

Identifying Unobserved Heterogeneity in Productivity

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Monte Carlo
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Application
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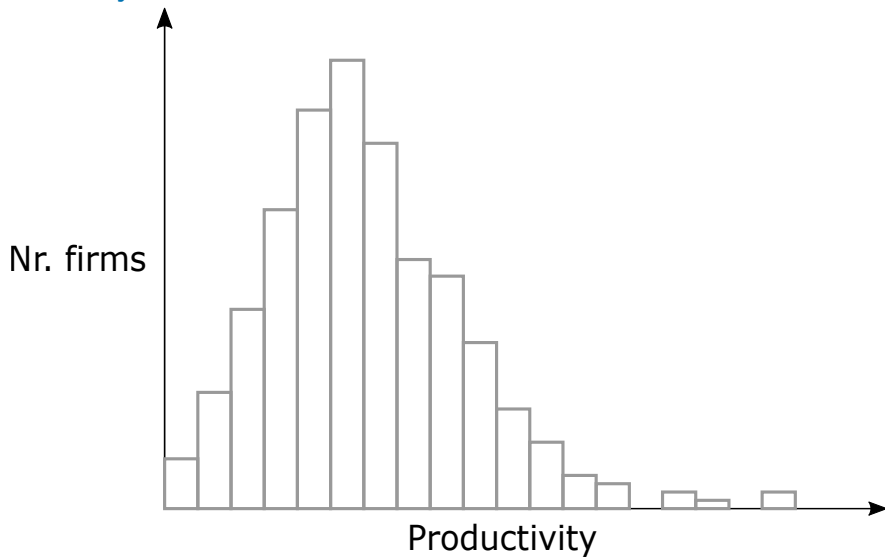
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Personal Introduction

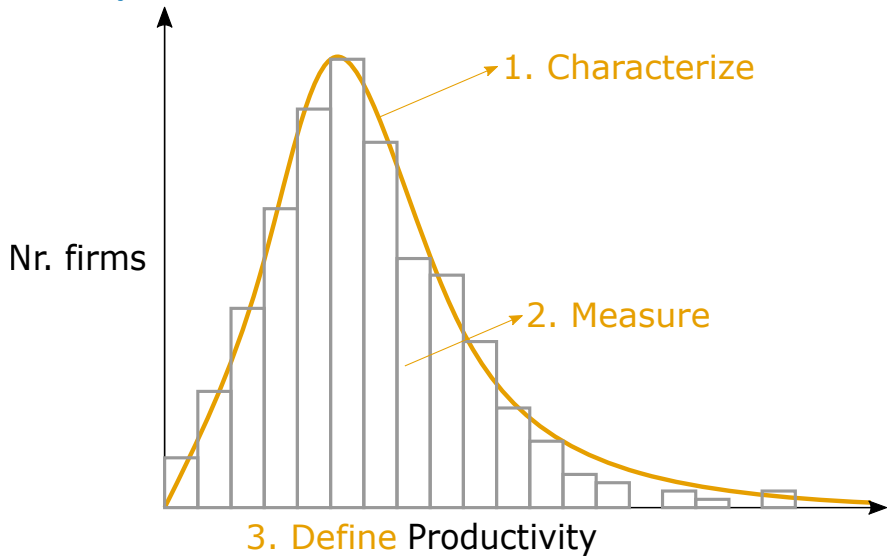
Who am I?

*As an international economist, I study the **firm-level** determinants of **technological progress** and quantify the consequences of **international trade** for technological progress and subsequent **economic prosperity**.*

