Identifying Unobserved Heterogeneity in Productivity

Ruben Dewitte[1], Catherine Fuss[2], Angelos Theodorakopoulos[3]

[1]Ghent University
Department of economics

[2]National Bank of Belgium
Research Department

[3]University of Oxford
Oxford Martin School

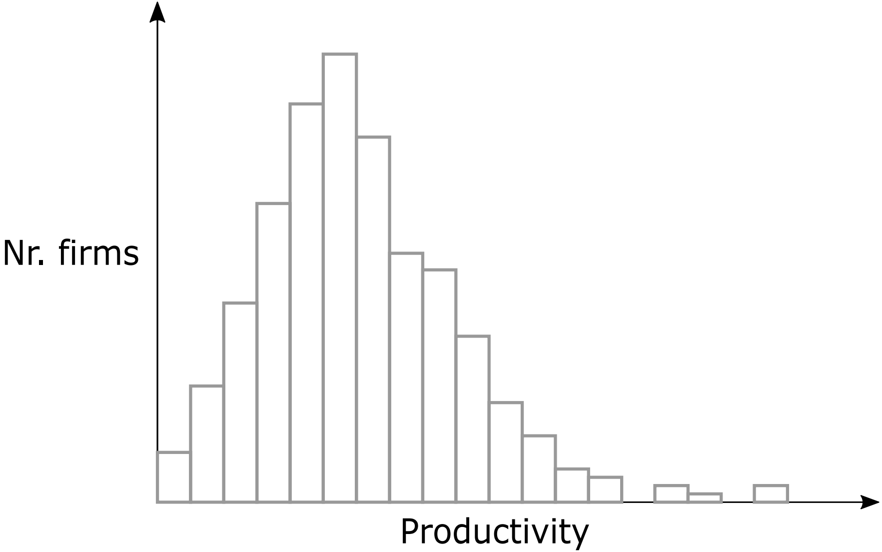
Job Market Seminar, University of Antwerp, 24th January 2022

Personal Introduction

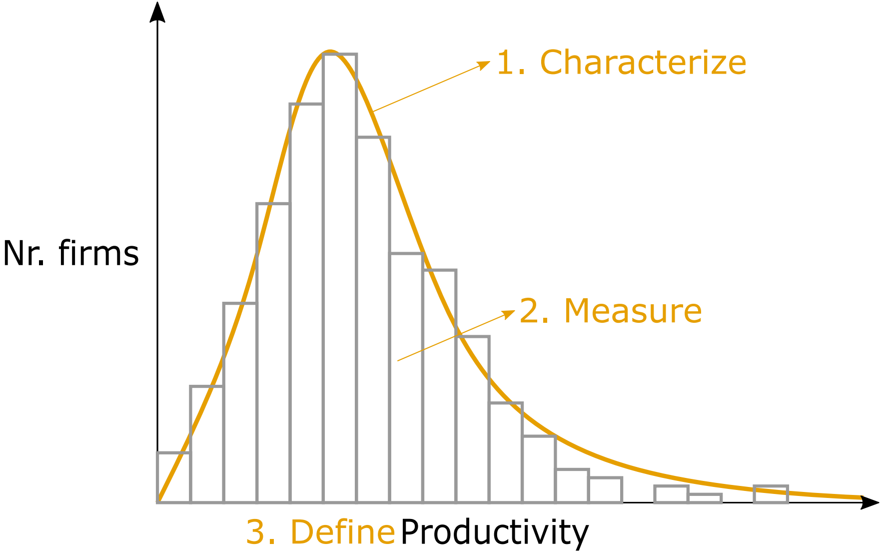
Who am I?

As an international economist, I study the firm-level determinants of technological progress and quantify the consequences of international trade for technological progress and subsequent economic prosperity.

Productivity distribution



Productivity distribution



Introduction

Motivation

Firms' productivity grows faster/slower depending on whether they . . .

- are financially constrained (Cabral and Mata, 2003; Albuquerque and Hopenhayn, 2004; Clementi and Hopenhayn, 2006; Angelini and Generale, 2008);
- trade (Kasahara and Rodrigue, 2008; De Loecker, 2013);
- innovate (Atkeson and Burstein, 2010; Bee et al., 2011; Maican et al., 2020);
- add or drop products (Klette and Kortum, 2004; Lentz and Mortensen, 2008);
- add or drop management layers (Bloom and Van Reenen, 2011; Caliendo and Rossi-Hansberg, 2012);
- incur specific market penetration costs (Arkolakis, 2016);
- have industry linkages (Luttmer, 2007);
- . . .

