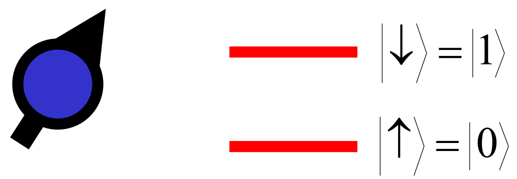
Leonardo DiCarlo

Superconducting quantum circuits: The transmon qubit

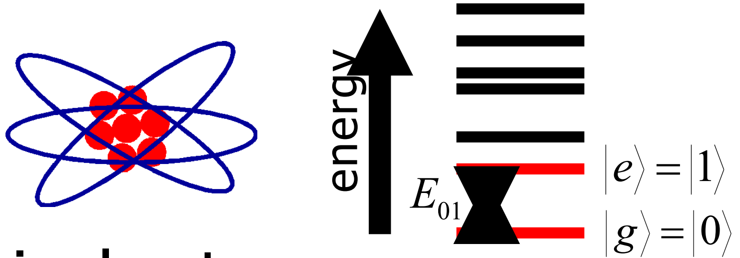
Nature’s quantum bits

true qubits



single spin-1/2

effective qubits



single atom

Manmade quantum bits

artificial atoms built from circuits

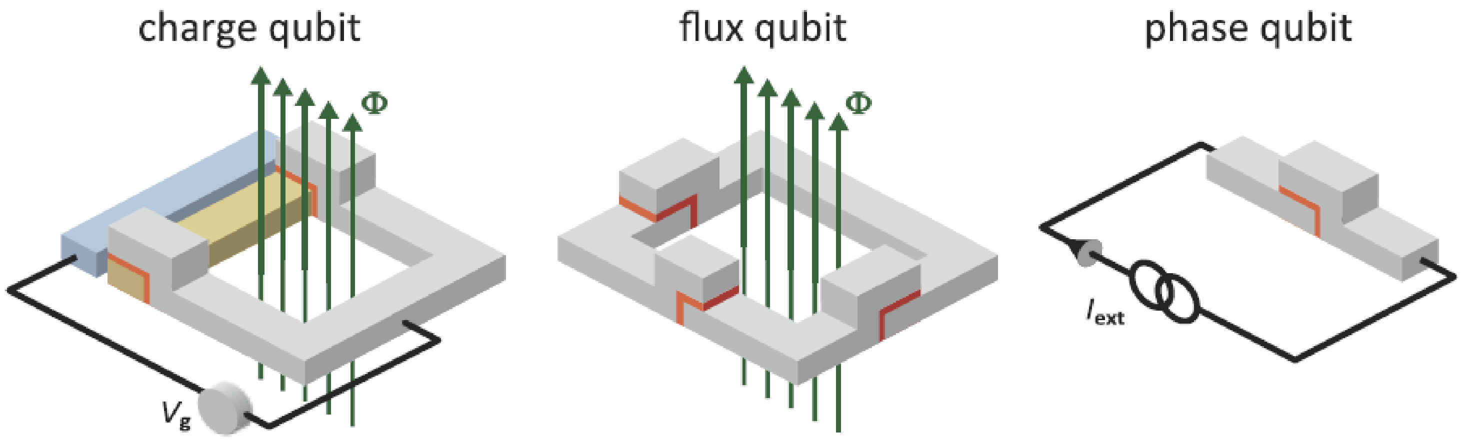
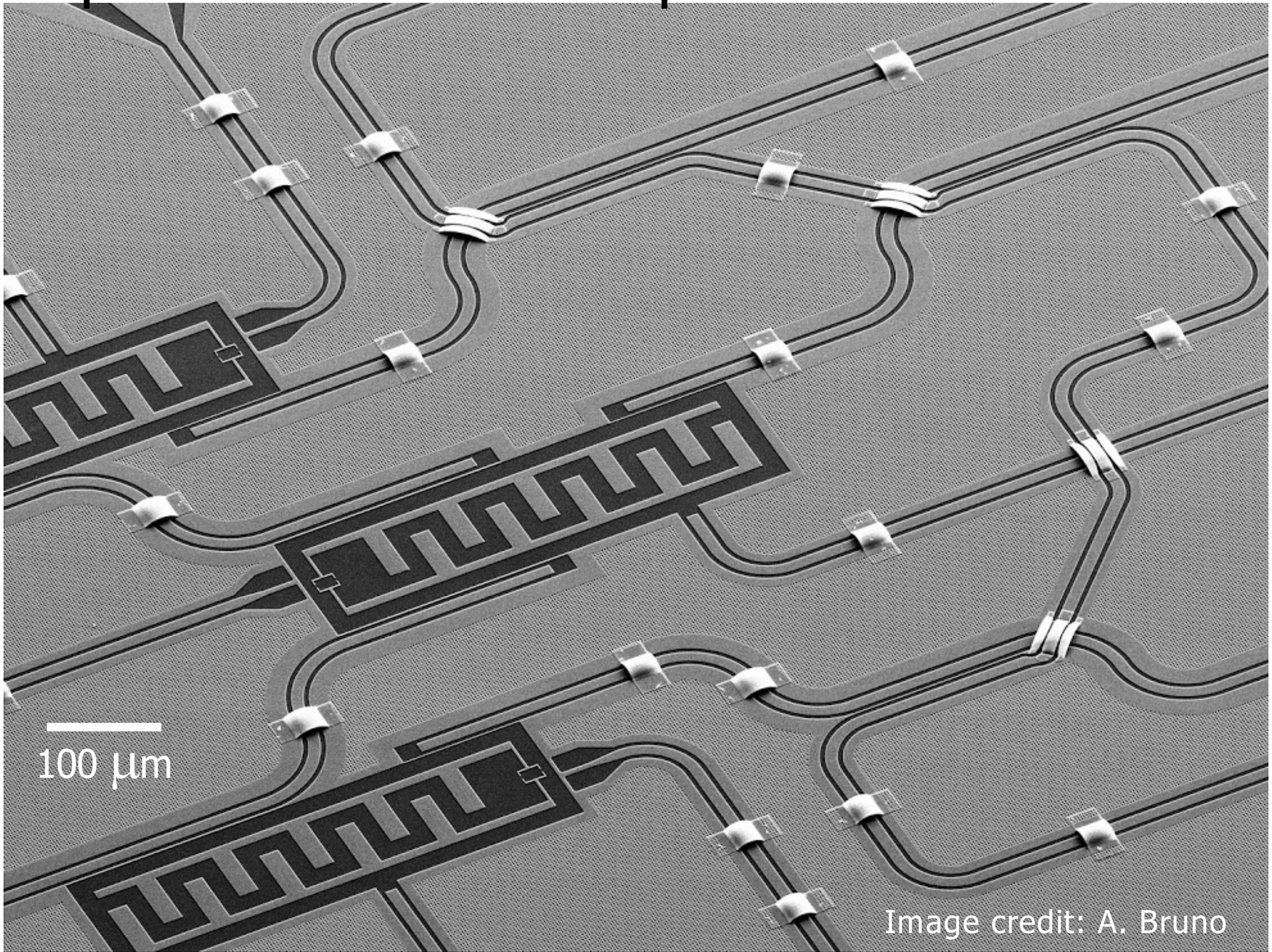
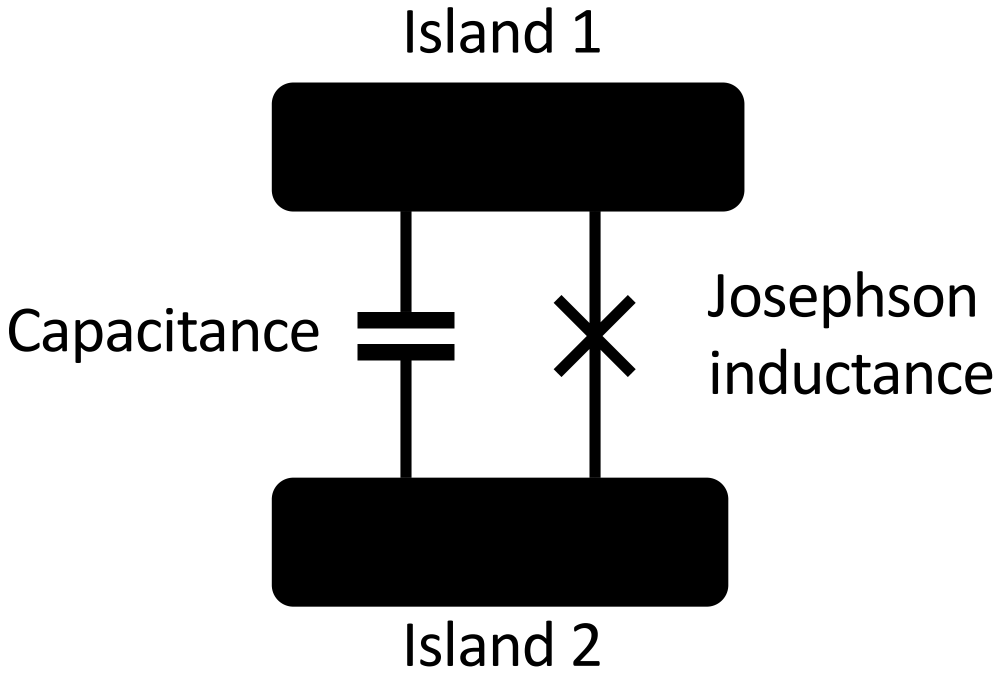


