Outline

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1. Non-monotonic Reasoning
   - Closed-World Assumption
   - Minimal entailment
   - Default Logic

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Strictness of FOL

To reason from $P(a)$ to $Q(a)$, need either

- facts about a itself
- universals, e.g. $\forall x(P(x) \supset Q(x))$
  - something that applies to all instances
  - all or nothing!

But most of what we learn about the world is in terms of $\underline{generics}$

- e.g., encyclopedia entries for ferris wheels, wildflowers, violins, turtles.

Properties are not strict for all instances, because of

- genetic / manufacturing varieties
  - early ferris wheels
- cases in exceptional circumstances
  - dried wildflowers
- borderline cases
  - toy violins
- imagined cases
  - flying turtles
- etc.

Generics vs. universals

✓ Violins have four strings.

vs.

✗ All violins have four strings.

vs.

? All violins that are not E_1 or E_2 or ... have four strings

- (exceptions usually cannot be enumerated)

Goal: be able to say a P is a Q in general, but not necessarily

- It is reasonable to conclude $Q(a)$ given $P(a)$, unless there is a good reason not to.

Here: qualitative version (no numbers)

Varieties of defaults (I)

General statements

- prototypical: The prototypical P is a Q.
  - Owls hunt at night.
- normal: Under typical circumstances, P's are Q's.
  - People work close to where they live.
- statistical: Most P's are Q's.
  - The people in the waiting room are growing impatient.

Lack of information to the contrary

- group confidence: All known P's are Q's.
  - Natural languages are easy for children to learn.
- familiarity: If a P was not a Q, you would know it.
  - an older brother
  - very unusual individual, situation or event

Varieties of defaults (II)

Conventional

- conversational: Unless I tell you otherwise, a P is a Q
  - "There is a gas station two blocks east"
    the default: the gas station is open.
- representational: Unless otherwise indicated, a P is a Q
  - the speed limit in a city

Persistence

- inertia: A P is a Q if it used to be a Q.
  - colours of objects
  - locations of parked cars (for a while!)

Here: we will use "Birds fly" as a typical default.

Closed-world assumption

Reiter's observation

- There are usually many more negative facts than positiveve facts!

Example

Airline flight guide provides

DirectConnect(cleveland,toronto)   DirectConnect(toronto,northBay)
DirectConnect(toronto,winnipeg)   ...

but not: ¬DirectConnect(cleveland,northBay)

Conversational default, called Closed World Assumption (CWA)

Only positive facts will be given, relative to some vocabulary

- But note: $KB \not\models$ -ve facts (would have to answer: "I don't know")

Proposal: a new version of entailment:

$$KB \models_c \alpha \text{ iff } KB \cup Negs \models \alpha$$

- where $Negs = \{\neg p \mid p \text{ atomic and } KB \not\models p\}$
- a common patern $KB' = KB \cup \Delta$

Properties of CWA

Closed World Assumption (CWA)

$$KB \models_c \alpha \text{ iff } KB \cup Negs \models \alpha$$

Gives: $KB \models_c$ positive facts and negative facts

CWA is an assumption about complete knowledge

Never any unknowns, relative to vocabulary

For every α (without quantifiers), $KB \models_c \alpha$ or $KB \models_c \neg\alpha$

- Why? Inductive argument:
  - immediately true for atomic sentences
  - push $\neg$ in, e.g. $KB \models \neg\neg\alpha$ iff $KB \models \alpha$
  - $KB \models (\alpha \land \beta)$ iff $KB \models \alpha$ and $KB \models \beta$
  - Say $KB \not\models_c (\alpha \lor \beta)$. Then $KB \not\models_c \alpha$ and $KB \not\models_c \beta$
    So by induction, $KB \models_c \neg\alpha$ and $KB \models_c \neg\beta$. Thus, $KB \models_c \neg(\alpha \lor \beta)$.

In general, a KB has *incomplete* knowledge.

