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Geometry and dynamics of one-norm geometric quantum discord

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Abstract We investigate the geometry of one-norm geometric quantum discord and present a geometric interpretation of one-norm geometric quantum discord for a class of two-qubit states. It is found that one-norm geometric quantum discord has geometric behavior different from that described in Lang and Caves (Phys Rev Lett 105:150501, 2010), Li et al. (Phys Rev A 83:022321, 2011) and Yao et al. (Phys Lett A 376:358–364, 2012). We also compare the dynamics of the one-norm geometric quantum discord and other measures of quantum correlations under correlated noise. It is shown that different decoherent channels bring different influences to quantum correlations measured by concurrence, entropic quantum discord and geometric quantum discord, which depend on the memory parameter and decoherence parameter. We lay emphasis on the behaviors such as entanglement sudden death and sudden transition of quantum discord. Finally, we study the dynamical behavior of one-norm geometric quantum discord in one-dimensional anisotropic XXZ model by utilizing the quantum renormalization group method. It is shown that the one-norm geometric quantum discord demonstrates quantum phase transition through renormalization group approach.

Keywords One-norm geometric quantum discord · Geometry · Correlated noise · Quantum phase transition · XXZ model

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1 Introduction

It is believed that entanglement is significant to quantum computation, communication and quantum information processing. However, entanglement is not the only type of quantum correlations. Separable states can have nonclassical correlations. Some measures have been proposed to quantify quantum correlations that have no entanglement. It has been found that many tasks can be carried out with quantum correlations other than entanglement [1–3]. It has been shown both theoretically and experimentally [4–9] that some separable states are more useful for certain tasks than their classical counterparts.

One kind of quantum correlation called entropic quantum discord has been introduced by Ollivier and Zurek [10] and independently by Henderson and Vedral [11] which have gained extensive attention recently [12–30].

The calculation of entropic quantum discord is a difficult task, even for two-qubit states, which is usually based on a numerical maximization procedure. Effective analytic ways are provided only for some specific classes of states [21–25].

The difficulty of calculating entropic quantum discord led Dakić et al. [26] to introduce a geometric measure of quantum discord which measures the quantum correlations of a state through the minimal Hilbert–Schmidt norm (or Schatten two-norm) between the given state and a zero-discord state. The calculation of this measure requires a simpler minimizing process. Subsequently, some authors [31–33] derived lower bound of the geometric measure of quantum discord for any bipartite states and an explicit expression for $2 \times n$ systems. However, in this way it is also difficult to calculate quantum correlations, and it has been pointed out recently [34–36] that the two-norm geometric quantum discord cannot be thought as a good measure for quantum correlations, which may increase under local operations on the unmeasured subsystem. The two-norm geometric quantum discord employs the Hilbert–Schmidt distance which lacks contractivity under trace-preserving maps. This is in contrast to the entropic quantum discord [10].

This fact results in a redefinition and further investigations of the geometric discord. One such geometric discord is the one-norm geometric quantum discord [37–42], which employs the Schatten one-norm that obeys the contractivity property and is invariant under unitary transformations, and it can distinguish between two states through a single measurement with the maximal probability. The distance measure is in connection with entanglement. It has been shown [41] that the entanglement counterpart of one-norm geometric quantum discord is negativity. Recently, closed analytical expressions are worked out for one-norm geometric quantum discord in Refs. [40–42].

Lately, Lang and Caves [20] demonstrated the level surfaces of entropic quantum discord for Bell-diagonal states and provided a pictorial approach which presented a complete interpretation of the structure of quantum discord and its dynamic behavior under decoherence. Interestingly, Li et al. [25] study the level surfaces of entropic quantum discord for a class of two-qubit states with parallel nonzero Bloch vectors, and the dynamic behavior of quantum discord under decoherence is considered. It is shown that a class of X states has sudden transition between classical and quantum correlations under decoherence. After that, Yao et al. [43] studied the level surfaces of two-norm geometric quantum discord and provided a pictorial interpretation of two-norm geometric quantum discord for Bell-diagonal states. They also observed its nonanalytic behavior under decoherence.

Recently, some authors also studied the dynamics of quantum correlations under the influence of noise channels [29,30,39,44–51]. It is shown that quantum discord may include sudden change in behavior and vanish at asymptotic time, and is more robust to noise than entanglement in comparison with entanglement sudden death [45,46]. It was also found that instantaneous disappearance of the quantum discord at some time points under non-Markovian environment [50,51].

