Gains From Trade: Demand, Supply and Idiosyncratic Uncertainty

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Abstract

Firm-level sales is often used as a proxy for productivity to quantify welfare Gains from Trade (GFT) using firm-level data. This ignores the existence of uncertainty and heterogeneity other than productivity in firm-level sales. We demonstrate, theoretically and empirically, that the existence of idiosyncratic uncertainty in firm-level sales results in a sizable bias in GFT. Conflating uncertainty with productivity, as proxied by firm-level sales, results in an over-dispersed distribution of productivity. Assigning this uncertainty-inflated productivity to the modeled economy's supply-side results in overestimated aggregate trade elasticities and GFT. We show the possibility to obtain unbiased productivity, aggregate trade elasticities, and GFT estimates by relying on the revenue production function.

Keywords: Productivity distribution, Trade Elasticity, Gravity, Gains From Trade

JEL Codes: L11, F11, F12

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1 Introduction

Quantifying Gains From Trade (GFT) has gained importance in recent years. As globalization and trade agreements attract increasing scrutiny, it becomes all the more important to provide policymakers with reliable information on the effects of trade across countries, industries and firms. In this light, the development of trade models that allow for heterogeneity at the firm level brought trade theory much closer to businesses and policy-makers (Cernat, 2014). In such models, a correct measurement of the aggregate trade elasticity, i.e. the response of aggregate trade flows to a change in trade costs, is paramount to obtain a correct evaluation of the effects of trade liberalization (Chaney, 2008; Arkolakis et al., 2012; Bas et al., 2017).

To calculate aggregate trade elasticities, one requires an approximation of the complete firm-level productivity distribution (Melitz and Redding, 2015; Bas et al., 2017). As firm-level productivity is usually unobserved by the researcher, the trade literature tends to rely on firm-level sales a a proxy for productivity (Bas et al., 2017; Nigai, 2017; Bee and Schiavo, 2018; Sager and Timoshenko, 2019). In doing so, however, the literature fails to account for uncertainty and heterogeneity other than productivity that might be captured by firm-level sales (di Giovanni et al., 2011; Amand and Pelgrin, 2016).[1]

This paper demonstrates that a productivity measure proxied by firm-level sales conflates at least two heterogeneity sources: productivity and transitory idiosyncratic uncertainty. The difference between both matters: whether a firm's competitive (dis)advantage is transitory (for instance, due to an unexpected cyber attack) or persistent (for instance, due to its geographical location and/or industrial affiliation) will likely affect its profit maximizing decisions. We investigate, theoretically and empirically, the bias in the aggregate trade elasticity and subsequent GFT calculations that arises from the presence of transitory idiosyncratic uncertainty in current measures of firm-level productivity. The premise is straightforward: *conflating transitory uncertainty with firm-level productivity, as proxied by firm-level sales, results in an over-dispersed distribution of productivity. Assigning this uncertainty-inflated productivity to the supply-side of the economy results in overestimated trade elasticities and GFT.* We show the possibility to obtain unbiased productivity, aggregate trade elasticity, and GFT estimates by relying on the revenue production function.

The existence of transitory uncertainty can bias aggregate trade elasticities and GFT through two channels: (i) mismeasurement of firm-level productivity, and (ii) theoretical model misspecification. We provide a general framework to identify and evaluate the importance of both channels. A dynamic, open economy heterogeneous firms model (Hopenhayn, 1992; Melitz, 2003) is presented that features transitory firm-level uncertainty in the production process originating from both demand and supply (Das et al., 2007; De Loecker, 2011; Kasahara and Lapham, 2013; Gandhi et al., 2020).

First, we show that current estimation techniques of aggregate trade elasticities result in biased measurements if transitory uncertainty is present. We propose a theoretically underpinned al-

[1] A similar argument relates to the deterministic bias in firm efficiency or productivity obtained from non-parametric data envelopment analysis or free disposable hull-estimations.

ternative identification strategy for aggregate trade elasticities that controls for idiosyncratic uncertainty. Aggregate trade elasticities consist of two components: a demand-side elasticity of substitution between varieties and a supply-side distribution of productivity (Chaney, 2008; Bas et al., 2017). We combine a firm-level production function with a CES demand system into a revenue production function which allows us to identify both components of the aggregate trade elasticity while controlling for transitory idiosyncratic uncertainty. We rely on structural production function estimation techniques (Klette and Griliches, 1996; De Loecker, 2011; Ackerberg et al., 2015) to control for endogeneity concerns and to obtain consistent parameter estimates.

Second, once unbiased measures of firm-level productivity are available, the framework allows us to theoretically evaluate the influence of transitory uncertainty on aggregate trade statistics such as the aggregate trade elasticity and GFT. We demonstrate that, under standard assumptions regarding the distribution of transitory uncertainty, modeled aggregate trade statistics are equivalent to those obtained from prevalent static heterogeneous firms models (Melitz, 2003) that do not feature uncertainty. Conditional on an unbiased measure of firm-level productivity, aggregate trade statistics from static heterogeneous firms models (Melitz, 2003) are thus not affected by the existence of transitory uncertainty. Hence, firm-level uncertainty results in biased aggregate trade statistics due to firm-level productivity mismeasurement, but not due to model misspecification

We evaluate the influence of this transitory uncertainty using French firm-level data over the years 1998–2006. We find that the variance of productivity increases by approximately 10% when productivity is conflated with transitory uncertainty. The impact of this residual is not homogeneously distributed, but is larger in the tails of the distribution. This results in an absolute aggregate trade elasticity estimate, when not controlling for transitory uncertainty, that is underestimated by about 11.6% in foreign markets where 25% of the domestic firms would be active, and about 13.8% in foreign markets where 10% of the domestic firms would be active. GFT, then, calculated as a shift from autarky to free trade ($\tau = 1$) in a stylized symmetric 2-country model, is overestimated with about 10.8% when transitory uncertainty is not controlled for. GFT from autarky to iceberg trade costs of $\tau = 1.25$, are overestimated by 23.7%. These large differences in GFT for different values of the iceberg trade costs can be attributed to the distribution of uncertainty. As transitory uncertainty mainly distorts the tails of the firm-level productivity distribution, a larger bias in aggregate trade statistics will be observed when foreign markets are less accessible, i.e. when exporting cutoffs are located more in the tails of this distribution. Therefore, we emphasize the importance of evaluating the relative differences in GFT for this stylized model. Overall, we find conclusive evidence that controlling for transitory idiosyncratic uncertainty is economically relevant when calculating the impact of trade costs on trade flows and welfare.

The remainder of the paper is organized as follows. In section 2, we provide an overview of the related literature. We present our theoretical framework in section 3. This framework allows us to define the identification strategy in section 4 and apply this strategy to French firm-level data. Section 5 evaluates the impact of transitory uncertainty on aggregate trade elasticities and GFT. Section 7 concludes.

2 Literature review

As stated in the introduction, the aggregate trade elasticity is identified once its two components, the demand-side elasticity of substitution between varieties and the supply-side productivity distribution, are determined (Bas et al., 2017). The identification of these components is burdened by the existence of firm-level uncertainty in realized demand and supply. Below, we discuss the prevalent approach of (i) recovering the supply-side productivity distribution parameters from the sales distribution and (ii) identifying the elasticity of substitution from firm-level gravity, in light of the possible existence of idiosyncratic uncertainty.

On the supply side, identifying the productivity distribution parameters is difficult as firm-level productivity is unobservable to the researcher. The trade literature, therefore, resorts to *sales as a proxy for productivity* (see, for instance Head et al. (2014); Nigai (2017); Bas et al. (2017)). Under the assumptions of the dominant heterogeneous firms model (Melitz, 2003) with productivity following a distribution that is closed under power-law transformations,[2] it can be shown there is an approximate one-to-one mapping between sales (x) and productivity (ω): $x \dot{\sim} e^{(\sigma-1)\omega}$, up to the elasticity of substitution (σ).[3] Acknowledging the existence of transitory firm-level uncertainty and measurement error, however, productivity can only be identified from firm-level sales up to a firm-level stochastic component (ε^T): $x \dot{\sim} e^{(\sigma-1)\omega+\varepsilon^T}$. This component is not expected to be negligible. Kasahara and Lapham (2013) calculated the residual (e^{ε^T}) to account for approximately 10% of the variation in firm-level sales vis-à-vis productivity. It is yet unclear how the presence of such residual impacts aggregate statistics in dominant heterogeneous firms models.

Moreover, possible additional sources of firm-level heterogeneity can originate from the definition of sales. Sales has been interpreted to signify total sales (Axtell, 2001), exporting sales (di Giovanni and Levchenko, 2013; Head et al., 2014), or domestic sales (Nigai, 2017). As soon as the country of study is an open economy with firm-level heterogeneity in the exporting/destination-decision, the one-to-one mapping between firm-level total sales and productivity disappears (di Giovanni et al., 2011; Amand and Pelgrin, 2016). Exporting sales only captures part of the firm population, with the size of that part depending on, among others, the size of the exporting destination and trade costs. The estimation of distribution parameters needs to be adapted accordingly, for instance by relying on truncated distributions (Sager and Timoshenko, 2019), and increases the possibility of finite sample biases. As for domestic sales, the productivity distribution can only be identified conditional on the correct identification of the elasticity of substitution (σ).

This second component of the aggregate trade elasticity, the elasticity of substitution, can be determined from the *firm-level gravity* equation. Specifically, it can be identified as the response

[2]Most common distributions used in the economic literature are closed under power-law transformations (see, for instance, Mrázová et al. (2021) and Dewitte et al. (forthcoming).

[3]This almost one-to-one mapping with the distribution of productivity also appears for prices, profits, output and employment (Melitz and Redding, 2014, p. 12).

of firm-level sales to cross-sectional and/or time variation in tariffs (Bas et al., 2017).[4] To do so, however, the researcher needs to capture the multilateral resistance terms accordingly and deal with other known issues as selection bias (the existence of firm-level zeros in trade data), heteroskedasticity and the difficulty in approximating trade costs (Yotov et al., 2016).[5] To date, we have no knowledge of gravity estimates that exploit the panel dimension of single-origin firm-level trade data while perfectly controlling for the multilateral resistance terms and/or dynamic productivity. As (Bas et al., 2017, footnote 11 on p.5) note, doing so with current techniques requires assumptions that are inconsistent with the underlying static trade theory. Bas et al. (2017), therefore, propose to rely on multiple-origin (minimum two) firm-level trade data and exploit solely the cross-sectional variation of the firm-level gravity equation to obtain theory-consistent elasticity of substitution estimates.

The proposed identification strategy in this paper circumvents the above described difficulties of prevalent identification strategies. Estimating a revenue production function allows us to exploit the panel dimension of firm-level data from a single country to identify both the productivity distribution and elasticity of substitution while controlling for idiosyncratic uncertainty.

This paper is also related to the literature that identifies aggregate trade elasticities from aggregate rather than firm-level data. Under the assumption of Pareto-distributed productivity, the two components of the aggregate trade elasticity collapse to a constant trade elasticity that can be identified from industry-level structural gravity equations.[6] Despite the popularity of this assumption, recent evidence exposes the superior performance of alternative distributional forms to capture firm-level heterogeneity (Head et al., 2014; Melitz and Redding, 2015; Nigai, 2017; Bee and Schiavo, 2018; Sager and Timoshenko, 2019). Adão et al. (2020), then, provide a general identification strategy for the aggregate trade elasticity using aggregate trade data, assuming that both the firm-level productivity distribution and elasticity of substitution are equal across countries. Additionally, the paper relates to the literature that studies the importance of the distributional assumption on trade forecasts (see, for instance di Giovanni et al. (2011); Head et al. (2014); Bas et al. (2017); Nigai (2017); Fernandes et al. (2018); Bee and Schiavo (2018); Sager and Timoshenko (2019)). However, distinct from this work, we consider the distribution of estimated productivity rather than using sales as a proxy for productivity. Egger et al. (2020) also relies on estimated productivity to recover firm-level heterogeneity, but do not focus on the importance of transitory uncertainty. Sager and Timoshenko (2020) focus on the importance of uncertainty originating from demand (without supply) on trade and welfare. They rely on a gravity framework to identify market-specific i.i.d. demand shocks. The framework of Sager and Timoshenko (2020) does not allow for dynamic productivity nor does it allow to identify the elasticity of substitution and, therefore, aggregate trade elasticities. Our paper can also be related to the granularity literature (Gabaix, 2011; di Giovanni and Levchenko, 2012; Eaton

[4] Note that one can also rely on the firm-level export demand equation to identify the elasticity of substitution from the variation in firm-level output due to variation in firm-level prices, if prices are correctly instrumented (see, for instance Fontagné et al. (2018); Fitzgerald and Haller (2018); Piveteau and Smagghue (2019)).

[5] Berthou and Fontagné (2016); Bas et al. (2017); Fitzgerald and Haller (2018) note that time variation in tariffs is small relative to the cross-sectional variation.

[6] See Arkolakis et al. (2012) for an exposition on the identification of aggregate trade elasticities from aggregate trade data under Pareto-distributed productivity and for a non-exhaustive list of works relying on this distributional assumption.

et al., 2012; Carvalho and Grassi, 2019), which discusses the transmission of firm-level shocks to the aggregate level. Our analysis demonstrates that transitory shocks have no aggregate implications in prevalent static heterogeneous firms models.

3 Heterogeneous firms model with idiosyncratic uncertainty[7]

We specify a dynamic open economy model with transitory firm-level uncertainty originating from demand and supply shocks. The core elements are a dynamic firm heterogeneous model (Melitz, 2003; Hopenhayn, 1992) augmented with uncertainty surrounding realized demand and supply, specified similarly to the structural revenue production function estimation literature (see for instance Das et al. (2007); De Loecker (2011); Kasahara and Lapham (2013); Gandhi et al. (2020)). This will provide us with a general framework to evaluate the empirical methods that deduce firm-level productivity from sales data.

Demand The preferences of a representative consumer in country $j \in J$ are defined over a continuum of horizontally differentiated varieties originating from country $i \in I$ ($\varpi \in \Omega^i$) and are assumed to take the Constant Elasticity of Substitution (CES) utility (U) form at time t,

$$U_t^j = \left(\sum_{i=1}^{I} \int_{\varpi \in \Omega^i} e^{\frac{1}{\sigma}\nu(\varpi)} y_t^{ij}(\varpi)^{\frac{\sigma-1}{\sigma}} d\varpi \right)^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

with σ the elasticity of substitution between varieties and $y_t^{ij}(\cdot)$ the quantity of a variety shipped from i that arrives in j. Utility maximization defines the optimal consumption and expenditure decisions over the individual varieties

$$\frac{y_t^{ij}(\varpi)}{Y_t^j} = \left[\frac{p_t^{ij}(\varpi)}{P_t^j} \right]^{-\sigma} e^{\nu_t(\varpi)}, \tag{2}$$

up to a variety-specific demand shock $e^{\nu_t(\varpi)}$ which is independent and identically distributed (i.i.d.) across varieties and time (see below for a discussion of this assumption). The set of varieties consumed is considered as an aggregate good $Y_t^j \equiv U_t^j$ (Melitz, 2003) and P_t^j is the CES aggregate price index.

Supply There is a continuum of businesses, or firms, ($b \in B$) which choose to supply a distinct horizontally-differentiated variety. They are heterogeneous in terms of their productivity $\omega_{bt} \in [0, \infty]$ drawn from the unconditional Cumulative Distribution Function (CDF) $G(\omega_{bt})$ after paying a fixed cost f_t^{ie} to enter the market.[8] The firm's productivity follows a Markov process

[7] See Appendix B for a detailed elaboration of the model.

[8] We follow Asker et al. (2017) in differentiating all fixed costs from factors of production. "In their financial statements, firms report overhead costs as Selling, General and Administrative Expenses (SG&A). These expenses are not directly related to production, and include sales, advertising, marketing, executive compensation, . . . and

independent across firms with conditional distribution $G(\omega_{bt+1}|\mathcal{I}_{bt})$ such that:[9]

$$e^{\omega_{bt+1}} = \mathbb{E}_{\omega}\left[e^{\omega_{bt+1}}|\mathcal{I}_{bt}\right] e^{\eta_{bt+1}}. \tag{3}$$

Productivity at time $t+1$ is specified as a function of the information set of the firm at time t, $\mathcal{I}_{bt}$ with $\omega_{bt} \in \mathcal{I}_{bt}$, and a productivity shock η_{bt+1}. The information set $\mathcal{I}_{bt}$ is a set of random variables that contains the information that the firm can use to solve its period t decisions problems. The productivity distribution is known to firms and stochastically increasing in ω_{bt} (Hopenhayn, 1992).