- Let KB be $(p \lor q)$.
  - Then $KB \models (p \lor q)$, but $KB \not\models p$, $KB \not\models \neg p$, $KB \not\models q$, $KB \not\models \neg q$
- With CWA, if $KB \models_c (\alpha \lor \beta)$, then $KB \models_c \alpha$ or $KB \models_c \beta$
  - similar argument to above

Query evaluation

Properties of entailment

With CWA, we can reduce queries (without quantifiers) to the atomic case:

- $KB \models_c (\alpha \wedge \beta)$ iff $KB \models_c \alpha$ and $KB \models_c \beta$
- $KB \models_c (\alpha \vee \beta)$ iff $KB \models_c \alpha$ or $KB \models_c \beta$
- $KB \models_c \neg(\alpha \wedge \beta)$ iff $KB \models_c \neg\alpha$ or $KB \models_c \neg\beta$
- $KB \models_c \neg(\alpha \vee \beta)$ iff $KB \models_c \neg\alpha$ and $KB \models_c \neg\beta$
- $KB \models_c \neg\neg\alpha$ iff $KB \models_c \alpha$

reduces any query about $KB \models_c \alpha$ to a set of queries $KB \models_c \rho$ about the literals ρ in α

If $KB \cup Negs$ is consistent, we get $KB \models_c \neg\alpha$ iff $KB \not\models_c \alpha$

- reduces to: $KB \models_c p$, where p is atomic

If atoms stored as a table, deciding if $KB \models_c \alpha$ is like DB-retrieval:

- reduce query to set of atomic queries
- solve atomic queries by table lookup

Different from ordinary logic reasoning (e.g. no reasoning by cases)

Consistency of CWA

Just because a KB is consistent, does not mean that $KB \cup Negs$ is also consistent.

Non-problematic cases

- If KB is a set of atoms, then $KB \cup Negs$ is always consistent
- Also works if KB has conjunctions and if KB has only negative disjunctions
  - If KB contains $(\neg p \vee \neg q)$. Add both $\neg p, \neg q$.

Problem

When $KB \models (\alpha \vee \beta)$, but $KB \not\models \alpha$ and $KB \not\models \beta$

- e.g. $KB = (p \vee q)$ $Negs = \{\neg p, \neg q\}$ $KB \cup Negs$ is inconsistent.

Solution: Generalised Closed World Assumption (GCWA)

Only apply CWA to atoms that are "uncontroversial".

- $Negs = \{\neg p \mid$ If $KB \models (p \vee q_1 \vee \cdots \vee q_n)$ then $KB \models (q_1 \vee \cdots \vee q_n)\}$

When KB is consistent, get:

- $KB \cup Negs$ consistent
- everything derivable is also derivable by CWA

Quantifiers and equality

Problem

So far, results do not extend to wffs with quantifiers

- can have $KB \not\models_c \forall x.\alpha$ and $KB \not\models_c \neg\forall x.\alpha$
  e.g. just because for every t, we have $KB \models_c \neg$DirectConnect(myHome, t)
  - does not mean that $KB \models_c \forall x[\neg$ DirectConnect(myHome, x)]

Solution

We may want to treat KB as providing complete information about what individuals exist
Define: $KB \models_{cd} \alpha$ iff $KB \cup Negs \cup Dc \models \alpha$

- where Dc is domain closure: $\forall x[x = c_1 \vee \cdots \vee x = c_n]$,
- and c_i are all the constants appearing in KB (assumed finite)

Get: $KB \models_{cd} \exists x.\alpha$ iff $KB \models_{cd} \alpha[x/c]$, for some c appearing in the KB
$KB \models_{cd} \forall x.\alpha$ iff $KB \models_{cd} \alpha[x/c]$, for all c appearing in the KB

- We have $KB \models_{cd} \alpha$ or $KB \models_{cd} \neg\alpha$, even with quantifiers

Then add: Un is unique names: $(c_i \neq c_j)$, for $i \neq j$
Get: $KB \models_{cdu} (c = d)$ iff c and d are the same constant

$\rightarrow$ full recursive query evaluation

Non-monotonicity

Ordinary entailment is monotonic

If $KB \models \alpha$, then $KB^* \models \alpha$, for any $KB \subseteq KB^*$

CWA entailment is *not* monotonic

Can have $KB \models_c \alpha$, $KB \subseteq KB'$, but $KB' \not\models_c \alpha$

- e.g. $\{p\} \models_c \neg q$, but $\{p, q\} \not\models_c \neg q$

Suggests study of non-monotonic reasoning

- start with explicit beliefs
- generate implicit beliefs non-monotonically, taking *defaults* into account
- implicit beliefs may not be uniquely determined (vs. monotonic case)

Will consider two approaches:

- minimal entailment: interpretations that minimize abnormality
- default logic: KB as facts + default rules of inference

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Minimizing abnormality

- CWA makes the extension of all predicates as small as possible
  - by adding negated literals
- Generalize: do this only for selected predicates
  - Ab predicates used to talk about typical cases