Image credit: W.D. Oliver & P.B. Welander, MRS Bulletin **38**, 816 (2013)

Transmon qubits embedded in a planar circuit

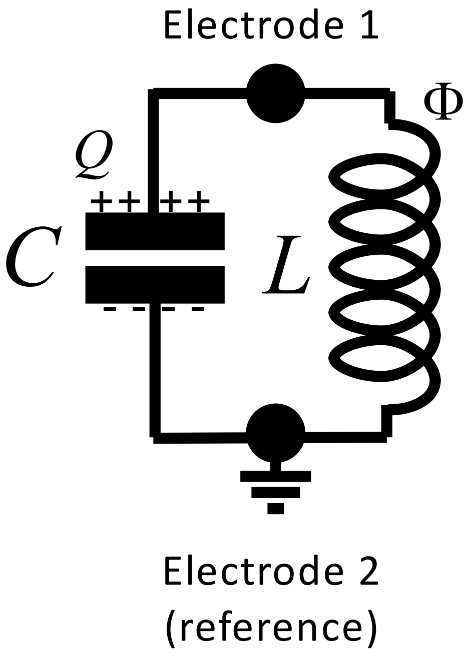


The Transmon qubit



Theory of the transmon: J. Koch *et al.*, Phys Rev. A **76**, 042319 (2007)

The quantized LC oscillator



Hamiltonian:

$$\hat{H}_{\mathrm{LC}} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L}$$

Capacitive term

Inductive term

Canonically conjugate variables:

$\hat{\Phi}$ = Flux through the inductor.

$\hat{Q}$ = Charge on capacitor plate.

$$\left[\hat{\Phi}, \hat{Q}\right] = i\hbar$$

M. Devoret, Les Houches Session LXIII (1995)

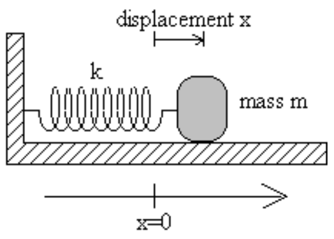
Correspondence with simple harmonic oscillator

$$\hat{H}_{\mathrm{LC}} = \frac{\hat{\Phi}^2}{2L} + \frac{\hat{Q}^2}{2C}$$

$$\left[\hat{\Phi},\hat{Q}\right] = i\hbar$$

$$\hat{H}_{\mathrm{SHO}} = \frac{k\hat{X}^2}{2} + \frac{\hat{P}^2}{2m}$$

$$\left[\hat{X},\hat{P}\right] = i\hbar$$



Correspondence:

$$\hat{\Phi} \leftrightarrow \hat{X} \qquad L \leftrightarrow \frac{1}{k}$$

$$\hat{Q} \leftrightarrow \hat{P} \qquad C \leftrightarrow m$$

$$\omega = \frac{1}{\sqrt{LC}} \leftrightarrow \sqrt{\frac{k}{m}}$$

Solve using ladder operators:

$$\hat{a} = \left(\frac{\hat{Q}}{Q_{\mathrm{zpf}}} - i\frac{\hat{\Phi}}{\Phi_{\mathrm{zpf}}}\right)$$