Production relies on a composite factor of production $\boldsymbol{A}^{ij}_{bt}(\boldsymbol{\beta})$ (Melitz and Redding, 2014) subject to shocks to the production function $e^{\epsilon_{bt}}$ which are i.i.d. across firms and time (see below for a discussion of this assumption):[10]

$$y^{ij}_{bt} = q^{ij}_{bt} e^{\epsilon_{bt}} = \boldsymbol{A}^{ij}_{bt}(\boldsymbol{\beta}) e^{\omega_{bt}+\epsilon_{bt}} \tag{4}$$

Supply of the production factor to the individual firm is perfectly elastic, so that firms are effectively price (W^i_t) takers on the input market. Firms from country i have to pay a fixed cost f^{ij}_t to produce goods destined for country j at time t.[11] The cost function of the firm involves the fixed production cost, iceberg trade costs $\tau^{ij}_t > 1$ and a constant marginal cost that depends on productivity and transitory shocks to production: $f^{ij}_t + \left(\frac{\tau^{ij}_t y^{ij}_{bt}}{e^{\omega_{bt}+\epsilon_{bt}}}\right) W^i_t$.

Profit maximization, then, results in an optimum quantity produced:

$$q^{ij}_{bt} = \left(\frac{\sigma-1}{\sigma}\frac{e^{\omega_{bt}}}{\tau^{ij}_t W^i_t}\right)^{\sigma} Y^j_t \left(P^j_t\right)^{\sigma} \mathbb{E}_{\varepsilon^T}\left[e^{\varepsilon^T_{bt}}\right]^{\sigma}. \tag{5}$$

where ε^T_{bt} gathers the transitory demand and supply shocks $\left(\varepsilon^T_{bt} = e^{\frac{\nu_{bt}}{\sigma}+\frac{\sigma-1}{\sigma}\epsilon_{bt}}\right)$. For each firm b at time t, the timing assumptions of the model can thus be summarized as follows:

1. Observe the vector of state variables $\mathcal{I}_{bt}$, with $\omega_{bt} \in \mathcal{I}_{bt}$;
2. Produce output q_{bt} and sell at a price determined by the demand curve;
3. Observe deviations from expectations regarding demand (ν_{bt}) and supply (ϵ_{bt});

The operational revenue for firms from country i selling in destination j at time t can be

can in part be interpreted as expenses on intangible capital." (Asker et al., 2017, p. 4). We assume all fixed cost expenses are equally distributed within the source market.

[9] $\mathbb{E}_x[\ldots] = \int \ldots f(x)dx$.

[10] This composite factor can, for instance, be a Constant Returns to Scale Cobb-Douglas function of Z fixed (F) and V variable (L) factors of production respectively: $\boldsymbol{A}^{ij}_{bt}(\boldsymbol{\beta}) = \prod_{z=1}^{Z}\prod_{v=1}^{V}(F^{ij}_{bzt})^{\beta^i_z}(L^{ij}_{bvt})^{\beta^i_v}$, where variable production factors can be adjusted every time period t after the realization of the information set $\mathcal{I}_{bt}$ while fixed production factors take one time period to adjust. See Online Appendix B for a model workout with such distinction between production factors.

[11] Similar to the fixed entry costs, fixed production costs are due in the domestic market in monetary terms rather than in production factors.

obtained as the product of this optimal quantity with the profit-maximizing price p_{bt}^{ij} given by eq. 2:

$$
\begin{aligned}
x_{bt}^{ij} = p_{bt}^{ij} y_{bt}^{ij} &= \left(y_{bt}^{ii}\right)^{\frac{\sigma-1}{\sigma}} \left(Y_t^i\right)^{\frac{1}{\sigma}} e^{\frac{\nu_{bt}}{\sigma}} P_t^j \\
&= \left(\frac{\sigma}{\sigma-1}\tau_t^{ij} W_t^i\right)^{1-\sigma} Y_t^j \left(P_t^j\right)^{\sigma} \mathbb{E}_{\varepsilon^T}\left[e^{\varepsilon_{bt}^T}\right]^{\sigma-1} e^{(\sigma-1)\omega_{bt}+\varepsilon_{bt}^T}. \qquad (6)
\end{aligned}
$$

Equation 6 shows that firm-level variation in sales originates from (at least) two different sources: productivity ω_{bt}, and transitory uncertainty ε_{bt}^T, such that $x_{bt}^{ij} \dot{\sim} e^{(\sigma-1)\omega_{bt}+\epsilon_{bt}^T}$. This stands in contrast with the prevalent method of using sales as a proxy for productivity which, as explained in the literature review above, attributes variation in sales solely to variation in productivity up to the elasticity of substitution: $x_{bt}^{ij} \dot{\sim} e^{(\sigma-1)\omega_{bt}}$.

The prevalent method thus ignores a component of stochastic variation present in firm-level sales that is economically relevant. Transitory uncertainty $\left(\varepsilon_{bt}^T = \frac{\nu_{bt}}{\sigma} + \frac{\sigma-1}{\sigma}\epsilon_{bt}\right)$ combines supply- and demand-side deviations that are not expected to be negligible. Kasahara and Lapham (2013) calculated this residual component to account for approximately 10% of the variation in firm-level sales vis-à-vis productivity $e^{\omega_{bt}}$.

The supply-side specification of these deviations, ϵ_{bt}, represents common practice in the productivity estimation literature to allow for deviations from productivity. Whereas productivity "[ω_{bt}] might represent variables such as the managerial ability of a firm, expected down-time due to machine breakdown, expected defect rates in a manufacturing process, soil quality, or the expected rainfall at a particular farm's location", "[ϵ_{bt}] might represent deviations from expected breakdown, defect, or rainfall amounts in a given year" (Ackerberg et al., 2015, p.2414).

Our demand-side specification of these deviations, ν_{bt}, follows De Loecker (2011) in assuming them to be firm-specific,[12] transitory and unobserved residual demand shocks. Notice that the demand-side uncertainty of our model is not necessarily specified more restrictively than the supply-side. Specifically, the model could be specified to allow for demand-side deviations correlated over time $\tilde{\nu}_{bt} \in \mathcal{I}_{bt}$ next to the i.i.d. demand shocks ν_{bt}. If the dynamics of these correlated demand shocks are similar to those of productivity (see De Loecker (2011); Gandhi et al. (2020)), the heterogeneity variable of relevance in equation 6 would become $\tilde{\omega}_{bt} = \omega_{bt} + \frac{\tilde{\nu}_{bt}}{(\sigma-1)\sigma}$. This variable is referred to as business conditions (Bloom, 2009) or profitability conditions (Sager and Timoshenko, 2020) rather than productivity ω_{bt}, and does not affect the main results of our model specification.[13]

[12] Note that each firm produces only one variety.

[13] The assumption on identically distributed deviations, on the other hand, is more difficult to be relaxed. It is a restrictive necessity for our theoretically underpinned identification strategy provided the data available (see Section 3) and to obtain equivalent aggregate trade statistics between a static version of this model and the prevalent heterogeneous firms model (see Section 5). Demand shocks could, for instance, be specified as partly consisting of serially correlated market-specific shocks (Sager and Timoshenko, 2020) without altering our main theoretical conclusions. The proposed estimation procedure, however, would require information on firm-level market-specific factor input use in that case.

4 Identification and Estimation

From the literature review in section 2, it was apparent that the current identification of aggregate trade elasticities based on firm-level trade data is hampered by uncertainty. Moreover, we have no knowledge of firm-level gravity estimation techniques that exploit single-origin panel data while allowing for dynamic productivity. In the productivity estimation literature, however, it is exactly the stochastic evolution of productivity that is relied upon to acquire identification of firm-level productivity and uncertainty (Olley and Pakes, 1996). Moreover, Klette and Griliches (1996) demonstrated that one can rely on a combination of the production function and the CES demand system to identify both the productivity distribution and the elasticity of substitution from a single estimation procedure. We rely on these insights to propose a theoretically underpinned alternative to the prevalent gravity/sales as proxy for productivity identification scheme. Combining the production function with the CES demand system allows us to simultaneously identify the two components of the aggregate trade elasticity, productivity and the elasticity of substitution, while controlling for idiosyncratic uncertainty.

4.1 Identification

We specify our estimation equation as the log-linearized combination of the domestic production function (eq. 4) with the domestic CES demand system (eq. 2) (Klette and Griliches, 1996; De Loecker, 2011):[14]

$$\begin{aligned} ln\left(\frac{x_{bt}^{ii}}{P_t^i}\right) = ln\left(\frac{p_{bt}^{ii}y_{bt}^{ii}}{P_t^i}\right) &= ln\left(\left(y_{bt}^{ii}\right)^{\frac{\sigma-1}{\sigma}}\left(Y_t^i\right)^{\frac{1}{\sigma}}e^{\frac{\nu_{bt}}{\sigma}}\right) \\ &= \frac{\sigma-1}{\sigma}\boldsymbol{A}_{bt}^{ii}(\boldsymbol{\beta}) + \frac{1}{\sigma}lnY_t^i + \frac{\sigma-1}{\sigma}\omega_{bt} + \varepsilon_{bt}^T. \end{aligned} \tag{7}$$

A domestic revenue function specification ($i = j$) avoids conflating productivity with heterogeneity originating from export output (Amand and Pelgrin, 2016; Nigai, 2017). Notice the difference between this 'production function specification' of the revenue equation and the 'gravity specification' of the revenue equation in equation 6.[15] The gravity specification relies on the profit-maximizing optimal input mix to rewrite firm-level revenue as a function of aggregate variables and two sources of firm-level heterogeneity: productivity ω_{bt}, and transitory uncertainty ε_{bt}^T. This production function specification, on the other hand, relies on actual firm-level heterogeneous input use.

The data on which we rely (see subsection 4.2) only contains information on total input use, not market-specific input use. Therefore, we follow Rivers (2010) in specifying the fraction of

[14] If the researcher has firm-level data on origin-destination specific output and input quantities, equation 4 could form the basis of our estimation procedure to recover firm-level productivity ω_{bt}. Unfortunately, most commonly available firm-level data consist of total firm-level revenue and input expenditures, conflating prices, quantities and destinations.

[15] Commonly, firm-level gravity is specified at the firm-product level. We do not account for the product dimension as this dimension is superfluous to our identification strategy. Upon data availability, our framework can be extended to account for multi-product firms (Bernard et al., 2009; De Loecker, 2011).

quantities sold on each market, which, under the assumption of equal demand elasticities across markets, equals the observable fraction of revenues sold in each market as $\theta_{bt}^{ij} = \frac{x_{bt}^{ij}}{\sum_{j=1}^{J} x_{bt}^{ij}} = \frac{q_{bt}^{ij}}{\sum_{j=1}^{J} q_{bt}^{ij}}$ (Rivers, 2010),[16] such that

$$\begin{aligned} ln\left(\frac{x_{bt}^{ii}}{P_t^i}\right) &= \frac{\sigma-1}{\sigma} ln\theta_{bt}^{ii} + lnx_{bt}^{i} \\ &= \frac{\sigma-1}{\sigma} ln\theta_{bt}^{ii} + \frac{\sigma-1}{\sigma}\boldsymbol{A}_{bt}^{i}(\boldsymbol{\beta}) + \frac{1}{\sigma} lnY_t^i + \frac{\sigma-1}{\sigma}\omega_{bt} + \varepsilon_{bt}^{T}. \end{aligned} \tag{8}$$

Equation 8 is our main estimating equation. The parameter identification for this equation is not straightforward, as productivity ω_{bt} is unobserved. A Nonlinear Least Squares (NLLS) estimation, for instance, will deliver biased coefficients as factor components of composite production factor $\boldsymbol{A}_{bt}^{i}(\boldsymbol{\beta})$ and the demand shifter Y_t^i are correlated with current productivity: $E\left[Y_t^i\left(\frac{\sigma-1}{\sigma}\omega_{bt} + \varepsilon_{bt}^{T}\right)\right] \neq 0$. Therefore, we rely on a structural productivity estimation technique like De Loecker (2011).[17] This technique uses the Ackerberg et al. (2015) proxy-variable approach (ACF) to separate the transitory uncertainty component from our main estimation equation (eq. 8) in a first stage. In line with the specified theoretical model (which does not feature intermediate inputs), Ackerberg et al. (2015) typically rely on intermediate inputs as a proxy variable and use a value-added specification of the production function. In a second stage, the estimation procedure relies on the Markov assumption for productivity (see eq. 3) to avert endogeneity problems and obtain consistent parameter and productivity estimates. Like Klette and Griliches (1996); De Loecker (2011); Halpern et al. (2015), we recover an estimate of the elasticity of substitution from variation in the demand shifter Y_t^i over time.

4.2 Data

For our empirical analysis, we rely on a large panel of French firms extracted from the Amadeus database by Bureau Van Dijk Electronic Publishing. The Amadeus database contains financial information (balance sheet and profit and loss account) as well as information on firms' location, activity, ownership, etc. We construct a sample covering the period 1998-2006 using multiple issues of the database (October releases from 1998 till 2015)[18]. Despite its wide geographical, time and sectoral range, the sample of firms can vary considerably. The providers of the database rely on national data sources, which are subject to change. In addition, for firms that do not provide information for more than three consecutive years, all (historical) information is removed. The estimation of firm productivity and the its subsequent analysis are therefore based on an extended version of Amadeus (now Orbis), as described in Merlevede et al. (2015) (see also (Kalemli-Ozcan et al., 2015) for a discussion on the construction of nationally representative firm-level data based on the Orbis database). Compiling annual versions of Amadeus, the

[16]If elasticities of demand are equal in both markets, firms allocate output such that prices received by firms in both are equal (Rivers, 2010). This equivalence breaks down if demand shocks are destination-specific.

[17]See Appendix C for an elaborate description of the estimation strategy.

[18]A single issue is only a snapshot of the ownership information and firms that exit are dropped from the next issue released. A single issue further only contains 10 years of financial data at maximum. In order to get a full overview of activity, location, ownership and financials through time, multiple issues are required.

extended database attenuates variability in the sample composition.

We restrict the dataset to manufacturing firms (NACE 2 class 10–33) that report positive total and domestic operating sales, tangible fixed assets, number of employees, costs of employees, material inputs and value added.[19] All monetary variables are deflated using the appropriate NACE 2-digit deflator from the EU-KLEMS database. Real output are sales deflated with producer price indices. Capital are tangible fixed assets deflated by the average of the deflators for five NACE 2-digit industries according to Javorcik (2004). Aggregate sales is constructed as a (market share) weighted sum of deflated sales (Rivers, 2010; De Loecker, 2011). Real material inputs are obtained by deflating material inputs with an intermediate input deflator as a weighted average of output deflators where the country-industry-time specific weights are based on intermediate input uses retrieved from input-output tables. Value added is then obtained as the difference between real output and real material inputs. Labor is simply the number of employees.

Our focus on France is motivated by the presence of information on firm-level exports in the Amadeus database for French firms. This allows us to use domestic firm-level sales, calculated as the difference between total firm-level sales and firm-level export sales, avoiding conflating firm-level productivity with heterogeneity in export output (see also Section 2). Furthermore, the trade literature has mainly focused on France concerning research on the characterization of the productivity distribution (see, for instance, di Giovanni and Levchenko (2013); Head et al. (2014); Nigai (2017); Bee and Schiavo (2018)). Our final database contains 379,765 observations from 86,959 unique French firms over the years 1998–2006. Summary statistics in Appendix Table 1 reveal that our database covers a wide range of the firm universe in France.

4.3 Estimation results

We apply both the Nonlinear Least Squares (NLLS) and the structural productivity estimation procedure (ACF) as described in Section 4.1 for a value-added Cobb-Douglas production function to our complete French firm-level dataset.[20] The resulting parameter estimates are displayed in Table 1. The capital and labor elasticities are slightly overestimated by the NLLS procedure compared to the ACF estimation procedure, which controls for endogeneity of productivity. Both estimators report increasing returns to scale, which is common when controlling for demand in the production function estimation (De Loecker, 2011). The ACF elasticity of substitution estimate takes a value of 4.59, which is in line with previously reported estimates obtained from variation in demand shifters on firm-level revenue (Rivers, 2010; De Loecker, 2011; Kasahara and Lapham, 2013; Halpern et al., 2015) as well as from variation in trade costs on firm-level exports (Bas et al., 2017).