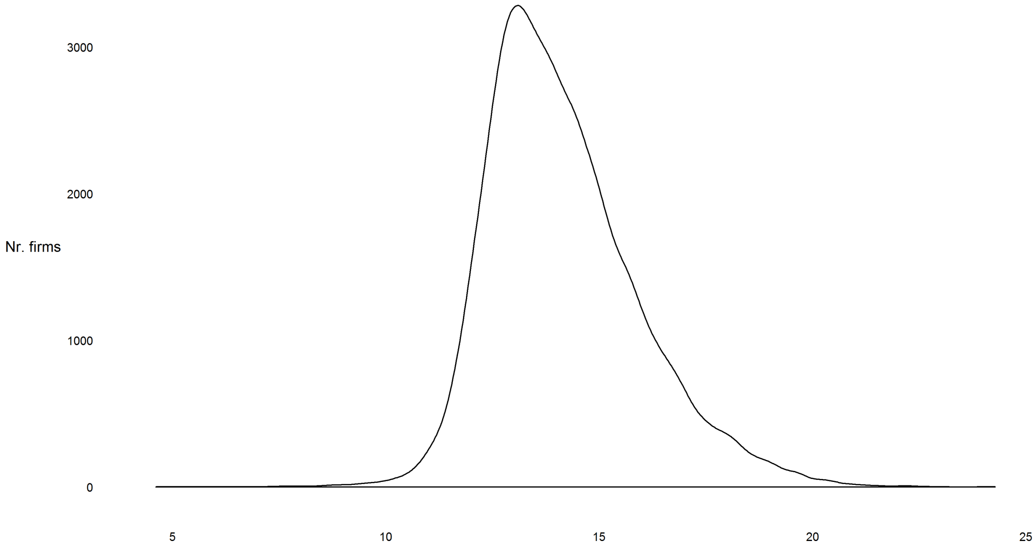
Motivation

"[M]any of the mechanisms in the literature undoubtedly contributed toward an explanation of establishment dynamics" (Rossi-Hansberg and Wright, 2007, p. 1641),

... but ...

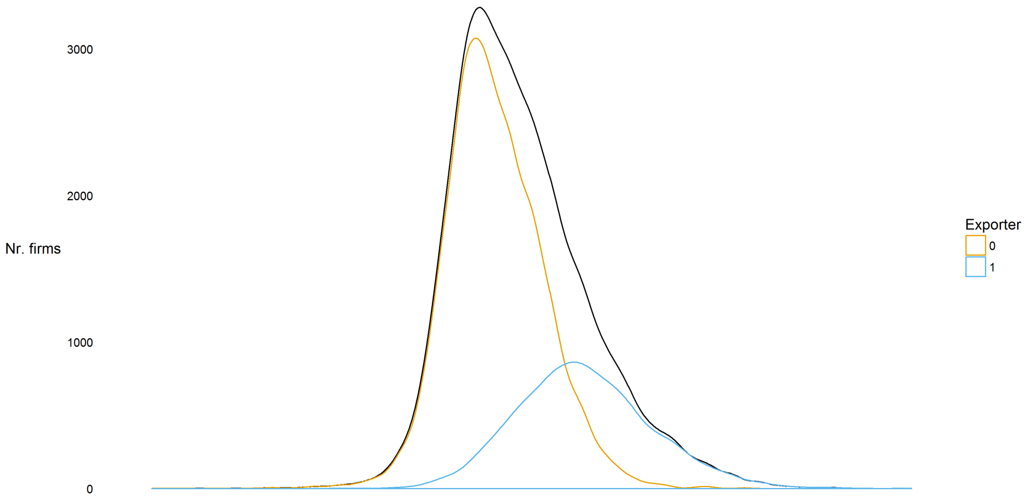
to date, it remains unclear which mechanism, or mechanisms, dominate.

Sales distribution



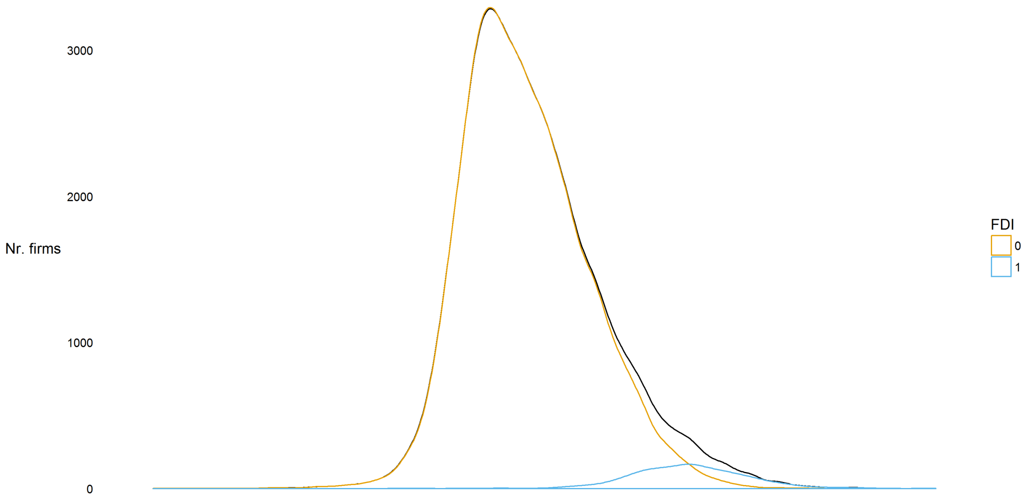
Sales distribution

. . . by export status



Sales distribution

. . . by FDI status (. . . of which 93% also export!)