On the other hand, dynamic behaviors of various quantum correlation measures for detecting quantum phase transition of the anisotropic spin-1/2 Heisenberg XXZ model [52–54] are explored by exploiting the quantum renormalization group method [55,56].

Inspired by the above works, it is natural to generalize the pictorial method to one-norm geometric quantum discord. In this paper, we study the level surfaces of one-norm geometric quantum discord for a class of two-qubit states and observe the nonanalytic behavior through the geometric picture. It is demonstrated that one-norm geometric quantum discord has different geometric structures from that described in Refs. [20,25,43].

Many literatures mainly consider memoryless channels in which consecutive signal transmissions are not correlated. In the correlated channels (i.e., the channels with memory), the noise acts on continuous usages of the channels. Therefore, in this work, we also study the one-norm geometric quantum discord under the influences of time-correlated noisy channels. We compare the decoherent dynamics of one-norm geometric quantum discord and other measures of quantum correlations. Our results show that decoherent evolutions do not lead to a sudden vanishing of quantum discord but sudden transition, in contrast to entanglement. In addition, we present other interesting behaviors of quantum discords under various time-correlated noisy channels.

Another work is the investigation of the evolution of one-norm geometric quantum discord in quantum phase transition via applying quantum renormalization group approach to the anisotropic spin-1/2 Heisenberg XXZ model, and critical behavior of the model is obtained.

Specifically, the relation of various measures of quantum correlations that we mention in this paper is shown in Fig. 1. Some concepts are defined in Sect. 2.

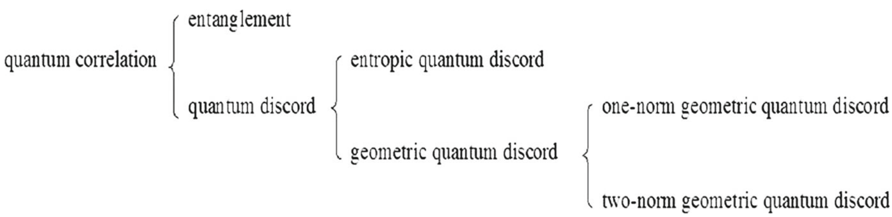

Fig. 1 The relations of various measures of quantum correlation

2 Various measures of quantum correlations

Let us consider a bipartite system AB in a Hilbert space $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$. For a bipartite quantum state ρ, entropic quantum discord is defined [10,11] as

$$\mathcal{Q}(\rho) = \mathcal{I}(\rho) - \mathcal{C}(\rho), \tag{1}$$

where $\mathcal{I}(\rho)$ is the quantum mutual information, which represents the total correlation of ρ, and $\mathcal{C}(\rho)$ is the measurement-based mutual information, which can be interpreted as the classical correlation of ρ. $\mathcal{I}(\rho)$ is defined by

$$\mathcal{I}(\rho) = \mathcal{S}(\rho_A) + \mathcal{S}(\rho_B) - \mathcal{S}(\rho), \tag{2}$$

where $\mathcal{S}(\rho) = -\mathrm{Tr}(\rho \log_2 \rho)$ is the Von Neumann entropy. Here, $\mathcal{C}(\rho)$ is given by

$$\mathcal{C}(\rho) = \sup_{\{\Pi_k\}} \left\{ \mathcal{S}(\rho_B) - \sum_k p_k \mathcal{S}(\rho_k) \right\}, \tag{3}$$

where $\rho_k = \frac{1}{p_k}(\Pi_k \otimes I)\rho(\Pi_k \otimes I)$ is the post-measurement state associated with the outcome k with the probability $p_k = \mathrm{Tr}[(\Pi_k \otimes I)\rho(\Pi_k \otimes I)]$. Projection operators $\{\Pi_k\}$ describe a Von Neumann measurement for subsystem A.

Closed analytic expressions of entropic quantum discord are provided only for some special two-qubit states, such as Bell-diagonal states [21] and X states [22].

A two-norm geometric quantum discord of a bipartite quantum state ρ is defined [26] as

$$\mathcal{D}_2(\rho) = \min_{\Omega_0} \|\rho - \rho_c\|_2^2, \tag{4}$$

where $\|X\|_2 = \sqrt{\mathrm{Tr}[X^\dagger X]}$ denotes the Hilbert–Schmidt norm (or Schatten two-norm) and Ω_0 is the set of classical-quantum states, whose general form is given by

$$\rho_c = \sum_k p_k \Pi_k^A \otimes \rho_k^B, \tag{5}$$

with $0 \leq p_k \leq 1$ ($\sum_k p_k = 1$), $\{\Pi_k^A\}$ denotes a set of orthogonal projectors for subsystem A and ρ_k^B being a general reduced density operator for subsystem B.

