Answer Set Programming

João Leite

Departamento de Informática
Faculdade de Ciências e Tecnologia
Universidade Nova de Lisboa, Portugal
jleite@fct.unl.pt

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Outline

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Classical Negation: Syntax

Generalisation

Extend the language of Logic Programs to allow classical negation $\neg$ (for atoms only!), besides default negation *not* (or $\sim$).

Definition (Language)

Given an alphabet $\mathcal{A}$ of atoms, let $\overline{\mathcal{A}} = \{\neg A \mid A \in \mathcal{A}\}$.

- We assume $\mathcal{A} \cap \overline{\mathcal{A}} = \emptyset$.
- The atoms A and $\neg A$ are complementary.
  - $\neg A$ is the classical negation of A, and vice versa.

Classical Negation: Semantics

Definition (Consistency)

A set X of atoms is consistent, if $X \cap \{\neg A \mid A \in (\mathcal{A} \cap X)\} = \emptyset$, and inconsistent, otherwise.

Definition (Answer Set)

A set X of atoms is an answer set of a logic program Π over $\mathcal{A} \cup \overline{\mathcal{A}}$ if X is an answer set of $\Pi \cup \{B \leftarrow A, \neg A \mid A \in \mathcal{A}, B \in (\mathcal{A} \cup \overline{\mathcal{A}})\}$

Proposition

For a normal or disjunctive logic program Π over $\mathcal{A} \cup \overline{\mathcal{A}}$, exactly one of the following two cases applies:

1. *All answer sets of Π are consistent or*
2. *$X = \mathcal{A} \cup \overline{\mathcal{A}}$ is the only answer set of Π.*

Classical Negation: Examples

Example

- $\Pi_1 = \{cross \leftarrow not\ train\}$
  - Answer set: $\{cross\}$
- $\Pi_2 = \{cross \leftarrow \neg train\}$
  - Answer set: $\emptyset$
- $\Pi_3 = \{cross \leftarrow \neg train,\ \neg train \leftarrow\}$
  - Answer set: $\{cross, \neg train\}$
- $\Pi_4 = \{cross \leftarrow \neg train,\ \neg train \leftarrow,\ \neg cross \leftarrow\}$
  - Answer set: $\{cross, \neg cross, train, \neg train\}$
- $\Pi_5 = \{cross \leftarrow \neg train,\ \neg train \leftarrow not\ train,\ \neg cross \leftarrow\}$
  - No answer set

Classical Negation: Translation

Definition ((Possibly inconsistent) answer sets)

For determining the (possibly inconsistent) answer sets of a logic program Π over $\mathcal{A} \cup \overline{\mathcal{A}}$ in the standard way, translate Π into Π' as follows:

$$\Pi' \quad = \quad \Pi \cup \{B \leftarrow A, \neg A \;\; \neg B \leftarrow A, \neg A \mid A \in \mathcal{A}, B \in \mathcal{A}, A \neq B\}$$

Definition (Consistent answer sets)

In order to determine the answer sets of a logic program Π over $\mathcal{A} \cup \overline{\mathcal{A}}$ in the standard way, translate Π (or $\mathcal{F}$) into Π'' (or $\mathcal{F}''$) as follows:

$$\Pi'' \quad = \quad \Pi \cup \{\leftarrow A, \neg A \mid A \in \mathcal{A}\}$$

Example

Example

- $\Pi = \{p \leftarrow,\ \neg p \leftarrow,\ q \leftarrow not\ r\}$
  $\Pi' = \Pi \cup \{A \leftarrow (B, \neg B),\ \neg A \leftarrow (B, \neg B) \mid A, B \in \{p, q, r\}\}$
  Answer set: $\{p, \neg p, q, \neg q, r, \neg r\}$
- $\Pi = \{p \,;\, q \leftarrow,\ r \leftarrow p,\ \neg r \leftarrow p\ \}$
  $\Pi' = \Pi \cup \{A \leftarrow (B, \neg B),\ \neg A \leftarrow (B, \neg B) \mid A, B \in \{p, q, r\}\}$
  Answer set: $\{q\}$
- $\Pi = \{p \,;\, not\ p \leftarrow \top,\ \neg p \,;\, not\ q \leftarrow \top,\ q \,;\, not\ q \leftarrow \top\}$
  $\Pi' = \Pi \cup \{A \leftarrow (B, \neg B),\ \neg A \leftarrow (B, \neg B) \mid A, B \in \{p, q\}\}$
  Answer sets: $\emptyset$, $\{p\}$, $\{\neg p\}$, $\{\neg p, q\}$, and $\{p, \neg p, q, \neg q\}$

Language Extensions

- The expressiveness of a language can be enhanced by introducing new constructs.
- To this end, we must address the following issues:
  - What is the syntax of the new language construct?
  - What is the semantics of the new language construct?
  - How to implement the new language construct?
- A way of providing semantics is to furnish a translation removing the new constructs, eg. classical negation.
- This translation might also be used for implementing the language extension. When is this feasible?