Motivation

Current literature on the structural identification of production functions assumes

1. Homogeneous productivity growth process for all firms;
2. Unobserved heterogeneity between clusters of firms according to à priori specified (categorical) proxy variables

⇒ Probability of omitted variable bias

⇒ Heavy data burden

This paper . . .

- Develops a production function estimator that allows for ànd identifies unobserved heterogeneity in productivity between clusters of firms using Finite Mixture Models
- Demonstrate the appropriateness via Monte Carlo
- Showcase the applicability on Belgian firm-level data
  - Strong evidence of heterogeneity in the evolution of productivity
  - Heterogeneity correlates with traditional firm-level characteristics, but unobserved heterogeneity remains

  $\Rightarrow$ Necessity to control for unobserved heterogeneity when comparing productivity across groups of firms, such as exporters vs. non-exporters, . . .

Related literature

- Heterogeneity in firm sales/productivity distribution (cf. before, Dewitte et al. (2020))
- Structural Production function estimation (Olley and Pakes, 1996; Levinsohn and Petrin, 2003; Ackerberg et al., 2015; Gandhi et al., 2020)

  . . . with unobserved heterogeneity (Lee et al., 2019; Gandhi et al., 2020; Ackerberg, 2021)

  . . . with Finite Mixture specification (Van Biesebroeck, 2003; Kasahara et al., 2017; Battisti et al., 2020)
- Finite Mixtures in SF literature (Beard et al., 1997; Greene, 2005; Orea and Kumbhakar, 2004; El-Gamal and Inanoglu, 2005)
- Mixture-of-experts models (Fruhwirth-Schnatter et al., 2019)

Outline

1. Behavioral framework
2. Production function estimation
3. Monte Carlo
4. Application to Belgian firm-level data
5. Conclusion

Behavioral Framework

Behavioral framework

Data and definitions

Dynamic heterogeneous firms model with cluster-dependent uncertainty in future, Hicks-neutral, productivity

- (Short) panel of firms $b = 1, \ldots, B$ over period $t = 1, \ldots, T$
- Output Y_{bt} and inputs $\{K_{bt}, L_{bt}, M_{bt}\}$ in perfectly competitive markets
- Information set $\mathcal{I}_{bt}$ such that generic input $X_{bt} \in \{K_{bt}, L_{bt}, M_{bt}\}$ is
  - *nonflexible* if predetermined $X_{bt} \in \mathcal{I}_{bt}$ or dynamic $X_{bt} = f(X_{bt-1})$
  - *flexible* if neither predetermined $X_{bt} \notin \mathcal{I}_{bt}$ nor dynamic $X_{bt} \neq f(X_{bt-1})$.

Behavioral framework

Production function and productivity

$$Y_{bt} = F^{klm}\left(K_{bt}, L_{bt}, M_{bt}\right) e^{\omega_{bt}+\varepsilon_{bt}} \qquad \Leftrightarrow$$
$$y_{bt} = f^{klm}\left(k_{bt}, l_{bt}, m_{bt}\right) + \omega_{bt} + \varepsilon_{bt},$$

- Productivity $\omega_{bt} \in \mathcal{I}_{bt}$ and ex-post productivity $\epsilon_{bt} \notin \mathcal{I}_{bt}$
- Furthermore, firm-level productivity ω_{bt} follows cluster-dependent first-order Markov process

$$p(\omega_{bt}|\mathcal{I}_{bt-1}) = p\left(\omega_{bt}|\omega_{bt-1}, z_b^s\right),$$

- Each firm b belongs to a certain cluster $s = 1, \ldots, S$, indicated by $z_b^i = \mathbb{I}_b\left(s = i\right), \forall i = 1, \ldots, S$

Behavioral framework

Firm's problem

- Optimal one-off decision rule with firm-specific cluster affinity $\varepsilon(z_b^s)$:

$$z_b^*\left(K_{b0}, L_{b0}, e^{\omega_{b0}}, \epsilon\right) = \arg\max_{z_b^s} \Bigg(\pi_{b0}\left(K_{b0}, L_{b0}, e^{\omega_{b0}}\right) + \epsilon(z_b^s) + E_\omega\left[\sum_{t=1}^{T} \beta^{t-1} \pi_{bt}\left(K_{bt}, L_{bt}, e^{\omega_{bt}}, z_b^s\right)\right] \Bigg)$$

$\Rightarrow$ Probability of cluster affiliation:

$$Pr(z_b^s | K_{b0}, L_{b0}, e^{\omega_{b0}}) = \int \mathbb{I}\left[z_b^*\left(K_{b0}, L_{b0}, e^{\omega_{b0}}\right) = z_b^s\right] f^\epsilon(\epsilon) d\epsilon.$$

Production Function Estimation

Parameter identification

Immediate identification of the production function parameters based on

$$y_{bt} = f^{klm}(k_{bt}, l_{bt}, m_{bt}) + \omega_{bt} + \varepsilon_{bt}$$

not possible due to simultaneity: $E\left[(\omega_{bt} + \varepsilon_{bt}) \,|\, k_{bt}, l_{bt}, m_{bt}\right] \neq 0$, and ω_{bt} unobserved.

