# Computational Game Theory

*Exercises on Bayesian Games*

## 1. War Game

Two opposed armies are poised to seize an island. Each army can either "attack" or "not-attack". Also, Army 1 is either "weak" or "strong" with probability p and (1−p), respectively. Army 2 is always "weak". Army's 1 type is known only to its general. An army can capture the island either by attacking when its opponent does not or by attacking when its rival does if it is strong and its rival is weak. If two armies of equal strength both attack, neither captures the island.
The payoffs are as follows: - The island is worth M if captured;- An army has a "cost" of fighting, which is equal to s>0 if it is strong and w>0 if it is weak (where s<w<M); - There is no cost of attacking if its rival does not attack.
These payoffs are pictured in the payoff matrices below:

Weak
(with probability p)

| 1/2 | Attack | Not-Attack |
|---|---|---|
| Attack | -w,-w | M,0 |
| Not-Attack | 0,M | 0,0 |

Strong
(with probability 1-p)

| 1/2 | Attack | Not-Attack |
|---|---|---|
| Attack | M-s,-w | M,0 |
| Not-Attack | 0,M | 0,0 |

When p=1/2, which is a pure strategy Bayesian equilibrium (there could be other equilibria that are not listed as one of the options): (1's type - 1's strategy; 2's strategy)

i. (Weak - Not-Attack, Strong - Attack; Attack);
ii. (Weak - Not-Attack, Strong - Attack; Not-Attack);
iii. (Weak - Attack, Strong - Attack; Attack);
iv. It does not exist.

## 2. Rock, Paper, Scissors

Consider the following variation to the Rock (R), Paper (P), Scissors (S) game:
Suppose that with probability p player 1 faces a Normal opponent and with probability 1−p, he faces a Simple opponent that will always play P. Player 2 knows whether he is Normal or Simple, but player 1 does not.
The payoffs are pictured in the payoff matrices below:

Normal
(with probability p)

| 1/2 | R | P | S |
|---|---|---|---|
| R | 0,0 | -1,1 | 1,-1 |
| P | 1,-1 | 0,0 | -1,1 |
| S | -1,1 | 1,-1 | 0,0 |

Simple
(with probability 1-p)

| 1/2 | P |
|---|---|
| R | -1,1 |
| P | 0,0 |
| S | 1,-1 |

a) Suppose p=1/3, select all pure strategy Bayesian equilibria (there may be more than one): (Form: 1's strategy; 2's type - 2's strategy)

i. (S; Normal - P, Simple - P)
ii. (S; Normal - R, Simple - P)
iii. (R; Normal - P, Simple - P)
iv. (P; Normal - P, Simple - P)

b) Suppose p=2/3, select all pure strategy Bayesian equilibria (there may be more than one): (Form: 1's strategy; 2's type - 2's strategy)

i. (R; Normal - P, Simple - P)
ii. (P; Normal - S, Simple - P)
iii. (S; Normal - R, Simple - P)

## 3. Work or Startup

An engineer has a talent t in {1,2} with equal probability (prob=1/2), and the value of t is private information to the engineer. The engineer's pure strategies are applying for a job or being an entrepreneur and doing a startup. The company's pure strategies are either hiring or not hiring the engineer. If the engineer applies for the job and the company does not hire, then the engineer becomes an entrepreneur and does a startup.
The utility of the engineer is t (talent) from being an entrepreneur, and w (wage) from being hired. The utility of the company is (t−w) from hiring the engineer and 0 otherwise.
These are pictured in the payoff matrices below, with the engineer being the row player and the company being the column player.

t=2 (with probability 1/2)

| Eng\Comp | Hire | Not |
|---|---|---|
| Startup | 2,0 | 2,0 |
| Work | w,2-w | 2,0 |

t=1 (with probability 1/2)

| Eng\Comp | Hire | Not |
|---|---|---|
| Startup | 1,0 | 1,0 |
| Work | w,1-w | 1,0 |

a) Suppose w=2, which of the below are pure strategy Bayesian equilibria? there may be more than one. (Form: Engineer's strategy, company's strategy)

i. (t=2 Startup, t=1 Work, Hire);
ii. (t=2 Startup, t=1 Work, Not);
iii. (t=2 Work, t=1 Work, Hire);
iv. (t=2 Work, t=1 Work, Not);

b) Suppose w=1, which of the below are pure strategy Bayesian equilibria? there may be more than one. (Form: Engineer's strategy, company's strategy)

i. (t=2 Work, t=1 Startup, Hire);
ii. (t=2 Work, t=1 Startup, Not);
iii. (t=2 Startup, t=1 Work, Hire);
iv. (t=2 Startup, t=1 Work, Not);

## 4. Battle of the Sexes

Consider a modified game of the Battle of Sexes to have incomplete information.
There are two possible types of player 2 (column):

- "Meet": player 2 wishes to be at the same movie as player 1, just as in the usual game. (This type has probability p)
- "Avoid": 2 wishes to avoid player 1 and go to the other movie. (This type has probability 1−p)

Player 2 knows her type, and 1 does not. They simultaneously choose P or L.
These payoffs are shown in the matrices below.

Meet (with probability p)

| 1\2 | L | P |
|---|---|---|
| L | 2,1 | 0,0 |
| P | 0,0 | 1,2 |

Avoid (with probability 1-p)

| 1\2 | L | P |
|---|---|---|
| L | 2,0 | 0,2 |
| P | 0,1 | 1,0 |

a) When p=1/2, which is a pure strategy Bayesian equilibrium: (1's strategy; 2's type - 2's strategy)

i. (L; Meet - L, Avoid - P);
ii. (P; Meet - P, Avoid - L);
iii. (L; Meet - P, Avoid - P);
iv. It does not exist.

b) When p=1/4, which is a pure strategy Bayesian equilibrium : (1's strategy; 2's type - 2's strategy)

i. (L; Meet - L, Avoid - P);
ii. (P; Meet - P, Avoid - L);
iii. (L; Meet - P, Avoid - P);
iv. It does not exist.