Example

$$Bird(chilly), \neg Flies(chilly),$$
$$Bird(tweety), (chilly \neq tweety),$$
$$\forall x[Bird(x) \wedge \neg Ab(x) \supset Flies(x)] \qquad \longleftarrow \textit{All birds that are normal fly}$$

Would like to conclude by default Flies(tweety), but $KB \not\models Flies(tweety)$

- because there is an interpretation $\mathfrak{I}$ where $I[tweety] \in I[Ab]$

- Solution: consider only interpretations where $I[Ab]$ is as small as possible, relative to KB
  - this is sometimes called "circumscription" since we circumscribe the Ab predicate.
  - for example, require that $I[chilly] \in I[Ab]$
- Generalizes to many Ab_i predicates

Minimal entailment

Definition

Given two interpretations over the same domain, $\mathfrak{I}_1$ and $\mathfrak{I}_2$

- $\mathfrak{I}_1 \leq \mathfrak{I}_2$ iff $I_1[Ab] \subseteq I_2[Ab]$, for every Ab predicate
- $\mathfrak{I}_1 < \mathfrak{I}_2$ iff $\mathfrak{I}_1 \leq \mathfrak{I}_2$ but not $\mathfrak{I}_2 \leq \mathfrak{I}_1$
    - read: $\mathfrak{I}_1$ is more normal than $\mathfrak{I}_2$

Definition (Minimal Entailment)

Define a new version of entailment, $\models_\leq$ as follows:
$KB \models_\leq \alpha$ iff for every $\mathfrak{I}$, if $\mathfrak{I} \models KB$ and no $\mathfrak{I}^* < \mathfrak{I}$ s.t $\mathfrak{I}^* \models KB$, then $\mathfrak{I} \models \alpha$

- With minimal entailment, α must be true in all interpretations satisfying KB that are *minimal* in abnormalities
- Get: $KB \models_\leq Flies(tweety)$
    - because if interpretation satisfies KB and is minimal, only $I[chilly]$ will be in $I[Ab]$
- Note: Minimization need not produce a *unique* interpretation:
    - $Bird(a), Bird(b), [\neg Flies(a) \vee \neg Flies(b)]$ yields two minimal interpretations
    - $KB \not\models_\leq Flies(a), KB \not\models_\leq Flies(b)$, but $KB \models_\leq Flies(a) \vee Flies(b)$

Different from the CWA: no inconsistency!
But stronger than GCWA: conclude a or b flies

Fixed and variable predicates

Example

Let's extend the previous example with

$$\forall x[Penguin(x) \supset Bird(x) \land \neg Flies(x)]$$

Get: $KB \models \forall x[Penguin(x) \supset Ab(x)]$
So minimizing Ab also minimizes penguins: $KB \models_{\leq} \forall x \neg Penguin(x)$

Definition (McCarthy's definition)

Let $\mathbf{P}$ and $\mathbf{Q}$ be sets of predicates. $\mathfrak{I}_1 \leq \mathfrak{I}_2$ iff they are over the same domain and

1. $I_1[P] \subseteq I_2[P]$, for every $P \in \mathbf{P}$ $\quad$ *Ab predicates*
2. $I_1[Q] = I_2[Q]$, for every $Q \in \mathbf{Q}$ $\quad$ *fixed predicates*

so only predicates in $\mathbf{Q}$ are allowed to vary

- $\models_{\leq}$ becomes parameterized by what is minimized *and* what is allowed to vary.
  - Previous example: minimize Ab, but allow only Flies to vary.
- Problems:
  - need to decide what to allow to vary
  - cannot conclude $\neg Flies(tweety)$ by default!
    - only get default $(\neg Penguin(tweety) \supset Flies(tweety))$

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Motivation

- We want to state something like "typically birds fly"
- ... and we want to reason with such statements
- Add non-logical inference rule:

$$\frac{bird\,(x)\ \ :\ \ can\_fly\,(x)}{can\_fly\,(x)}$$

  with the intended meaning:
  If x is a bird and if it is consistent to assume that x can fly, then conclude that x can fly.