Productivity distribution



Productivity distribution



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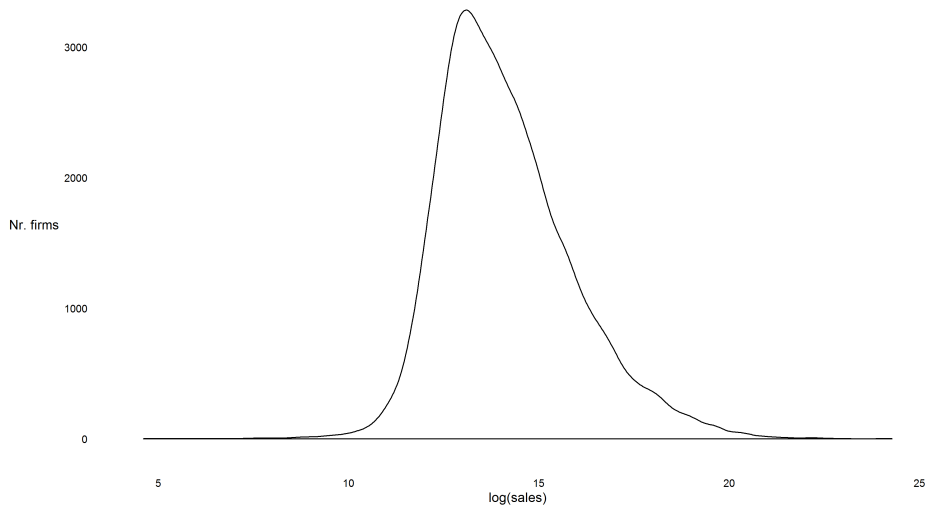
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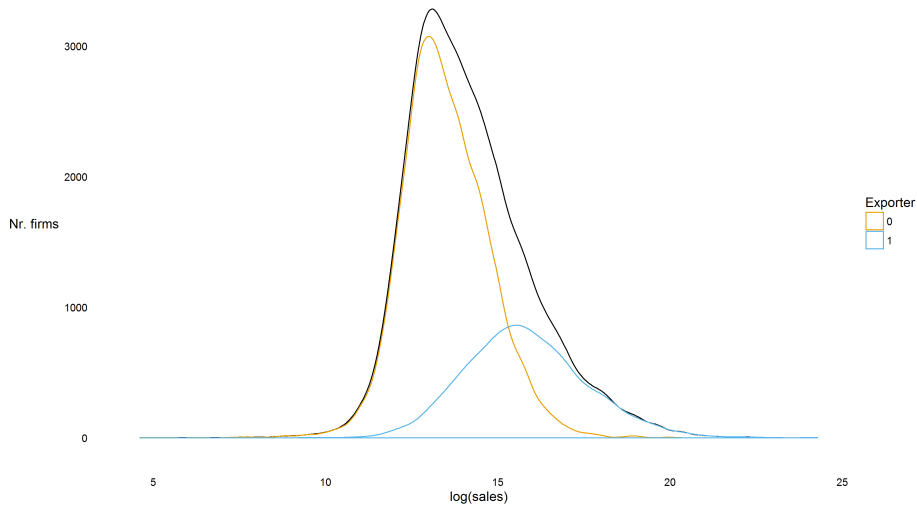
Motivation

Sales distribution



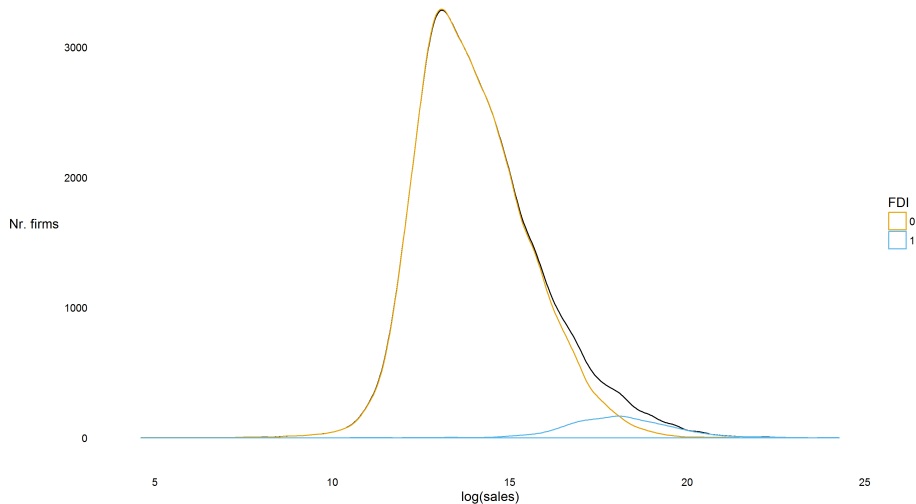
Motivation

Sales distribution . . . by export status



Motivation

Sales distribution ... by FDI status (... of which 93% also export!)



Motivation

Current literature on the structural identification of production functions assumes

1. Homogeneous productivity growth process for all firms
2. Unobserved heterogeneity between clusters of firms according to à priori specified (categorical) proxy variables

⇒ Probability of **omitted variable bias**

⇒ Heavy data burden

Motivation

Current approach ... a recipe for omitted variable bias?

