Theory of Computer Science

G. Röger
S. Eriksson
Spring Term 2023

University of Basel
Computer Science

Exercise Sheet 5

Due: Wednesday, April 5, 2023

Exercise 5.1 (Chomsky Normal Form; 2 Points)
Specify a grammar G' in Chomsky normal form that generates the same language as the context-free grammar $G = \langle V, \Sigma, P, \mathrm{S}\rangle$ with $V = \{\mathrm{S}, \mathrm{X}, \mathrm{Y}, \mathrm{Z}\}$, $\Sigma = \{\mathtt{a}, \mathtt{b}\}$, and the following rules in P:

$$\mathrm{S} \to \varepsilon \qquad \mathrm{S} \to \mathrm{XY} \qquad \mathrm{S} \to \mathrm{Z} \qquad \mathrm{X} \to \mathrm{Y} \qquad \mathrm{X} \to \mathtt{a}\mathrm{Z}\mathtt{b}$$
$$\mathrm{Y} \to \mathrm{X} \qquad \mathrm{Y} \to \mathtt{b}\mathrm{Y} \qquad \mathrm{Z} \to \mathtt{bb} \qquad \mathrm{Z} \to \mathrm{Z}\mathtt{a}$$

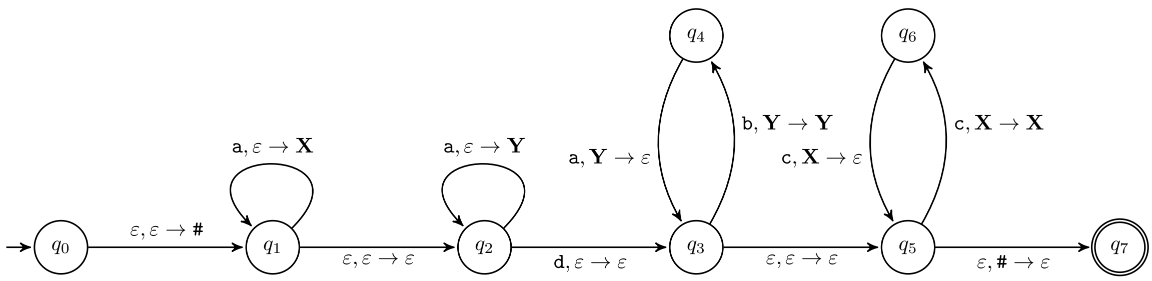
Specify sufficient intermediate steps, so your construction is understandable.

Exercise 5.2 (Length of Derivations in Chomsky Normal Form; 2 Points)
Let G be a grammar in Chomsky normal form and $w \in \mathcal{L}(G)$ a non-empty word ($w \neq \varepsilon$), which is generated by G. Show that every derivation of w from the start variable of G consists of exactly $2|w| - 1$ steps.

Exercise 5.3 (Push-down Automata; 1.5+1.5 points)
Consider the push-down automaton (PDA) $M = \langle Q, \Sigma, \Gamma, \delta, q_0, \{q_7\}\rangle$ with

- $Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$,
- $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{d}\}$,
- $\Gamma = \{\mathbf{X}, \mathbf{Y}, \mathtt{\#}\}$,

and the following transition function δ:



(a) Show that the automaton accepts the word $w = \mathtt{aaadbababacc}$. Remember that specifying a sequence of states is not enough, you also need to specify which input is read and what the status of the stack is in each step.

(b) What language does this automaton accept? Describe the language in set-builder notation.

Exercise 5.4 (Context-free Languages Closure and Decidability, 1+1+1 points)
Are the following statements true? Briefly justify your answer

(a) If language L is context-free, then $\bar{L}$ is not context-free.

(b) The language $L = \mathtt{a}^n\mathtt{b}^n\mathtt{b}^m\mathtt{a}^m$ with $n, m \in \mathbb{N}_0$ is context-free.

(c) Given two context-free grammars G_1 and G_2, the question "Is $w \in \mathcal{L}(G_1) \cap \mathcal{L}(G_2)$?" is decidable.