- Exceptions can be represented using simple logical implications:

$$\begin{aligned} \forall x : penguin\,(x) &\supset \neg can\_fly\,(x) \\ \forall x : emu\,(x) &\supset \neg can\_fly\,(x) \\ \forall x : kiwi\,(x) &\supset \neg can\_fly\,(x) \end{aligned}$$

- FOL with classical logical consequence relation $\models$ and deductive closure Cn such that $Cn\,(E) = \{A \mid E \models A\}$

Definition (Default)

A Default d is an expression

$$\frac{A \ : \ B_1, ..., B_n}{C}$$

where A, B_i and C are formulas in first order logic.

A : Prerequisite must be true before rule can be applied

B_i : Consistency Condition the negation should not be true

C : Consequence will be concluded

- A default rule is called closed if it does not contain free variables.
- We denote A, $\{B_1, ..., B_n\}$ and C, by $pre\,(d)$, $just\,(d)$ and $cons\,(d)$, respectively.

Definition ((Closed) Default Theory)

A (closed) default theory is a pair (D, W), where D is a countable set of (closed) defaults and W is a countable set of sentences in first order logic.
We interpret non-closed defaults as schemata representing all of their ground instances.

Extensions

- Default theories extend the theories given by W using the default rules

$$D \rightsquigarrow Extensions.$$

Example

$$W = \{a, \neg b \vee \neg c\}$$
$$D = \left\{ \;\frac{a:b}{b} \quad \frac{a:c}{c}\; \right\}$$

One possible extension should contain b, another one c. Having them together is impossible.

- Intuitively: An extension is a belief context resulting from W and D.
- In general, a default theory can have more than one extension.

- What do we do if we have more than one extension?

Credulous Reasoning If φ holds in one extension, we accept φ as a credulous default conclusion.

Skeptical Reasoning If φ holds in all extensions, we accept φ as a skeptical default conclusion.

Choice Reasoning We compute one arbitrary extension and stick to it.

Extensions - Informally

Desirable properties of an extension E of (D, W):

- Contains all facts W i.e. $W \subseteq E$.
- Is deductively closed i.e. $Cn\,(E) = E$.
- All applicable default rules are applied:

  If
  1. $\frac{A \,:\, B_1, \ldots, B_n}{C} \in D$
  2. $A \in E$
  3. $\neg B_i \notin E$

  Then $C \in E$.
- Some condition of groundedness: each formula in an extension needs sufficient reasons to be there.

  Question Would minimality wrt the previous requirements be enough?

Groundedness

Desirable properties of an extension E of (D, W):

- Contains all facts W i.e. $W \subseteq E$.
- Is deductively closed i.e. $Cn(E) = E$.
- All applicable default rules are applied:

If
1. $\frac{A \,:\, B_1, \ldots, B_n}{C} \in D$
2. $A \in E$
3. $\neg B_i \notin E$

Then $C \in E$.

Example

Consider

$$D = \left\{ \frac{:a}{b} \right\} \quad W = \emptyset$$

$Cn(\{\neg a\})$ is a minimal set satisfying the previous properties but the theory (D, W) gives no support for $\neg a$.

Extensions

Reiter's proposal

- Rests on the observation that given a set S of formulas to use to test for consistency of justifications, there is a unique least theory, say $\Gamma(S)$, containing W, closed under classical provability and also under defaults (in a certain sense determined by S).
- For theory S to be grounded in (D, W), S must be precisely what (D, W) implies, given that S is used to test the consistency of justifications.

Definition (Default Extension)

Let (D, W) be a default theory. The operator Γ assigns to every set S of formulas the smallest set of formulas such that:

1. $W \subseteq \Gamma(S)$.
2. $Cn(\Gamma(S)) = \Gamma(S)$.
3. If $\frac{A \,:\, B_1, \ldots, B_n}{C} \in D$ and $\Gamma(S) \models A, S \not\models \neg B_i, 1 \leq i \leq n$, then $C \in \Gamma(S)$.

A set E of formulas is an extension of (D, W) iff $E = \Gamma(E)$.

How to use this definition?