Study	Export	Import	R&D	FDI	Others (industry, location, ...)
Olley and Pakes (1996)					Age, telecommunications industry
Javorcik (2004)				x	Manufacturing (plant-ind-location-time FE)
Amiti and Konings (2007)		x			Manufacturing
Das et al. (2007)	x				2-digit industry
Blalock and Gertler (2008)				x	Manufacturing (ind-location-time FE)
Kasahara and Rodrigue (2008)		x			Manufacturing
Aw et al. (2011)	x		x		Electronics industry
De Loecker (2013)	x				2-digit industry, investment
Doraszelski and Jaumandreu (2013)			x		2-digit industry, investment
Kasahara and Lapham (2013)	x	x			3-and 4- digit industry

This paper ...

- Develops a production function estimator that allows for and identifies unobserved heterogeneity in productivity between **clusters of firms** using **Finite Mixture Models**
 - Demonstrates the appropriateness via Monte Carlo
 - Showcases the applicability on Belgian firm-level data
 - Strong evidence of heterogeneity in the evolution of productivity
 - Heterogeneity correlates with traditional firm-level characteristics, but **unobserved heterogeneity** remains
- ⇒ Necessity to control for unobserved heterogeneity when comparing productivity across groups of firms, such as exporters vs. non-exporters, ...

Related literature

- Heterogeneity in firm sales/productivity distribution (cf. before, Dewitte et al. (2020))
- Structural Production function estimation (Olley and Pakes, 1996; Levinsohn and Petrin, 2003; Akerberg et al., 2015; Gandhi et al., 2020)
 - ... with unobserved heterogeneity (Lee et al., 2019; Gandhi et al., 2020; Akerberg, 2021)
 - ... with **Finite Mixture** specification (Van Biesebroeck, 2003; Kasahara et al., 2017; Battisti et al., 2020)
- Finite Mixtures in SF literature (Beard et al., 1997; Greene, 2005; Orea and Kumbhakar, 2004; El-Gamal and Inanoglu, 2005)
- Mixture-of-experts models (Fruhwirth-Schnatter et al., 2019)

Outline

1. Behavioral framework
2. Production function estimation
3. Monte Carlo
4. Application to Belgian firm-level data
5. Conclusion

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Behavioral Framework

Behavioral framework

Data and definitions

Dynamic heterogeneous firms model with **cluster-dependent** uncertainty in future, **Hicks-neutral**, productivity

- (Short) panel of firms $b = 1, \dots, B$ over period $t = 1, \dots, T$
- Output Y_{bt} and inputs $\{K_{bt}, L_{bt}, M_{bt}\}$ in **perfectly competitive markets**
- Information set \mathcal{I}_{bt} such that generic input $X_{bt} \in \{K_{bt}, L_{bt}, M_{bt}\}$ is
 - **nonflexible** if predetermined $X_{bt} \in \mathcal{I}_{bt}$ or dynamic $X_{bt} = f(X_{bt-1})$
 - **flexible** if neither predetermined $X_{bt} \notin \mathcal{I}_{bt}$ nor dynamic $X_{bt} \neq f(X_{bt-1})$.

Behavioral framework

Production function and productivity

$$Y_{bt} = F^{klm}(K_{bt}, L_{bt}, M_{bt}) e^{\omega_{bt} + \varepsilon_{bt}} \quad \Leftrightarrow$$
$$y_{bt} = f^{klm}(k_{bt}, l_{bt}, m_{bt}) + \omega_{bt} + \varepsilon_{bt},$$

- Productivity $\omega_{bt} \in \mathcal{I}_{bt}$ and ex-post productivity $\varepsilon_{bt} \notin \mathcal{I}_{bt}$
- Furthermore, firm-level productivity ω_{bt} follows **cluster-dependent** first-order Markov process

$$p(\omega_{bt} | \mathcal{I}_{bt-1}) = p(\omega_{bt} | \omega_{bt-1}, z_b^s),$$

- Each firm b belongs to a certain cluster $s = 1, \dots, S$, indicated by $z_b^i = \mathbb{I}_b(s = i), \forall i = 1, \dots, S$

