Computational Game Theory

Exercises on Dominance, Maxmin and Correlated Equilibrium

1. Dominance

Consider the following payoff matrix:

1\2	X	Y	Z
A	1,2	2,2	5,1
B	4,1	3,5	3,3
C	5,2	4,4	7,0
D	2,3	0,4	3,0

a) Find the strictly dominant strategy.
b) Find a weakly dominant strategy that is not strictly dominant.
c) When player 1 plays D, what is player 2's best response?
d) Find all strategy profiles that form pure Nash equilibria (there may be more than one, or none).

2. Nash Equilibrium – Bargaining

There are 2 players that have to decide how to split one euro. The bargaining process works as follows. Players simultaneously announce the share they would like to receive s1 and s2, with 0≤ s1,s2 ≤1. If s1+s2 ≤1, then the players receive the shares they named and if s1+s2 >1, then both players fail to achieve an agreement and receive zero.

a) Which of the following is a strictly dominant strategy?
i) 1 ii) 0.5 iii) 0 iv) Neither

3. Voting

Three voters vote over two candidates (A and B), and each voter has two pure strategies: vote for A and vote for B. When A wins, voter 1 gets a payoff of 1, and 2 and 3 get payoffs of 0; when B wins, 1 gets 0 and 2 and 3 get 1. Thus, 1 prefers A, and 2 and 3 prefer B. The candidate getting 2 or more votes is the winner (majority rule).

a) Find all very weakly **dominant** strategies (there may be more than one, or none).

4. Iterated Removal of Strictly Dominated Strategies

Consider the following payoff matrix:

1\2	Left	Middle	Right
Top	3,8	2,0	1,2
Bottom	0,0	1,7	8,2

We say that a game is dominance solvable, if iterative deletion of strictly dominated strategies yields a unique outcome.

a) Is the previous game dominance solvable? Consider both pure strategies and mixed strategies.

5. Iterated removal of weakly dominated strategies

Show that the iterated elimination of weakly dominated strategies is not order-independent.

6. Minimax

Consider the matching-pennies game:

1\2	Left	Right
Left	2,-2	-2,2
Right	-2,2	2,-2

a) Which is the maxmin strategy for player 1?

b) Apply the Minimax theorem presented in the lectures to find the payoff that any player must receive in any Nash Equilibrium.

7. Correlated Equilibrium

Consider the following payoff matrix:

1\2	B	F
B	2,1	0,0
F	0,0	1,2

Consider the following assignment device (for example a fair coin): with probability 1/2 it tells players 1 and 2 to play B, and with probability 1/2 it tells them to play F. Both players know that the device will follow this rule.

What is the expected payoff of each player when both players follow the recommendations made by the device? If one of players follows the recommendation, does the other player have an incentive to follow the recommendation as well?