- The definition does not tell us how to construct an extension
- However, it tells us how to check whether a set is an extension
  1. Guess a set S
  2. Now construct a minimal set $\Gamma(S)$ by starting with W
  3. Add conclusions from default rules D when necessary
  4. If, in the end, when no more conclusions can be added, $S = \Gamma(S)$, then S must be an extension of (D, W)

Examples

$$D = \left\{ \frac{a:b}{b}, \frac{b:a}{a} \right\} \qquad W = \{a \vee b\}$$

$$D = \left\{ \frac{a:b}{\neg b} \right\} \qquad W = \emptyset$$

$$D = \left\{ \frac{a:b}{\neg b} \right\} \qquad W = \{a\}$$

$$D = \left\{ \frac{:a}{a}, \frac{:b}{b}, \frac{:c}{c} \right\} \qquad W = \{b \supset \neg a \wedge \neg c\}$$

$$D = \left\{ \frac{:c}{\neg d}, \frac{:d}{\neg e}, \frac{:e}{\neg f} \right\} \qquad W = \emptyset$$

$$D = \left\{ \frac{:c}{\neg d}, \frac{:d}{\neg c} \right\} \qquad W = \emptyset$$

$$D = \left\{ \frac{a:b}{c}, \frac{a:d}{e} \right\} \qquad W = \{a, (\neg b \vee \neg d)\}$$

Questions

- Can we say something about the existence of extensions?
- Is it possible to characterise the set of extensions more intuitively?
- How do the different extensions relate to each other?
  - Can one extension be a subset of another one?
  - Are extensions pairwise incompatible (i.e. jointly inconsistent)?
- Is that possible that an extension is inconsistent?

A more intuitive characterisation of extensions:

Theorem

Let (D, W) be a default theory and E a set of formulas. Let:

$$E_0 = W$$
$$E_{k+1} = Cn\,(E_k) \cup \left\{ C \mid \frac{A \;:\; B_1, ..., B_n}{C} \in D, E_k \models A, E \not\models \neg B_i, 1 \leq i \leq n \right\}$$

- *Then,* $\Gamma\,(E) = \bigcup_{k=0}^{\infty} E_k$.
- *Moreover, a set* E *of formulas is an extension of* (D, W) *iff*

$$E = \bigcup_{k=0}^{\infty} E_k$$

Question Why is this characterisation non-constructive?

Another Important Result

Definition

Let E be a set of formulas. A default d is generating for E if $E \models pre(d)$ and, for every $B_i \in just(d)$, $E \not\models \neg B_i$. If D is a set of defaults, we write $GD(D, E)$ for the set of defaults in D that are generating for E.

Theorem

Let E be an extension of a default theory (D, W). Then

$$E = Cn(W \cup \{cons(d) \mid d \in GD(D, E)\})$$

This result turns out to be fundamental for algorithms to compute extensions.

Corollary

Let (D, W) be a default theory.

1. *If W is inconsistent, then (D, W) has a single extension which consists of all formulas in the language.*
2. *If W is consistent and every default in D has at least one justification, then every extension of (D, W) is consistent.*

Theorem

If E and F are extensions of (D, W) such that $E \subseteq F$ then $E = F$.

Proof sketch.

$E = \bigcup_{k=0}^{\infty} E_k$ and $F = \bigcup_{k=0}^{\infty} F_k$. It suffices to show that $F_k \subseteq E_k$.
Induction:

- *Trivially $E_0 = F_0$.*
- *Assume $C \in F_{k+1}$.*
    - *$C \in Cn(F_k)$ implies $C \in Cn(E_k)$ (because $F_k \subseteq E_k$) i.e., $C \in E_{i+1}$.*
    - *Otherwise $\frac{A : B_1, \ldots, B_n}{C} \in D, F_k \models A, F \not\models \neg B_i, 1 \le i \le n$. However, then we have $E_k \models A$ (because $F_k \subseteq E_k$) and $E \not\models \neg B_i, 1 \le i \le n$ (because $E \subseteq F$), i.e., $C \in E_{i+1}$.*

$\square$

Normal Default Theories

Definition

A default is normal if it has the form $\frac{A\ :\ B}{B}$

Theorem

Let (D, W) be a normal default theory.

1. (D, W) *has at least one extension.*
2. *if E and F are extensions of (D, W) and $E \neq F$, then then $E \cup F$ is inconsistent.*
3. *if E is an extension of (D, W), then for every set D' of normal defaults, the normal default theory $(D \cup D', W)$ has an extension E' such that $E \subseteq E'$.*

The last property is often called semi-monotonicity of normal default logic. It asserts that adding normal defaults to a normal default theory does not destroy existing extensions but possibly only augments them.

Theorem

Let (D, W) be a normal default theory.

2 *if E and F are extensions of (D, W) and $E \neq F$, then then $E \cup F$ is inconsistent.*

Proof sketch.