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Choice Rules

Idea

Choices over subsets.

Syntax

$$\{A_1, \ldots, A_m\} \leftarrow A_{m+1}, \ldots, A_n, not\ A_{n+1}, \ldots, not\ A_o,$$

Informal meaning

If the body is satisfied in an answer set, then any subset of $\{A_1, \ldots, A_m\}$ can be included in the answer set.

Example

The program $\Pi = \{\ \{a\} \leftarrow b,\ b \leftarrow\}$ has two answer sets: $\{b\}$ and $\{a, b\}$.

Implementation

lparse/gringo + smodels/cmodels/nomore/clasp

Choice Rules: Embedding in normal logic programs

Definition (Embedding of Choice Rules in normal logic programs)

A choice rule of form

$$\{A_1, \ldots, A_m\} \leftarrow A_{m+1}, \ldots, A_n, not\ A_{n+1}, \ldots, not\ A_o$$

can be translated into $2m + 1$ rules

$$\begin{array}{ccc} & A \leftarrow A_{m+1}, \ldots, A_n, not\ A_{n+1}, \ldots, not\ A_o. & \\ A_1 \leftarrow A, not\ \overline{A_1}. & \ldots & A_m \leftarrow A, not\ \overline{A_m}. \\ \overline{A_1} \leftarrow not\ A_1. & \ldots & \overline{A_m} \leftarrow not\ A_m \end{array}$$

by introducing new atoms $A, \overline{A_1}, \ldots, \overline{A_m}$.

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Cardinality constraints

Syntax

A (positive) cardinality constraint is of the form $l \ \{A_1, \ldots, A_m\} \ u$

Informal meaning

A cardinality constraint is satisfied in an answer set X, if the number of atoms from $\{A_1, \ldots, A_m\}$ satisfied in X is between l and u (inclusive).
More formally, if $l \leq |\{A_1, \ldots, A_m\} \cap X| \leq u$.

Conditions

$l \ \{A_1 : B_1, \ldots, A_m : B_m\} \ u$ where $B_1, \ldots, B_m$ are used for restricting instantiations of variables occurring in $A_1, \ldots, A_m$.

Example

2 {hd(a),...,hd(m)} 4

Implementation

lparse/gringo + smodels/cmodels/nomore/clasp

n-colorability revisited (with $n = 3$)

Example (n-colorability (with $n = 3$))

C(I)	vertex(1) $\leftarrow$ edge(1,2) $\leftarrow$ vertex(2) $\leftarrow$ edge(2,3) $\leftarrow$ vertex(3) $\leftarrow$ edge(3,1) $\leftarrow$
C(P)	color(r) $\leftarrow$ color(b) $\leftarrow$ color(g) $\leftarrow$ 1 {colored(V,C) : color(C)} 1 $\leftarrow$ vertex(V) $\leftarrow$ edge(V,U),color(C), colored(V,C),colored(U,C)
Answer set	{ colored(1,r), colored(2,b), colored(3,g), ... }

Outline

Cardinality rules

Idea

Control cardinality of subsets.

Syntax

$$A_0 \leftarrow l\ \{A_1, \ldots, A_m, not\ A_{m+1}, \ldots, not\ A_n\}$$

Informal meaning

If at least l elements of the “body” are true in an answer set, then add A_0 to the answer set. l is a lower bound on the “body”

Example

The program $\Pi = \{\ a \leftarrow 1\{b, c\},\ b \leftarrow\}$ has one answer set: $\{a, b\}$.

Implementation

lparse/gringo + smodels/cmodels/nomore/clasp

Cardinality Rules: Embedding in normal logic programs

Definition (Embedding of Cardinality Rules in normal logic programs)

Replace each cardinality rule

$$A_0 \leftarrow l\ \{A_1, \ldots, A_m\} \qquad \text{by} \qquad A_0 \leftarrow cc(1, l)$$

where atom $cc(i, j)$ represents the fact that at least j of the atoms in $\{A_i, \ldots, A_m\}$, that is, of the atoms that have an equal or greater index than i, are in a particular answer set.
The definition of $cc(i, j)$ is given by the rules

$$\begin{array}{rcl} cc(i, j+1) & \leftarrow & cc(i+1, j), A_i \\ cc(i, j) & \leftarrow & cc(i+1, j) \\ cc(m+1, 0) & \leftarrow & \end{array}$$

What about space complexity? The problem is that if the set $\{A_1, ..., A_m\}$ is big, then for this quadratic translation the resulting set of rules is rather large, requiring $O(nl)$ new atoms to be introduced. Moreover, the size of the translation grows towards $O(n^2)$ with the value of l.

