

## Transcript Majorana fermions and where to find them Michael Wimmer

In the next few videos we want to introduce you to topological quantum computing.

To understand what topological quantum computing is, we first need to introduce you to the basic building blocks which are the so called Majorana fermions or Majorana bound states.

To understand Majorana fermions it is useful to first have a quick glimpse at high energy physics.

In high energy physics we know that for every particle there is always an antiparticle.

For example, there is an electron and positron, a proton and antiproton and so forth.

These are examples of what we call Dirac fermions.

However, there is also a separate kind of fermion in high energy physics which is called a Majorana fermion.

It is special because it is a fermion which is its own antiparticle.

The concept was actually first introduced by Ettore Majorana whose picture you can see [here](#).

Interestingly, to this date there are no Majorana fermions known in nature as elementary particles.

This is actually very similar to Majorana's life story itself.

He himself vanished during traveling on a ship, nobody knows for sure what happened to him.

Majorana fermions also could exist in principle, but nobody knows for sure if they do.

The definition of a Majorana fermion is that it is a particle that is its own antiparticle.

In physical terms this means that the creation operator equals the annihilation operator.

There are some candidates for these Majorana fermions.

For example, in high energy physics neutrinos are generally described as Dirac fermions, but there is an extension in which they could be Majorana fermions.

But people have also thought about how one could effectively realize Majorana fermions as quasi-particles in condensed matter systems.

This will be the topic of this lecture.

There is a little caveat here; what we will find in condensed matter are not quite the Majorana Fermions that high energy physicists are looking for.

What we find are *states*, and it's actually more appropriate to call them Majorana bound states, which is the term I will be using from now on – or I will simply refer to them as Majoranas.

To introduce Majorana bound states, we can first do a simple mathematical trick.

In condensed matter physics, a state can be filled, this is an electron, or it can be empty, in which case we have a hole.

These are the equivalent of particle and antiparticle, respectively.

They are described by fermionic operators: creation operator  $c^\dagger$  for the electron and annihilation operator  $c$  for the hole.

Now, I can do a simple linear superposition of operators: an equal superposition of a creation and annihilation operator.

It is very easy to see, I urge you to do the mathematics yourself, that those operators are Majorana operators:  $\gamma_1$  equals  $\gamma_1^\dagger$ , and  $\gamma_2$  equals  $\gamma_2^\dagger$ .

Graphically, you can see that these states are at the same time occupied and unoccupied.

Of course, this was just a mathematical trick: a transformation to go from one basis to another one.

What you see here is that still, one ordinary Fermionic operator can always be described by two Majorana operators.

However, once you can separate the two Majorana bound states, and they become distinguishable from each other, then things become interesting.

We can summarize at this point: First, two Majoranas form one fermionic state.

Second, as quasiparticles in condensed matter systems, they always come in pairs – in condensed matter the building blocks are always ordinary fermions.

Now, what is the connection to qubits?

With fermionic states, we can encode qubits.

If the state is empty this will be the state 0 of the qubit, and if it is filled it is the state 1.

An example for this is charge qubits.

The problem is that these are usually very sensitive to local perturbations.

But now Majoranas come to the rescue!

As I just told you, 2 Majorana's correspond to 1 fermion.

But if I have 2 spatially separated Majoranas I can encode one fermionic degree of freedom in a very nonlocal way, protected from any local perturbation.

This is our topological qubit: two Majorana bound states form one topological qubit, which is protected against almost all sorts of perturbations and is expected to have a very long coherence time.

For that reason it is very interesting to look for Majorana Bound states in condensed matter.

Where should we look for Majoranas in condensed matter?

We can again start from the concept of particle and antiparticle.

As I told you before, the equivalent of particles and antiparticles in condensed matter are electrons and holes.

A Majorana is a superposition of those two things.

The problem is however that electron and hole have opposite charge.

It is usually impossible to form a superposition of them.

It however turns out that a good system for Majoranas are superconductors.

In superconductors we have a sea of Cooper pairs.

Cooper pairs are states consisting of 2 electrons.

Suppose now we have a single hole in our system.

If I take just one Cooper pair out of the vast sea of Cooper pairs, then this Cooper pair together with the hole just looks like a single electron.

The distinction between electron and a hole are thus effectively blurred in a superconductor.

Hence, we can form a linear superposition of electrons and holes there!

It is thus very natural to look for Majorana states in superconductors.