1 Notations

Given current stock price S_0, strike price K, risk-free rate r, dividend yield q, time to maturity T, volatility σ, and

$$\begin{aligned} d_1 &= \frac{\ln(S_0/K) + \left(r - q + \sigma^2/2\right)T}{\sigma\sqrt{T}} && (1) \\ d_2 &= \frac{\ln(S_0/K) + \left(r - q - \sigma^2/2\right)T}{\sigma\sqrt{T}} && (2) \\ \phi(x) &= \frac{1}{\sqrt{2\pi}}e^{-x^2/2} && (3) \\ N(x) &= \int_{-\infty}^{x}\phi(s)ds \;=\; \frac{1}{2}\mathrm{erfc}\left(\frac{-x}{\sqrt{2}}\right) && (4) \end{aligned}$$

2 Option Price

$$\begin{aligned} \mathrm{Payoff}_{\mathrm{Call}} &= (S_T - K)^+ && (5) \\ C(S_0; K, r, q, \sigma, T) &= S_0 e^{-qT}N(d_1) - Ke^{-rT}N(d_2) && (6) \end{aligned}$$

$$\begin{aligned} \mathrm{Payoff}_{\mathrm{Put}} &= (K - S_T)^+ && (7) \\ P(S_0; K, r, q, \sigma, T) &= e^{-rT}KN(-d_2) - S_0 e^{-qT}N(-d_1) && (8) \end{aligned}$$

3 Greeks

$$\begin{array}{rcccl} \Delta_C &=& \frac{\partial C}{\partial S_0} &=& e^{-qT}N(d_1) \\ \Delta_P &=& \frac{\partial P}{\partial S_0} &=& -e^{-qT}N(-d_1) \\ \Gamma &=& \frac{\partial^2 C}{\partial S_0^2} &=& \frac{\partial^2 P}{\partial S_0^2} \;=\; e^{-qT}\frac{\phi(d_1)}{S_0\sigma\sqrt{T}} \\ \Lambda &=& \frac{\partial \ln C}{\partial \ln S_0} &=& \frac{\partial \ln P}{\partial \ln S_0} \;=\; \Delta_C \times \frac{S_0}{C} \;=\; \Delta_P \times \frac{S_0}{P} \end{array} \qquad (9)$$

4 Taylor Series Expansion

$$\begin{aligned} C(S_0) &= C(a) + \frac{C'(a)}{1!}(S_0 - a) + \frac{C''(a)}{2!}(S_0 - a)^2 + \frac{C'''(a)}{3!}(S_0 - a)^3 + \cdots && (10) \\ P(S_0) &= P(a) + \frac{P'(a)}{1!}(S_0 - a) + \frac{P''(a)}{2!}(S_0 - a)^2 + \frac{P'''(a)}{3!}(S_0 - a)^3 + \cdots && (11) \end{aligned}$$

1st order

$$\begin{aligned} C(S_0) &= C(a) + \Delta_C(S_0 - a) && (12) \\ &= \Delta_C \cdot S_0 + \left(C(a) - \Delta_C \cdot a\right) && (13) \end{aligned}$$

$$\begin{aligned} P(S_0) &= P(a) + \Delta_P(S_0 - a) && (14) \\ &= \Delta_P \cdot S_0 + \left(P(a) - \Delta_P \cdot a\right) && (15) \end{aligned}$$

2nd order

$$C(S_0) = C(a) + \Delta_C(S_0 - a) + \frac{\Gamma(a)}{2}(S_0 - a)^2 \tag{16}$$

$$= \frac{\Gamma(a)}{2}S_0^2 + \big(\Delta_C - \Gamma(a) \cdot a\big)S_0 + \left(C(a) - \Delta_C \cdot a + \frac{\Gamma(a)}{2} \cdot a^2\right) \tag{17}$$

$$P(S_0) = P(a) + \Delta_P(S_0 - a) + \frac{\Gamma(a)}{2}(S_0 - a)^2 \tag{18}$$

$$= \frac{\Gamma(a)}{2}S_0^2 + \big(\Delta_P - \Gamma(a) \cdot a\big)S_0 + \left(P(a) - \Delta_P \cdot a + \frac{\Gamma(a)}{2} \cdot a^2\right) \tag{19}$$