Computational Game Theory

Exercises on Repeated Games

1. Repeated Games with single NE in stage game

Two players play the following normal form game:

1\2	Left	Middle	Right
Left	4,2	3,3	1,2
Middle	3,3	5,5	2,6
Right	2,1	6,2	3,3

a) Which is the pure strategy Nash equilibrium of this stage game (if it is played only once)?

b) Suppose that the game is repeated for two periods. What is the outcome from the subgame perfect Nash equilibrium of the whole game:

i. (Left, Left) is played in both periods
ii. (Right, Right) is played in both periods
iii. (Middle, Middle) is played in the first period, followed by (Left, Left)
iv. (Middle, Middle) is played in the first period, followed by (Right, Right)

c) Suppose that there is a probability p that the game continues next period and a probability (1−p) that it ends. What is the threshold p^* such that when $p \geq p^*$ (Middle, Middle) is sustainable as a subgame perfect equilibrium by grim trigger strategies, but when $p < p^*$ playing Middle in all periods is not a best response? [Here the grim strategy is: play Middle if the play in all previous periods was (Middle, Middle); play Right otherwise.]

i. 1/2 ii. 1/3 iii. 1/4 iv. 2/5

2. Repeated Games with multiple NE in stage game

Two players play the following normal form game:

1\2	Left	Middle	Right
Left	1,1	5,0	0,0
Middle	0,5	4,4	0,0
Right	0,0	0,0	3,3

a) Which are the pure strategy Nash equilibria of this stage game (if it is played only once)? There can be more than one.

b) Suppose that the game is repeated for two periods. Which of the following outcomes could occur in some subgame perfect equilibrium? (There might be more than one).

i. (Left, Left) is played in both periods
ii. (Right, Right) is played in both periods
iii. (Middle, Middle) is played in the first period, followed by (Right, Right)

3. Tit for Tat

In an infinitely repeated Prisoner's Dilemma, a version of what is known as a "tit for tat" strategy of a player i is described as follows:

- There are two "statuses" that player i might be in during any period: "normal" and "revenge";
- In a normal status player i cooperates;
- In a revenge status player i defects;
- From a normal status, player i switches to the revenge status in the next period only if the other player defects in this period;
- From a revenge status player i automatically switches back to the normal status in the next period regardless of the other player's action in this period.

Consider an infinitely repeated game so that with probability p that the game continues to the next period and with probability (1−p) it ends.

1\2	Cooperate (C)	Defect (D)
Cooperate (C)	4,4	0,5
Defect (D)	5,0	1,1

a) True or False: When player 1 uses the above-described "tit for tat" strategy and starts the first period in a revenge status (thus plays defect for sure), in any infinite payoff maximizing strategy, player 2 plays defect in the first period.

b) What is the payoff for player 2 from always cooperating when player 1 uses this tit for tat strategy and begins in a normal status? How about always defecting when 1 begins in a normal status?

i. $4+4p+4p^2+4p^3+\ldots$; $5+p+p^2+p^3+\ldots$

ii. $4+4p+4p^2+4p^3+\ldots$; $5+p+5p^2+p^3+\ldots$

iii. $5+4p+4p^2+4p^3+\ldots$; $4+4p+4p^2+4p^3+\ldots$

iv. $5+4p+4p^2+4p^3+\ldots$; $5+p+p^2+p^3+\ldots$

c) What is the threshold p^* such that when $p \geq p^*$ always cooperating by player 2 is a best response to player 1 playing tit for tat and starting in a normal status, but when $p < p^*$ always cooperating is not a best response?

i. 1/2

ii. 1/3

iii. 1/4

iv. 1/5