Parameter identification

Solution? Resort to two-stage procedure.

Stage 1: Rely on flexible input m_{bt} (Ackerberg et al. (2015); Gandhi et al. (2017),...) to obtain

$$\phi_{bt} = f^{kl}(k_{bt}, l_{bt}) + \omega_{bt},$$

where ϕ_{bt} represents non-flexible output variation.

Parameter identification

Stage 2: Rely on Markov property $\omega_{bt} = \qquad [g(\omega_{bt-1}) + \eta_{bt}]$:

$$\phi_{bt} = f^{kl}(k_{bt}, l_{bt}) + \qquad \left[g\left(\phi_{bt-1} - f^{kl}(k_{bt-1}, l_{bt-1})\right) + \eta_{bt}\right].$$

$\Rightarrow$ Moment conditions that allow parameter estimation with GMM:

$$E\left[\qquad \eta_{bt} \middle| k_{bt}, l_{bt(-1)}, \phi_{bt-1} \right] = 0$$

Parameter identification with unobserved heterogeneity

Stage 2: Rely on Markov property $\omega_{bt} = \sum_{s=1}^{S} z_b^s \left[g^s(\omega_{bt-1}) + \eta_{bt}^s\right]$:

$$\phi_{bt} = f^{kl}\left(k_{bt}, l_{bt}\right) + \sum_{s=1}^{S} z_b^s \left[g^s\left(\phi_{bt-1} - f^{kl}\left(k_{bt-1}, l_{bt-1}\right)\right) + \eta_{bt}^s\right].$$

$\Rightarrow$ Moment conditions that contain unobserved heterogeneity:

$$E\left[\sum_{s=1}^{S} z_b^s \eta_{bt}^s \middle| k_{bt}, l_{bt(-1)}, \phi_{bt-1}\right] = 0$$

Parameter estimation with unobserved heterogeneity

Unobserved heterogeneity can be accounted for trough a Likelihood specification:

1. Reduced-form multinomial logit for cluster affiliation

$$Pr(z_b^s | k_{b0}, l_{b0}, \omega_{b0}; \gamma^1, \ldots, \gamma^s) = \frac{e^{\gamma_0^i + \gamma_k^i k_{b0} + \gamma_l^i l_{b0} + \gamma_\omega^i \omega_{b0}}}{\sum_{s=1}^S e^{\gamma_0^s + \gamma_k^s k_{b0} + \gamma_l^s l_{b0} + \gamma_\omega^s \omega_{b0}}}, \qquad \forall i = 1, \ldots, S.$$

Parameter estimation with unobserved heterogeneity

2. Limited Information Likelihood conditional on cluster affiliation

- Productivity follows a Gaussian Mixture (Dewitte et al., 2020)

$$\eta_{bt}^{s} = \phi_{bt} - f^{kl}\left(k_{bt}, l_{bt}; \boldsymbol{\beta}\right) - g(\phi_{bt-1}, l_{bt-1}, k_{bt-1}; \boldsymbol{\beta}, \boldsymbol{\alpha^s}) \sim \mathcal{N}\left(0, (\sigma_{\eta}^{s})^2\right)$$

- Reduced-form instrumental equation for endogenous labor

$$\zeta_{bt}^{s} = l_{bt} - \delta_0 - \delta_1 k_{bt} - \delta_2^s \phi_{bt-1} - \delta_3^s k_{bt-1} - \delta_4^s l_{bt-1} \sim \mathcal{N}\left(0, (\sigma_{\zeta}^{s})^2\right)$$

$\Rightarrow$ Bivariate normal specification

$$p^o(\phi_{bt}, l_{bt}|\cdot) \sim \mathcal{N}\left(\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}(\sigma_{\eta}^{s})^2 & \sigma_{\eta,\zeta}\\ \sigma_{\eta,\zeta} & (\sigma_{\zeta}^{s})^2\end{bmatrix}\right)$$