[19] We clean the data both on levels and on growth rates to prevent effects of extreme outliers and extreme noise on the analysis. Specifically, we limit the sample to observations with a labor use large than 1 and limit deflated turnover, deflated materials and deflated capital to values larger than 1,000 euro. Further, we removed the yearly lowest and highest percentile of the included variables and dropped observations with yearly growth rates of included variables higher than 100 in absolute values.

[20] We estimate an aggregate rather than industrial production function, allowing us to easily obtain the complete productivity distribution and compare the results with the trade literature on distributions (see, for instance, Head et al. (2014); Bas et al. (2017); Nigai (2017); Bee and Schiavo (2018)).

Table 1: Production function estimation results

	Capital	Labor	Returns to Scale	Elasticity of Substitution
NLLS	0.177	1.072	1.249	5.482
	(0.002)	(0.007)	(0.008)	(0.157)
ACF	0.166	1.045	1.212	4.590
	(0.002)	(0.007)	(0.007)	(0.110)

Notes: Standard errors displayed between brackets are obtained from wild bootstrap clustered at the firm level with 99 repliations. Estimates obtained from French firm-level database over the years 1998–2006 with 379,765 observations from 86,959 firms.

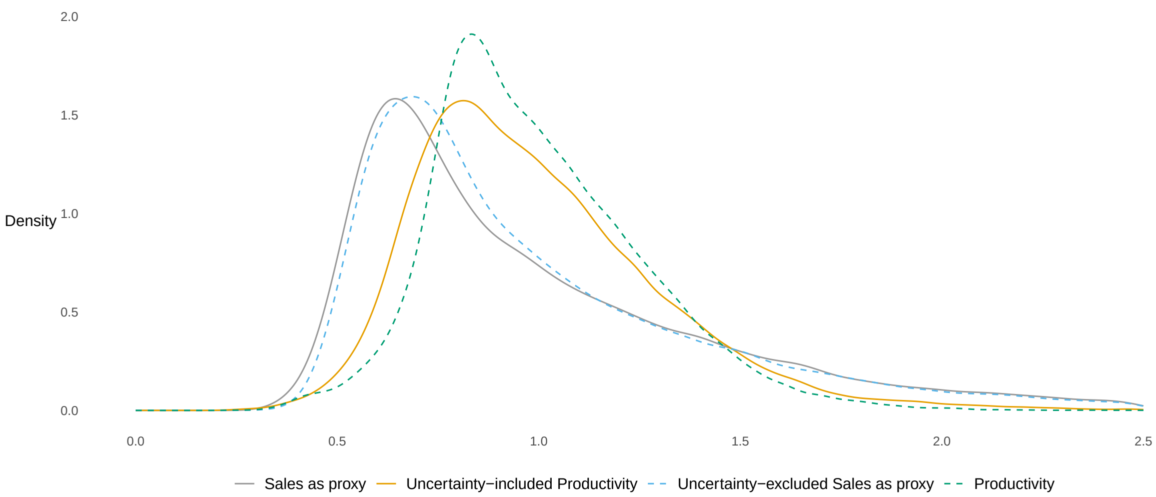

Figure 1: Nonparametric kernel density of domestic sales as a proxy for productivity, uncertainty-excluded sales as a proxy for productivity, uncertainty-included productivity and productivity in the year 2006.
Note: All variables are demeaned.

With consistent estimates of the elasticity of substitution, productivity, and uncertainty at hand, we turn our attention to the distribution of productivity and the influence of uncertainty on this distribution. Next to estimated *productivity* $\left(e^{\hat{\omega}_{bt}}\right)$, which is free from uncertainty, we construct three additional measures of productivity based on equation 6. These additional measures will allow for a straightforward comparison with measures of productivity currently used in the literature.

We augment productivity with transitory uncertainty to obtain *uncertainty-included productivity* $\left(e^{\hat{\omega}_{bt}+\frac{\epsilon_{bt}^{T}}{\hat{\sigma}-1}}\right)$. Additionally, we consider domestic *sales as a proxy* for productivity $\left(\left(x_{bt}^{ii}\right)^{\frac{1}{\hat{\sigma}-1}}\right)$ in line with the trade literature (see Section 2). This measure of productivity is inflated with uncertainty and, thanks to our identification procedure, can now be compared to *uncertainty-excluded sales as a proxy* for productivity $\left(\left(x_{bt}^{ii}\right)^{\frac{1}{\hat{\sigma}-1}} e^{-\frac{\epsilon_{bt}^{T}}{\hat{\sigma}-1}}\right)$.

We display the nonparametric kernel densities of all four measures of heterogeneity for the year 2006 in Figure 1.[21] We observe that accounting for uncertainty affects the shapes of the density distributions, seemingly resulting in a less dispersed distribution. The exact influence is difficult to gauge from this Figure, however. Therefore, we report the variance and ratios of the 75-25, 90-10, 95-5, and 99-1 quantiles of the respective heterogeneity variables in Table 2. The reported variance in this Table increases by ±7% (for sales as a proxy) or ±53% (for estimated productivity) when productivity is conflated with uncertainty. This confirms the numbers reported by Kasahara and Lapham (2013) on the importance of uncertainty when measuring heterogeneity. Moreover, the overdispersion induced by these residuals grows towards the tails of the distribution. There are larger differences in the tail ratio (the 99-1 quantile) than the 75-25 ratio.

Table 2: Variance and quantile ratios for sales as a proxy for productivity, uncertainty-excluded sales as a proxy for productivity, uncertainty-augmented productivity, and productivity in the year 2006.

Variable	Variance	75/25	90/10	95/5	99/1
Sales as proxy $\left(\left(x_{bt}^{ii}\right)^{\frac{1}{\hat{\sigma}-1}}\right)$	0.224	1.852	2.949	3.848	5.905
Uncertainty-excluded Sales as proxy $\left(\left(x_{bt}^{ii}\right)^{\frac{1}{\hat{\sigma}-1}} e^{-\frac{\epsilon_{bt}^{T}}{\hat{\sigma}-1}}\right)$	0.209	1.766	2.813	3.633	5.499
Uncertainty-included Productivity $\left(e^{\hat{\omega}_{bt}+\frac{\epsilon_{bt}^{T}}{\hat{\sigma}-1}}\right)$	0.101	1.495	2.093	2.566	4.158
Productivity $\left(e^{\hat{\omega}_{bt}}\right)$	0.066	1.415	1.865	2.248	3.538

Notes: Values obtained from sample of 34,339 French firms in 2006.

Overall, our theoretically underpinned identification strategy provides sensible parameter es-

[21] Notice that using sales as a proxy for productivity results in a density with a heavier right tail relative to estimated productivity. This difference can be attributed to the differing underlying assumptions. Whereas firm-level sales relies on profit maximization to assume away firm-level heterogeneous input expenditures, the revenue production explicitly controls for these expenditures. Therefore, if, for instance, firm-level distortions are present (Hsieh and Klenow, 2009), the observed input expenditures will deviate from those expected under profit maximization without taking distortions into account. Additionally, sources of heterogeneity can originate from a wrong production function specification or firm-level heterogeneity in the demand function (for instance markups) that affects the firm-level sales specification differently than the firm-level revenue production specification.

timates and the estimation results confirm the first part of our premise: conflating uncertainty with productivity results in an overdispersed distribution of measured productivity.

5 Aggregate implications

Having established that transitory idiosyncratic uncertainty exists, is sizable and results in overdispersed productivity, we want to evaluate its influence on aggregate trade statistics such as the trade elasticity and GFT. The existence of uncertainty can bias aggregate trade statistics through two channels: (i) mismeasurement of firm-level productivity, and (ii) theoretical model misspecification. This section demonstrates that, under certain assumptions regarding the distribution of uncertainty, modeled aggregate trade statistics are equivalent to those obtained in prevalent static heterogeneous firms models (Melitz, 2003) that do not feature uncertainty. Thus, the influence of transitory uncertainty on currently used aggregate trade statistics can be ascribed to firm-level productivity mismeasurement rather than model misspecification: conditional on a unbiased measurement of firm-level productivity, aggregate statistics obtained from static heterogeneous firms models (Melitz, 2003) are not affected by the existence of transitory uncertainty. We conclude by quantifying the bias in the aggregate trade elasticity and subsequent GFT that results from the in the previous section established bias in firm-level productivity measurement.

5.1 Aggregating with certainty in future productivity

To allow comparison with the current literature, we reduce our presented framework (see Section 3) to a static productivity specification: $G(\omega_{bt+1}|\mathcal{I}_t)$ is such that $\omega_{bt+1} = \omega_{bt} = \omega_b$ (Melitz, 2003), while still allowing for uncertainty in realized supply and demand.[22] Eliminating the dynamics as such results in a clear analytical expressions for the equilibrium variables. Moreover, it allows us to demonstrate the influence of transitory uncertainty on the trade elasticity and GFT compared to the predominant Melitz (2003)-model (see for instance Head et al. (2014); Melitz and Redding (2015); Nigai (2017); Bee and Schiavo (2018)) in a straightforward manner.

In a static setting, the productivity cutoffs for serving each market (ω^{ij*}) are determined by the expected zero-profit conditions and by the free entry condition, requiring the probability of successful entry times the expected future value of entry conditional upon successful entry to equal the sunk entry cost:

$$0 = \mathbb{E}_{\omega,\varepsilon^T}\left[\pi^{ij}(\omega^{ij*})|\mathcal{I}_b\right] \tag{9}$$

$$f^{ie} = \left[1 - G(\omega^{ii*})\right]\mathbb{E}_{\omega,\varepsilon^T}\left[\sum_{j=1}^{j}\pi^{ij}(\omega_b)\right]. \tag{10}$$

With the productivity cutoffs determined, we can sum equation 6 across all active firms to

[22]For a discussion on the implications of productivity dynamics on the economy, see Impullitti et al. (2013); Alessandria and Choi (2014); Ruhl and Willis (2017).

obtain an expression for aggregate trade between country i and j:

$$\begin{aligned} x^{ij} &= \frac{M^{ij}}{1-G(\omega^{ij*})}\int_{\omega^{ij*}}^{\infty}\int_{-\infty}^{\infty} x_b^{ij}\,dG(\omega_b)dG(\varepsilon_b^T) \\ &= \frac{M^{ij}}{1-G(\omega^{ij*})}\left(\frac{\sigma}{\sigma-1}\tau^{ij}W^i\right)^{1-\sigma} Y^j\left(P^j\right)^{\sigma}\mathbb{E}_{\varepsilon^T}\left[e^{\varepsilon_b^T}\right]^{\sigma}\int_{\omega^{ij*}}^{\infty} e^{(\sigma-1)\omega_b}dG(\omega_b). \end{aligned} \tag{11}$$

From this aggregate revenue expression, we observe that transitory uncertainty-induced firm-level heterogeneity aggregates up to a constant $\mathbb{E}_{\varepsilon^T}\left[e^{\varepsilon_b^T}\right]$.

The partial sensitivity of aggregate trade to changes in variable trade costs, the aggregate trade elasticity, can then be defined as (Chaney, 2008; Arkolakis et al., 2012; Melitz and Redding, 2014; Bas et al., 2017):[23]

$$\begin{aligned} \gamma^{ij} \equiv \left.\frac{\partial lnX^{ij}}{\partial ln\tau^{ij}}\right|_{\omega^{ii*}} &= 1-\sigma-\frac{e^{\sigma\omega^{ij*}}g(\omega^{ij*})}{\int_{\omega^{ij*}}^{\infty}e^{(\sigma-1)\omega_b}dG(\omega_b)} \\ &= \underbrace{1-\sigma}_{\text{intensive margin}} - \underbrace{\frac{e^{(\sigma-1)\omega^{ij*}}}{\int_{\omega^{ij*}}^{\infty}e^{(\sigma-1)\omega_b}dG(\omega_b)}}_{\text{weights}} \times \underbrace{\frac{d\ln M^{ij}}{dln\tau^{ij}}}_{\text{extensive margin}}, \end{aligned} \tag{12}$$

where $\frac{dlnM^{ij}}{dln\tau^{ij}} = \frac{e^{\omega^{ij*}}g\left(\omega^{ij*}\right)}{1-G(\omega^{ij*})}$. It can be observed that, if productivity ω_b is unbiased, the trade elasticity is independent of transitory idiosyncratic uncertainty.

Similarly, the changes in welfare from a change in variable trade costs ($\boldsymbol{\tau} \rightarrow \boldsymbol{\tau}'$) can, upon choosing the composite wage as the numeraire $W^i = 1$, be written as a ratio of the aggregate price indices. If expectations remain constant over time, non-persistent heterogeneity has no influence on the outcome:[24]

$$\frac{(\mathbb{W}^i)'}{\mathbb{W}^i} = \frac{P^i}{(P^i)'}. \tag{13}$$

Overall, the intuition for the role of non-persistent idiosyncratic uncertainty in a model with certainty in future productivity is as follows. As shocks to supply and demand are assumed i.i.d., the expectation of these shocks reduce to a constant. As a result, all aggregate variables in the model are determined up to a constant. A comparison of the modeled impact of exogenous developments (that do not change the nature of this idiosyncratic uncertainty) cancel out and will, conditional on an unbiased measurement of productivity, be equal to the impact from a model without idiosyncratic uncertainty.

[23] Aggregate trade elasticity is here defined as the direct response of aggregate trade to variable trade costs, keeping the indirect effect trough the price index via its impact on the domestic cutoff fixed (Melitz and Redding, 2015).

[24] This reasoning mimics the logic related to the need to identify the productivity distribution up to a constant only (Bee and Schiavo, 2018).

5.2 Aggregate trade elasticity

Quantifying the aggregate trade elasticity (eq. 12) requires a value for the elasticity of substitution and a fitted parametric distribution function. We assume the previously obtained value for the elasticity of substitution of 4.59 and fit a Lognormal distribution to four specifications of firm-level productivity: sales as a proxy for productivity $\left(\left(x_{bt}^{ii}\right)^{\frac{1}{\hat{\sigma}-1}}\right)$, transitory uncertainty-excluded sales as a proxy for productivity $\left(\left(x_{bt}^{ii}\right)^{\frac{1}{\hat{\sigma}-1}} e^{-\frac{\epsilon_{bt}^{T}}{\hat{\sigma}-1}}\right)$, transitory uncertainty-included productivity $\left(e^{\hat{\omega}_{bt}+\frac{\epsilon_{bt}^{T}}{\hat{\sigma}-1}}\right)$ and estimated productivity $\left(e^{\hat{\omega}_{bt}}\right)$. We plot the resulting trade elasticities γ^{ij} in function of the probabilities of being active in market j: $1 - G(\omega^{ij*})$ in Figure 2. Two conclusions can be drawn from this Figure.

First, for a given ease of market access, heterogeneity variables augmented with uncertainty (that is, sales as a proxy and uncertainty-included productivity) result in trade elasticities that are lower in absolute value relative to their uncertainty-free counterparts. This phenomenon can be ascribed to the larger variance of heterogeneity variables augmented with uncertainty (see Table 2). The larger the variance of a variable used to calculate the trade elasticity, the smaller the extensive margin reaction, the addition of a marginal exporter, relative to the intensive margin reaction for a given change in trade costs. Neglecting the existence of uncertainty therefore results in underestimated trade elasticities (in absolute values).

Moreover, the bias in calculating trade elasticities increases as the ease of market access decreases. At a survival probability of 0.25 for a domestic firm in a foreign market, for instance, the distribution of uncertainty-included productivity results in an absolute trade elasticity about 11.6% (6.13/5.49) lower than when calculated based on the productivity distribution. This difference increases to an approximate 13.8% difference at a survival probability of 0.1. Such results are in line with results reported in Table 2: heterogeneity differences between variables augmented with and purged from uncertainty are larger in the tail of the distribution. Trade elasticities obtained from uncertainty-free productivity measures therefore diverge, as market access becomes increasingly restrictive, from the intensive margin of trade $(1 - \hat{\sigma})$ increasingly faster than their uncertainty-augmented counterparts.