Behavioral framework

Firm's problem

- Optimal **one-off** decision rule with firm-specific cluster affinity $\epsilon(z_b^s)$:

$$z_b^* (K_{b0}, L_{b0}, e^{\omega_{b0}}, \epsilon) = \arg \max_{z_b^s} \left(\pi_{b0} (K_{b0}, L_{b0}, e^{\omega_{b0}}) + \epsilon(z_b^s) + E_{\omega} \left[\sum_{t=1}^T \beta^{t-1} \pi_{bt} (K_{bt}, L_{bt}, e^{\omega_{bt}}, z_b^s) \right] \right)$$

⇒ Probability of cluster affiliation:

$$Pr(z_b^s | K_{b0}, L_{b0}, e^{\omega_{b0}}) = \int \mathbb{I} [z_b^* (K_{b0}, L_{b0}, e^{\omega_{b0}}) = z_b^s] f^{\epsilon}(\epsilon) d\epsilon.$$

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Production Function Estimation

Parameter identification

Immediate identification of the production function parameters based on

$$y_{bt} = f^{klm}(k_{bt}, l_{bt}, m_{bt}) + \omega_{bt} + \varepsilon_{bt}$$

not possible due to simultaneity: $E[(\omega_{bt} + \varepsilon_{bt}) | k_{bt}, l_{bt}, m_{bt}] \neq 0$, and ω_{bt} unobserved.

Parameter identification

Solution? Resort to two-stage procedure.

Stage 1: Rely on flexible input m_{bt} (Akerberg et al. (2015); Gandhi et al. (2020),...) to obtain

$$\phi_{bt} = f^{kl}(k_{bt}, l_{bt}) + \omega_{bt},$$

where ϕ_{bt} represents non-flexible output variation.

Parameter identification

Stage 2: Rely on Markov property $\omega_{bt} = [g(\omega_{bt-1}) + \eta_{bt}]$:

$$\phi_{bt} = f^{kl}(k_{bt}, l_{bt}) + \left[g(\phi_{bt-1} - f^{kl}(k_{bt-1}, l_{bt-1})) + \eta_{bt} \right].$$

⇒ Moment conditions that allow parameter estimation with GMM:

$$E \left[\eta_{bt} \middle| k_{bt}, l_{bt(-1)}, \phi_{bt-1} \right] = 0$$

Parameter identification with unobserved heterogeneity

Stage 2: Rely on Markov property $\omega_{bt} = \sum_{s=1}^S z_b^s [g^s(\omega_{bt-1}) + \eta_{bt}^s]$:

$$\phi_{bt} = f^{kl}(k_{bt}, l_{bt}) + \sum_{s=1}^S z_b^s \left[g^s(\phi_{bt-1} - f^{kl}(k_{bt-1}, l_{bt-1})) + \eta_{bt}^s \right].$$

⇒ Moment conditions that contain unobserved heterogeneity:

$$E \left[\sum_{s=1}^S z_b^s \eta_{bt}^s \middle| k_{bt}, l_{bt(-1)}, \phi_{bt-1} \right] = 0$$

Parameter estimation with unobserved heterogeneity

Unobserved heterogeneity can be accounted for through a Likelihood specification:

1. Reduced-form **multinomial logit** for cluster affiliation:

$$Pr(z_b^s | k_{b0}, l_{b0}, \omega_{b0}; \gamma^1, \dots, \gamma^s) = \frac{e^{\gamma_0^i + \gamma_k^i k_{b0} + \gamma_l^i l_{b0} + \gamma_\omega^i \omega_{b0}}}{\sum_{s=1}^S e^{\gamma_0^s + \gamma_k^s k_{b0} + \gamma_l^s l_{b0} + \gamma_\omega^s \omega_{b0}}}, \quad \forall i = 1, \dots, S.$$

Parameter estimation with unobserved heterogeneity

2. Limited Information Likelihood conditional on cluster affiliation

- Productivity follows a **Gaussian Mixture** (Dewitte et al., 2020)

$$\eta_{bt}^s = \phi_{bt} - f^{kl}(k_{bt}, l_{bt}; \beta) - g(\phi_{bt-1}, l_{bt-1}, k_{bt-1}; \beta, \alpha^s) \sim \mathcal{N}(0, (\sigma_\eta^s)^2)$$

- Reduced-form instrumental equation for endogenous labor

$$\zeta_{bt}^s = l_{bt} - \delta_0 - \delta_1 k_{bt} - \delta_2^s \phi_{bt-1} - \delta_3^s k_{bt-1} - \delta_4^s l_{bt-1} \sim \mathcal{N}(0, (\sigma_\zeta^s)^2)$$

