

Knowledge Representation and Reasoning

Exercises on Advanced ASP

1 Cardinality Rules

Consider the following cardinality constraint in the head of a rule: $1\{a, b, c\}2$.

- a) Compile the cardinality constraint into cardinality rules of the form

$$a_0 \leftarrow l\{a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n\}$$

along with normal and choice rules as well as integrity constraints.

- b) Compile the logic program P resulting from the previous subtask into a program P' with normal and choice rules as well as integrity constraints only, using the $cc(i, j)$ construction from the lecture slides.
- c) Determine the stable models of P and the corresponding stable models of P' .

2 Weight Rules

Consider the following weight constraint in the head of a rule: $4\{1 : b_1, 1 : b_2, 2 : c_1, 2 : c_2\}5$.

- a) Compile the weight constraint into weight rules of the form

$$a_0 \leftarrow l\{w_1 : a_1, \dots, w_m : a_m, w_{m+1} : \sim a_{m+1}, \dots, w_n : \sim a_n\}$$

along with normal rules and integrity constraints.

- b) Generalize (and simplify) the scheme used for cardinality constraints before, and compile the logic program P resulting from the previous subtask into a program P' with normal and choice rules as well as integrity constraints only.

3 Extended Programs

Find the stable models of the following extended programs:

$$\begin{array}{llll} a) P = \{ & 1\{p, q\} \leftarrow & 1\{r, s\}1 \leftarrow \{p, q\}1\} \\ b) P = \{ & 1\{p, q, r\}2 \leftarrow & 2\{p, q, s\}2 \leftarrow 1\{q, r, s\}2\} \\ c) P = \{ & 2\{p, q, r\} \leftarrow & \{p, q\}1 \leftarrow s & s \leftarrow q, r\} \\ d) P = \{ & p \leftarrow 2\{q, r, s\} & 1\{q, r, s\}2 \leftarrow \sim p & 2\{r, s\} \leftarrow \sim q\} \\ e) P = \{ & p \leftarrow 2\{q, r, s\} & 2\{p, q, r\} \leftarrow \sim s & 2\{r, s\} \leftarrow p\} \end{array}$$

4 Extended Encodings

What well-known NP-Problems are described by the following extended encodings (where instances are represented by facts)?

$$\begin{array}{ll} a) & P = \left\{ \begin{array}{l} \{t(X)\} \leftarrow v(X) \\ \leftarrow c(C), \{t(X) : p(C, X), \sim t(X) : n(C, X)\}0 \end{array} \right\} \\ b) & P = \left\{ \begin{array}{l} \{t(X)\} \leftarrow v(X) \\ \leftarrow h(S), \{t(X) : c(S, X)\}0 \\ \leftarrow h(S), 2\{t(X) : c(S, X)\} \end{array} \right\} \end{array}$$

5 Programs with Aggregates

Determine the stable models of the following logic programs P with aggregates, check whether the contained aggregates are monotone, anti-monotone, or non-monotone, and provide appropriate translations of the aggregates to propositional formulas.

$$\begin{aligned}
 \text{a) } P &= \left\{ \begin{array}{l} p \leftarrow \text{sum}\{1 : p, 1 : q\} \neq 1 \\ p \leftarrow q \\ q \leftarrow p \end{array} \right\} \\
 \text{b) } P &= \left\{ \begin{array}{l} p \leftarrow \text{sum}\{1 : p, 1 : q\} < 1 \\ p \leftarrow \text{sum}\{1 : p, 1 : q\} > 1 \\ p \leftarrow q \\ q \leftarrow p \end{array} \right\} \\
 \text{c) } P &= \left\{ \begin{array}{l} \{p\} \\ \{q\} \\ s \leftarrow \text{sum}\{1 : p, 1 : q, 2 : s\} \neq 3 \end{array} \right\} \\
 \text{d) } P &= \left\{ \begin{array}{l} \{p\} \\ \{q\} \\ s \leftarrow \text{sum}\{1 : p, 1 : q, 2 : s\} < 3 \\ s \leftarrow \text{sum}\{1 : p, 1 : q, 2 : s\} > 3 \end{array} \right\}
 \end{aligned}$$