One can write an arbitrary two-qubit state in the Bloch representation

$$\rho = \frac{1}{4}\left(I \otimes I + \sum_i^3 x_i \sigma_i \otimes I + \sum_i^3 I \otimes y_i \sigma_i + \sum_{i,j=1}^3 t_{ij} \sigma_i \otimes \sigma_j \right), \tag{6}$$

with $x_i = \mathrm{Tr}(\rho \sigma_i \otimes I)$, $y_i = \mathrm{Tr}(\rho I \otimes \sigma_i)$, $t_{ij} = \mathrm{Tr}(\rho \sigma_i \otimes \sigma_j)$ being real parameters and σ_i being Pauli matrices. According to the above equation, two-norm geometric quantum discord can be calculated [26]

$$\mathcal{D}_2(\rho) = \frac{1}{4}\left(\|\mathbf{x}\|^2 + \|T\|_2^2 - \lambda_{\max}\right), \tag{7}$$

where $\mathbf{x} = (x_1, x_2, x_3)^t$ is a column vector, $\|\mathbf{x}\|^2 = \sum_i x_i^2$, $T = (t_{ij})$ is a matrix, and $\lambda_{\max}$ is the largest eigenvalue of the matrix $\mathbf{x}\mathbf{x}^t + TT^t$.

Actually, there is an alternative compact form of $\mathcal{D}_2(\rho)$, reading [49]

$$\mathcal{D}_2(\rho) = \frac{1}{4}\left(\sum_i \lambda_i^2 - \max_i \lambda_i^2\right), \tag{8}$$

with λ_i being the singular values of the matrix $T' = (\mathbf{x}, T)$, a 3×4 matrix.

The one-norm geometric quantum discord of a bipartite quantum state ρ is defined [40–42] as

$$\mathcal{D}_1(\rho) = \min_{\Omega_0} \|\rho - \rho_c\|_1, \tag{9}$$

where $\|X\|_1 = \mathrm{Tr}[\sqrt{X^\dagger X}]$ denotes the trace norm (or Schatten one-norm) and Ω_0 is the set of classical-quantum states.

A two-qubit X state has the matrix form

$$\rho_X = \frac{1}{4}\begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{pmatrix}, \tag{10}$$

which describes a quantum state subjected to the unit trace and positivity conditions $\sum_{i=1}^4 \rho_{ii} = 1$, $\rho_{22}\rho_{33} \geqslant |\rho_{23}|^2$, and $\rho_{11}\rho_{44} \geqslant |\rho_{14}|^2$. Without loss of generality, off-diagonal entries ρ_{32} and ρ_{41} can be taken as positive. The $\mathcal{D}_1(\rho)$ of X states is given [42] by

$$\mathcal{D}_1(\rho_X) = \sqrt{\frac{\gamma_1^2 \max\{\gamma_3^2, \gamma_2^2 + x_3^2\} - \gamma_2^2 \min\{\gamma_3^2, \gamma_1^2\}}{\max\{\gamma_3^2, \gamma_2^2 + x_3^2\} - \min\{\gamma_3^2, \gamma_1^2\} + \gamma_1^2 - \gamma_2^2}}, \tag{11}$$

where

$$\begin{aligned} &\gamma_1 = 2\left(\rho_{32} + \rho_{41}\right), \quad \gamma_2 = 2\left(\rho_{32} - \rho_{41}\right), \quad \gamma_3 = 1 - 2\left(\rho_{22} + \rho_{33}\right), \\ &x_3 = 2\left(\rho_{11} + \rho_{22}\right) - 1. \end{aligned} \tag{12}$$

Note that Eq. (11) holds when $|\gamma_1| \geqslant |\gamma_2|$ and is the same as the corresponding equation in Ref. [42] except for a factor of $\frac{1}{2}$.