Parameter estimation with unobserved heterogeneity

Complete log-likelihood

$$\mathcal{L}^c\left(\boldsymbol{\Theta}, \boldsymbol{z}\right) = \sum_{b=1}^{B}\sum_{s=1}^{S} z_b^s log\Big(Pr(z_b^s | k_{b0}, l_{b0}, \omega_{b0}; \gamma^s) \times \prod_{t=1}^{T} p(\phi_{bt}, l_{bt} | k_{bt}, l_{bt}, \phi_{bt-1}, l_{bt-1}, k_{bt-1}, z_b^s; \boldsymbol{\theta}^s)\Big)$$

$\Rightarrow$ Estimate with Expectation-Maximization algorithm

Comparison with alternative identification strategies

Example

Assume Cobb-Douglas production function, AR(1) productivity and two clusters:

$$\begin{aligned}\phi_{bt} = \beta_k k_{bt} + \beta_l l_{bt} &+ \mathbb{I}\left(EXP_b = 0\right)\left(\alpha_0^1 + \alpha_1^1\left(\phi_{bt-1} - \beta_k k_{bt-1} - \beta_l l_{bt-1}\right) + \eta_{bt}^1\right) \\ &+ \mathbb{I}\left(EXP_b = 1\right)\left(\alpha_0^2 + \alpha_1^2\left(\phi_{bt-1} - \beta_k k_{bt-1} - \beta_l l_{bt-1}\right) + \eta_{bt}^2.\right).\end{aligned}$$

When imposing a unitary prior:

$$\phi_{bt} = \beta_k k_{bt} + \beta_l l_{bt} + \alpha_0^* + \alpha_1^*\left(\phi_{bt-1} - \beta_k k_{bt-1} - \beta_l l_{bt-1}\right) + \eta_{bt}^*.$$

If $\alpha_{0,1}^s \gtrless 0$, by definition, the omitted cluster indicator is correlated with the remaining explanatory variables and will positively/negatively bias the estimated coefficients (see, f.i., De Loecker (2013)).

Comparison with alternative identification strategies

1. Unitary cluster affiliation: $E\left[\eta^{*}_{bt} \middle| k_{bt}, l_{bt(-1)}, \phi_{bt-1}\right] \neq 0$

2. Deterministic cluster affiliation: proxy variable $E\tilde{X}P_b$ for EXP_b:

$$E\left[\sum_{s=0}^{1} \mathbb{I}\left(E\tilde{X}P_b = s\right)\eta^{s}_{bt} \middle| k_{bt}, l_{bt(-1)}, \phi_{bt-1}\right] = 0.$$

3. Random cluster affiliation:

$$E\left[\sum_{s=1}^{2} Pr(z^s_b | \boldsymbol{k}_b, \boldsymbol{l}_b, \phi_b, E\tilde{X}P_b; \hat{\boldsymbol{\Theta}})\eta^{s}_{bt} \middle| k_{bt}, l_{bt(-1)}, \phi_{bt-1}\right] = 0.$$

Monte Carlo

Setup

Adaptation of Ackerberg et al. (2015)'s Monte Carlo exercise with heterogeneity in productivity between groups of firms

- 100 simulated datasets of 1,000 firms over 10 years
- Value-added production technology with endogenous labor
- Productivity follows a Finite Mixture AR(1)-process

$$\omega_{bt} = \sum_{s=1}^{2} z_b^s \left[\alpha_0^s + \alpha_1^s \omega_{bt-1} + \eta_{bt}^s\right], \tag{1}$$

with $Pr(z_b^1) = 0.8$, $Pr(z_b^2) = 0.2$ and $\eta_{bt}^s \sim \mathcal{N}\left(0, \sigma_\eta^s\right)$.

Monte Carlo results

DGP 2

Methodology	β_k	β_l	α_0^1	α_1^1	σ_η^1	α_0^2	α_1^2	σ_η^2	$Pr(z_b^1)$	$Pr(z_b^2)$
True coefficients	0.40	0.60	1.00	0.70	0.21	0.80	0.77	0.25	0.80	0.20
Uni. GMM	0.45	0.60	0.83	0.71	0.22	-	-	-	1.00	1.00
	(0.01)	(0.01)	(0.03)	(0.01)	(0.00)	(-)	(-)	(-)	(0.00)	(0.00)
1-comp. LIML	0.45	0.60	0.83	0.71	0.22	-	-	-	1.00	1.00
	(0.01)	(0.01)	(0.03)	(0.01)	(0.00)	(-)	(-)	(-)	(0.00)	(0.00)
2-comp. LIML	0.40	0.60	0.99	0.70	0.21	0.80	0.76	0.25	80.53	19.47
	(0.02)	(0.01)	(0.05)	(0.01)	(0.00)	(0.08)	(0.02)	(0.01)	(3.51)	(3.51)

Monte Carlo results

Overview

Omitted variable bias is sizable, but our proposed estimator can correct this bias

- . . . even with endogeneity present
- . . . even without proxy variables
- . . . almost perfectly with proxy variables
- . . . even with noisy proxy variables

