

Computational Game Theory

Exercises on Mechanism Design

1. Mechanism Design as an Optimization Problem

Consider a voting with 3 players (1, 2, 3) and 2 alternatives (a, b). Each player has a strict preference, and prefers a or b with equal probabilities. Each player gets 5 if his/her top choice is selected and 0 otherwise.

Consider the following 2 voting rules with transferable utility: in both cases, each player submits one vote $v_i \in \{a, b\}$ ($i \in \{1, 2, 3\}$).

- The winning alternative w is uniformly randomly chosen from 3 votes submitted by 3 players, and there is no transfer.
- w is chosen under the same rule as in (i). Players with $v_i = w$ pay 1, and players with $v_i \neq w$ get 1. For example under (ii), if player 1 prefers a, votes a, and $v_i = a$ is selected to be the winning alternative, player 1 gets 5 because a wins, pays 1 because $v_1 = a$, and thus gets a net payoff 4.

- Which voting rules are truthful?
- Which voting rules are efficient?
- Which voting rules are budget balanced?
- Which voting rules are ex interim individual rational?
- Which voting rule earns a higher revenue?
- Which voting rule gives a higher maxmin fairness?

2. Single-item VCG

Suppose that we wish to use VCG to allocate a block of bandwidth to one of three companies: X, Y, and Z. X and Y are competitors in the same industry; Z is in a different industry. Their payoffs for the three possible allocations are:

Agent	U(Allocate to X)	U(Allocate to Y)	U(Allocate to Z)
X	10	-15	0
Y	-12	5	0
Z	0	0	4

- To which company will VCG allocate the block?
- What will be the winning company's payment?
- Does this environment exhibit choice set monotonicity?
- Does this environment exhibit no negative externalities?
- Does this environment exhibit no single-agent effect?

3. VCG with Incomplete Reports

Suppose that we are using VCG to allocate a pair of shoes among three agents. Each agent can receive one shoe, both shoes, or neither shoe. The feasible outcomes are those where each shoe is allocated to at most one agent.

The agents value the shoes as follows:

Agent	$u(\{\text{left}\})$	$u(\{\text{right}\})$	$u(\{\text{left, right}\})$
X	2	4	6
Y	0	0	7
Z	4	2	6

However, instead of reporting completely, the agents report only their utility for getting both shoes, as follows:

Agent	$\hat{u}(\{\text{left}\})$	$\hat{u}(\{\text{right}\})$	$\hat{u}(\{\text{left, right}\})$
X	0	0	6
Y	0	0	7
Z	0	0	6

- Are the agents playing a dominant strategy?
- Are the agents playing a Nash equilibrium?
- What allocation will VCG choose given the reported preferences?
- What is the efficient allocation given the true preferences?