⇒ Bivariate normal specification

$$p^o(\phi_{bt}, l_{bt} | \cdot) \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} (\sigma_\eta^s)^2 & \sigma_{\eta, \zeta} \\ \sigma_{\eta, \zeta} & (\sigma_\zeta^s)^2 \end{bmatrix}\right)$$

Parameter estimation with unobserved heterogeneity

Complete log-likelihood

$$\mathcal{L}^c(\boldsymbol{\Theta}, \mathbf{z}) = \sum_{b=1}^B \sum_{s=1}^S z_b^s \log \left(\Pr(z_b^s | k_{b0}, l_{b0}, \omega_{b0}; \boldsymbol{\gamma}^s) \right. \\ \left. \times \prod_{t=1}^T p(\phi_{bt}, l_{bt} | k_{bt}, l_{bt}, \phi_{bt-1}, l_{bt-1}, k_{bt-1}, z_b^s; \boldsymbol{\theta}^s) \right)$$

⇒ Estimate with Expectation-Maximization algorithm

Comparison with alternative identification strategies

Example

Assume Cobb-Douglas production function, AR(1) productivity and two clusters:

$$\begin{aligned}\phi_{bt} = & \beta_k k_{bt} + \beta_l l_{bt} + \mathbb{I}(EXP_b = 1) (\alpha_0^1 + \alpha_1^1 (\phi_{bt-1} - \beta_k k_{bt-1} - \beta_l l_{bt-1}) + \eta_{bt}^1) \\ & + \mathbb{I}(EXP_b = 2) (\alpha_0^2 + \alpha_1^2 (\phi_{bt-1} - \beta_k k_{bt-1} - \beta_l l_{bt-1}) + \eta_{bt}^2).\end{aligned}$$

When imposing a unitary process:

$$\phi_{bt} = \beta_k k_{bt} + \beta_l l_{bt} + \alpha_0^* + \alpha_1^* (\phi_{bt-1} - \beta_k k_{bt-1} - \beta_l l_{bt-1}) + \eta_{bt}^*.$$

If $\alpha_{0,1}^* \geq 0$, by definition, the omitted cluster indicator is correlated with the remaining explanatory variables and will positively/negatively bias the estimated coefficients (see, f.i., De Loecker (2013)).

Comparison with alternative identification strategies

Posterior specification

1. Unitary cluster affiliation: $E \left[\eta_{bt}^* \middle| k_{bt}, l_{bt(-1)}, \phi_{bt-1} \right] \neq 0$.

2. Deterministic cluster affiliation: proxy variable $E\tilde{X}P_b$ for EXP_b :

$$E \left[\sum_{s=1}^2 \mathbb{I} \left(E\tilde{X}P_b = s \right) \eta_{bt}^s \middle| k_{bt}, l_{bt(-1)}, \phi_{bt-1} \right] = 0.$$

3. Random cluster affiliation:

$$E \left[\sum_{s=1}^S Pr(z_b^s | \mathbf{k}_b, \mathbf{l}_b, \phi_b, E\tilde{X}P_b; \hat{\Theta}) \eta_{bt}^s \middle| k_{bt}, l_{bt(-1)}, \phi_{bt-1} \right] = 0.$$

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Monte Carlo

Setup

Adaptation of Akerberg et al. (2015)'s Monte Carlo exercise with heterogeneity in productivity between groups of firms

- 100 simulated datasets of 1,000 firms over 10 years
- Value-added production technology with endogenous labor
- Productivity follows a Finite Mixture AR(1)-process

$$\omega_{bt} = \sum_{s=1}^2 z_b^s [\alpha_0^s + \alpha_1^s \omega_{bt-1} + \eta_{bt}^s], \quad (1)$$

with $Pr(z_b^1) = 0.8$, $Pr(z_b^2) = 0.2$ and $\eta_{bt}^s \sim \mathcal{N}(0, \sigma_\eta^s)$.