Application to Belgian firm-level data

Estimation framework

Production function estimation

- Belgian firm-level data on the manufacturing industry between 2008-2018
  - Focus on rubber and plastic products sector (sector 22)
- **First stage**: Gross-output and value-added CD/Translog specification
- **Second stage:**
  - GMM without additional heterogeneity in the AR(1) process
  - LIML with increasing heterogeneity (nr. clusters) in the AR(1) process

Production function results

Value Added specification with endogenous labor for the rubber and plastic products sector (sector 22)

	GMM	LIML					
Description		1-comp.	2-comp.	3-comp.	4-comp.	5-comp.	6-comp.
Capital	0.130	0.118	0.124	0.124	0.124	0.124	0.124
	(0.016)	(0.017)	(0.017)	(0.018)	(0.016)	(0.020)	(0.020)
Labor	0.879	0.860	0.854	0.866	0.860	0.852	0.857
	(0.020)	(0.045)	(0.023)	(0.027)	(0.028)	(0.027)	(0.029)
RTS	1.009	0.978	0.979	0.990	0.984	0.977	0.981
	(0.015)	(0.038)	(0.018)	(0.020)	(0.023)	(0.026)	(0.025)
Std. Dev.	0.180	0.159	0.156	0.157	0.156	0.155	0.156
	(0.016)	(0.030)	(0.019)	(0.017)	(0.018)	(0.019)	(0.016)
Nr. parameters	7	20	37	54	71	88	105
NLL		-6835	-8735	-9134	-9321	-9528	-9647
BIC		-13505	-17166	-17824	-18060	-18335	-18434
ICLbic		-13505	-17121	-17714	-17903	-18163	-18246

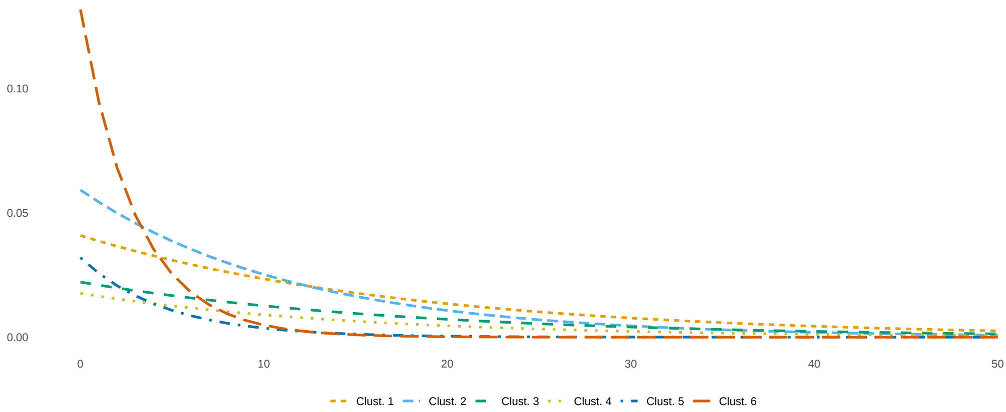
Heterogeneity in the productivity evolution

Coefficients of the Markov process

Cluster description	Prop. (%)	α_0	α_1	σ_η	μ_ω	σ_ω
Cluster 1	24.973	0.677	0.946	0.041	12.522	0.126
	(2.685)	(0.147)	(0.010)	(0.005)	(0.673)	(0.020)
Cluster 2	20.824	1.029	0.918	0.059	12.575	0.149
	(3.612)	(0.120)	(0.011)	(0.008)	(0.683)	(0.015)
Cluster 3	20.000	0.679	0.945	0.022	12.400	0.068
	(4.788)	(0.117)	(0.010)	(0.004)	(0.676)	(0.015)
Cluster 4	18.626	0.805	0.935	0.018	12.367	0.050
	(2.108)	(0.215)	(0.018)	(0.002)	(0.679)	(0.009)
Cluster 5	9.808	2.424	0.805	0.032	12.400	0.054
	(2.272)	(0.481)	(0.039)	(0.004)	(0.681)	(0.008)
Cluster 6	5.769	3.532	0.719	0.132	12.568	0.190
	(0.556)	(0.610)	(0.039)	(0.015)	(0.668)	(0.027)

