Knowledge Representation and Reasoning

Exercises on Advanced ASP

1 Cardinality Rules

Consider the following cardinality constraint in the head of a rule: $1\{a, b, c\}2$.

a) Compile the cardinality constraint into cardinality rules of the form

$$a_0 \leftarrow l\{a_1, \ldots, a_m, \sim a_{m+1}, \ldots, \sim a_n\}$$

along with normal and choice rules as well as integrity constraints.

b) Compile the logic program P resulting from the previous subtask into a program P' with normal and choice rules as well as integrity constraints only, using the $cc(i, j)$ construction from the lecture slides.

c) Determine the stable models of P and the corresponding stable models of P'.

Answer: a) P:

$$\begin{gathered} \{a, b, c\} \\ x \leftarrow 1\{a, b, c\} \\ y \leftarrow 3\{a, b, c\} \\ z \leftarrow x, \sim y \\ \leftarrow \sim z \end{gathered}$$

b) P':

$$\begin{array}{cccc}
\{a, b, c\} & cc(1,1) \leftarrow cc(2,0), a & cc(2,1) \leftarrow cc(3,0), b & cc(3,1) \leftarrow cc(4,0), c \\
x \leftarrow cc(1,1) & cc(1,0) \leftarrow cc(2,0) & cc(2,0) \leftarrow cc(3,0) & cc(3,0) \leftarrow cc(4,0) \\
y \leftarrow cc(1,3) & cc(1,2) \leftarrow cc(2,1), a & cc(2,2) \leftarrow cc(3,1), b & [cc(3,2) \leftarrow cc(4,1), c] \\
z \leftarrow x, \sim y & cc(1,1) \leftarrow cc(2,1) & cc(2,1) \leftarrow cc(3,1) & [cc(3,1) \leftarrow cc(4,1)] \\
\leftarrow \sim z & cc(1,3) \leftarrow cc(2,2), a & [cc(2,3) \leftarrow cc(3,2), b] & [cc(3,3) \leftarrow cc(4,2), c] \\
 & cc(1,2) \leftarrow cc(2,2) & [cc(2,2) \leftarrow cc(3,2)] & [cc(3,2) \leftarrow cc(4,2)] \\
 & [cc(1,4) \leftarrow cc(2,3), a] & [cc(2,4) \leftarrow cc(3,3), b] & [cc(3,4) \leftarrow cc(4,3), c] \\
cc(4,0) & [cc(1,3) \leftarrow cc(2,3)] & [cc(2,3) \leftarrow cc(3,3)] & [cc(3,3) \leftarrow cc(4,3)]
\end{array}$$

c)

P	P'
$\{a, x, z\}$	$\{a, x, z, cc(1,0), cc(2,0), cc(3,0), cc(4,0), cc(1,1)\}$
$\{b, x, z\}$	$\{b, x, z, cc(1,0), cc(2,0), cc(3,0), cc(4,0), cc(1,1), cc(2,1)\}$
$\{c, x, z\}$	$\{c, x, z, cc(1,0), cc(2,0), cc(3,0), cc(4,0), cc(1,1), cc(2,1), cc(3,1)\}$
$\{a, b, x, z\}$	$\{a, b, x, z, cc(1,0), cc(2,0), cc(3,0), cc(4,0), cc(1,1), cc(2,1), cc(1,2)\}$
$\{a, c, x, z\}$	$\{a, c, x, z, cc(1,0), cc(2,0), cc(3,0), cc(4,0), cc(1,1), cc(2,1), cc(3,1), cc(1,2)\}$
$\{b, c, x, z\}$	$\{b, c, x, z, cc(1,0), cc(2,0), cc(3,0), cc(4,0), cc(1,1), cc(2,1), cc(3,1), cc(1,2), cc(2,2)\}$

2 Weight Rules

Consider the following weight constraint in the head of a rule: $4\{1 : b_1, 1 : b_2, 2 : c_1, 2 : c_2\}5$.

a) Compile the weight constraint into weight rules of the form

$$a_0 \leftarrow l\{w_1 : a_1, \ldots, w_m : a_m, w_{m+1} :\sim a_{m+1}, \ldots, w_n :\sim a_n\}$$

along with normal rules and integrity constraints.

b) Generalize (and simplify) the scheme used for cardinality constraints before, and compile the logic program P resulting from the previous subtask into a program P' with normal and choice rules as well as integrity constraints only.