Monte Carlo results

DGP 2

Methodology	β_k	β_l	α_0^1	α_1^1	σ_η^1	α_0^2	α_1^2	σ_η^2	$Pr(z_b^1)$	$Pr(z_b^2)$
True coefficients	0.40	0.60	1.00	0.70	0.21	0.80	0.77	0.25	80	20
GMM	0.45 (0.01)	0.60 (0.01)	0.83 (0.03)	0.71 (0.01)	0.22 (0.00)	- (-)	- (-)	- (-)	100 (0.00)	100 (0.00)
LIML	0.45 (0.01)	0.60 (0.01)	0.83 (0.03)	0.71 (0.01)	0.22 (0.00)	- (-)	- (-)	- (-)	100 (0.00)	100 (0.00)
2-comp. LIML	0.40 (0.02)	0.60 (0.01)	0.99 (0.05)	0.70 (0.01)	0.21 (0.00)	0.80 (0.08)	0.76 (0.02)	0.25 (0.01)	80.53 (3.51)	19.47 (3.51)

Monte Carlo results

Overview

Omitted variable bias exists, but our proposed estimator can correct this bias

- ... even with endogenous labor;
- ... even without proxy variables;
- ... almost perfectly with proxy variables;
- ... even with noisy proxy variables.

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Application to Belgian firm-level data

Estimation framework

Production function estimation

- Belgian firm-level data on the manufacturing industry between 2008-2018
 - Focus on rubber and plastic products sector (sector 22): 4,399 observations from 626 firms.
- **First stage:** Value-added Translog specification (Akerberg et al., 2015)
- **Second stage:**
 - GMM without additional heterogeneity in the AR(1) process
 - LIML with increasing heterogeneity (nr. clusters) in the AR(1) process

Production function results

Value Added specification with endogenous labor for sector 22

Posteriors

Description	GMM	LIML					
		1-comp.	2-comp.	3-comp.	4-comp.	5-comp.	6-comp.
Capital	0.130 (0.016)	0.118 (0.017)	0.124 (0.017)	0.124 (0.018)	0.124 (0.016)	0.124 (0.020)	0.124 (0.020)
Labor	0.879 (0.020)	0.860 (0.045)	0.854 (0.023)	0.866 (0.027)	0.860 (0.028)	0.852 (0.027)	0.857 (0.029)
RTS	1.009 (0.015)	0.978 (0.038)	0.979 (0.018)	0.990 (0.020)	0.984 (0.023)	0.977 (0.026)	0.981 (0.025)
Std. Dev.	0.180 (0.016)	0.159 (0.030)	0.156 (0.019)	0.157 (0.017)	0.156 (0.018)	0.155 (0.019)	0.156 (0.016)
Nr. parameters	7	20	37	54	71	88	105
NLL		-6835	-8735	-9134	-9321	-9528	-9647
BIC		-13505	-17166	-17824	-18060	-18335	-18434
ICLbic		-13505	-17121	-17714	-17903	-18163	-18246

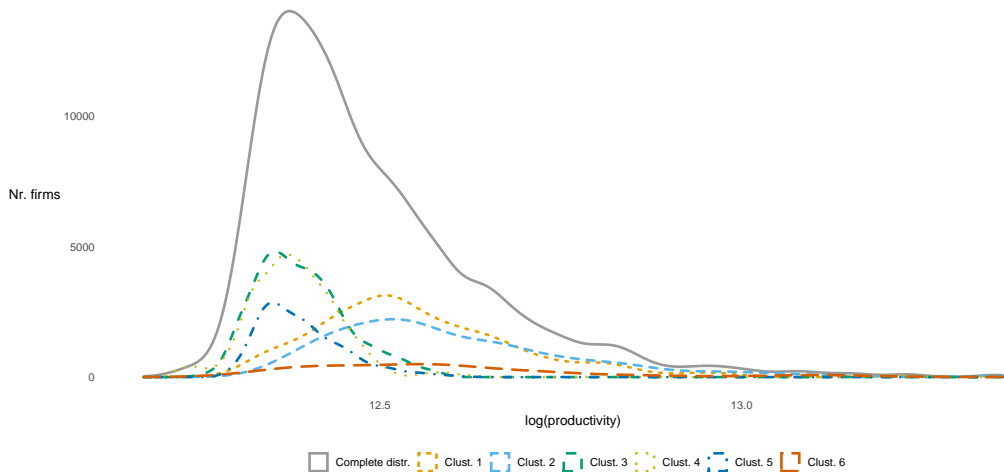
Heterogeneity in the productivity evolution

Coefficients of the Markov process [Visualization](#)

