Knowledge Representation and Reasoning

Exercises on Description Logics

1 Relationship with First-Order Logic

Translate the following $\mathcal{ALC}$ concepts into English and then into FOL:

1. $Father \sqcap \forall .child. (Doctor \sqcup Manager)$
2. $\exists manages. (Company \sqcap \exists employs.Doctor)$
3. $Father \sqcap \forall .child. (Doctor \sqcup \exists manages. (Company \sqcap \exists employs.Doctor))$

2 Knowledge Representation in $\mathcal{ALC}$

Let Man, $Woman$, $Male$, $Female$, and $Human$ be concept names, and let $has\text{-}child$, $is\text{-}brother\text{-}of$, $is\text{-}sister\text{-}of$, and $is\text{-}married\text{-}to$ be role names. Construct a TBox that contains definitions for $Mother$, $Grandfather$, $Niece$, $Father$, $Aunt$, $Nephew$, $Grandmother$, $Uncle$, and $Mother\text{-}of\text{-}at\text{-}least\text{-}one\text{-}male$.

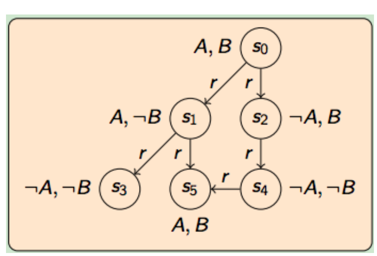
3 Knowledge Representation in $\mathcal{ALC}$

Express the following sentences in terms of the description logic $\mathcal{ALC}$.

1. All employees are humans.
2. A mother is a female who has a child.
3. A parent is a mother or a father.
4. A grandmother is a mother who has a child who is a parent.
5. Only humans have children that are humans.

4 Semantics of $\mathcal{ALC}$

Let $\mathcal{I}$ be the following $\mathcal{ALC}$ interpretation on the domain $\Delta^{\mathcal{I}} = \{s_0, s_1, ..., s_5\}$.



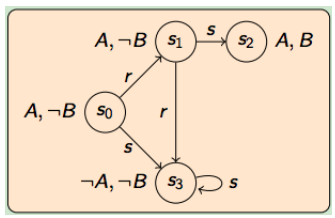
Determine the interpretation of the following concepts:

1. $\top^{\mathcal{I}}$.
2. $\bot^{\mathcal{I}}$.
3. $A^{\mathcal{I}}$.

4. $B^{\mathcal{I}}$.
5. $(A \sqcap B)^{\mathcal{I}}$.
6. $(A \sqcup B)^{\mathcal{I}}$.
7. $(\neg A)^{\mathcal{I}}$.
8. $(\exists r.A)^{\mathcal{I}}$.
9. $(\forall r.\neg B)^{\mathcal{I}}$.
10. $(\forall r.(A \sqcup B))^{\mathcal{I}}$.

5 Semantics of $\mathcal{ALC}$

Let $\mathcal{I}$ be the following $\mathcal{ALC}$ interpretation on the domain $\Delta^{\mathcal{I}} = \{s_0, s_1, ..., s_3\}$.



Determine the interpretation of the following concepts:

1. $(A \sqcup B)^{\mathcal{I}}$.
2. $(\exists s.\neg A)^{\mathcal{I}}$.
3. $(\forall s.A)^{\mathcal{I}}$.
4. $(\exists s.\exists s.\exists s.\exists s.A)^{\mathcal{I}}$.
5. $(\neg \exists r.(\neg A \sqcup \neg B))^{\mathcal{I}}$.
6. $(\exists s.(A \sqcup \forall s.\neg B) \sqcup \neg \forall r.\exists r.(A \sqcup \neg A))^{\mathcal{I}}$.

