Computational Game Theory

Exercises on Social Choice

1. Voting Scheme

Consider the following situation:

400 agents: A > B > D > C
300 agents: D > C > B > A
200 agents: B > D > C > A
100 agents: C > A > B > D
two agents: C > D > A > B

a) For the plurality systems, who would be declared the winner? Assume ties are broken alphabetically (for example A defeats B in a tie).
b) Who would be declared the winner by the Plurality with Elimination system?
c) Who would be declared the winner by the Borda Rule System?
d) Who would be declared the winner by the Pairwise Elimination system with ordering D, B, A, C?

2. Social Welfare

Let us adapt the Pairwise Elimination social choice voting scheme into a Social Welfare function. Define this welfare function as follows:
Take the preference ordering of voter #1 to be the agenda.
While outcomes remain to be ordered, do the following:

- Perform pairwise elimination with the current agenda to determine a winner.
- Remove the winner from the agenda.

For example, if voter #1 submits B > D > C > A, then we take B,D,C,A to be the agenda. We perform pairwise elimination with this agenda. Let us say that C wins. We then perform pairwise elimination again, but now with the agenda B,D,A. Let us say A wins. Then we use the agenda B,D. If D now wins, then we output C,A,D,B as our social ordering.

a) Does this social welfare function satisfy pareto efficiency?
b) Does this social welfare function satisfy IIA?
c) Does this social welfare function satisfy Dictatorship?
d) Does this social welfare function satisfy The Condorcet Condition ("If a Condorcet Winner exists, it will win.")?

3. Benign Dictatorship

Consider the following social choice function, which we'll call benign dictatorship (BD). Under this social choice function, each voter submits a strict total ordering over the candidates, and the winning candidate is determined as follows. For each candidate, count the number of times that it appears at the top of a voter's ordering. If there is a single candidate with the maximum count, then it is declared the winner. Otherwise, there must be a tie. The tie is broken according to the preferences of voter #1 (who is, thus, the benign dictator). Note that in case of a tie, the winner need not be #1's top choice. Thus, BD is really a plurality rule with dictatorial tie breaking.
a) Does BD satisfy Weak Pareto efficiency?
b) Does BD satisfy Dictatorship?
c) Does BD satisfy monotonicity?
d) Does BD satisfy The Condorcet Condition ("If a Condorcet Winner exists, it will win.")?
e) Is it ever rational for any voter to lie? More formally, assume that the utility that a player receives from a given winner (of the K candidates) is based on the player's true preferences, e.g. K for the player's first choice, K-1 for the player's second choice, down to 1 for the player's last choice. Is there some situation in which a player would receive more utility from lying than from submitting their true preference ordering?
f) Is it ever rational for voter #1 (the "benign dictator") to lie?