Computational Game Theory

Exercises on Coalitional Game Theory

1. Core

Three players together can obtain 1 to share, any two players can obtain 0.8, and one player by herself can obtain zero. Then, N=3 and v(1)=v(2)=v(3)=0, v(1,2)=v(2,3)=v(3,1)=0.8, v(1,2,3)=1.
Which allocation is in the core of this coalitional game?

i. (0,0,0); ii. (0.4, 0.4, 0); iii. (1/3, 1/3, 1/3); iv. The core is empty;

2. Buyers and Sellers

There is a market for an indivisible good with B buyers and S sellers. Each seller has only one unit of the good and has a reservation price of 0. Each buyer wants to buy only one unit of the good and has a reservation price of 1. Thus v(C)=min(B_C,S_C) where B_C and S_C are the number of buyers and sellers in coalition C (and so, for instance, v(i)=0 for any single player, and v(i,j)=1 if i,j are a pair of a buyer and seller).

a) If the number of buyers and sellers is B=2 and S=1, respectively, which allocations are in the core? [There might be more than one]

i. Each seller receives 1 and each buyer receives 0.
ii. Each seller receives 0 and each buyer receives 1.
iii. Each seller receives 1/2 and each buyer receives 1/2.

b) Now assume that competition among sellers increases, so that B=2 and S=2. Which allocations are in the core? [There might be more than one]

i. Each seller receives 1 and each buyer receives 0.
ii. Each seller receives 0 and each buyer receives 1.
iii. Each seller receives 1/2 and each buyer receives 1/2.

3. Core and Shapley Value

The instructor of a class allows the students to collaborate and write up together a particular problem in the homework assignment. Points earned by a collaborating team are divided among the students in any way they agree on. There are exactly three students taking the course, all equally talented, and they need to decide which of them if any should collaborate. The problem is so hard that none of them working alone would score any points. Any two of them can score 4 points together. If all three collaborate, they can score 6 points.

a) Which allocation is in the core of this coalitional game?

i. (2, 2, 2); ii. (0,0,0); iii. (2, 2, 0); iv. The core is empty;

b) What is the Shapley value of each player?

i. ϕ=(0,0,0) ii. ϕ=(2,0,2) iii. ϕ=(1/3,1/3,1/3) iv. ϕ=(2,2,2)

4. Production

There is a single capitalist (c) and a group of 2 workers (w1 and w2). The production function is such that total output is 0 if the firm (coalition) is composed only of the capitalist or of the workers (a coalition between the capitalist and a worker is required to produce positive output).
The production function satisfies:

$F(c \cup w1)=F(c \cup w2)=3$ $F(c \cup w1 \cup w2)=4$

a) Which allocations are in the core of this coalitional game? [There might be more than one]

i. $x_c=2$, $x_{w1}=1$, $x_{w2}=1$;   ii. $x_c=2.5$, $x_{w1}=0.5$, $x_{w2}=1$;   iii. $x_c=4$, $x_{w1}=0$, $x_{w2}=0$;

b) What is the Shapley value of the capitalist?

i. 3;   ii. 4;   iii. 7/3;   iv. 7;

c) What is the Shapley value of each worker?

i. 1;   ii. 5/6;   iii. 3/4;   iv. 1/2;

d) True or False: If there was an additional 3rd worker that is completely useless (i.e., his marginal contribution is 0 in every coalition), then the sum of the Shapley Values of the capitalist and the first two workers will remain unchanged.