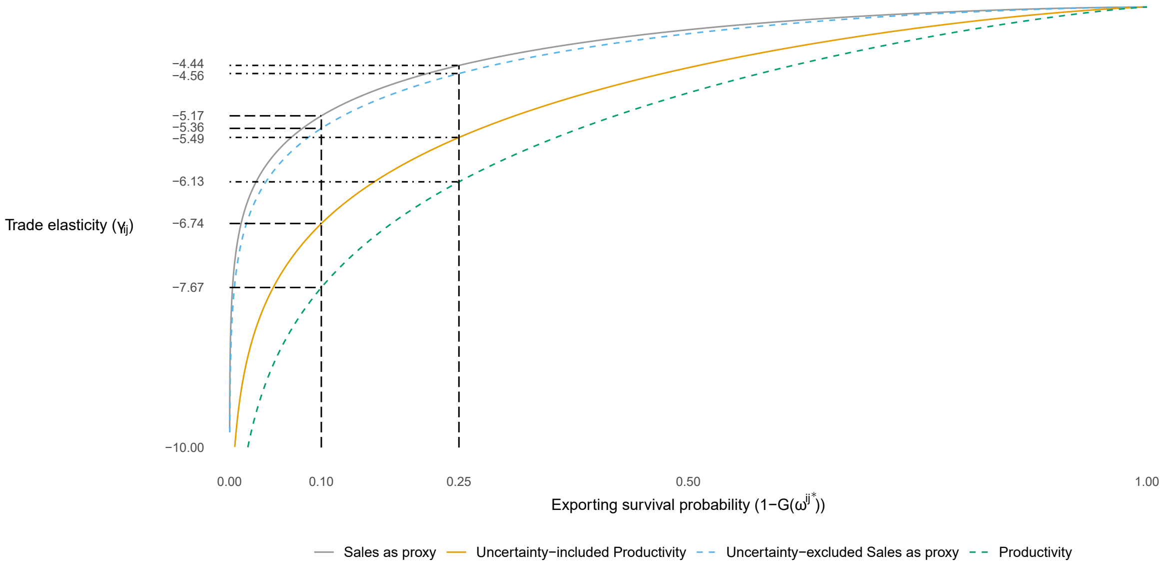

Figure 2: Trade elasticities for sales as a proxy for productivity, uncertainty-excluded sales as a proxy for productivity, uncertainty-included productivity and productivity in the year 2006.
Note: Trade elasticities calculated assuming a fitted Lognormal distribution function and an elasticity of substitution of 4.59.

5.3 Gains From Trade

We extend the results found for the (partial) trade elasticity in the previous subsection to the full trade elasticity and the accompanying welfare implications in this subsection. For simplicity, we rely on a stylized two-country symmetric heterogeneous firms model. This allows us to perform a GFT exercise in line with the current literature investigating the importance of distributional assumptions on GFT (Head et al., 2014; Melitz and Redding, 2015; Bee and Schiavo, 2018) and investigate the influence of our alternative estimation procedure on welfare predictions.

The parameterization of our model is standard (Head et al., 2014; Melitz and Redding, 2015; Bee and Schiavo, 2018). We work with two symmetric countries i and j and choose labor in one country as the numeraire, so that $W^i = W^j = 1$. We choose fixed entry costs $f^e = 0.545$ and set fixed costs equal to 1 ($f^{ii} = f^{ij} = 1$). Domestic variable trade costs also equal one ($\tau^{ii} = 1$). The value of the elasticity of substitution remains at the value of 4.59, $\hat{\sigma} = 4.59$,[25] and we continue to parameterize all heterogeneity variables assuming a Lognormal distribution for productivity.

We calculate the percentage changes in welfare from a reduction in variable trade costs relative to autarky ($\tau^{ij} = 10 \rightarrow \tau^{ij\prime} = 1.96, 1.75, 1.5, 1.25, 1$). In perspective, a variable trade cost calibrated to match the average fraction of exports in firm sales in French manufacturing of 0.08, amounts to 1.96.[26] The resulting percentage GFT are displayed in Table 3. We can immediately

[25]Note that this value for the elasticity of substitution will results in low GFT estimates relative to studies that rely on the typical elasticity of substitution with a value of 4 (Melitz and Redding, 2015; Bee and Schiavo, 2018).

[26]In a two-country symmetric heterogeneous firms model with $\tau^{ii} = 1$, we have that $\frac{X^{ij}}{X^{ii}+X^{ij}} = \frac{\tau^{1-\sigma}}{1+\tau^{1-\sigma}}$ (Melitz and Redding, 2015).

observe that the ranking in terms of heterogeneity and in terms of trade elasticities is preserved for GFT. Conflating productivity with firm-level noise results in overestimated GFT. GFT obtained using estimated productivity amounts to 10.8%, about 0.92 percentage points (or 7.9%) lower than the GFT predicted from uncertainty-augmented productivity.[27] Moreover, these welfare gains from increased market access are realized faster for uncertainty-included productivity. GFT estimates are overestimated by 23.7% for a reduction in variable trade costs to the calibrated value of 1.25 and by 54.2% for a reduction in variable trade costs to 1.5 for uncertainty-included productivity. This is in line with results reported for the partial trade elasticity, which approaches its intensive margin faster, as market access improves, for transitory uncertainty-included productivity than pure estimated productivity. A similar picture emerges for sales as a proxy for productivity.

Table 3: Percentage welfare gains from a reduction in variable trade costs relative to Autarky ($\tau = 10 \rightarrow \tau'$)

Variable	$\tau'_{cal} = 1.96$	$\tau' = 1.75$	$\tau' = 1.5$	$\tau' = 1.25$	$\tau' = 1$
Sales as proxy $\left(\left(x_{bt}^{ii}\right)^{\frac{1}{\hat{\sigma}-1}}\right)$	0.295	0.665	1.810	5.048	14.059
Uncertainty-excluded sales as proxy $\left(\left(x_{bt}^{ii}\right)^{\frac{1}{\hat{\sigma}-1}} e^{-\frac{\epsilon_{bt}^{T}}{\hat{\sigma}-1}}\right)$	0.246	0.578	1.649	4.784	13.710
Uncertainty-included productivity $\left(e^{\hat{\omega}_{bt}+\frac{\epsilon_{bt}^{T}}{\hat{\sigma}-1}}\right)$	0.062	0.205	0.846	3.328	11.722
Productivity $\left(e^{\hat{\omega}_{bt}}\right)$	0.024	0.101	0.549	2.685	10.797

6 Robustness

It is possible that the size of uncertainty is affected by our modeling choices. Specifically, firm-level noise can be affected by a misspecified production function or data cleaning.[28] Additionally, it is possible that the heterogeneity in productivity is insufficiently captured assuming a Lognormal distribution. We attenuate these concerns with the following three robustness tests.

Firstly, we evaluate the importance of uncertainty assuming a Translog production function. Results, available in Figures 1 and 3 as well as Tables 2, 4, and 6, reveal that assuming a Translog production function rather than a Cobb-Douglas specification does not alter the main results. Secondly, we re-evaluate our analysis with an uncleaned data sample, to ensure the results are not influenced by data cleaning. The results are displayed in Figures 2 and 4 as well as Tables 3, 5, and 7. The analysis on the uncleaned data sample results in a relatively high estimated elasticity of subsitution, probably due to the presence of relatively large firms in the uncleaned data sample that might exert market power, but does not alter our conclusions

[27] A comparison in percentage rather than absolute differences is preferred due to the stylized model this calibration exercise relies on. Absolute differences are likely more sensitive to model specification and parametrization. See Costinot and Rodríguez-Clare (2014) for a discussion on the sensitivity of GFT on model specifications.

[28] Additionally, firm-level noise can also increase if our demand function is misspecified. As this assumption of a CES demand function is shared with current practices of measuring aggregate trade elasticities (see Section 2), we do not attempt to control for a possible misspecification of demand.

regarding the importance of uncertainty. Lastly, it is possible the heterogeneity in productivity is insufficiently captured assuming a Lognormal distribution. We therefore also calculate aggregate trade elasticities and GFT assuming a Weibull and Gamma distribution (see Figure 5 and Table 8). Again, our main results stand: not controlling for transitory uncertainty results in an underestimation of aggregate trade elasticities (in absolute values) and an overestimation of GFT.

7 Conclusion

This paper identifies and evaluates firm-level uncertainty as a source of bias in the measurement of firm-level productivity and the aggregate trade elasticities and Gains From Trade (GFT) that are derived from it. If productivity is conflated with transitory idiosyncratic uncertainty, we obtain an overdispersed distribution of measured productivity, underestimated aggregate trade elasticities (in absolute values) and overestimated GFT. In light of this uncertainty, prevalent methods to identify aggregate trade elasticities result in biased measurements. We propose a theoretically underpinned alternative that estimates both components of the aggregate trade elasticity (elasticity of substitution and productivity distribution parameters) from a revenue production function. An empirical application to French firm-level data proves our identified source of mismeasurement to be economically relevant.

Our work further highlights the possibilities and advantages of relying on the revenue production function to calculate aggregate trade elasticities and resulting Gains From Trade. Estimating a revenue production function allows us to exploit the panel dimension of firm-level data from a single country to identify both the productivity distribution and elasticity of substitution while controlling for idiosyncratic uncertainty. Moreover, data to estimate such a production function is easily accessible for multiple countries through, for instance, the Orbis database.

The theoretical elaborations in this paper demonstrate that, under relatively light assumptions on the distribution of uncertainty, the existence of firm-level uncertainty is not problematic for prevalent static firm-level heterogeneous models that do not feature this uncertainty. But, when quantifying these models, one does need to ensure firm-level uncertainty is controlled for in order to obtain unbiased model inputs.

The results in this paper also point to future elaborations on the influence of firm-level uncertainty on aggregate trade statics. Whereas we demonstrate the role of *transitory* uncertainty in a static framework to allow comparison with current literature, an extension to a dynamic framework would allow to investigate the influence of *persistent* uncertainty in productivity on aggregate trade outcomes. Further research could also focus on the specification of uncertainty. One could, for instance, extend the current definition of uncertainty to allow for product-market specific deviations. Bas et al. (2017), for instance, specify their demand-side deviation as a firm-market specific cost to reach a market originating from, among others, differences in internal knowledge on how to reach consumers in that market. Sager and Timoshenko (2019, 2020) argue that the demand-side deviation represents, observed or unobserved, variety-specific demand that firms need to learn over time through market participation. The current specification captures the firm-level average of such market-specific shocks. Lastly, data limitations prohibit us to differentiate between supply and demand shocks and does not allow for firm-level markups. All these possibilities set out interesting research paths for expanding the current methodology to continuously improve our measurements and understanding of firm-level trade elasticities using firm-level data.

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Online Appendix to "Gains From Trade: Demand, Supply and Idiosyncratic Uncertainty"

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Appendix A Additional Figures and Tables

A.1 Figures

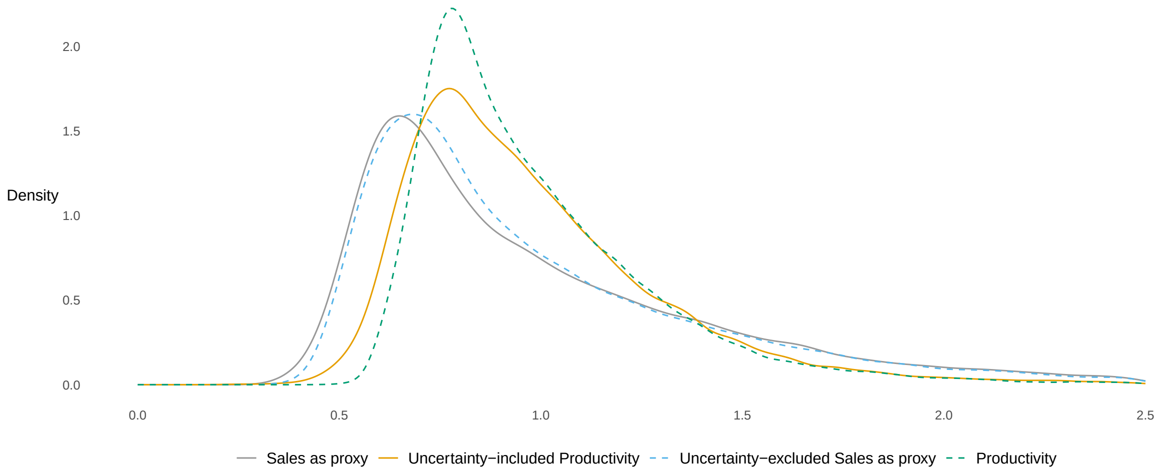

Figure 1: Nonparametric kernel density of domestic sales as a proxy for productivity, uncertainty-excluded sales as a proxy for productivity, uncertainty-included productivity and productivity in the year 2006 obtained from assuming a Translog production function.
Note: All variables are demeaned.

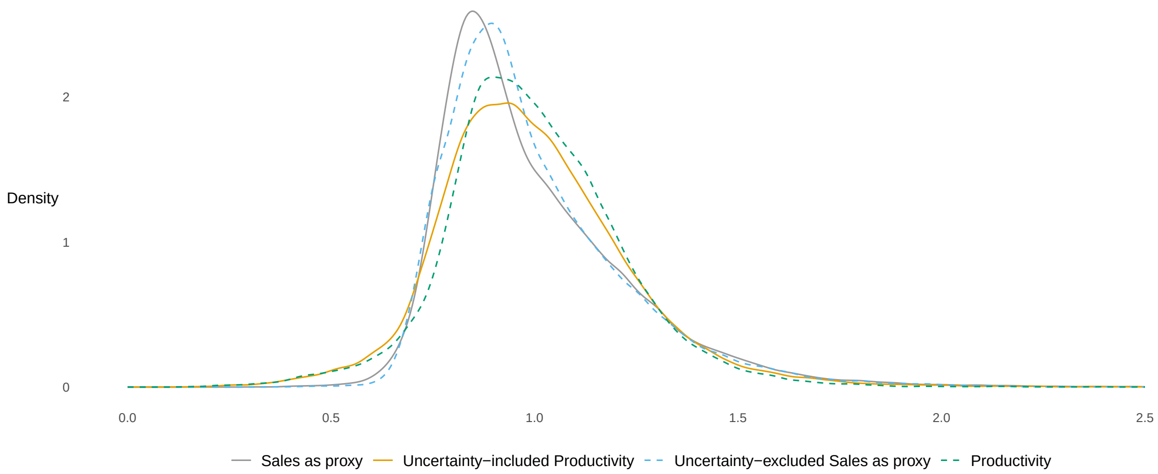

Figure 2: Nonparametric kernel density of domestic sales as a proxy for productivity, (transitory) uncertainty-excluded sales as a proxy for productivity, (transitory) uncertainty-included productivity and productivity in the year 2006 obtained from an uncleaned data sample.
Note: All variables are demeaned.

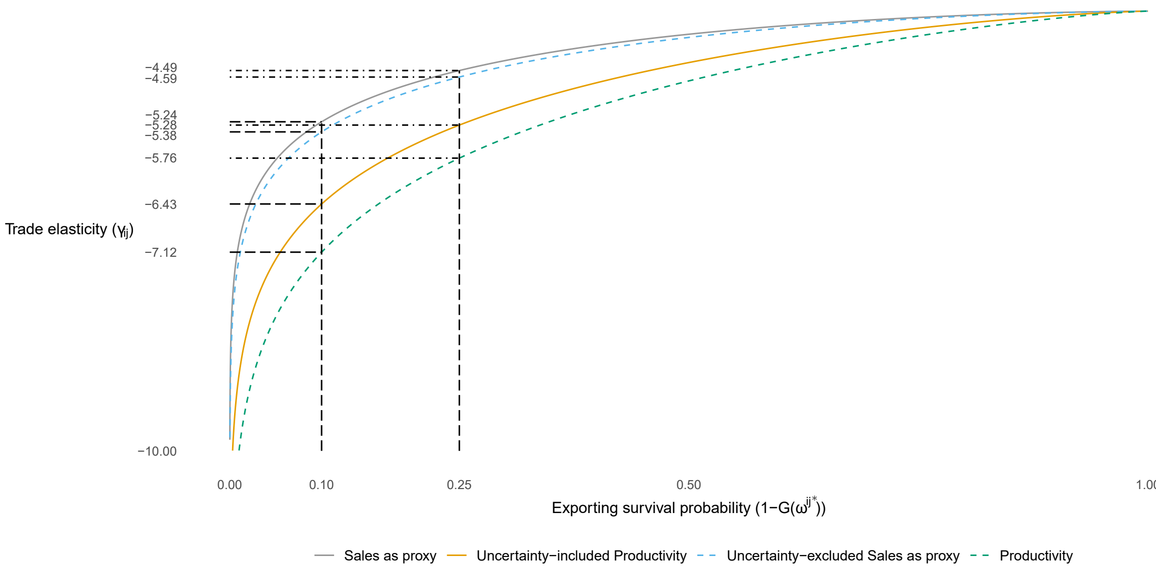

Figure 3: Trade elasticities for sales as a proxy for productivity, uncertainty-excluded sales as a proxy for productivity, uncertainty-included productivity and productivity in the year 2006 obtained from assuming a Translog production function.

