Majorana bound states in superconductors

Let us now look a little bit more in detail, how we can find Majorana bound states in superconductors. I showed you in the last video that Majorana bound states can appear quite naturally there. The blurring of electron and hole can be described mathematically as a so-called particle-hole symmetry. For a particle at energy E you must have an antiparticle at energy minus E. That is given by this formula here, with the annihilation operator gamma at energy E equals the creation operator at energy minus E.

For zero energy we then immediately get Majorana bound states! In this case the Majorana creation operator, gamma dagger, equals the annihilation operator gamma. So, all we need to do is look for states in superconductors with zero energy!

Interestingly, the particle hole symmetry also helps to protect these Majorana bound states. We say that they are topologically protected. Let me show you now in a simple picture what this actually means. Particle-hole symmetry means that the energy spectrum must be symmetric around zero energy. This spectrum has a superconducting gap, this is shown as a white region in the slides. If you have one state at zero energy, one Majorana bound state, then this state is protected and has to remain at zero energy - regardless of what kind of perturbations you do to your systems. If the Majorana state were to move away from 0 energy, the system would not have particle hole symmetry anymore. That symmetry is fundamental, so this is not allowed. This way the particle hole symmetry protects the Majorana bound states. We call these states symmetry protected topological states.

Now I told you before, that in real condensed matter systems Majorana bound states always come in pairs. Any normal fermionic state can be described with two Majorana states. In this case a perturbation can actually move them symmetrically in the spectrum. So that of course is not protected. However, true Majorana bound states are spatially separated, far apart and cannot talk to each other. Each of those is then again protected by the particle hole symmetry.

We can thus distinguish two kinds of superconductors: either there are Majorana bound states - and then they are protected). Or there are no Majoranas at all. A superconductor that has Majoranas we call a topological superconductor. A superconductor without Majorana fermions we call trivial superconductor.

Now, there is actually an interesting aspect about Majorana bound states being at 0 energy. You can have multiple Majorana pairs and the states that you can make out of these Majorana pairs all have zero energy, too. So, with N Majorana bound state pairs, you actually have a 2^N fold degenerate ground state, because each pair can be occupied or not occupied. In a topological superconductors we thus generally have a gap, and at 0 energy a 2^N fold degenerate ground state. This will be important in a later stage, as this allows for topologically protected operations on Majorana bound states. This will be covered in a separate lecture.

In reality, we cannot separate the Majorana bound states infinitely far from each other. Hence there is a small overlap left over. But this overlap is then exponentially small, so the states will be exponentially close to zero energy, which is good enough.

At this point it might seem easy to find Majoranas: we just have to find states in superconductors with zero energy. But this is actually not as easy as it seems.

Because, how could one get states at small energies in a superconductor? After all, there is the superconducting gap. We could consider though the situation where there is a vortex in the superconductor. In a vortex, magnetic flux penetrates the superconductor and locally suppresses the superconducting gap Delta. The suppressed gap is shown here by a black line. Still, if you calculate the bound states of this system, you find that there is only a state at a finite energy. The reason for this, is the quantum mechanical zero-point motion.

To get rid of the zero-point motion one needs to consider unconventional superconductor, such as so called P-wave superconductors. In that case, there is an additional Berry phase of Pi that can cancel the zero-point motion. We get exactly then one state at zero energy, which is a Majorana bound state.

It turns out that instead of going to vortices, which are actually hard to control, we can go to one dimensional systems: nanowires. If we make a nanowire out of P-wave superconductor, you will also get Majorana states at the ends of the wire.

So, a P-wave superconductor would be nice to have, but it turns out that all the superconductors in nature that we know of are just trivial superconductors. There are some candidates that might be P-wave superconductors, but nobody knows for sure. The most promising approach is thus to engineer the P-wave superconductor out of normal, ordinary trivial materials.

I want to focus here on one particular example. It was shown that a semiconducting nanowire with spin-orbit interaction in proximity to a s-wave superconductor in a finite magnetic field can support Majoranas. Now one has to put all of these ingredients together, but this is not enough, one also has to tune some parameters to get to the topological phase. In particular, in this case we need to tune the magnetic field so that the Zeeman splitting exceeds the superconducting gap. Additionally, we also have to tune the chemical potential into the Zeeman gap. If we can do this, for example, with a gate, then Majorana bound states will appear at the right gate settings and magnetic field.

This is now a system you can make in the lab. All the ingredients are in principle known experimentally. This was done for the first time in 2012 in Delft, and this is what in practice the experimental system looks like.