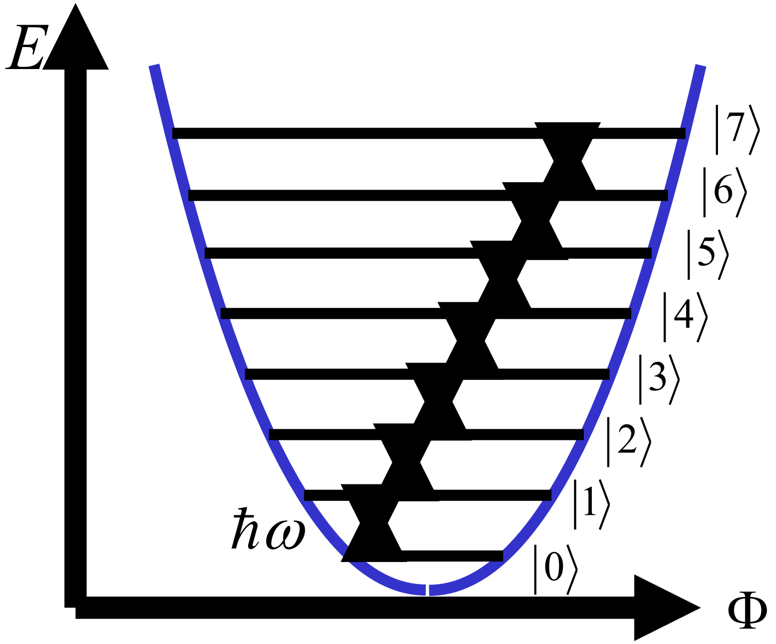
$$\hat{a}^\dagger = \left(\frac{\hat{Q}}{Q_{\mathrm{zpf}}} + i\frac{\hat{\Phi}}{\Phi_{\mathrm{zpf}}}\right)$$

$$\Phi_{\mathrm{zpf}} = \sqrt{2\hbar Z}$$

$$Q_{\mathrm{zpf}} = \sqrt{2\hbar / Z}$$

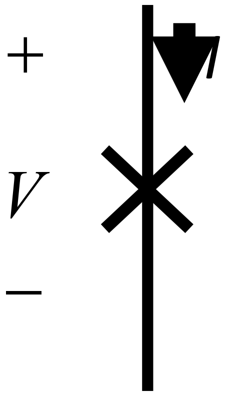
$$Z = \omega L = \frac{1}{\omega C} = \sqrt{\frac{L}{C}}$$

$$\hat{H}_{\mathrm{LC}} = \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) \qquad \left[\hat{a}_{\mathrm{r}},\hat{a}_{\mathrm{r}}^\dagger\right] = 1$$