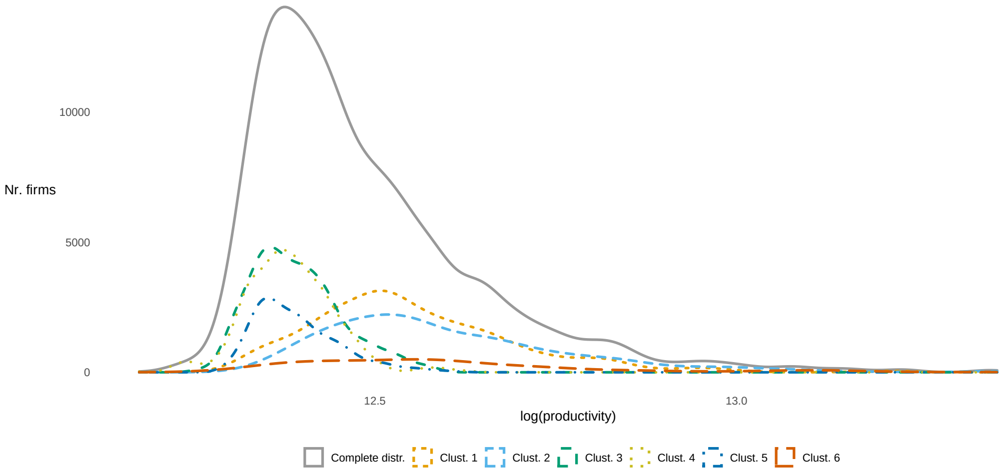
Heterogeneity in the productivity evolution

Impulse-Response to a one std. dev. productivity shock for sector 22



Cluster identification

Productivity density by clusters for sector 22



Cluster characterization

Available firm-level characteristics do not improve nor explain cluster affiliation

$$\frac{Pr(z_b^i|\ldots;\boldsymbol{\gamma}^i)}{Pr(z_b^1|\ldots;\boldsymbol{\gamma}^1)} = \gamma_0^i + \gamma_1^i k_{b0} + \gamma_2^i l_{b0} + \gamma_3^i \omega_{b0} + \gamma_4^i age_{b0}$$
$$+ \gamma_5^i ExportStatus_b + \gamma_6^i ImportStatus_b$$
$$+ \gamma_7^i FDIStatus_b, \qquad \forall i = 2, \ldots, S$$

Specification	Log-likelihood	BIC	ICLbic
Base specification	9,647.45	-18,433.93	-18,245.55
Augmented specification	9,658.52	-18,292.07	-18,112.65
Augmented specification without initial conditions	9,602.63	-18,262.30	-18,063.17

Cluster characterization

Summary statistics

	Overall	Clust. 1	Clust. 2	Clust. 3	Clust. 4	Clust. 5	Clust. 6
Cluster proportions (%)	100.00	24.97	20.82	20.00	18.63	9.81	5.77
log(Initial output)	15.17	16.05	14.81	14.65	16.10	13.67	14.78
log(Inital capital)	13.28	13.86	12.86	12.91	14.12	12.42	12.89
log(Initial labour)	2.78	3.37	2.05	2.67	3.97	1.80	2.03
log(Initial productivity)	12.49	12.56	12.62	12.39	12.38	12.38	12.61
Inital age	24.80	26.59	21.18	26.13	29.41	20.25	22.10
Exporter prop. (%)	65.13	78.87	59.83	51.79	81.63	46.15	60.78
Importer prop. (%)	80.68	91.55	86.32	69.64	90.82	58.46	70.59
FDI prop. (%)	10.26	14.79	3.42	7.14	21.43	3.08	7.84

- → Large and international firms group in clusters 1 and 4
- → Initial productivity correlates strongly with stationary productivity
- → Younger firms correlate with less persistent productivity processes

Exporter characterization

Average productivity premia for sector 22

Necessity to control for unobserved heterogeneity when comparing productivity across groups of firms, for instance for exporters vs. non-exporters:

Method	Unitary	Deterministic	Exhaustive
GMM	-1.493 (0.836)	0.276 (1.058)	0.760 (2.388)
LIML	2.559 (5.960)	3.857 (8.824)	2.854 (5.465)
Finite Mixture LIML	2.063 (2.108)	2.092 (2.153)	2.011 (2.116)

Conclusion

Conclusion

- General extension of state-of-the-art production function estimation procedures to control for, and identify, unobserved heterogeneity in the evolution of productivity.
- Strong evidence of heterogeneity in the evolution of productivity
  - Positively correlated with the initial conditions of a firm, especially with initial productivity.
  - Export, import, and FDI status correlated with multiple clusters $\Rightarrow$ heterogeneity beyond what is captured by these observed firm-level characteristics
- Contrary to existing methods, our estimator maintains its performance in the face of supplementary information

Future research

- What drives these clusters?
- Allow for regime-switching (Van Biesebroeck, 2003)
- Non-Hicks neutral productivity
- Firm-level fixed effects

Looking forward to working with you!

www.rubendewitte.be

rubenl.dewitte@ugent.be

@ruben0dewitte
ruben0dewitte

Bibliography I

Ackerberg, D. A. (2021). Comment on "olley and pakes-style production function estimators with firm fixed effects". *Oxford Bulletin of Economics and Statistics 83*(3), 836–840.

Ackerberg, D. A., K. Caves, and G. Frazer (2015). Identification Properties of Recent Production Function Estimators. *Econometrica 83*(6), 2411–2451.

Albuquerque, R. and H. A. Hopenhayn (2004). Optimal lending contracts and firm dynamics. *The Review of Economic Studies 71*(2), 285–315.