Normal Rules: Embedding in Cardinality Rules

Definition (Normal Rules: Embedding in Cardinality Rules)

A normal rule

$$A_0 \leftarrow A_1, \ldots, A_m, not\ A_{m+1}, \ldots, not\ A_n,$$

can be represented by the cardinality rule

$$A_0 \leftarrow n + m\ \{A_1, \ldots, A_m, not\ A_{m+1}, \ldots, not\ A_n\}.$$

Cardinality Rules with upper bounds

Definition (Embedding of Cardinality Rules with upper bounds in normal logic programs)

A rule of the form

$$A_0 \leftarrow l \; \{A_1, \ldots, A_m, not\ A_{m+1}, \ldots, not\ A_n\} \; u$$

stands for

$$\begin{aligned} A_0 &\leftarrow B, not\ C \\ B &\leftarrow l \; \{A_1, \ldots, A_m, not\ A_{m+1}, \ldots, not\ A_n\} \\ C &\leftarrow u+1 \; \{A_1, \ldots, A_m, not\ A_{m+1}, \ldots, not\ A_n\} \end{aligned}$$

Cardinality Constraints as heads

Definition (Embedding of Cardinality Constraints as heads in normal logic programs)

A rule of the form

$$l\ \{A_1, \ldots, A_m\}\ u \leftarrow A_{m+1}, \ldots, A_n, not\ A_{n+1}, \ldots, not\ A_o,$$

stands for

$$\begin{array}{rcl} B & \leftarrow & A_{m+1}, \ldots, A_n, not\ A_{n+1}, \ldots, not\ A_o \\ \{A_1, \ldots, A_m\} & \leftarrow & B \\ C & \leftarrow & l\ \{A_1, \ldots, A_m\}\ u \\ & \leftarrow & B, not\ C \end{array}$$

Full-fledged Cardinality Rules

Definition (Embedding of Cardinality Rules in normal logic programs)

A rule of the form

$$l_0\ S_0\ u_0 \leftarrow l_1\ S_1\ u_1, \ldots, l_n\ S_n\ u_n$$

stands for $0 \leq i \leq n$

$$\begin{array}{rcl} B_i & \leftarrow & l_i\ S_i \\ C_i & \leftarrow & u_i + 1\ S_i \\ A & \leftarrow & B_1, \ldots, B_n, not\ C_1, \ldots, not\ C_n \\ & \leftarrow & A, not\ B_0 \\ & \leftarrow & A, C_0 \\ S_0 \cap \mathcal{A} & \leftarrow & A \end{array}$$

where $\mathcal{A}$ is the underlying alphabet.

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Weight constraints

Syntax

$$l\ [A_1 = w_1, \ldots, A_m = w_m, not\ A_{m+1} = w_{m+1}, \ldots, not\ A_n = w_n]\ u$$

Informal meaning

A weight constraint is satisfied in an answer set X, if

$$l \leq \left(\sum_{1 \leq i \leq m, A_i \in X} w_i + \sum_{m < i \leq n, A_i \notin X} w_i\right) \leq u\ .$$

Generalization of cardinality constraints.

Example

80 [hd(a)=50,. . . ,hd(m)=100] 400

Implementation

lparse/gringo + smodels/cmodels/nomore/clasp

Optimization statements

Idea

Compute optimal answer sets by minimizing or maximizing a weighted sum of given atoms, respectively.

Syntax

$$minimize\ [A_1 = w_1, \dots, A_m = w_m, not\ A_{m+1} = w_{m+1}, \dots, not\ A_n = w_n]$$
$$maximize\ [A_1 = w_1, \dots, A_m = w_m, not\ A_{m+1} = w_{m+1}, \dots, not\ A_n = w_n]$$

Several optimization statements are interpreted lexicographically.

Example

- minimize [hd(a)=30,. . . ,hd(m)=50]
- minimize [road(X,Y) : length(X,Y,L) = L]

Implementation

lparse/gringo + smodels/cmodels/nomore/clasp

Weak integrity constraints

Syntax

$$:\sim A_1, \ldots, A_m, not\ A_{m+1}, \ldots, not\ A_n\ [w : l]$$

Informal meaning

1. minimize the sum of weights of violated constraints in the highest level;
2. minimize the sum of weights of violated constraints in the next lower level;
3. etc

Implementation

dlv

Conditional literals in `gringo`

- We often want to encode the contents of a (multi-)set rather than enumerating each of the elements.
- To support this, `lparse` and `gringo` allow for conditional literals.

Syntax

$$A_0 : A_1 : \ldots : A_m : not\ A_{m+1} : \ldots : not\ A_n$$

Informal meaning

List all ground instances of A_0 such that corresponding instances of $A_1, \ldots, A_m, not\ A_{m+1}, \ldots, not\ A_n$ are true.