Cluster description	Prop. (%)	α_0	α_1	σ_η	μ_ω	σ_ω
Cluster 1	24.973 (2.685)	0.677 (0.147)	0.946 (0.010)	0.041 (0.005)	12.522 (0.673)	0.126 (0.020)
Cluster 2	20.824 (3.612)	1.029 (0.120)	0.918 (0.011)	0.059 (0.008)	12.575 (0.683)	0.149 (0.015)
Cluster 3	20.000 (4.788)	0.679 (0.117)	0.945 (0.010)	0.022 (0.004)	12.400 (0.676)	0.068 (0.015)
Cluster 4	18.626 (2.108)	0.805 (0.215)	0.935 (0.018)	0.018 (0.002)	12.367 (0.679)	0.050 (0.009)
Cluster 5	9.808 (2.272)	2.424 (0.481)	0.805 (0.039)	0.032 (0.004)	12.400 (0.681)	0.054 (0.008)
Cluster 6	5.769 (0.556)	3.532 (0.610)	0.719 (0.039)	0.132 (0.015)	12.568 (0.668)	0.190 (0.027)

Cluster identification

Productivity density by clusters for sector 22



Cluster characterization

Available firm-level characteristics do **not improve nor explain** cluster affiliation

$$\frac{Pr(z_b^i | \dots; \gamma^i)}{Pr(z_b^1 | \dots; \gamma^1)} = \gamma_0^i + \gamma_1^i k_{b0} + \gamma_2^i l_{b0} + \gamma_3^i \omega_{b0} + \gamma_4^i age_{b0} \\ + \gamma_5^i Exp_b + \gamma_6^i Imp_b \\ + \gamma_7^i FDI_b, \quad \forall i = 2, \dots, S$$

Specification	Log-likelihood	BIC	ICLbic
Base specification	9,647.45	-18,433.93	-18,245.55
Augmented specification	9,658.52	-18,292.07	-18,112.65
Augmented specification without initial conditions	9,602.63	-18,262.30	-18,063.17

Cluster characterization

Summary statistics **Visualization**

	Overall	Clust. 1	Clust. 2	Clust. 3	Clust. 4	Clust. 5	Clust. 6
Cluster proportions (%)	100.00	24.97	20.82	20.00	18.63	9.81	5.77
log(Initial output)	15.17	16.05	14.81	14.65	16.10	13.67	14.78
log(Initial capital)	13.28	13.86	12.86	12.91	14.12	12.42	12.89
log(Initial labour)	2.78	3.37	2.05	2.67	3.97	1.80	2.03
log(Initial productivity)	12.49	12.56	12.62	12.39	12.38	12.38	12.61
Initial age	24.80	26.59	21.18	26.13	29.41	20.25	22.10
Exporter prop. (%)	65.13	78.87	59.83	51.79	81.63	46.15	60.78
Importer prop. (%)	80.68	91.55	86.32	69.64	90.82	58.46	70.59
FDI prop. (%)	10.26	14.79	3.42	7.14	21.43	3.08	7.84

- Younger firms correlate with less persistent productivity processes
- Initial productivity correlates strongly with stationary productivity
- Large and international firms group in clusters 1 and 4

Exporter characterization

Average productivity premia for sector 22

Necessity to control for unobserved heterogeneity when comparing productivity across groups of firms, for instance for exporters vs. non-exporters:

$$\omega_{bt} = \alpha_0 + \alpha_1 \omega_{bt-1} + \alpha_2 \text{Exp}_b + \alpha_3 \omega_{bt-1} \text{Exp}_b + \alpha_4 \text{Age}_{b0} + \alpha_5 \text{Imp}_b + \alpha_6 \omega_{bt-1} \text{Imp}_b + \alpha_7 \text{FDI}_b + \alpha_8 \omega_{bt-1} \text{FDI}_b + \eta_{bt}.$$

Method	No control	Deterministic	Exhaustive
GMM	-1.493 (0.836)	0.276 (1.058)	0.760 (2.388)
LIML	2.559 (5.960)	3.857 (8.824)	2.854 (5.465)
Method	Base spec.	Deterministic	Exhaustive
6 - comp. LIML	2.063 (2.108)	2.092 (2.153)	2.011 (2.116)

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Conclusion

- General extension of state-of-the-art production function estimation procedures to control for, and identify, **unobserved heterogeneity** in the evolution of productivity.
- Strong evidence of heterogeneity in the evolution of productivity
 - Positively correlated with the initial conditions of a firm, especially with **initial productivity**.
 - Export, import, and FDI status correlated with multiple clusters \Rightarrow heterogeneity beyond what is captured by these observed firm-level characteristics
- Contrary to existing methods, our estimator **maintains its performance in the face of supplementary information**

Future research

- What drives these clusters?
- Allow for regime-switching (Van Biesebroeck, 2003)
- Non-Hicks neutral productivity
- ...

