mid XX$ $Z \to BB \mid ZA$ $X \to AZB \mid BX$
$A \to a$
$B \to b$

Step 4: Shorten
To get our CNF we have to shorten the right-hand side to exactly two variables per rule. Since $X \to AZB$ is too long, we add $V \to ZB$ and get our final Grammar $G' = \langle S, V, X, Z, a, b, P, S \rangle$ with:

P: $S \to BB \mid ZA \mid XX$ $Z \to BB \mid ZA$ $X \to AV \mid BX$ $V \to ZB$
$A \to a$
$B \to b$

Exercise 5.2

Proof. In the case $|w| = 1$ we can only have one terminal Rule $S \to a$ because $S \to AB$ would be replaced by at least two terminal rules and therefore be $|w| \geq 2$.

With $S \to AB$ we can generate any word w with $|w| \geq 2$ in the language G by replacing the A or B.

$S \to AB \to ACD \to ACEF \to \ldots \to X_1 \ldots X_n$

By the definition of the Chomsky Normal Form, the Grammar has either a terminal or two Variables on the right-hand side. Therefore with every Step, exactly one Variable can be added.
So to generate $X_1 \ldots X_n$ exactly $n - 1$ steps are needed.

Now we have to replace every Variable X with a terminal symbol and because again by definition of the CNF, we can only replace one X at a time. Therefore we need n more steps to generate our final word w.

In total we need exactly $2n - 1$ steps and since $n = |w|$ the statement is true. □

Exercise 5.3

(a)

Σ	step	Stack
ϵ	$q_0 \to q_1$	$\#$
a	$q_1 \to q_1$	$\#X$
ϵ	$q_1 \to q_2$	$\#X$
a	$q_2 \to q_2$	$\#XY$
a	$q_2 \to q_2$	$\#XYY$
d	$q_2 \to q_3$	$\#XYY$
b	$q_3 \to q_4$	$\#XYY$
a	$q_4 \to q_3$	$\#XY$
b	$q_3 \to q_4$	$\#XY$
a	$q_4 \to q_3$	$\#X$
ϵ	$q_3 \to q_5$	$\#X$
c	$q_5 \to q_6$	$\#X$
c	$q_6 \to q_5$	$\#$
ϵ	$q_5 \to q_7$	ϵ

(b)

This PDA accepts the following language: $w = \{\, a^x\ d\ (ba)^y\ (cc)^z \mid a = y + z, \quad x, y, z \in \mathbb{N}_0 \,\}$
The self-loops q_1 and q_2 stack a's. Then follows a single d and the loop $q_3 \to q_4 \to q_3$ (ab) which removes anything stacked by q_1 followed by the loop $q_5 \to q_6 \to q_5$ (cc) which removes anything stacked by q_2. So in the end the visits of $q_1 + q_2$ have to equal the visits of $q_3 + q_5$.