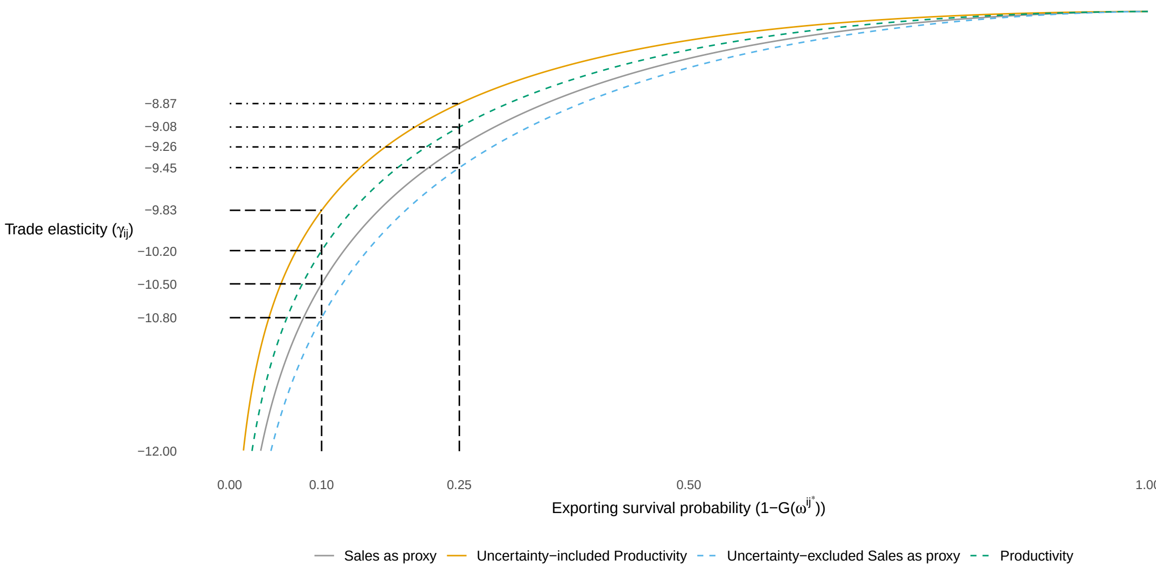

Figure 4: Trade elasticities for sales as a proxy for productivity, transitory uncertainty-excluded sales as a proxy for productivity, transitory uncertainty-included productivity and productivity in the year 2006 obtained from an uncleaned data sample.

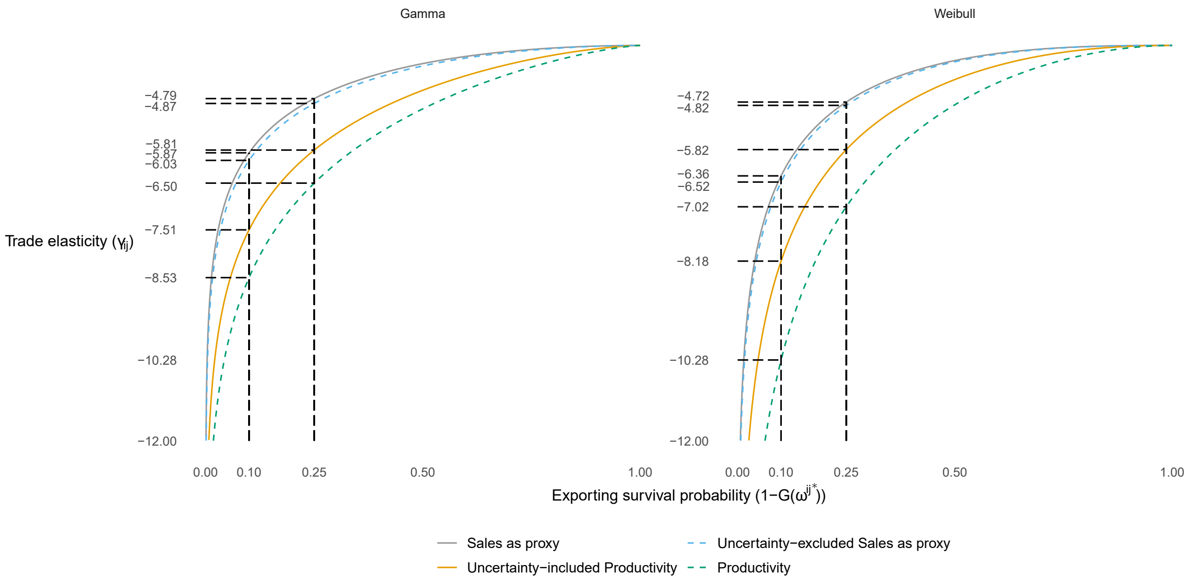

Figure 5: Trade elasticities for sales as a proxy for productivity, transitory uncertainty-excluded sales as a proxy for productivity, transitory uncertainty-included productivity and productivity in the year 2006 for different distributional assumptions.

A.2 Tables

Table 1: Summary statistics

Variable (in logs)	Obs.	Nr. Firms	mean	sd	min	max
Sales	379765	86959	14.13	1.46	10.57	19.72
Domestic sales^{a}	379765	86959	14.02	1.40	10.56	18.67
Value added^{b}	379765	86959	13.66	1.39	4.35	19.64
Domestic value added^{a,b}	379765	86959	13.54	1.33	4.35	18.50
Capital	379765	86959	11.37	1.85	6.91	17.09
Labor	379765	86959	2.50	1.21	0.69	6.46
Materials	379765	86959	12.85	1.82	7.41	18.40
Aggregate Domestic sales^{c}	379765	86959	16.19	0.14	15.95	16.42

Notes: All financial variables are displayed in real terms. ^{a}Domestic variables are obtained as the difference between the total and the exporting share of those variables. ^{b}Value added is obtained as the difference between sales and material inputs. ^{c}Aggregate variables are constructed as the market-share weighted sum of the underlying variables.

Table 2: Translog production function estimation results.

	Capital	Labor	Returns to Scale	Elasticity of Substitution
NLLS	0.183	1.051	1.233	5.836
	(0.002)	(0.007)	(0.007)	(0.169)
ACF	0.181	1.027	1.208	4.633
	(0.002)	(0.007)	(0.007)	(0.109)

Notes: Standard errors displayed between brackets are obtained from wild bootstrap clustered at the firm level with 99 repliations. Estimates obtained from French firm-level database over the years 1998–2006 with 379,765 observations from 86,959 firms.

Table 3: Production function estimation results from an uncleaned data sample.

	Capital	Labor	Returns to Scale	Elasticity of Substitution
NLLS	0.187	0.969	1.156	7.364
	(0.002)	(0.006)	(0.006)	(0.254)
ACF	0.174	0.923	1.097	9.046
	(0.002)	(0.004)	(0.005)	(0.324)

Notes: Standard errors displayed between brackets are obtained from wild bootstrap clustered at the firm level with 99 repliations. Estimates obtained from French firm-level database over the years 1998–2006 with 474,044 observations from 99,465 firms.

Table 4: Variance and quantile ratios for sales as a proxy for productivity, uncertainty-excluded sales as a proxy for productivity, uncertainty-augmented productivity, and productivity in the year 2006 obtained from assuming a Translog production function.

Variable	Variance	75/25	90/10	95/5	99/1
Sales as proxy $\left((x_{bt}^{ii})^{\frac{1}{\hat{\sigma}-1}}\right)$	0.218	1.838	2.911	3.786	5.780
Uncertainty-excluded Sales as proxy $\left((x_{bt}^{ii})^{\frac{1}{\hat{\sigma}-1}} e^{-\frac{\epsilon_{bt}^{T}}{\hat{\sigma}-1}}\right)$	0.208	1.767	2.811	3.627	5.410
Uncertainty-included Productivity $\left(e^{\hat{\omega}_{bt}+\frac{\epsilon_{bt}^{T}}{\hat{\sigma}-1}}\right)$	0.161	1.505	2.135	2.656	4.598
Productivity $\left(e^{\hat{\omega}_{bt}}\right)$	0.122	1.447	1.953	2.377	3.653

Notes: Values obtained from sample of 34,339 French firms in 2006.

Table 5: Variance and quantile ratios for sales as a proxy for productivity, uncertainty-excluded sales as a proxy for productivity, uncertainty-augmented productivity, and productivity in the year 2006 from an uncleaned data sample.

Variable	Variance	75/25	90/10	95/5	99/1
Sales as proxy $\left((x_{bt}^{ii})^{\frac{1}{\hat{\sigma}-1}}\right)$	0.056	1.337	1.700	1.978	2.743
Uncertainty-excluded Sales as proxy $\left((x_{bt}^{ii})^{\frac{1}{\hat{\sigma}-1}} e^{-\frac{\epsilon_{bt}^{T}}{\hat{\sigma}-1}}\right)$	0.051	1.312	1.688	1.953	2.580
Uncertainty-included Productivity $\left(e^{\hat{\omega}_{bt}+\frac{\epsilon_{bt}^{T}}{\hat{\sigma}-1}}\right)$	0.059	1.333	1.727	2.088	3.567
Productivity $\left(e^{\hat{\omega}_{bt}}\right)$	0.046	1.301	1.646	1.993	3.402

Notes: Values obtained from sample of 34,339 French firms in 2006.

Table 6: Percentage welfare gains from a reduction in variable trade costs relative to Autarky ($\tau = 10 \to \tau'$) for data obtained from assuming a Translog production function.

Variable	$\tau'_{cal} = 1.95$	$\tau' = 1.75$	$\tau' = 1.5$	$\tau'_{cal} = 1.25$	$\tau' = 1$
Sales as proxy $\left((x_{bt}^{ii})^{\frac{1}{\hat{\sigma}-1}}\right)$	0.285	0.627	1.735	4.917	13.879
Uncertainty-excluded sales as proxy $\left((x_{bt}^{ii})^{\frac{1}{\hat{\sigma}-1}} e^{-\frac{\epsilon_{bt}^{T}}{\hat{\sigma}-1}}\right)$	0.247	0.560	1.611	4.710	13.605
Uncertainty-included productivity $\left(e^{\hat{\omega}_{bt}+\frac{\epsilon_{bt}^{T}}{\hat{\sigma}-1}}\right)$	0.088	0.255	0.967	3.559	12.038
Productivity $\left(e^{\hat{\omega}_{bt}}\right)$	0.044	0.150	0.696	3.007	11.256

Table 7: Percentage welfare gains from a reduction in variable trade costs relative to Autarky ($\tau = 10 \to \tau'$) obtained from an uncleaned data sample.

Variable	$\tau' = 2$	$\tau' = 1.75$	$\tau' = 1.5$	$\tau'_{cal} = 1.35$	$\tau' = 1$
Sales as proxy $\left((x_{bt}^{ii})^{\frac{1}{\hat{\sigma}-1}}\right)$	0.000	0.003	0.043	0.215	6.499
Uncertainty-excluded sales as proxy $\left((x_{bt}^{ii})^{\frac{1}{\hat{\sigma}-1}} e^{-\frac{\epsilon_{bt}^{T}}{\hat{\sigma}-1}}\right)$	0.000	0.002	0.035	0.187	6.362
Uncertainty-included productivity $\left(e^{\hat{\omega}_{bt}+\frac{\epsilon_{bt}^{T}}{\hat{\sigma}-1}}\right)$	0.001	0.006	0.068	0.292	6.838
Productivity $\left(e^{\hat{\omega}_{bt}}\right)$	0.000	0.004	0.053	0.246	6.645

Table 8: Percentage Gains from a reduction in variable trade costs relative to Autarky ($\tau = 10 \rightarrow \tau'$) for different distributional forms.

Distribution	Variable	$\tau'_{cal} = 1.96$	$\tau' = 1.75$	$\tau' = 1.5$	$\tau' = 1.25$	$\tau' = 1$
Gamma	Sales as proxy $\left(\left(x_{bt}^{ii}\right)^{\frac{1}{\hat{\sigma}-1}}\right)$	0.111	0.338	1.226	4.160	12.945
Gamma	Uncertainty-excluded sales as proxy $\left(\left(x_{bt}^{ii}\right)^{\frac{1}{\hat{\sigma}-1}} e^{-\frac{\epsilon_{bt}^{T}}{\hat{\sigma}-1}}\right)$	0.094	0.300	1.138	3.998	12.724
Gamma	Productivity $\left(e^{\hat{\omega}_{bt}}\right)$	0.006	0.042	0.362	2.303	10.302
Gamma	Uncertainty-included productivity $\left(e^{\hat{\omega}_{bt}+\frac{\epsilon_{bt}^{T}}{\hat{\sigma}-1}}\right)$	0.019	0.096	0.577	2.849	11.110
Weibull	Sales as proxy $\left(\left(x_{bt}^{ii}\right)^{\frac{1}{\hat{\sigma}-1}}\right)$	0.043	0.193	0.945	3.765	12.494
Weibull	Uncertainty-excluded sales as proxy $\left(\left(x_{bt}^{ii}\right)^{\frac{1}{\hat{\sigma}-1}} e^{-\frac{\epsilon_{bt}^{T}}{\hat{\sigma}-1}}\right)$	0.036	0.172	0.890	3.662	12.356
Weibull	Productivity $\left(e^{\hat{\omega}_{bt}}\right)$	0.000	0.004	0.152	1.865	9.828
Weibull	Uncertainty-included productivity $\left(e^{\hat{\omega}_{bt}+\frac{\epsilon_{bt}^{T}}{\hat{\sigma}-1}}\right)$	0.003	0.037	0.415	2.650	10.964

Appendix B Heterogeneous firms model

We specify an open economy model with persistent firm-level uncertainty in initial and future productivity as well as non-persistent firm-level uncertainty in the production process originating from demand and supply. The core elements are Melitz (2003) augmented with the stochastic evolution of firm productivity as in Hopenhayn (1992), while the introduction of uncertainty surrounding realized demand and supply is embedded in the structural estimation literature (see for instance Das et al. (2007); De Loecker (2011); Kasahara and Lapham (2013); Gandhi et al. (2020)). This will provide us with a general framework to evaluate empirical methods that deduce firm-level heterogeneity from sales data.

In a second stage, we reduce the productivity dynamics to certainty in future productivity (Melitz, 2003) while featuring uncertainty in realized supply and demand.[1] Eliminating the dynamics as such has the advantage of resulting in clear analytical expressions for the equilibrium variables. Moreover, it allows us to easily demonstrate the influence of transitory uncertainty on the trade elasticity and GFT compared to the predominant Melitz (2003)-model (see, for instance Head et al. (2014); Melitz and Redding (2015); Nigai (2017); Bee and Schiavo (2018)).