Example

`gringo` instantiates the program:

```
p(1). p(2). p(3).  q(2).
{r(X) : p(X) : not q(X)}.
```

to:

```
p(1). p(2). p(3).  q(2).
{r(1), r(3)}.
```

Domain predicates in `gringo`

- The predicates of literals on the right-hand side of a colon (:) must be defined from facts without any negative recursion.
- Such domain predicates are fully evaluated by `gringo`.

Example

```
p(1). p(2).
q(X) :- p(X), not p(X+1).
q(X) :- p(X), q(X+1).
r(X) :- p(X), not r(X+1).
```

- `p/1` and `q/1` are domain predicates because none of them negatively depends on itself.
- `r/1` is not a domain predicate because it is defined in terms of `not r(X+1)`.

See `gringo` documentations for further details.

Outline

Aggregates: Motivation

- Aggregates provide a general way to obtain a single value from a collection of input values given as a set, a bag, or a list.
- Popular aggregate (functions):
  - Average
  - Count
  - Maximum
  - Minimum
  - Sum
- Cardinality and Weight constraints rely on Count and Sum aggregates.

Aggregates: Syntax

Definition (Aggregate)

An aggregate has the form:

$$F \langle A_1 = w_1, \ldots, A_m = w_m, not\ A_{m+1} = w_{m+1}, \ldots, not\ A_n = w_n \rangle \prec k$$

where

- F stands for a function mapping multi-sets of $\mathbb{Z}$ to $\mathbb{Z} \cup \{+\infty, -\infty\}$,
- $\prec$ stands for a relation between $\mathbb{Z} \cup \{+\infty, -\infty\}$ and $\mathbb{Z}$,
- k is an integer,
- A_i is an atom, and
- w_i are integers

for $1 \leq i \leq n$.

Example

$sum \langle hd(a) = 30, \ldots, hd(m) = 50 \rangle \leq 300$

Aggregates: Semantics

Definition (Semantics of Aggregates)

- A (positive) aggregate $F\ \langle A_1 = w_1, \ldots, A_n = w_n \rangle \prec k$ can be represented by the formula:

$$\bigwedge_{I \subseteq \{1,\ldots,n\}, F\langle w_i | i \in I \rangle \not\prec k} \left(\bigwedge_{i \in I} A_i \rightarrow \bigvee_{i \in \bar{I}} A_i \right)$$

  where $\bar{I} = \{1, \ldots, n\} \setminus I$ and $\not\prec$ is the complement of $\prec$.
- Then, $F\ \langle A_1 = w_1, \ldots, A_n = w_n \rangle \prec k$ is true in X iff the above formula is true in X.

Aggregates: An example

Example

- Consider $sum\langle p = 1, q = 1\rangle \neq 1$
  - i.e, $A_1 = p$, $A_2 = q$ and $w_1 = 1$, $w_2 = 1$

I	$\langle w_i \mid i \in I\rangle$	$sum\langle w_i \mid i \in I\rangle$	$sum\langle w_i \mid i \in I\rangle = 1$
$\emptyset$	$\langle\rangle$	0	*false*
$\{1\}$	$\langle 1\rangle$	1	*true*
$\{2\}$	$\langle 1\rangle$	1	*true*
$\{1,2\}$	$\langle 1,1\rangle$	2	*false*

- We get $(p \rightarrow q) \wedge (q \rightarrow p)$
- Analogously, we obtain $(p \vee q) \wedge \neg(p \wedge q)$ for $sum\langle p = 1, q = 1\rangle = 1$.

Recall

$$\bigwedge_{I\subseteq\{1,\ldots,n\},F\langle w_i | i\in I\rangle \not\prec k} \left(\bigwedge_{i\in I} A_i \rightarrow \bigvee_{i\in \bar{I}} A_i \right)$$

Aggregates: Monotonicity

Monotone aggregates

- For instance,
  - $body^+(r)$
  - $sum\langle p = 1, q = 1\rangle > 1$ amounts to $q \wedge p$
- We get a simpler characterization: $\bigwedge_{I\subseteq\{1,\ldots,n\}, F\langle w_i \mid i\in I\rangle \not\prec k} \bigvee_{i\in\bar{I}} A_i$

Anti-monotone aggregates

- For instance,
  - $body^-(r)$
  - $sum\langle p = 1, q = 1\rangle < 1$ amounts to $\neg p \wedge \neg q$
- We get a simpler characterization: $\bigwedge_{I\subseteq\{1,\ldots,n\}, F\langle w_i \mid i\in I\rangle \not\prec k} \neg \bigwedge_{i\in I} A_i$

Non-monotone aggregates

- For instance, $sum\langle p = 1, q = 1\rangle \neq 1$ is non-monotone.

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