Angelini, P. and A. Generale (2008). On the evolution of firm size distributions. *The American Economic Review 98*(1), 426–438.

Arkolakis, C. (2016). A Unified Theory of Firm Selection and Growth. *The Quarterly Journal of Economics 131*(1), 89.

Atkeson, A. and A. Burstein (2010). Innovation, firm dynamics, and international trade. *Journal of Political Economy 118*(3), 433–484.

Bibliography II

Battisti, M., F. Belloc, and M. Del Gatto (2020). Labor productivity and firm-level tfp with technology-specific production functions. *Review of Economic Dynamics 35*, 283–300.

Beard, T. R., S. B. Caudill, and D. M. Gropper (1997). The diffusion of production processes in the us banking industry: A finite mixture approach. *Journal of Banking & Finance 21*(5), 721–740.

Bee, M., M. Riccaboni, and S. Schiavo (2011, Aug). Pareto versus lognormal: A maximum entropy test. *Phys. Rev. E 84*, 026104.

Bloom, N. and J. Van Reenen (2011). Human resource management and productivity. In *Handbook of labor economics*, Volume 4, pp. 1697–1767. Elsevier.

Cabral, L. M. B. and J. Mata (2003, September). On the evolution of the firm size distribution: Facts and theory. *American Economic Review 93*(4), 1075–1090.

Bibliography III

Caliendo, L. and E. Rossi-Hansberg (2012). The impact of trade on organization and productivity. *The Quarterly Journal of Economics 127*(3), 1393–1467.

Clementi, G. L. and H. A. Hopenhayn (2006, 02). A Theory of Financing Constraints and Firm Dynamics*. *The Quarterly Journal of Economics 121*(1), 229–265.

De Loecker, J. (2013, August). Detecting learning by exporting. *American Economic Journal: Microeconomics 5*(3), 1–21.

Dewitte, R., M. Dumont, G. Rayp, and P. Willemé (2020). Unobserved heterogeneity in the productivity distribution and gains from trade. MPRA Paper 102711, Ghent University.

El-Gamal, M. A. and H. Inanoglu (2005). Inefficiency and heterogeneity in turkish banking: 1990–2000. *Journal of Applied Econometrics 20*(5), 641–664.

Fruhwirth-Schnatter, S., G. Celeux, and C. P. Robert (2019). *Handbook of mixture analysis*. CRC press.

Bibliography IV

Gandhi, A., S. Navarro, and D. Rivers (2017). How Heterogeneous is Productivity? A Comparison of Gross Output and Value Added? Working paper.

Gandhi, A., S. Navarro, and D. A. Rivers (2020). On the identification of gross output production functions. *Journal of Political Economy 128*(8), 2973–3016.

Greene, W. (2005). Reconsidering heterogeneity in panel data estimators of the stochastic frontier model. *Journal of Econometrics 126*(2), 269–303.

Kasahara, H. and J. Rodrigue (2008). Does the use of imported intermediates increase productivity? plant-level evidence. *Journal of Development Economics 87*(1), 106–118.

Kasahara, H., P. Schrimpf, and M. Suzuki (2017). Identification and estimation of production function with unobserved heterogeneity. *mimeo*.

Klette, T. and S. Kortum (2004). Innovating firms and aggregate innovation. *Journal of Political Economy 112*(5), 986–1018.

Bibliography V

Lee, Y., A. Stoyanov, and N. Zubanov (2019). Olley and pakes-style production function estimators with firm fixed effects. *Oxford Bulletin of Economics and Statistics 81*(1), 79–97.

Lentz, R. and D. T. Mortensen (2008). An empirical model of growth through product innovation. *Econometrica 76*(6), 1317–1373.

Levinsohn, J. and A. Petrin (2003). Estimating productin functions using inputs to control for unobservables. *The Review of Economic Studies 70*, 317–341.

Luttmer, E. G. (2007). Selection, growth, and the size distribution of firms. *The Quarterly Journal of Economics 122*(3), 1103–1144.

Maican, F., M. Orth, M. J. Roberts, V. A. Vuong, et al. (2020). The Dynamic Impact of Exporting on Firm R&D Investment. Working Paper 27986, NBER.

Olley, S. and A. Pakes (1996). The dynamics of productivity in the telecommunications equipment industry. *Econometrica 64*(6), 1263–1297.

Bibliography VI

Orea, L. and S. C. Kumbhakar (2004). Efficiency measurement using a latent class stochastic frontier model. *Empirical Economics 29*(1), 169–183.

Rossi-Hansberg, E. and M. L. J. Wright (2007, December). Establishment size dynamics in the aggregate economy. *American Economic Review 97*(5), 1639–1666.

Van Biesebroeck, J. (2003). Productivity Dynamics with Technology Choice: An Application to Automobile Assembly. *The Review of Economic Studies 70*(1), 167–198.