M. Devoret, Les Houches Session LXIII (1995)

The Josephson junction

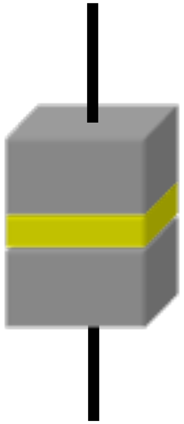


$$I = I_c \sin\left(2\pi \frac{\Phi}{\Phi_o}\right)$$

$$V = \dot{\Phi}$$

$$\Phi_o = \frac{h}{2e}$$

flux quantum

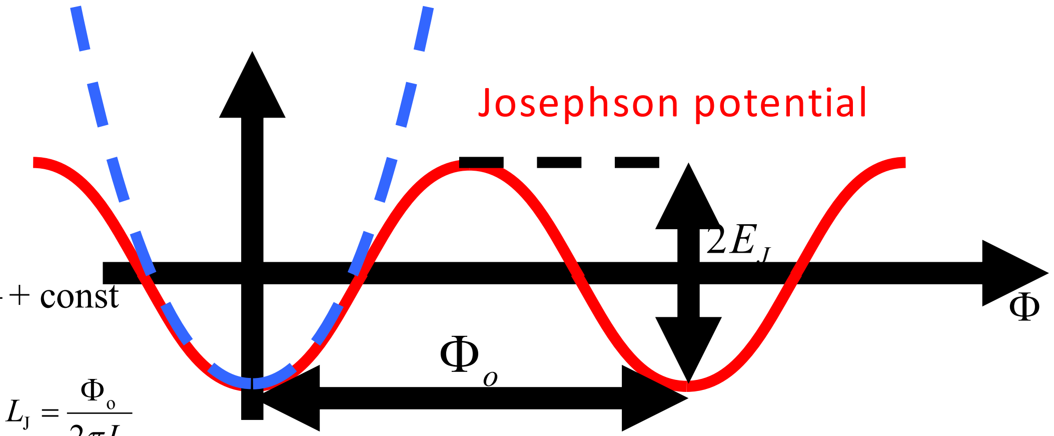


S superconductor-
I insulator-
S superconductor
tunnel junction

$$I_c = \frac{\pi}{2e}\frac{\Delta}{R}$$

$$E_{\text{stored}} = E_J\left(1-\cos\left(2\pi \frac{\Phi}{\Phi_o}\right)\right)$$

