Computing minmax and maxmin

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Maxmin Strategies

- Player 1's maxmin strategy is a strategy that maximizes 1's worst-case payoff, in the situation where player 2 happen to play the strategy which causes the greatest harm to 1.
- The maxmin value (or safety level) of the game for player 1 is that minimum amount of payoff guaranteed by a maxmin strategy.

Definition (Maxmin, 2-player)

The maxmin strategy for player 1 is $\arg\max_{s_1} \min_{s_2} u_1(s_1, s_1)$, and the maxmin value for player 1 is $\max_{s_1} \min_{s_2} u_1(s_1, s_2)$.

Minmax Strategies

- Player 1's minmax strategy against player 2 in a 2-player game is a strategy that minimizes 2's best-case payoff, and the minmax value for 1 against 2 is 2's payoff.

Definition (Minmax, 2-player)

In a two-player game, the minmax strategy for player 1 against player 2 is $\arg\min_{s_1} \max_{s_2} u_2(s_1, s_2)$, and player 2's minmax value is $\min_{s_1} \max_{s_2} u_2(s_1, s_2)$.

Minmax Theorem

Theorem (Minimax theorem (von Neumann, 1928))

In any finite, two-player, zero-sum game, in any Nash equilibrium each player receives a payoff that is equal to both his maxmin value and his minmax value.

Computing a maxmin strategy

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How to compute a maxmin strategy

- Given the game $G = (\{1, 2\}, A_1 \times A_2, (u_1, u_2))$, construct the zero-sum game $G' = (\{1, 2\}, A_1 \times A_2, (u_1, -u_1))$.
- By the minmax theorem, since G' is zero-sum, every strategy for player 1 which is part of a Nash equilibrium strategy profile for G' is a maxmin strategy for player 1 in G'.
- Notice that by definition, player 1's maxmin strategy is independent of player 2's utility function.
- Thus, player 1's maxmin strategy is the same in G and in G'.
- So, finding a maxmin strategy in G reduces to finding a Nash equilibrium of G'.

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