B.1 Setup

Demand Consumer preferences in country $j \in J$ are defined over a continuum of horizontally differentiated varieties originating from country $i \in I$ ($\varpi \in \Omega^i$) and are assumed to take the Constant Elasticity of Substitution (CES) utility (U) form at time t,

$$U_t^j = \left(\sum_{i=1}^{I} \int_{\varpi \in \Omega^i} e^{\frac{1}{\sigma}\nu_t(\varpi)} y_t^{ij}(\varpi)^{\frac{\sigma-1}{\sigma}} \, d\varpi \right)^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

with σ the elasticity of substitution between varieties and $e^{\nu_t(\varpi)}$ a variety-specific demand shock independently and identically distributed across varieties and time. Let the aggregate expenditure in country j be R^j, and the price of a good p_t^{ij}, then the utility maximixation problem is

$$\begin{aligned} \max_{y_t^{ij}(\varpi)} U_t^j = \left(\sum_{i=1}^{I} \int_{\varpi \in \Omega^i} e^{\frac{1}{\sigma}\nu_t(\varpi)} y_t^{ij}(\varpi)^{\frac{\sigma-1}{\sigma}} \, d\varpi \right)^{\frac{\sigma}{\sigma-1}} \\ s.t. \quad \sum_{i=1}^{I} \int_{\varpi \in \Omega^i} p_t^{ij}(\varpi) y_t^{ij}(\varpi) d\varpi \leq R^j. \end{aligned} \tag{2}$$

The Lagrangian is:

[1] For a discussion on the implications of productivity dynamics on the economy, see Impullitti et al. (2013); Ruhl and Willis (2017).

$$\mathcal{L} = \left(\sum_{i=1}^{I} \int_{\varpi \in \Omega^i} e^{\frac{1}{\sigma}\nu_t(\varpi)} y_t^{ij}(\varpi)^{\frac{\sigma-1}{\sigma}} d\varpi \right)^{\frac{\sigma}{\sigma-1}} - \lambda \left(\sum_{i=1}^{I} \int_{\varpi \in \Omega^i} p_t^{ij}(\varpi) y_t^{ij}(\varpi) d\varpi - R^j \right), \tag{3}$$

and First Order Conditions (FOC) are:

1.

$$\begin{aligned} &\frac{\partial \mathcal{L}}{\partial y_t^{ij}(\varpi)} = 0 \\ &\Leftrightarrow \quad \left(\sum_{i=1}^{I} \int_{\varpi \in \Omega^i} e^{\frac{1}{\sigma}\nu_t(\varpi)} y_t^{ij}(\varpi)^{\frac{\sigma-1}{\sigma}} d\varpi \right)^{\frac{1}{\sigma-1}} e^{\frac{1}{\sigma}\nu_t(\varpi)} y_t^{ij}(\varpi)^{\frac{-1}{\sigma}} = \lambda p_t^{ij}(\varpi) \end{aligned} \tag{4}$$

2.

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \quad \Leftrightarrow \quad \sum_{i=1}^{I} \int_{\varpi \in \Omega^i} p_t^{ij}(\varpi) y_t^{ij}(\varpi) d\varpi = R^j \tag{5}$$

Exponentiating the first FOC by $(1-\sigma)$ and aggregating, we obtain

$$\begin{aligned} &\left(\sum_{i=1}^{I} \int_{\varpi \in \Omega^i} e^{\frac{1}{\sigma}\nu_t(\varpi)} y_t^{ij}(\varpi)^{\frac{\sigma-1}{\sigma}} d\varpi \right)^{-1} e^{\frac{1-\sigma}{\sigma}\nu_t(\varpi)} y_t^{ij}(\varpi)^{\frac{\sigma-1}{\sigma}} = \lambda^{1-\sigma} p_t^{ij}(\varpi)^{1-\sigma} \\ &\Leftrightarrow \\ &\left(\sum_{i=1}^{I} \int_{\varpi \in \Omega^i} e^{\frac{1}{\sigma}\nu_t(\varpi)} y_t^{ij}(\varpi)^{\frac{\sigma-1}{\sigma}} d\varpi \right)^{-1} \sum_{i=1}^{I} \int_{\varpi \in \Omega^i} e^{\frac{1}{\sigma}\nu_t(\varpi)} y_t^{ij}(\varpi)^{\frac{\sigma-1}{\sigma}} d\varpi = \lambda^{1-\sigma} \sum_{i=1}^{I} \int_{\varpi \in \Omega^i} e^{\nu_t(\varpi)} p_t^{ij}(\varpi)^{1-\sigma} d\varpi \\ &\Leftrightarrow \\ &(P_t^j)^{-1} = \lambda, \end{aligned} \tag{6}$$

where and P_t^j is the CES aggregate price index in country j at time t:

$$(P_t^j)^{1-\sigma} = \sum_{i=1}^{I} \int_{\varpi \in \Omega^i} e^{\nu_t(\varpi)} p_t^{ij}(\varpi)^{1-\sigma} d\varpi. \tag{7}$$

Plugging the expression for λ back in the FOC provides us with the optimal consumption and expenditure decisions over the individual varieties:

$$\frac{y_t^{ij}(\varpi)}{Y_t^j} = \left(\frac{p_t^{ij}(\varpi)}{P_t^j} \right)^{-\sigma} e^{\nu_t(\varpi)}, \tag{8}$$

where the set of varieties consumed is considered as an aggregate good $Y_t^j \equiv U_t^j$.

Supply There is a continuum of businesses ($b \in B$) which choose to supply a distinct horizontally-differentiated variety. They are heterogeneous in terms of their productivity $\omega_{bt} \in [0, \infty]$ drawn from the unconditional Cumulative Distribution Function (CDF) $G(\omega_{bt})$ after paying a fixed cost f_t^{ie} to enter the market.[2] The firm's productivity follows a Markov process follows a Markov process independent across firms with conditional distribution $G(\omega_{bt+1}|\mathcal{I}_{bt})$ such that[3]

$$e^{\omega_{bt+1}} = \mathbb{E}_{\omega}\left[e^{\omega_{bt+1}}|\mathcal{I}_{bt}\right] e^{\eta_{bt+1}}. \tag{9}$$

Productivity at time $t+1$ is thus a function of the information set of the firm at time t, $\mathcal{I}_{bt}$ with $\{\omega_{bt}\} \in \mathcal{I}_{bt}$, and a productivity shock $\eta_{bt}+1$ (see below for a summary of the timing assumptions of the model). The productivity distribution is known to firms and stochastically increasing in ω_{bt}.

Production relies a composite factor of production $\boldsymbol{A}_{bt}^{ij}(\boldsymbol{\beta})$ (Melitz and Redding, 2014) consisting of Z fixed (F) and V variable (L) factors of production respectively:
$\{F_{1t}, \ldots, F_{Zt}, L_{1t}, \ldots L_{Vt}\}$. Variable production factors can be adjusted every time period after the observation set $\mathcal{I}_{bt-1}$ is observed. Fixed production factors, on the other hand, take one time period to adjust.[4] These production factors are combined under Cobb-Douglas Constant Returns to Scale technology with respective factor intensities β_{iz}, β_{iv}:

$$y_{bt}^{ij} = q_{bt}^{ij} e^{\epsilon_{bt}} = \prod_{z=1}^{Z}\prod_{v=1}^{V} (F_{bzt}^{ij})^{\beta_z^i} (L_{bvt}^{ij})^{\beta_v^i} e^{\omega_{bt}+\epsilon_{bt}}, \qquad \sum_{z=1}^{Z} \beta_{iz} + \sum_{v=1}^{V} \beta_{iv} = 1, \tag{10}$$

subject to shocks to the production function $e^{\epsilon_{bt}}$, which are independent and identically distributed across firms and time. Supply of the production factors to the individual firm is perfectly elastic, so that firms are effectively price (W_{zt}^i, W_{vt}^i) takers on the input markets. Firms from country i have to pay a fixed cost f_t^{ij} to produce goods destined for country j at time t.[5]

The variable cost minimization problem of the firm is then:

$$\begin{aligned} \min_{F_{b1t}^{ij},\ldots F_{bZt}^{ij}, L_{b1t}^{ij},\ldots L_{bVt}^{ij}} \Gamma_t^i = \sum_{z=1}^{Z} W_{zt}^i F_{bzt}^{ij} + \sum_{v=1}^{V} W_{vt}^i F_{bvt}^{ij} & \\ s.t. \quad y_{bt}^{ij} = \prod_{z=1}^{Z}\prod_{v=1}^{V} (F_{bzt}^{ij})^{\beta_z^i} (L_{bvt}^{ij})^{\beta_v^i} e^{\omega_{bt}+\epsilon_{bt}}. & \end{aligned} \tag{11}$$

[2] We follow Asker et al. (2017) in differentiating all fixed costs from factors of production. "In their financial statements, firms report overhead costs as Selling, General and Administrative Expenses (SG&A). These expenses are not directly related to production, and include sales, advertising, marketing, executive compensation, ... and can in part be interpreted as expenses on intangible capital." (Asker et al., 2017, p. 4). We assume all fixed cost expenses are equally distributed within the source market.

[3] $\mathbb{E}_x[\ldots] = \int \ldots f(x)dx$

[4] For simplicity, we assume fixed production factors have no dynamic implications.

[5] Similar to the fixed entry costs, fixed production costs are due in monetary terms rather than in terms of production factors.

The corresponding Lagrangian is:

$$\mathcal{L} = \sum_{z=1}^{Z} W_{zt}^{i} F_{bzt}^{ij} + \sum_{v=1}^{V} W_{vt}^{i} F_{bvt}^{ij} + \lambda \left(y_{bt}^{ij} - \prod_{z=1}^{Z} \prod_{v=1}^{V} (F_{bzt}^{ij})^{\beta_z^i} (L_{bvt}^{ij})^{\beta_v^i} e^{\omega_{bt}+\epsilon_{bt}} \right), \tag{12}$$

with the following FOC

1.
$$\forall z = 1, \ldots, Z : \frac{\partial \mathcal{L}}{\partial F_{bzt}^{ij}} = 0 \Leftrightarrow W_{zt}^{i} = \lambda \beta_z^i \frac{y_{bt}^{ij}}{F_{bzt}}; \tag{13}$$

2.
$$\forall v = 1, \ldots, V : \frac{\partial \mathcal{L}}{\partial F_{bzt}^{ij}} = 0 \Leftrightarrow W_{vt}^{i} = \lambda \beta_v^i \frac{y_{bt}^{ij}}{L_{bvt}}; \tag{14}$$

3.
$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Leftrightarrow y_{bt}^{ij} = \prod_{z=1}^{Z} \prod_{v=1}^{V} (F_{bzt}^{ij})^{\beta_z^i} (L_{bvt}^{ij})^{\beta_v^i} e^{\omega_{bt}+\epsilon_{bt}}. \tag{15}$$

Solving for F_{bzt}^{ij} from FOC one and two:

$$\frac{W_{zt}^{i}}{W_{1t}^{i}} = \frac{\beta_z^i}{\beta_1^i} \frac{L_{b1t}^{ij}}{F_{bzt}^{ij}} \Leftrightarrow F_{bzt}^{ij} = \frac{\beta_z^i}{\beta_1^i} \frac{W_{1t}^{i}}{W_{zt}^{i}} L_{b1t}^{ij}, \tag{16}$$

and substituting in the production function, we obtain:

$$\begin{aligned}
y_{bt}^{ij} &= (L_{b1t}^{ij})^{\beta_1^i} \prod_{z=1}^{Z} \prod_{v=2}^{V} \left(\frac{\beta_z^i}{\beta_1^i} \frac{W_{1t}^{i}}{W_{zt}^{i}} L_{b1t}^{ij} \right)^{\beta_z^i} \left(\frac{\beta_v^i}{\beta_1^i} \frac{W_{1t}^{i}}{W_{vt}^{i}} L_{b1t}^{ij} \right)^{\beta_v^i} e^{\omega_{bt}+\epsilon_{bt}} \\
&= L_{b1t}^{ij} \prod_{z=1}^{Z} \prod_{v=2}^{V} \left(\frac{\beta_z^i}{\beta_1^i} \frac{W_{1t}^{i}}{W_{zt}^{i}} \right)^{\beta_z^i} \left(\frac{\beta_v^i}{\beta_1^i} \frac{W_{1t}^{i}}{W_{vt}^{i}} \right)^{\beta_v^i} e^{\omega_{bt}+\epsilon_{bt}} \\
L_{b1t}^{ij} &= \frac{y_{bt}^{ij}}{e^{\omega_{bt}+\epsilon_{bt}}} \prod_{z=1}^{Z} \prod_{v=2}^{V} \left(\frac{\beta_z^i}{\beta_1^i} \frac{W_{1t}^{i}}{W_{zt}^{i}} \right)^{-\beta_z^i} \left(\frac{\beta_v^i}{\beta_1^i} \frac{W_{1t}^{i}}{W_{vt}^{i}} \right)^{-\beta_v^i}.
\end{aligned} \tag{17}$$

Substituting this expression in equation 16, we completely solve for the fixed production factors:

$$F_{bzt}^{ij} = \frac{\beta_z^i}{\beta_1^i} \frac{W_{1t}^{i}}{W_{zt}^{i}} \frac{y_{bt}^{ij}}{e^{\omega_{bt}+\epsilon_{bt}}} \prod_{z=1}^{Z} \prod_{v=2}^{V} \left(\frac{\beta_z^i}{\beta_1^i} \frac{W_{1t}^{i}}{W_{zt}^{i}} \right)^{-\beta_z^i} \left(\frac{\beta_v^i}{\beta_1^i} \frac{W_{1t}^{i}}{W_{vt}^{i}} \right)^{-\beta_v^i}, \tag{18}$$

and similarly for the variable production factors:

$$L_{bvt}^{ij} = \frac{\beta_v^i}{\beta_1^i} \frac{W_{1t}^{i}}{W_{vt}^{i}} \frac{y_{bt}^{ij}}{e^{\omega_{bt}+\epsilon_{bt}}} \prod_{z=1}^{Z} \prod_{v=2}^{V} \left(\frac{\beta_z^i}{\beta_1^i} \frac{W_{1t}^{i}}{W_{zt}^{i}} \right)^{-\beta_z^i} \left(\frac{\beta_v^i}{\beta_1^i} \frac{W_{1t}^{i}}{W_{vt}^{i}} \right)^{-\beta_v^i}. \tag{19}$$

Substituting these obtained expressions for the production factors back in the variable cost function, we rewrite the variable cost function:

$$
\begin{aligned}
\Gamma_t^i &= \sum_{z=1}^{Z} W_{zt}^i \frac{\beta_z^i}{\beta_1^i}\frac{W_{1t}^i}{W_{zt}^i}\frac{y_{bt}^{ij}}{e^{\omega_{bt}+\epsilon_{bt}}}\prod_{z=1}^{Z}\prod_{v=2}^{V}\left(\frac{\beta_z^i}{\beta_1^i}\frac{W_{1t}^i}{W_{zt}^i}\right)^{-\beta_z^i}\left(\frac{\beta_v^i}{\beta_1^i}\frac{W_{1t}^i}{W_{vt}^i}\right)^{-\beta_v^i} \\
&\qquad + \sum_{v=1}^{V} W_{vt}^i \frac{\beta_v^i}{\beta_1^i}\frac{W_{1t}^i}{W_{vt}^i}\frac{y_{bt}^{ij}}{e^{\omega_{bt}+\epsilon_{bt}}}\prod_{z=1}^{Z}\prod_{v=2}^{V}\left(\frac{\beta_z^i}{\beta_1^i}\frac{W_{1t}^i}{W_{zt}^i}\right)^{-\beta_z^i}\left(\frac{\beta_v^i}{\beta_1^i}\frac{W_{1t}^i}{W_{vt}^i}\right)^{-\beta_v^i} \\
&= \sum_{z=1}^{Z}\frac{y_{bt}^{ij}}{e^{\omega_{bt}+\epsilon_{bt}}}\prod_{z=1}^{Z}\prod_{v=1}^{V}\frac{(W_{vt}^i)^{\beta_v^i}(W_{zt}^i)^{\beta_z^i}}{(\beta_v^i)^{\beta_v^i}(\beta_z^i)^{\beta_z^i-1}} + \sum_{z=1}^{Z}\frac{y_{bt}^{ij}}{e^{\omega_{bt}+\epsilon_{bt}}}\prod_{v=1}^{V}\prod_{v=1}^{V}\frac{(W_{vt}^i)^{\beta_v^i}(W_{zt}^i)^{\beta_z^i}}{(\beta_v^i)^{\beta_v^i-1}(\beta_z^i)^{\beta_z^i}} \qquad (20) \\
&= \frac{y_{bt}^{ij}}{e^{\omega_{bt}+\epsilon_{bt}}}\prod_{z=1}^{Z}\prod_{v=1}^{V}\frac{(W_{vt}^i)^{\beta_v^i}(W_{zt}^i)^{\beta_z^i}}{(\beta_v^i)^{\beta_v^i}(\beta_z^i)^{\beta_z^i-1}}\left[\sum_{z=1}^{Z}\beta_z^i + \sum_{v=1}^{V}\beta_v^i\right] \\
&= \frac{y_{bt}^{ij}}{e^{\omega_{bt}+\epsilon_{bt}}}\prod_{z=1}^{Z}\prod_{v=1}^{V}\frac{(W_{vt}^i)^{\beta_v^i}(W_{zt}^i)^{\beta_z^i}}{(\beta_v^i)^{\beta_v^i}(\beta_z^i)^{\beta_z^i-1}}.
\end{aligned}
$$

The total cost function of the firm involves the fixed production cost, iceberg trade costs $\tau_t^{ij} > 1$ and a constant marginal cost that depends on its persistent productivity and transitory shocks to production: $f_t^{ij} + \left(\frac{\tau_t^{ij} y_{bt}^{ij}}{e^{\omega_{bt}+\epsilon_{bt}}}\right) W_t^i$, with $W_t^i = \prod_{z=1}^{Z}\prod_{v=1}^{V}\frac{(W_{vt}^i)^{\beta_v^i}(W_{zt}^i)^{\beta_z^i}}{(\beta_v^i)^{\beta_v^i}(\beta_z^i)^{\beta_z^i}}$.