$$E_J = \frac{I_c \Phi_o}{2\pi}$$ Josephson Energy



For small flux: $E_J \approx \frac{\Phi^2}{2L_J} + \text{const}$

Josephson inductance $L_J = \frac{\Phi_o}{2\pi I_c}$

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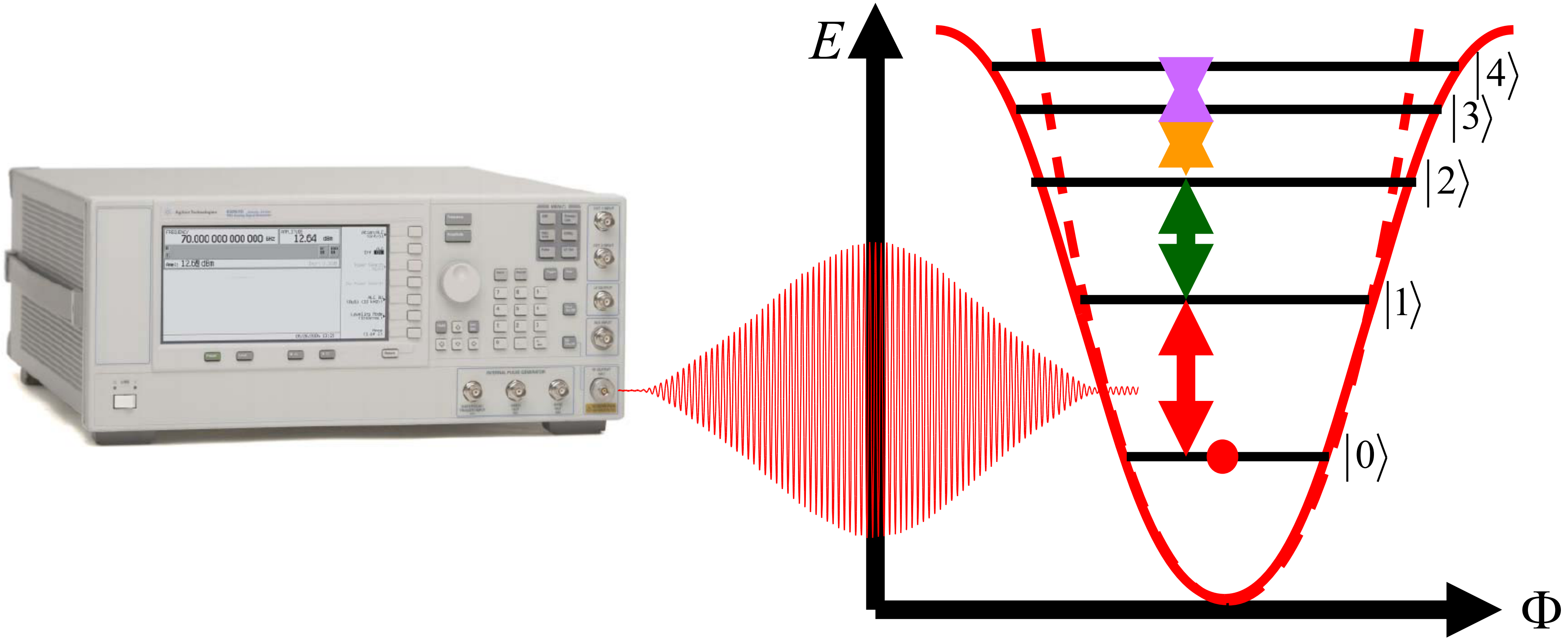
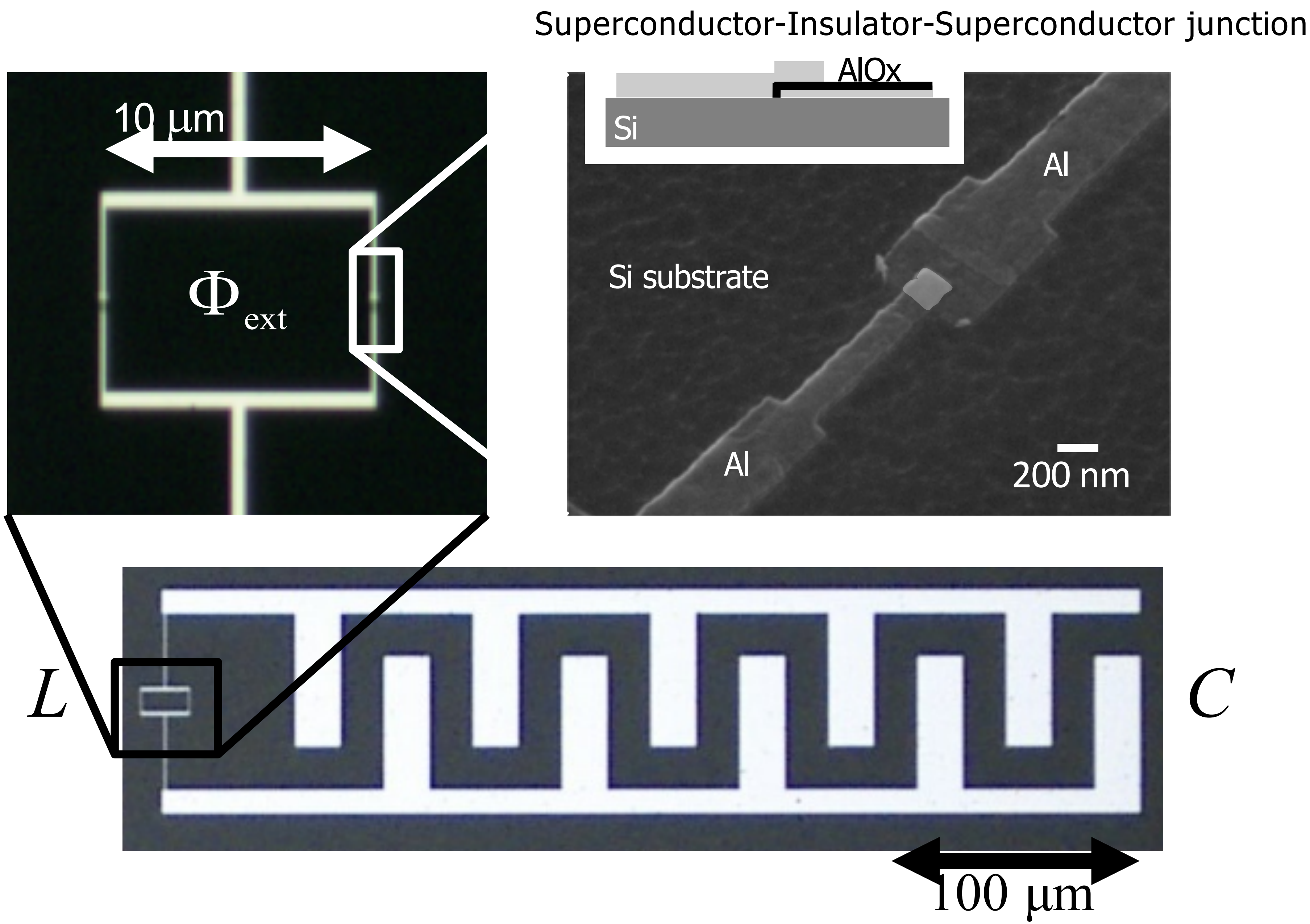
Transmon energy spectrum



Two-junction transmon



Flux control of transmon frequency

Short-circuited transmission line