A fact about maximally entangled states - Ben Criger

Often when doing quantum information, ket-notation can fail you. This happens a lot when you have a densely packed vector full of a bunch of different coefficients and there's no obvious structure. Now, hopefully when that happens to you, you're only going to deal with a few qubits. Otherwise, you are going to writing out coefficients for a very long time.

Here, we can see a not too bad example with two qubits where we're going to prove that for any maximally entangled state of the form psi-star-psi plus psi-perp-star-psi-perp, it's always equal to the Bell state that's fully correlated that we know and love: 1 over root 2 (times) 0-0 plus 1-1. And this is regardless of what psi is.

So, if we take an arbitrary wavevector psi which is two complex coefficients alpha and beta, we can define psi-star, which is just the complex conjugate of that state, which is also a valid state, psi-perp, which is some state which is orthogonal to psi, which I encourage you to check for yourself by taking the inner product, and psi-perp-star, which is the complex conjugate of the orthogonal state.

And if we write out all of our tensor products using the formula that we learned earlier, we obtain a pair of densely packed vector full of those coefficients, which would be very awkward in ket-notation. But we have here alpha-star-alpha, alpha-star-beta, beta-star-alpha and beta-star-beta. And we're going to add to that: beta-beta-star, minus beta-alpha-star, minus alpha-beta-star and alpha-alpha-star.

Now, beta-star-beta plus alpha-alpha-star is just the magnitude of alpha squared plus the magnitude of beta squared, which is 1. And we see that same thing in the top term here, alpha-star-alpha plus beta-beta-star, that's 1. And then these inner terms cancel. Because you have alpha-star-beta minus beta-alpha-star, that has just been flipped here. And (for) beta-star-alpha minus alpha-beta-star, if I flipped these two, it becomes obvious that they are equal and opposite, so they cancel.

This implies for example, that for example if Alice and Bob are a Bell state, and Alice measure in a 0-1 basis, she can get a state 0 or 1 and she can tell that Bob has the same state. Now she measures instead in the basis psi-star psi-star-perp, she can tell that Bob has the state psi or psi-perp depending on her measurement result.