As production factors differ in their timing of adjustment, we differentiate between long- and short-run profit maximization. The long-run expected profit maximization optimizes the quantity of fixed production factors for time t based on the information provided in $t-1$, $\mathcal{I}_{bt-1}$:

$$
\begin{aligned}
\max_{\boldsymbol{F}_{bzt}^{ij}} \mathbb{E}_{\eta,\nu,\epsilon}\left[\pi_{bt}^{ij}|\mathcal{I}_{bt-1}\right] &= \max_{\boldsymbol{F}_{bzt}^{ij}} \mathbb{E}_{\eta,\nu,\epsilon}\left[p_{bt}^{ij}y_{bt}^{ij} - f_t^{ij} - \tau_t^{ij}\left(\sum_{z=1}^{Z}F_{bzt}^{ij}W_{zt}^i - \sum_{v=1}^{V}L_{bvt}^{ij}W_{vt}^i\right)\middle|\mathcal{I}_{bt-1}\right] \\
&= \max_{\boldsymbol{F}_{bzt}^{ij}}\left(Y_t^j\right)^{\frac{1}{\sigma}} P_t^j \mathbb{E}_\eta\left[\left(q_{bt}^{ij}\right)^{\frac{\sigma-1}{\sigma}}\middle|\mathcal{I}_{bt-1}\right]\mathbb{E}_{\nu,\epsilon}\left[e^{\frac{\nu_{bt}}{\sigma}+\frac{\sigma-1}{\sigma}\epsilon_{bt}}\middle|\mathcal{I}_{bt-1}\right] \\
&\qquad - f_t^{ij} - \tau_t^{ij}\left(\sum_{z=1}^{Z}F_{bzt}^{ij}W_{zt}^i - \sum_{v=1}^{V}L_{bvt}^{ij}W_{vt}^i\right). \qquad (21)
\end{aligned}
$$

From the First-order conditions, we obtain the optimal quantity of fixed production factors:

$$
\begin{aligned}
0 &= \frac{\partial \mathbb{E}_{\eta,\nu,\epsilon}\left[\pi_{bt}^{ij}|\mathcal{I}_{bt-1}\right]}{\partial F_{bzt}^{ij}} \\
&= \frac{\sigma-1}{\sigma}\mathbb{E}_\eta\left[\left(q_{bt}^{ij}\right)^{-\frac{1}{\sigma}}\middle|\mathcal{I}_{bt-1}\right]\frac{\beta_z^i \mathbb{E}_\eta\left[q_{bt}^{ij}\middle|\mathcal{I}_{bt-1}\right]}{F_{bzt}^{ij}}\left(Y_t^j\right)^{\frac{1}{\sigma}} P_t^j \mathbb{E}_{\nu,\epsilon}\left[e^{\frac{\nu_{bt}}{\sigma}+\frac{\sigma-1}{\sigma}\epsilon_{bt}}\middle|\mathcal{I}_{bt-1}\right] - \tau_t^{ij}W_{zt}^i \\
&\Leftrightarrow
\end{aligned}
$$

$$
F_{bzt}^{ij} = \frac{\sigma-1}{\sigma}\mathbb{E}_\eta\left[\left(q_{bt}^{ij}\right)^{-\frac{1}{\sigma}}\middle|\mathcal{I}_{bt-1}\right]\frac{\beta_z^i \mathbb{E}_\eta\left[q_{bt}^{ij}\middle|\mathcal{I}_{bt-1}\right]}{\tau_t^{ij}W_{zt}^i}\left(Y_t^j\right)^{\frac{1}{\sigma}} P_t^j \mathbb{E}_{\nu,\epsilon}\left[e^{\frac{\nu_{bt}}{\sigma}+\frac{\sigma-1}{\sigma}\epsilon_{bt}}\middle|\mathcal{I}_{bt-1}\right]. \qquad (22)
$$

The short-run expected profit maximization, then, optimizes the quantity of variable production factors for time t given the fixed production factors ($\overline{F}_{bvt}^{ij}$) and based on the information provided in t, $\mathcal{I}_{bt}$:

$$\begin{aligned}\max_{\boldsymbol{L}_{bvt}^{ij}} \mathbb{E}_{\nu,\epsilon}\left[\pi_{bt}^{ij}|\mathcal{I}_{bt}\right] &= \max_{\boldsymbol{L}_{bvt}^{ij}} \mathbb{E}_{\nu,\epsilon}\left[p_{bt}^{ij}\overline{y}_{bt}^{ij} - f_t^{ij} - \tau_t^{ij}\left(\sum_{z=1}^{Z}\overline{F}_{bzt}^{ij}W_{zt}^{i} - \sum_{v=1}^{V} L_{bvt}^{ij}W_{vt}^{i}\right)\middle| \mathcal{I}_{bt}\right] \\ &= \max_{\boldsymbol{L}_{bvt}^{ij}} \left(\overline{q}_{bt}^{ij}\right)^{\frac{\sigma-1}{\sigma}} \left(Y_t^j\right)^{\frac{1}{\sigma}} P_t^j \mathbb{E}_{\nu,\epsilon}\left[e^{\frac{\nu_{bt}}{\sigma}+\frac{\sigma-1}{\sigma}\epsilon_{bt}}\middle| \mathcal{I}_{bt}\right] \\ &\quad - f_t^{ij} - \tau_t^{ij}\left(\sum_{z=1}^{Z}\overline{F}_{bzt}^{ij}W_{zt}^{i} - \sum_{v=1}^{V} L_{bvt}^{ij}W_{vt}^{i}\right). \qquad (23)\end{aligned}$$

The first-order conditions allow us to deduce optimal quantity of variable production factors:

$$\begin{aligned}0 &= \frac{\partial \mathbb{E}_{\nu,\epsilon}\left[\pi_{bt}^{ij}|\mathcal{I}_{bt}\right]}{\partial L_{bvt}^{ij}} \\ &= \frac{\sigma-1}{\sigma}\left(\overline{q}_{bt}^{ij}\right)^{-\frac{1}{\sigma}} \frac{\beta_v^i \overline{q}_{bt}^{ij}}{L_{bvt}^{ij}} \left(Y_t^j\right)^{\frac{1}{\sigma}} P_t^j \mathbb{E}_{\nu,\epsilon}\left[e^{\frac{\nu_{bt}}{\sigma}+\frac{\sigma-1}{\sigma}\epsilon_{bt}}\middle| \mathcal{I}_{bt}\right] - \tau_t^{ij}W_{zt}^{i} \\ &\Leftrightarrow \\ L_{bvt}^{ij} &= \frac{\sigma-1}{\sigma}\left(\overline{q}_{bt}^{ij}\right)^{-\frac{1}{\sigma}} \frac{\beta_v^i \overline{q}_{bt}^{ij}}{\tau_t^{ij}W_{vt}^{i}} \left(Y_t^j\right)^{\frac{1}{\sigma}} P_t^j \mathbb{E}_{\nu,\epsilon}\left[e^{\frac{\nu_{bt}}{\sigma}+\frac{\sigma-1}{\sigma}\epsilon_{bt}}\middle| \mathcal{I}_{bt}\right]. \qquad (24)\end{aligned}$$

Completing the production function (eq. 10) with the optimal input mix allows us to obtain an expression for the optimal quantity produced:[6,7]

[6]Note that if all inputs would be variable, this optimum quantity would reduce to

$$q_b^{ij} = \left(\frac{\sigma-1}{\sigma}\frac{e^{\omega}}{\tau^{ij}W^i}\right)^{\sigma} Y^j \left(P^j\right)^{\sigma} \mathbb{E}_{\nu,\epsilon}\left[e^{\epsilon_b^T}\right]^{\sigma}$$

[7]We rely on the i.i.d. nature of uncertainty to reduce the expectation term to a constant $\mathbb{E}_{\nu,\epsilon}\left[e^{\varepsilon_{bt}^T}\middle|\mathcal{I}_{bt}\right] = \mathbb{E}_{\nu,\epsilon}\left[e^{\varepsilon_{bt}^T}\right]$, $\mathbb{E}_{\eta}\left[e^{\varepsilon_{bt}^P}\middle|\mathcal{I}_{bt}\right] = \mathbb{E}_{\eta}\left[e^{\varepsilon_{bt}^P}\right]$. See Gandhi et al. (2020) for a discussion on the empirical consequences of assuming full independence.

$$
\begin{aligned}
q_{bt}^{ij} &= \prod_{z=1}^{Z}\prod_{v=1}^{V}(F_{bzt}^{ij})^{\beta_z^i}(L_{bvt}^{ij})^{\beta_v^i}e^{\omega_{bt}} \\
&= \frac{\sigma-1}{\sigma}\left(Y_t^j\right)^{\frac{1}{\sigma}}P_t^j\frac{1}{\tau_t^{ij}}\left(\prod_{z=1}^{Z}\prod_{v=1}^{V}\frac{(\beta_v^i)^{\beta_v^i}(\beta_z^i)^{\beta_z^i}}{(W_{vt}^i)^{\beta_v^i}(W_{zt}^i)^{\beta_z^i}}\right) \\
&\quad\times\mathbb{E}_{\nu,\epsilon}\left[e^{\frac{\nu_{bt}}{\sigma}+\frac{\sigma-1}{\sigma}\epsilon_{bt}}\middle|\mathcal{I}_{bt-1}\right]^{\sum_{z=1}^{Z}\beta_z^i}\mathbb{E}_{\nu,\epsilon}\left[e^{\frac{\nu_{bt}}{\sigma}+\frac{\sigma-1}{\sigma}\epsilon_{bt}}\middle|\mathcal{I}_{bt}\right]^{\sum_{v=1}^{V}\beta_v^i} \\
&\quad\times\mathbb{E}_{\eta}\left[\left(q_{bt}^{ij}\right)^{\frac{\sigma-1}{\sigma}}\middle|\mathcal{I}_{bt-1}\right]^{\sum_{z=1}^{Z}\beta_z^i}\left[\left(q_{bt}^{ij}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\sum_{v=1}^{V}\beta_v^i}e^{\omega_{bt}} \\
&= \frac{\sigma-1}{\sigma}\left(Y_t^j\right)^{\frac{1}{\sigma}}P_t^j\frac{(\beta_v^i)^{\sum_{v=1}^{V}\beta_v^i}(\beta_z^i)^{\sum_{z=1}^{Z}\beta_z^i}}{\tau_t^{ij}(W_{vt}^i)^{\sum_{v=1}^{V}\beta_v^i}(W_{zt}^i)^{\sum_{z=1}^{Z}\beta_z^i}} \\
&\quad\times\mathbb{E}_{\nu,\epsilon}\left[e^{\frac{\nu_{bt}}{\sigma}+\frac{\sigma-1}{\sigma}\epsilon_{bt}}\middle|\mathcal{I}_{bt}\right] \\
&\quad\times\left(\frac{\mathbb{E}_{\eta}\left[e^{\eta_{bt}}\middle|\mathcal{I}_{bt-1}\right]}{e^{\eta_{bt}}}\right)^{\frac{\sigma-1}{\sigma}\sum_{z=1}^{Z}\beta_z^i}\left(q_{bt}^{ij}\right)^{\frac{\sigma-1}{\sigma}}e^{\omega_{bt}} \\
&= \left(\frac{\sigma-1}{\sigma}\frac{e^{\omega_{bt}}}{\tau_t^{ij}W_t^i}\left(Y_t^j\right)^{\frac{1}{\sigma}}(P_t^j)\mathbb{E}_{\nu,\epsilon}\left[e^{\frac{\nu_{bt}}{\sigma}+\frac{\sigma-1}{\sigma}\epsilon_{bt}}\middle|\mathcal{I}_{bt}\right]\right)^{\sigma}\left(\frac{\mathbb{E}_{\eta}\left[e^{\eta_{bt}}\middle|\mathcal{I}_{bt-1}\right]}{e^{\eta_{bt}}}\right)^{(\sigma-1)\sum_{z=1}^{Z}\beta_z^i} \\
&= \left(\frac{\sigma-1}{\sigma}\frac{e^{\omega_{bt}}}{\tau_t^{ij}W_t^i}\right)^{\sigma}Y_t^j(P_t^j)^{\sigma}\mathbb{E}_{\varepsilon^T}\left[e^{\varepsilon_{bt}^T}\middle|\mathcal{I}_{bt}\right]^{\sigma}\frac{\mathbb{E}_{\varepsilon^P}\left[e^{\varepsilon_{bt}^P}\middle|\mathcal{I}_{bt-1}\right]^{\sigma}}{e^{\sigma\varepsilon_{bt}^P}} \\
&= \left(\frac{\sigma-1}{\sigma}\frac{e^{\omega_{bt}}}{\tau_t^{ij}W_t^i}\right)^{\sigma}Y_t^j(P_t^j)^{\sigma}\mathbb{E}_{\varepsilon^T}\left[e^{\varepsilon_{bt}^T}\right]^{\sigma}\frac{\mathbb{E}_{\varepsilon^P}\left[e^{\varepsilon_{bt}^P}\right]^{\sigma}}{e^{\sigma\varepsilon_{bt}^P}},
\end{aligned}
\tag{25}
$$

where the wages are summarized as $W_t^i = \prod_{z=1}^{Z}\prod_{v=1}^{V}\frac{(\beta_v^i)^{\beta_v^i}(\beta_z^i)^{\beta_z^i}}{(W_{vt}^i)^{\beta_v^i}(W_{zt}^i)^{\beta_z^i}}$. ε_{bt}^T gathers the transitory demand and supply shocks ($\varepsilon_{bt}^T = e^{\frac{\nu_{bt}}{\sigma}+\frac{\sigma-1}{\sigma}\epsilon_{bt}}$), while ε_{bt}^P gathers the productivity shocks with permanent implications $\left(\varepsilon_{bt}^P = \frac{\sigma-1}{\sigma}\left(\sum_{z=1}^{Z}\beta_z^i\right)\eta_{bt}\right)$.

For each firm b at time t, the timing assumptions of the model can be summarized as follows:

1. Observe the vector of state variables $\mathcal{I}_{bt}$, with $\omega_{bt}\in\mathcal{I}_{bt}$;
2. Choose freely adjustable inputs optimally for each market;
3. Produce output q_{bt} and sell at a price determined by the demand curve;
4. Observe deviations from expectations regarding demand (ν_{bt}) and supply (ϵ_{bt});
5. Decide optimally on next-period fixed production factors for each market.