Two-qubit Bell-diagonal states have the matrix form

$$\rho_{BD} = \frac{1}{4}\begin{pmatrix} 1+c_3 & 0 & 0 & c_1-c_2 \\ 0 & 1-c_3 & c_1+c_2 & 0 \\ 0 & c_1+c_2 & 1-c_3 & 0 \\ c_1-c_2 & 0 & 0 & 1+c_3 \end{pmatrix}, \tag{13}$$

which are special cases of X states. The one-norm geometric discord of Bell-diagonal states can be obtained explicitly [40,42] as

$$\mathcal{D}_1(\rho_{BD}) = \operatorname{int}\{|c_1|, |c_2|, |c_3|\}, \tag{14}$$

where $\operatorname{int}\{|c_1|, |c_2|, |c_3|\}$ denotes the intermediate value among the absolute values of c_1, c_2 and c_3.

The entanglement of formation is a monotonically increasing function of the concurrence [57]. For two-qubit mixed states, the concurrence is defined as

$$\mathcal{C}(\rho) = \max\left\{0, 2\max\left\{\sqrt{\lambda_1}, \sqrt{\lambda_2}, \sqrt{\lambda_3}, \sqrt{\lambda_4}\right\} - \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\right\}, \tag{15}$$

where λ_i are the eigenvalues of $\rho\tilde{\rho}$ in decreasing order, and $\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$ with ρ^* as the complex conjugated density matrix.

For X states, the concurrence is given as $\mathcal{C}(\rho_X) = \max(0, C_1, C_2)$ with $C_1 = 2(|\rho_{41}| - \sqrt{\rho_{33}\rho_{22}})$ and $C_2 = 2(|\rho_{32}| - \sqrt{\rho_{44}\rho_{11}})$.

3 Level surfaces of one-norm geometric quantum discord

Under appropriate local unitary transformations, any two-qubit state ρ can be written [58] as

$$\rho = \frac{1}{4}\left(I \otimes I + \mathbf{r}\cdot\sigma \otimes I + I \otimes \mathbf{s}\cdot\sigma + \sum_{i=1}^{3} c_i\sigma_i \otimes \sigma_i\right), \tag{16}$$

where $\mathbf{r} = (r_1, r_2, r_3)$ and $\mathbf{s} = (s_1, s_2, s_3)$ are Bloch vectors and σ_i are the standard Pauli matrices.

In the following, we assume that the Bloch vectors are z directional, that is, $\mathbf{r} = (0, 0, r)$ and $\mathbf{s} = (0, 0, s)$. One can also change them to be x or y directional through an appropriate local unitary transformation without losing its diagonal property of the correlation term [58]. In this case, the matrix form of Eq. (16) is given as

$$\rho_{SX} = \frac{1}{4}\begin{pmatrix} 1+r+s+c_3 & 0 & 0 & c_1-c_2 \\ 0 & 1+r-s-c_3 & c_1+c_2 & 0 \\ 0 & c_1+c_2 & 1-r+s-c_3 & 0 \\ c_1-c_2 & 0 & 0 & 1-r-s+c_3 \end{pmatrix}, \tag{17}$$

which is a special case of X states.

When $\mathbf{r} = \mathbf{s} = \mathbf{0}$, ρ_{SX} reduces to Bell-diagonal states ρ_{BD}, which has four eigenvalues

$$
\begin{aligned}
\lambda_1 &= \frac{1}{4}\left(1 - c_1 - c_2 - c_3\right), \\
\lambda_2 &= \frac{1}{4}\left(1 - c_1 + c_2 + c_3\right), \\
\lambda_3 &= \frac{1}{4}\left(1 + c_1 - c_2 + c_3\right), \\
\lambda_4 &= \frac{1}{4}\left(1 + c_1 + c_2 - c_3\right).
\end{aligned} \tag{18}
$$

Note that each state ρ_{BD} is associated with a 3-tuple (c_1, c_2, c_3), and if ρ_{BD} describes a physical state (positive operator), then $0 \leqslant \lambda_i \leqslant 1$ and $\sum_{i=1}^{4} \lambda_i = 1$. In this condition, the region of Bell-diagonal states must be restricted to a tetrahedron $\mathcal{T}$ with vertices $(-1, -1, -1)$, $(-1, 1, 1)$, $(1, -1, 1)$, and $(1, 1, -1)$ (see Fig. 1 in Ref. [20]). The Bell states locate at the four vertices of $\mathcal{T}$. A Bell-diagonal state is separable if and only if its partial transpose is positive [59]. Partial transposition only changes the sign of c_2, so operators with positive partial transpose $\left(\rho_{BD}^{T}\right)$ are the reflection of $\mathcal{T}$ through the plane $c_2 = 0$. The region of separable Bell-diagonal states is the intersection of the two tetrahedra ρ_{BD}^{T} and ρ_{BD}, which forms a octahedron $\mathcal{O}$ with vertices $(\pm 1, 0, 0)$, $(0, \pm 1, 0)$ and $(0, 0, \pm 1)$ (see Fig. 1 in Ref. [20]). There are four entangled regions outside $\mathcal{O}$, one in each vertex of $\mathcal{T}$.

