Computational Game Theory

Exercises on Extensive Form Games

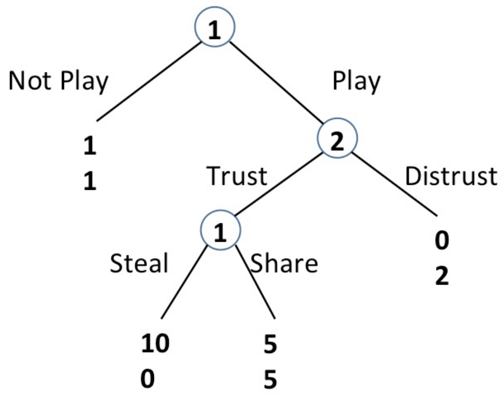
1. Splitting Coins

Two players have to share 50 coins (of equal value). Players' payoffs are the number of coins they each get. First, player 1 splits the coins into 2 piles. Second, player 2 chooses one pile for him/herself and gives the other pile to player 1.
What is agent 1's strategy in a backward induction solution?

i) Splitting coins into 25/25.
ii) Splitting coins into 0/50.
iii) Splitting coins into 15/35.
iv) Splitting coins into 1/49.

2. Trust Game

Consider the following game.



a) Find all of the pure strategy Nash Equilibria. There can be more than one equilibria. [Here ((Not Play,Steal),(Trust)) indicates that player 1 chooses Not Play at the first decision node and Steal at the second decision node, and 2 chooses Trust at his unique decision node.]

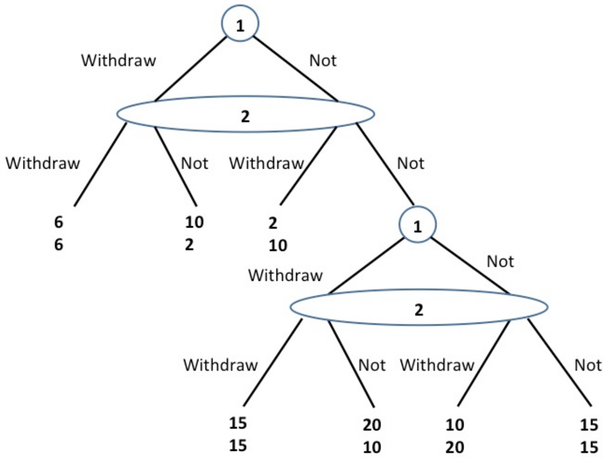
i) ((Not play, Steal),(Distrust))
ii) ((Not play, Share), (Distrust))
iii) ((Not play, Steal), (Trust))
iv) ((Play, Steal), (Distrust))
v) ((Play, Share), (Trust))

b) Which is the Subgame Perfect Equilibrium of this game?

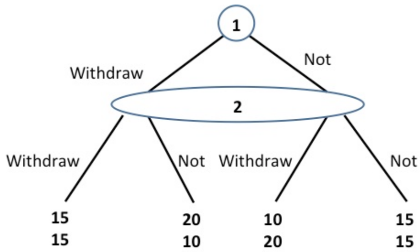
i) ((Not play, Steal),(Distrust))
ii) ((Not play, Share), (Distrust))
iii) ((Not play, Steal), (Trust))
iv) ((Play, Steal), (Distrust))
v) ((Play, Share), (Trust))

3. Investors

- There are 2 investors. Each has deposited \$10 in the same bank.
- The bank invested both deposits in a single long-term project.
  - If the bank wants to end the project before its completion, a total of \$12 can be recovered (out of the \$20 invested).
  - If the bank waits until the project is completed, it will receive a total of \$30. Investors can withdraw money from their bank accounts at only 2 periods: before the project is completed and after.
- The extensive from representation of the game between both investors is depicted below:



In order to find the subgame perfect Nash equilibrium of the whole game first focus on the subgame that starts with investor 1's second decision node:



a) Which is a pure strategy Nash equilibrium of this subgame?

i) (Withdraw, Withdraw)
ii) (Withdraw, Not)
iii) (Not, Withdraw)
iv) (Not, Not)

b) What are the subgame perfect Nash equilibria of the whole game? There might be more than one. [Hint: ((Withdraw, Not),(Not, Withdraw)) are the first and second investors' strategies in their first and second decision nodes, respectively. So, ((Withdraw, Not),(Not, Withdraw)) indicates that the first investor withdraws at her first decision node but not at her second, while the second invest does not withdraw at his first decision node but does at his second decision node.]

i) ((Withdraw, Withdraw), (Withdraw, Withdraw))
ii) ((Withdraw, Withdraw), (Not, Withdraw))
iii) ((Not, Withdraw), (Withdraw, Withdraw))
iv) ((Not, Withdraw), (Not, Withdraw))

4. Pirates' Games

- Five pirates have obtained 100 gold coins and have to divide up the loot. The pirates are all extremely intelligent, treacherous and selfish (especially the captain) each wanting to maximize the number of coins that he gets.
- It is always the captain who proposes a distribution of the loot. All pirates vote on the proposal, and if half the crew or more go "Aye", the loot is divided as proposed.
- If the captain fails to obtain support of at least half his crew (which includes himself), all pirates turn against him and make him walk the plank. The pirates then start over again with the next most senior pirate as captain (the pirates have a strict order of seniority denoted by A, B, C, D and E).
- Pirates' preferences are ordered in the following way. First of all, each pirate wants to survive. Second, given survival, each pirate wants to maximize the number of gold coins he receives. Finally, each pirate would prefer to throw another overboard in the case of indifference.

What is the maximum number of coins that the original captain gets to keep across all subgame perfect equilibria of this game? (Hint, work by backward induction to reason what the split will be if three captains have been forced to walk the plank and there are only two pirates left. Just one vote is enough to approve the split among the two pirates. Then use that to solve for what happens when two have walked the plank and there are three pirates left, and so forth.)

i) 100
ii) 0
iii) 50
iv) 98