The realized revenue expression for firms from country i selling in destination j at time t can now, relying on the profit-maximizing price and the profit-maximizing output, be expressed as:[8]

$$
\begin{aligned}
x_{bt}^{ij} = p_{bt}^{ij} y_{bt}^{ij} &= \left(y_{bt}^{ij}\right)^{\frac{\sigma-1}{\sigma}} \left(Y_t^j\right)^{\frac{1}{\sigma}} e^{\frac{\nu_{bt}}{\sigma}} P_t^j \\
&= \left[\left(\frac{\sigma-1}{\sigma} \frac{e^{\omega_{bt}}}{\tau_t^{ij} W_t^i} \right)^{\sigma} Y_t^j (P_t^j)^{\sigma} \mathbb{E}_{\varepsilon^P, \varepsilon^T} \left[e^{\varepsilon_{bt}^T + \varepsilon_{bt}^P} \right]^{\sigma} \frac{1}{e^{\sigma \varepsilon_{bt}^P}} \right]^{\frac{\sigma-1}{\sigma}} \left(Y_t^j\right)^{\frac{1}{\sigma}} P_t^j e^{\sigma \varepsilon_{bt}^T} \\
&= \left(\frac{\sigma}{\sigma-1} \tau_t^{ij} W_t^i \right)^{1-\sigma} Y_t^j \left(P_t^j\right)^{\sigma} \mathbb{E}_{\varepsilon^P, \varepsilon^T} \left[e^{\varepsilon_{bt}^T + \varepsilon_{bt}^P} \right]^{\sigma-1} e^{(\sigma-1)\omega_{bt} + \varepsilon_{bt}^T - (\sigma-1)\epsilon_{bt}^P} \qquad (26)
\end{aligned}
$$

B.2 Operating decisions and aggregation with certainty in future productivity

Going further, we reduce the dynamics of the model specifying $G(\omega_{bt}|\mathcal{I}_{bt-1})$ such that $\omega_{bt} = \omega_{bt-1} = \omega_b$. The model thus simplifies to a heterogeneous firms model which features certainty in future productivity after entry but uncertainty in the realized supply and demand. Eliminating the dynamics as such has the advantage of resulting in clear analytical expressions for the equilibrium variables. Moreover, it allows us to focus on the influence of transitory uncertainty when computing the trade elasticity and GFT compared to the predominant heterogeneous firms model without transitory uncertainty (see for instance Head et al. (2014); Melitz and Redding (2015); Nigai (2017); Bee and Schiavo (2018)). The assumption of certainty in future productivity does imply, however, that there is no role for permanent uncertainty in the model from here onwards.

The productivity cutoffs for serving each market are determined by the expected zero-profit conditions and the by the free entry condition, requiring the probability of successful entry times the expected future value of entry conditional upon successful entry to equal the sunk entry cost:

$$
0 = \mathbb{E}_{\omega, \varepsilon^T} \left[\pi^{ij}(\omega^{ij*}) | \mathcal{I}_b \right]; \qquad (27)
$$

$$
f^{ie} = \left[1 - G(\omega^{ii*})\right] \mathbb{E}_{\omega, \varepsilon^T} \left[\sum_{j=1}^{j} \pi^{ij}(\omega_b) \right] \qquad (28)
$$

With the productivity cutoffs determined, we can sum equation 8 across all active firms to obtain an expression for aggregate trade between country i and j:

[8] Note that if all inputs would be variable, the realized revenue expression would simplify to

$$
x_{bt}^{ij} = \left(\frac{\sigma}{\sigma-1} \tau_t^{ij} W_t^i \right)^{1-\sigma} Y_t^j \left(P_t^j\right)^{\sigma} \mathbb{E}_{\varepsilon^T} \left[e^{\varepsilon_{bt}^T} \right]^{\sigma-1} e^{(\sigma-1)\omega_{bt} + \varepsilon_{bt}^T}
$$

$$
\begin{aligned}
x^{ij} &= \frac{M^{ij}}{1-G(\omega^{ij*})}\int_{\omega^{ij*}}^{\infty}\int_{-\infty}^{\infty} x_b^{ij}\, dG(\omega_b)dG(\varepsilon_b^T) \\
&= \frac{M^{ij}}{1-G(\omega^{ij*})}\left(\frac{\sigma}{\sigma-1}\tau^{ij}W^i\right)^{1-\sigma} Y^j \left(P^j\right)^{\sigma} \mathbb{E}_{\varepsilon^T}\left[e^{\varepsilon_b^T}\right]^{\sigma-1}\int_{\omega^{ij*}}^{\infty}\int_{-\infty}^{\infty} e^{(\sigma-1)\omega_b+\varepsilon_b^T} dG(\omega_b)dG(\varepsilon_b^T) \\
&= \frac{M^{ij}}{1-G(\omega^{ij*})}\left(\frac{\sigma}{\sigma-1}\tau^{ij}W^i\right)^{1-\sigma} Y^j \left(P^j\right)^{\sigma} \mathbb{E}_{\varepsilon^T}\left[e^{\varepsilon_b^T}\right]^{\sigma-1}\int_{\omega^{ij*}}^{\infty} e^{(\sigma-1)\omega_b}\int_{-\infty}^{\infty} e^{\varepsilon_b^T} dG(\omega_b)dG(\varepsilon_b^T) \\
&= \frac{M^{ij}}{1-G(\omega^{ij*})}\left(\frac{\sigma}{\sigma-1}\tau^{ij}W^i\right)^{1-\sigma} Y^j \left(P^j\right)^{\sigma} \mathbb{E}_{\varepsilon^T}\left[e^{\varepsilon_b^T}\right]^{\sigma}\int_{\omega^{ij*}}^{\infty} e^{(\sigma-1)\omega_b} dG(\omega_b). \qquad (29)
\end{aligned}
$$

From the aggregate revenue expression (eq. 29), we observe that transitory uncertainty-induced firm-level heterogeneity reduces to a constant. The partial sensitivity of aggregate trade to changes in variable trade costs, the aggregate trade elasticity, is then defined as (Chaney, 2008; Arkolakis et al., 2012; Melitz and Redding, 2014; Bas et al., 2017):[9]

$$
\begin{aligned}
\gamma^{ij} \equiv \left.\frac{\partial ln x^{ij}}{\partial ln \tau^{ij}}\right|_{\omega^{ii*}} &= 1-\sigma + \left.\frac{dln\int_{\omega^{ij*}}^{\infty} e^{(\sigma-1)\omega_b}dG(\omega_b)}{dln\omega^{ij*}}\frac{\partial ln\omega^{ij*}}{\partial ln\tau^{ij}}\right|_{\omega^{ii*}} \\
&= 1-\sigma + \left.\frac{d\int_{\omega^{ij*}}^{\infty} e^{(\sigma-1)\omega_b}dG(\omega_b)}{d\omega^{ij*}}\frac{e^{\omega^{ij*}}}{\int_{\omega^{ij*}}^{\infty} e^{(\sigma-1)\omega_b}dG(\omega_b)}\frac{\partial ln\omega^{ij*}}{\partial ln\tau^{ij}}\right|_{\omega^{ii*}} \\
&= 1-\sigma - \frac{e^{\sigma\omega^{ij*}}g(\omega^{ij*})}{\int_{\omega^{ij*}}^{\infty} e^{(\sigma-1)\omega_b}dG(\omega_b)}, \qquad (30)
\end{aligned}
$$

which is independent of transitory idiosyncratic uncertainty.

The mass of firms is specified as the ratio of aggregate over average revenue:

$$
M^i = \left[1-G(\omega^{ii*})\right] M^{ie} = \frac{x^i}{\mathbb{E}_{\omega,\epsilon^T}\left[x^i\right]}. \qquad (31)
$$

We can rewrite the mass of firms using the free entry condition, goods and labor market clearing $\left(x^i = \tilde{W}^i L^i + M^{ie} f^{ie} + \sum_{j=1}^{J} M^{ij} f^{ij}\right)$, with $\tilde{W}_t^i = W_t^i \prod_{z=1}^{Z}\prod_{v=1}^{V}(\beta_v^i)^{\beta_v^i}(\beta_z^i)^{\beta_z^i}$, as a function of exogenous variables:

[9] Aggregate trade elasticity is here defined as the direct response of aggregate trade to trade costs, keeping the indirect effect trough the price index via its impact on the domestic cutoff fixed (Melitz and Redding, 2015).

$$
\begin{aligned}
M^i &= \frac{\tilde{W}^i L^i + M^{ie} f^{ie} + \sum_{j=1}^{J} M^{ij} f^{ij}}{\sigma\left(\frac{f^{ie}}{1-G(\omega^{ii*})} + \sum_{j=1}^{J} \frac{1-G(\omega^{ij*})}{1-G(\omega^{ii*})} f^{ij}\right)} \\
&= \frac{\tilde{W}^i L^i + M^i \frac{f^{ie}}{1-G(\omega^{ii*})} + M^i \sum_{j=1}^{J} \frac{1-G(\omega^{ij*})}{1-G(\omega^{ii*})} f^{ij}}{\sigma\left(\frac{f^{ie}}{1-G(\omega^{ii*})} + \sum_{j=1}^{J} \frac{1-G(\omega^{ij*})}{1-G(\omega^{ii*})} f^{ij}\right)} \\
&= \frac{\sigma}{\sigma-1} \frac{\tilde{W}^i L^i}{\sigma\left(\frac{f^{ie}}{1-G(\omega^{ii*})} + \sum_{j=1}^{J} \frac{1-G(\omega^{ij*})}{1-G(\omega^{ii*})} f^{ij}\right)} \\
&= \frac{\tilde{W}^i L^i}{(\sigma-1)\left(\frac{f^{ie}}{1-G(\omega^{ii*})} + \sum_{j=1}^{J} \frac{1-G(\omega^{ij*})}{1-G(\omega^{ii*})} f^{ij}\right)}
\end{aligned}
\tag{32}
$$

Assuming a two-country symmetric economy and setting the wage of the composite factor as the numeraire, welfare can be calculated as the inverse of the price index

$$
\mathbb{W}^i = (P^i)^{-1}, \tag{33}
$$

with

$$
\begin{aligned}
(P^i)^{1-\sigma} &= \sum_{j=1}^{J} M^{ij} \mathbb{E}_{\nu,\epsilon}\left[e^{\frac{\nu_b}{\sigma} + \frac{\sigma-1}{\sigma}\epsilon_b} \middle| \mathcal{I}_b \right]^{\sigma-1} \mathbb{E}_{\nu,\epsilon}\left[e^{\frac{\nu_b}{\sigma} + \frac{\epsilon_b}{\sigma}} \right]^{1-\sigma} \int_{\omega^{ij*}}^{\infty} \left(\frac{\sigma}{\sigma-1} \frac{\tau^{ij}}{e^{\omega_b}} W^i \right)^{1-\sigma} dG(\omega_b) \\
&= M^i \sum_{j=1}^{J} \frac{1-G(\omega^{ij*})}{1-G(\omega^{ii*})} \mathbb{E}_{\varepsilon^T}\left[e^{(\sigma-1)\varepsilon_b^T} \right] \mathbb{E}_{\nu,\epsilon}\left[e^{\frac{1-\sigma}{\sigma}(\nu_b+\epsilon_b)} \right] \left(\frac{\sigma}{\sigma-1} \tau^{ij} W^i \right)^{1-\sigma} \int_{\omega^{ij*}}^{\infty} e^{(\sigma-1)\omega_b} dG(\omega_b)
\end{aligned}
\tag{34}
$$

The changes in welfare from a change in variable trade costs ($\boldsymbol{\tau} \to \boldsymbol{\tau}'$) are then simply a ratio of the aggregate price indices such that, if expectations remain constant over time, non-persistent heterogeneity has no influence on the outcome:

$$
\frac{(\mathbb{W}^i)'}{\mathbb{W}^i} = \frac{P^i}{(P^i)'}. \tag{35}
$$

Appendix C Structural productivity estimation

This section describes in detail the structural productivity estimation technique relied upon in this paper for a value-added Cobb-Douglas production function. The estimation strategy consist of two stages.[10] The first stage relies on the Ackerberg et al. (2015) proxy-variable approach to separate transitory uncertainty (ε_{bt}^{T}) from our main estimation equation (eq. 8). In the second stage, we identify the parameters of interest building on moment conditions with respect to the stochastic shocks of productivity. As such, we avert endogeneity problems and obtain consistent parameter estimates.

C.1 First stage

The first stage consists of separating the transitory uncertainty (ε_{bt}^{T}) from the main estimating equation (eq. 8). We follow Ackerberg et al. (2015) in dividing the set of variable production factors $\boldsymbol{L}_{bt}^{i}$ into a factor decided upon at time t, the proxy variable $\left\{L_{1bt}^{i}\right\}$, and a set of the remaining variable production factors also decided upon at time t, but before the proxy variable is decided upon $\boldsymbol{L}_{bt}^{i} \setminus \left\{L_{1bt}^{i}\right\}$).[11] As a result, the proxy demand function can be written as a function of all state variables and variable production factors, including unobserved (for the researcher) productivity:

$$L_{b1t}^{i} = h\left(\mathcal{I}_{bt}, \boldsymbol{L}_{bt}^{i} \setminus \left\{L_{1bt}^{i}\right\}\right). \tag{36}$$

Based on this proxy input demand equation and assuming strict monotonicity between this input demand and productivity, we can write:

$$\omega_{bt} = h^{-1}\left(\mathcal{I}_{bt} \setminus \omega_{bt}, \boldsymbol{L}_{bt}^{i}\right). \tag{37}$$

This inverse of the proxy demand function forms the basis of a control function approach that allows us to estimate the main estimation equation (eq. 8) and identify the transitory uncertainty:

$$\begin{aligned}
ln\hat{\phi}_{bt}^{ii} &= lnx_{bt}^{ii} - ln\theta_{bt}^{ii}L_{b1t}^{i} - \varepsilon_{bt}^{T} \\
&= \tilde{h}\left(\theta_{bt}^{ii}, F_{b1t}^{i}, \ldots, F_{bZt}^{i}, L_{b2t}^{i}, \ldots, L_{bVt}^{i}, Y_{t}^{i}, h^{-1}\left(\mathcal{I}_{bt} \setminus \omega_{bt}, \boldsymbol{L}_{bt}^{i}\right)\right), \\
&= \frac{\sigma-1}{\sigma}ln\theta_{bt}^{ii} + \frac{\sigma-1}{\sigma}\left(\sum_{z=1}^{Z}\beta_{z}^{i}lnF_{bzt}^{i} + \sum_{v=2}^{V}\beta_{v}^{i}lnL_{bvt}^{i}\right) + \frac{1}{\sigma}lnY_{t}^{i} + \frac{\sigma-1}{\sigma}\omega_{bt},
\end{aligned} \tag{38}$$

[10]Due to data availability, we do not control for the selection bias as defined by Olley and Pakes (1996). Our framework can easily be extended to include an extra estimation stage as specified in Olley and Pakes (1996). However, following on their results for unbalanced panels, this extra stage is expected to have little influence on the results.

[11]See Ackerberg et al. (2015) for a discussion on the possible Data Generating Processes that could generate such differences in timing decisions for factor inputs.

where $ln\phi_{bt}^{ii} = lnx_{bt}^{ii} - ln\theta_{bt}^{ii} L_{b1t}^{i}$ denotes domestic sales minus the domestic share of the proxy variable input factor with unit output elasticity.

C.2 Second stage

The second stage, then, aims to obtain consistent estimates for the parameters in equation 38. We parametrize the assumed Markov process for productivity as a first-order auto-regressive process

$$\omega_{bt} = \alpha_0 + \alpha_1 \omega_{bt-1} + \eta_{bt}, \tag{39}$$

where, by construction $E\left[\eta_{bt}|\mathcal{I}_{bt-1}\right] = 0$. Consistent parameter estimates for equation 38 can then be obtained based on the following moment conditions:

$$E\left(\eta_{bt}\begin{bmatrix} ln\boldsymbol{F}_{t}^{i} \\ ln\boldsymbol{L}_{bt}^{i} \setminus \left\{L_{1bt}^{i}\right\} \\ lnY_{t-1}^{i} \end{bmatrix}\right) = 0. \tag{40}$$

Current state parameter variables are exogenous to productivity shocks if decided upon at time $t-1$, while those that are at a later time can be identified from lagged observations. The elasticity of substitution parameter is identified from lagged values of the aggregate demand shifter.[12] Consistent estimates of the productivity Markov process and revenue production function parameters at hand, we are also capable of backing out an estimate of persistent uncertainty $\left(\varepsilon_{bt}^{P} = \frac{\hat{\sigma}-1}{\hat{\sigma}}\left(\sum_{z=1}^{Z}\hat{\beta}_{z}^{i}\right)\eta_{bt}\right)$.

[12] Note that, as already mentioned in Klette and Griliches (1996), this methodology does not allow for a time trend or time fixed effects, as the inclusion of a time variable would leave little to no variation for the demand shifter to identify the elasticity of substitution.

Appendix References

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