Figure 2 plots level surfaces of one-norm geometric quantum discord for Bell-diagonal states. Compared with level surfaces of entropic quantum discord depicted in Refs. [20,25] and two-norm geometric quantum discord depicted in Ref. [43], the level surfaces of one-norm geometric quantum discord are composed of three identical intersecting "cuboids" instead of irregular "tubes" [20,25] and regular "cylinders" [43] (see Fig. 2). The cuboids surfaces are running along the three Cartesian axes, and their ends are cut off by state tetrahedron $\mathcal{T}$. The cuboids shrink toward the Cartesian axes as one-norm geometric quantum discord becomes smaller. We can see that if one-norm geometric quantum discord increases, the cuboids cut off by state tetrahedron $\mathcal{T}$ become four identical pieces reaching to the vertices of $\mathcal{T}$, which stand for four Bell states, the maximally entangled states. The case is similar to geometrical picture of quantum discord [20]. We easily analyze that the symmetrical cuboid structure is due to the function $\mathrm{int}\{|c_1|, |c_2|, |c_3|\}$. Actually, from the function, we know that one-norm geometric quantum discord is the intermediate value of absolute values of c_1, c_2 and c_3. Because of requiring to compute absolute values, the graph of the function presents symmetrical structure naturally. For example, provided that the intermediate value is $|c_1| = 0.15$, then the graph of the function $|c_1| = 0.15$ is composed of two symmetrical planes being perpendicular to c_1–c_2 plane. Combining with the graph of function $|c_3| = 0.15$, we obtain one cuboid being perpendicular to c_1–c_3 plane center. Plus the graph of function $|c_2| = 0.15$, all the intersecting graphs cut off by state tetrahedron $\mathcal{T}$ develop the level surface of Fig. 2a. Other cases of Fig. 2 are similar. It is also not difficult to see that the one-norm geometric quantum discord does not vanish

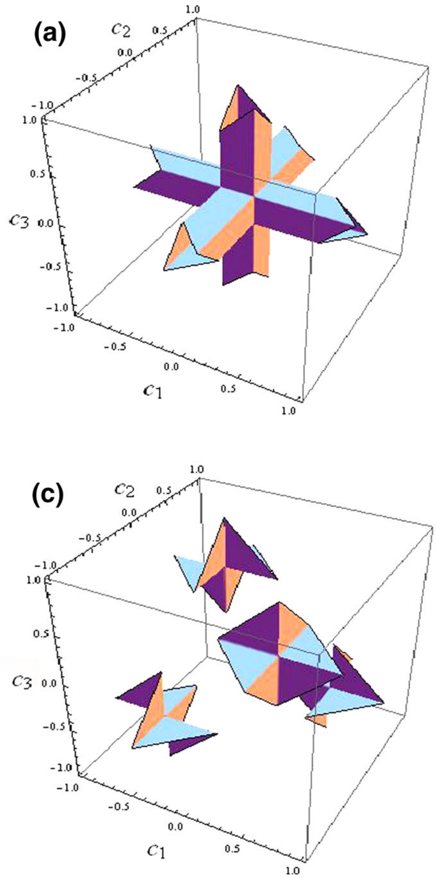

Fig. 2 Level surfaces of constant one-norm geometric quantum discord cut off by the state tetrahedron $\mathcal{T}$ at their ends: **a** $\mathcal{D}_1(\rho_{BD}) = 0.15$, **b** $\mathcal{D}_1(\rho_{BD}) = 0.25$, **c** $\mathcal{D}_1(\rho_{BD}) = 0.5$

even if states are in octahedron $\mathcal{O}$, the set of separable states, where entanglement vanishes. Thus, it is clear that discord is quite different from entanglement. The one-norm geometric quantum discord is away from the Cartesian axes, in line with the notion of trace distance from classical-quantum states.