6 (Un)Satisfiability and Validity of $\mathcal{ALC}$

For each of the following formulas, indicate if it is valid, satisfiable or unsatisfiable. If it is not valid, provide a model that falsifies it:

1. $\forall r.(A \sqcap B) \equiv \forall r.A \sqcap \forall r.B$.
2. $\forall r.(A \sqcup B) \equiv \forall r.A \sqcup \forall r.B$.
3. $\exists r.(A \sqcap B) \equiv \exists r.A \sqcap \exists r.B$.
4. $\exists r.(A \sqcup B) \equiv \exists r.A \sqcup \exists r.B$.

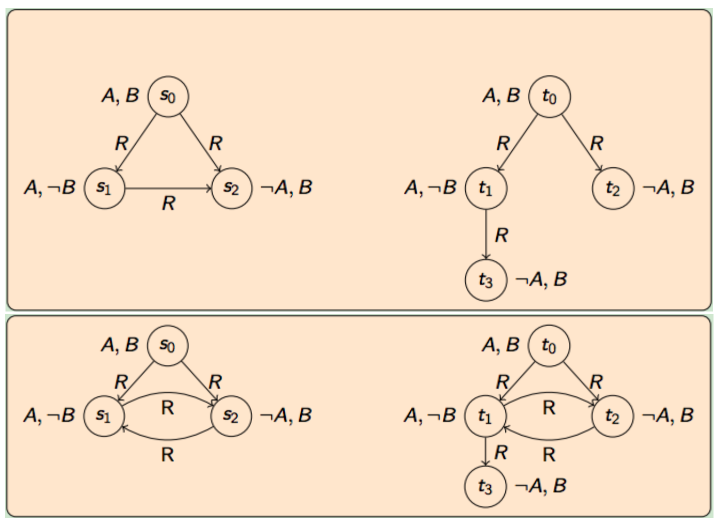
7 (Un)Satisfiability and Validity of $\mathcal{ALC}$

For each of the following concepts, indicate if it is valid, satisfiable or unsatisfiable. If it is valid, or unsatisfiable, provide a proof. If it is satisfiable (and not valid), then exhibit a model that interprets the concept in a non-empty set:

1. $\neg(\forall r.A \sqcup \exists r.(\neg A \sqcap \neg B))$.
2. $\exists r.(\forall s.C) \sqcap \forall r.(\exists s.\neg C)$.
3. $(\exists s.C \sqcap \exists s.D) \sqcap \forall s.(\neg C \sqcup \neg D)$.
4. $\exists s.(C \sqcap D) \sqcap (\forall s.\neg C \sqcup \forall s.\neg D)$.
5. $C \sqcap \exists r.A \sqcap \exists r.B \sqcap \neg\exists r.(A \sqcap B)$

8 Bissimulation

For each of the following pairs of models, check if they are bisimular. If yes, find the bisimulation relation, if not find a formula that is true in the first model and false in the second.



9 $\mathcal{ALC}$ Tableaux

Check by means of tableaux whether the following subsumption is valid:

1. $\neg\forall r.A \sqcap \forall r.((\forall r.B) \sqcup A) \sqsubseteq \forall r.\neg(\exists r.A) \sqcup \exists r.(\exists r.B)$

10 $\mathcal{ALC}$ Tableaux

Which of the following statements are true? Explain your answer.

1. $\forall r.(A \sqcap B) \sqsubseteq \forall r.A \sqcap \forall r.B$
2. $\forall r.A \sqcap \forall r.B \sqsubseteq \forall r.(A \sqcap B)$
3. $\forall r.A \sqcup \forall r.B \sqsubseteq \forall r.(A \sqcup B)$
4. $\forall r.(A \sqcup B) \sqsubseteq \forall r.A \sqcup \forall r.B$
5. $\exists r.(A \sqcap B) \sqsubseteq \exists r.A \sqcap \forall r.B$
6. $\exists r.(A \sqcup B) \sqsubseteq \exists r.A \sqcup \forall r.B$

7. $\exists r.A \sqcup \forall r.B \sqsubseteq \exists r.(A \sqcup B)$
8. $\exists r.A \sqcap \forall r.B \sqsubseteq \exists r.(A \sqcap B)$