Let $E = \bigcup_{k=0}^{\infty} E_k$ and $F = \bigcup_{k=0}^{\infty} F_k$ with

$$E_0 = W$$

$$E_{k+1} = Cn\,(E_k) \cup \left\{ B \mid \frac{A\ :\ B}{B} \in D, E_k \models A, E \not\models \neg B_i, 1 \leq i \leq n \right\} \text{ for } k \geq 0$$

and the same for F_k. Since $E \neq F$, there must exist a smallest k such that $E_k \neq F_k$. This means that there exists $\frac{A\ :\ B}{B} \in D$ with $E_k = F_k \models A$ but $B \in E_{k+1}$ and $B \notin F_{k+1}$. This is only possible if $\neg B \in F$ (so that $F \models \neg B$). This means that $B \in E$ and $\neg B \in F$, i.e., $E \cup F$ is inconsistent. □

This property is often called orthogonality of normal default logic.

Question Can we have top-down goal driven reasoning?

Example

Consider the default theory

$$D = \left\{ d_1 = \frac{p : q}{r}, d_2 = \frac{r : q}{s}, d_3 = \frac{:}{\neg q} \right\} \quad W = \{p\}$$

and suppose we are interested in testing whether s is supported (for now we take this to be equivalent to existence of an extension that contains s) by the default theory. An argument could be:

1. s is the consequent of d_2 so let's try to derive its prerequisite r.
2. r is the consequent of d_1 so let's try to derive its prerequisite p.
3. p is included in W so we are done.

We did not pay attention to the consistency, but this should not be a problem because there are no conflicts among W, d_1 and d_2.
So, we could be tempted to answer the question positively.
However, the only extension is $Cn\left(\{p, \neg q\}\right)$ which does not include s.

Fortunately, the previous problem cannot arise in normal default theories.

Definition (Default Proofs)

A default proof of B in a normal default theory (D, W) is a finite sequence of defaults $\left(d_i = \frac{A_i \,:\, B_i}{B_i}\right)_{i=1,\ldots,n}$ such that:

- $W \cup \{B_1, ..., B_n\} \models B$
- $W \cup \{B_1, ..., B_n\}$ is consistent
- $W \cup \{B_1, ..., B_k\} \models A_{k+1}$, for $0 \leq k \leq n-1$

Theorem

A formula B has a default proof in a normal default theory (D, W) iff there exists an extension E of (D, W) such that $B \in E$.

Default Proofs in Normal Default Theories

Example

Consider the default theory (D, W) with $W = \{q \wedge r \supset p\}$ and $D = \{d_1, d_2, d_3, d_4, d_5, d_6\}$ with

$$d_1 = \frac{:d}{d} \quad d_2 = \frac{d:\neg c \wedge b}{\neg c \wedge b} \quad d_3 = \frac{d:c}{c} \quad d_4 = \frac{:a}{a} \quad d_5 = \frac{a \wedge b:q}{q} \quad d_6 = \frac{\neg c:r}{r}$$

We want to know whether p is included in some extension of (D, W).
One default proof is $d1, d2, d4, d6, d5$.

Example

Consider the default theory (D, W) with $W = \emptyset$ and $D = \{d_1, d_2, d_3\}$ with

$$d_1 = \frac{q:p}{p} \quad d_2 = \frac{\neg p:q}{q} \quad d_3 = \frac{:\neg p}{\neg p}$$

Question Why isn't $d3, d2, d1$ a default proof for p?

Answer Because $W \cup cons\,(d3) \cup cons\,(d2) \cup cons\,(d1) = W \cup \{p, q, \neg p\}$ is inconsistent.

Limitations of Normal Default Theories

Example

Suppose we are given the information: Bill is a high school dropout. Typically, high school dropouts are adults. Typically, adults are employed.
These facts are naturally represented by the default theory (D, W) with $W = \{dropout\,(bill)\}$ and

$$D = \left\{ \frac{dropout\,(X) : adult\,(X)}{adult\,(X)}, \frac{adult\,(X) : employed\,(X)}{employed\,(X)} \right\}$$

which has the single extension $Cn\,(\{dropout\,(bill)\,, adult\,(bill)\,, employed\,(bill)\})$.
It is counterintuitive to assume that Bill is employed! Whereas the second default seems accurate on its own, we want to prevent its application in case the adult X is a dropout i.e.

$$\frac{adult\,(X) : employed\,(X) \wedge \neg dropout\,(X)}{employed\,(X)}$$

Question? Why not simply add $\neg dropout\,(X)$ to the prerequisite of the default to keep it normal?