Looking forward to working with you!



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Appendix

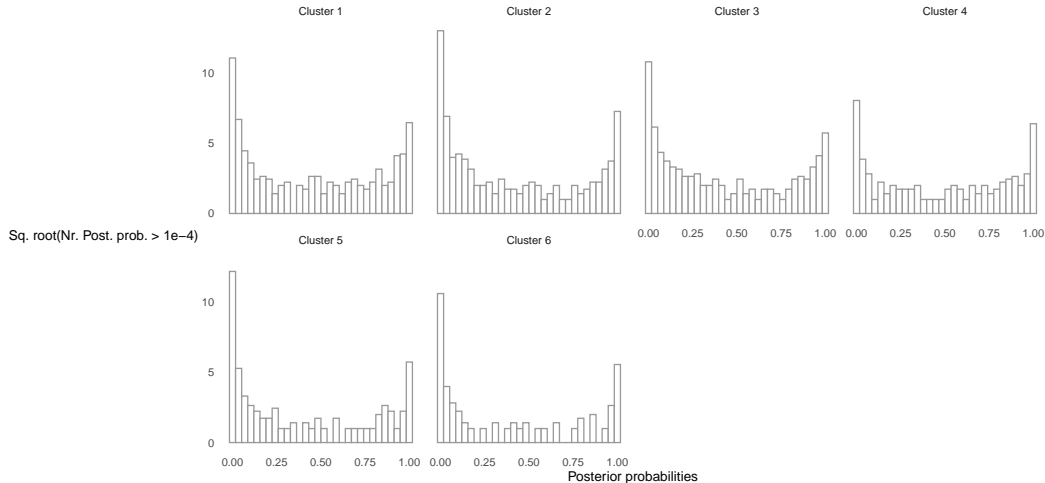
Posterior specification [Go back](#)

From Bayes' theorem:

$$\hat{z}_b^s = Pr(z_b^s | \mathbf{k}_b, \mathbf{l}_b, \phi_b; \Theta) = \frac{Pr(z_b^s | k_{b0}, l_{b0}, \omega_{b0}; \gamma^s) p^\circ(\phi_b, \mathbf{l}_b | \mathbf{k}_b, \mathbf{l}_b, \phi_b, z_b^s; \theta^s)}{p^\circ(\phi; \Theta)}.$$

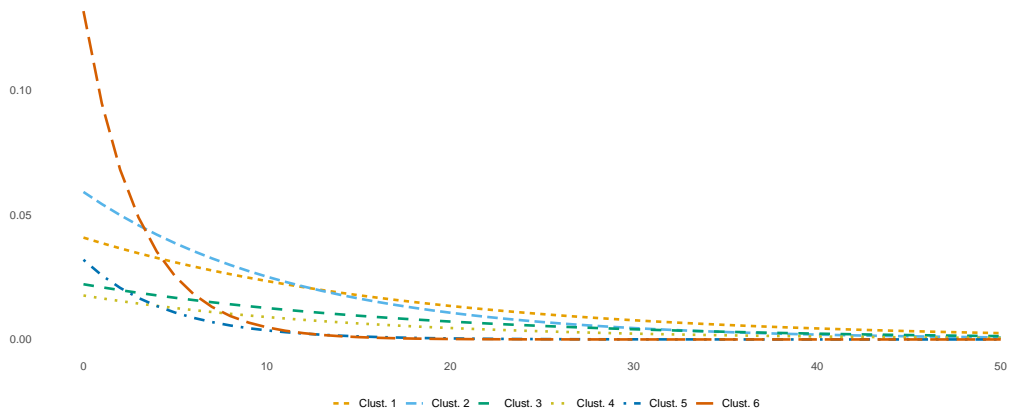
Production function estimation results

Histogram of posterior probabilities for a 6-cluster production function

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Heterogeneity in the productivity evolution

Impulse-Response to a one std. dev. productivity shock for sector 22

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Exporter characterization

Cluster affiliation probability conditional **only** on initial productivity and exporting

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