Computational Game Theory

Exercises on Nash Equilibrium in Normal Form Games

1. Pure Strategy Nash Equilibrium

Consider the following payoff matrices:

	Left	Right
Top	5,8	3,4
Bottom	2,2	7,3

	Left	Right
Top	2,3	5,3
Bottom	5,4	3,3

	Left	Right
Top	4,2	5,1
Bottom	6,0	3,3

a) For each game, find all strategy profiles that form pure strategy Nash equilibria.

2. Nash Equilibrium – Bargaining

There are 2 players that have to decide how to split one euro. The bargaining process works as follows. Players simultaneously announce the share they would like to receive s1 and s2, with 0≤ s1,s2 ≤1. If s1+s2 ≤1, then the players receive the shares they named and if s1+s2 >1, then both players fail to achieve an agreement and receive zero.

a) Which of the following strategy profiles is a pure strategy Nash equilibrium?

i) (0.3, 0.7) ii) (0.5,0.5) iii) (1,1) iv) All

3. Bertrand Duopoly

Two firms produce identical goods, with a production cost of c>0 per unit. Each firm sets a nonnegative price (p1 and p2). All consumers buy from the firm with the lower price, if pi≠pj. Half of the consumers buy from each firm if pi=pj. D is the total demand.

Profit of firm i is:

0 if pi>pj (no one buys from firm i);

D(pi-c)/2 if pi=pj (Half of customers buy from firm i);

D(pi-c) if pi<pj (All customers buy from firm i);

Find the pure strategy Nash equilibrium:

i) Both firms set p=0
ii) Both firms set p=c
iii) Firm 1 sets p=0 and firm 2 sets p=c.
iv) No pure strategy Nash equilibrium exists

4. Voting

Three voters vote over two candidates (A and B), and each voter has two pure strategies: vote for A and vote for B. When A wins, voter 1 gets a payoff of 1, and 2 and 3 get payoffs of 0; when B wins, 1 gets 0 and 2 and 3 get 1. Thus, 1 prefers A, and 2 and 3 prefer B. The candidate getting 2 or more votes is the winner (majority rule).

b) Find all pure strategy Nash equilibria (there may be more than one, or none)?

i) All voting for A.
ii) All voting for B.
iii) Voter 1 voting for A, and 2 and 3 voting for B.
iv) Voter 1 and 2 voting for A, and 3 voting for B.

5. Mixed Strategy Nash Equilibrium

Consider the following payoff matrices:

	Left	Right
Top	5,8	3,4
Bottom	2,2	7,3

	Left	Right
Top	2,3	5,3
Bottom	5,4	3,3

	Left	Right
Top	4,2	5,1
Bottom	6,0	3,3

For each game, find all strategy profiles that form mixed-strategy Nash equilibria.

6. Comparative Statics

Consider the following payoff matrix:

1\2	Left	Right
Top	**x**,2	0,0
Bottom	0,0	2,2

In a mixed strategy Nash equilibrium where player 1 plays Top with probability p and player 2 plays Left with probability q. How do p and q change as **x** is increased (x>1)?

i) p is the same, q decreases.
ii) p increases, q increases.
iii) p decreases, q decreases.
iv) p is the same, q increases.

7. Employment

There are 2 firms, each advertising an available job opening. Firms offer different wages:

- Firm 1 offers w1=4 and 2 offers w2=6.
- There are two unemployed workers looking for jobs.
- They simultaneously apply to either of the firms.
- If only one worker applies to a firm, then he/she gets the job.
- If both workers apply to the same firm, the firm hires a worker at random and the other worker remains unemployed (and receives a payoff of 0).

Find a mixed strategy Nash Equilibrium where p is the probability that worker 1 applies to firm 1 and q is the probability that worker 2 applies to firm 1.

i) p=q=1/4
ii) p=q=1/3
iii) p=q=1/2
iv) p=q=1/5

8. Treasure

A king is deciding where to hide his treasure, while a pirate is deciding where to look for the treasure. The payoff to the king from successfully hiding the treasure is 5 and from having it found is 2. The payoff to the pirate from finding the treasure is 9 and from not finding it is 4. The king can hide it in location X, Y or Z.

a) Suppose the pirate has two pure strategies: inspect both X and Y (they are close together), or just inspect Z (it is far away).
Find a mixed strategy Nash equilibrium where p is the probability the treasure is hidden in X or Y and 1-p that it is hidden in Z (treat the king as having two strategies) and q is the probability that the pirate inspects X and Y:

i) p=1/2, q=1/2
ii) p=4/9, q=2/5
iii) p=5/9, q=3/5
iv) p=2/5, q=4/9

b) Suppose instead that the pirate can investigate any two locations, so has three pure strategies: inspect XY or YZ or XZ.
Find a mixed strategy Nash equilibrium where the king mixes over three locations (X, Y, Z) and the pirate mixes over (XY, YZ, XZ). Which of following probabilities (king),(pirate) form an equilibrium?

i) (1/3, 1/3, 1/3), (4/9, 4/9, 1/9);
ii) (4/9, 4/9, 1/9), (1/3, 1/3, 1/3);
iii) (1/3, 1/3, 1/3), (2/5, 2/5, 1/5);
iv) (1/3, 1/3, 1/3), (1/3, 1/3, 1/3);