Answer:
a) P:

$$\begin{array}{l} \{b_1, b_2, c_1, c_2\} \\ x \leftarrow 4\{1 : b_1, 1 : b_2, 2 : c_1, 2 : c_2\} \\ y \leftarrow 6\{1 : b_1, 1 : b_2, 2 : c_1, 2 : c_2\} \\ z \leftarrow x, \sim y \\ \leftarrow \sim z \end{array}$$

b)

$$\begin{array}{llll} \{b_1, b_2, c_1, c_2\} & cc(4,2) \leftarrow c_2 & cc(2,5) \leftarrow cc(3,4), b_2 & cc(1,6) \leftarrow cc(2,5), b_1 \\ x \leftarrow cc(1,4) & cc(3,2) \leftarrow c_1 & cc(2,4) \leftarrow cc(3,4) & cc(1,5) \leftarrow cc(2,5) \\ y \leftarrow cc(1,6) & cc(3,4) \leftarrow cc(4,2), c_1 & cc(2,3) \leftarrow cc(3,2), b_2 & cc(1,5) \leftarrow cc(2,4), b_1 \\ z \leftarrow x, \sim y & cc(3,2) \leftarrow cc(4,2) & & cc(1,4) \leftarrow cc(2,4) \\ \leftarrow \sim z & & & cc(1,4) \leftarrow cc(2,3), b_1 \end{array}$$

Note that this has been further simplified compared to the version presented during the class.

3 Extended Programs

Find the stable models of the following extended programs:

$$\begin{array}{llll} a) P = \{ & 1\{p, q\} \leftarrow & 1\{r, s\}1 \leftarrow \{p, q\}1\} & \\ b) P = \{ & 1\{p, q, r\}2 \leftarrow & 2\{p, q, s\}2 \leftarrow 1\{q, r, s\}2\} & \\ c) P = \{ & 2\{p, q, r\} \leftarrow & \{p, q\}1 \leftarrow s & s \leftarrow q, r\} \\ d) P = \{ & p \leftarrow 2\{q, r, s\} & 1\{q, r, s\}2 \leftarrow \sim p & 2\{r, s\} \leftarrow \sim q\} \\ e) P = \{ & p \leftarrow 2\{q, r, s\} & 2\{p, q, r\} \leftarrow \sim s & 2\{r, s\} \leftarrow p\} \end{array}$$

Answer:
a) $\{p, r\}$, $\{p, s\}$, $\{q, r\}$, $\{q, s\}$, and $\{p, q\}$
b) $\{p\}$, $\{p, q\}$, $\{p, s\}$, $\{q, s\}$, $\{p, r, s\}$, and $\{q, r, s\}$
c) $\{p, q\}$, $\{p, r\}$, and $\{q, r, s\}$
d) $\{q\}$ and $\{p, r, s\}$
e) none

4 Extended Encodings

What well-known NP-Problems are described by the following extended encodings (where instances are represented by facts)?

a) $$P = \left\{ \begin{array}{l} \{t(X)\} \leftarrow v(X) \\ \qquad\quad \leftarrow c(C), \{t(X) : p(C,X), \sim t(X) : n(C,X)\}0 \end{array} \right\}$$

b) $$P = \left\{ \begin{array}{l} \{t(X)\} \leftarrow v(X) \\ \qquad\quad \leftarrow h(S), \{t(X) : c(S,X)\}0 \\ \qquad\quad \leftarrow h(S), 2\{t(X) : c(S,X)\} \end{array} \right\}$$

Answer:
Left as an exercise.

5 Programs with Aggregates

Determine the stable models of the following logic programs P with aggregates, check whether the contained aggregates are monotone, anti-monotone, or non-monotone, and provide appropriate translations of the aggregates to propositional formulas.

a) $$P = \left\{ \begin{array}{l} p \leftarrow sum\{1 : p, 1 : q\} \neq 1 \\ p \leftarrow q \\ q \leftarrow p \end{array} \right\}$$

b) $$P = \left\{ \begin{array}{l} p \leftarrow sum\{1 : p, 1 : q\} < 1 \\ p \leftarrow sum\{1 : p, 1 : q\} > 1 \\ p \leftarrow q \\ q \leftarrow p \end{array} \right\}$$

c) $$P = \left\{ \begin{array}{l} \quad \{p\} \\ \quad \{q\} \\ s \leftarrow sum\{1 : p, 1 : q, 2 : s\} \neq 3 \end{array} \right\}$$

d) $$P = \left\{ \begin{array}{l} \quad \{p\} \\ \quad \{q\} \\ s \leftarrow sum\{1 : p, 1 : q, 2 : s\} < 3 \\ s \leftarrow sum\{1 : p, 1 : q, 2 : s\} > 3 \end{array} \right\}$$

Answer:
a) $\{p, q\}$; non-monotone; $(p \rightarrow q) \wedge (q \rightarrow p)$
b) none; antimonotone and monotone; $(\neg p \wedge \neg q)$ and $(p \wedge q)$
c) $\{s\}$ and $\{p, q, s\}$; non-monotone; $((p \wedge s) \rightarrow q) \wedge ((q \wedge s) \rightarrow p)$
d) $\{s\}$; antimonotone and monotone; $\neg s \vee (\neg p \wedge \neg q)$ and $(s \wedge p \wedge q)$