State decomposition - Ben Criger

Often in quantum computation, it is necessary to express one state in terms of a basis of some other states. I am going to go ahead and call this state decomposition, although you can see it under a variety of names and usually it's clear from the context that they mean expressing one state in terms of a basis that is more familiar or convenient to.

Now the rule of state decomposition is that any state phi can be expressed as a sum over k terms, where each term consists of an inner product of phi - the trial state – with one of the basis states phi_k, and the states phi_k. Number times vector. And this is the same as decomposing a vector in a basis if you are already familiar with linear algebra. As an example of this, let's take a look at a trial wave function of zero, and a basis which is the Hadamard basis which we reviewed in the lecture. The sum over psi_k, phi, psi_k is then the inner product of plus and zero times plus and the inner product of minus and zero times minus.

Now here I figured out those inner product. The inner product of plus zero is one over root two times the dot product of these two vectors, which is just one over root two. And the calculation for minus zero is very similar, there is a minus sign here, but it gets multiplied by zero, making no difference, and we end up with one over root two again. Therefore, zero can be decomposed as one over root two, plus + minus, just as the plus state can be decomposed as one over root two, zero + one.