Now, let us consider the ρ_{SX} with nonzero Bloch vectors [see Eq. (17)] which is a two-qubit X state. The eigenvalues of ρ_{SX} are given by

$$\mu_\pm = \frac{1}{4}\left[1 - c_3 \pm \sqrt{(r-s)^2 + (c_1+c_2)^2}\right],$$
$$\nu_\pm = \frac{1}{4}\left[1 + c_3 \pm \sqrt{(r+s)^2 + (c_1-c_2)^2}\right]. \tag{19}$$

Then the geometrical deformation $\mathcal{F}$ can be obtained from the positivity condition of ρ_{SX} [58]. The condition of the deformation turns into

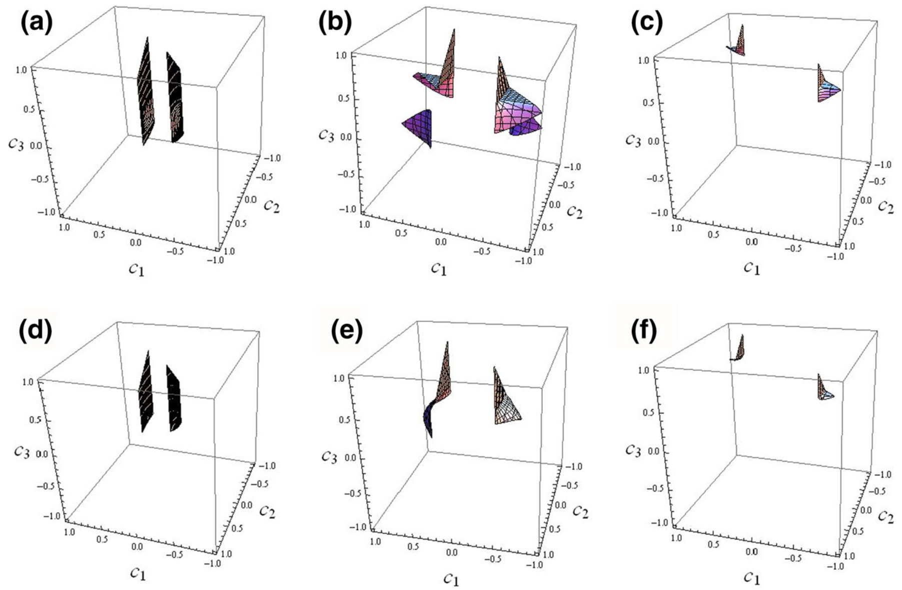

Fig. 3 Level surfaces of constant one-norm geometric quantum discord of state ρ_{SX} cut off by deformation $\mathcal{F}$: **a** $r = s = \frac{1}{3}$, $\mathcal{D}_1(\rho_{SX}) = 0.2$, **b** $r = s = \frac{1}{3}$, $\mathcal{D}_1(\rho_{SX}) = 0.4$, **c** $r = s = \frac{1}{3}$, $\mathcal{D}_1(\rho_{SX}) = 0.7$, **d** $r = s = \frac{1}{2}$, $\mathcal{D}_1(\rho_{SX}) = 0.2$, **e** $r = s = \frac{1}{2}$, $\mathcal{D}_1(\rho_{SX}) = 0.4$, **f** $r = s = \frac{1}{2}$, $\mathcal{D}_1(\rho_{SX}) = 0.7$

$$\min(\mu_-, \nu_-) = 0. \tag{20}$$

Through Eq. (12), we can obtain the following equation:

$$\gamma_1 = c_1, \quad \gamma_2 = c_2, \quad \gamma_3 = c_3, \quad x_3 = r. \tag{21}$$

By the above equation, the one-norm geometric quantum discord of ρ_{SX} reduces to

$$\mathcal{D}_1(\rho_{SX}) = \frac{1}{2}\sqrt{\frac{c_1^2 \max\left\{c_3^2, c_2^2 + r^2\right\} - c_2^2 \min\left\{c_3^2, c_1^2\right\}}{\max\left\{c_3^2, c_2^2 + r^2\right\} - \min\left\{c_3^2, c_1^2\right\} + c_1^2 - c_2^2}}, \tag{22}$$

which is independent of s. We assume that $|c_1| \geqslant |c_2|$ [42].

In Fig. 3, we plot the level surfaces of one-norm geometric quantum discord of state ρ_{SX}. One can see that the level surfaces of one-norm geometric quantum discord are quite different from the cases of Bell-diagonal states, which are composed of two separable mirror-symmetrical plots. In fact, this is closely related to the parameter r of Eq. (22). When $r = s \neq 0$, for example $r = \frac{1}{3}$, the graph of $\mathcal{D}_1(\rho_{SX}) = 0.2$ is composed of two separable symmetrical surfaces which are perpendicular to c_1–c_3 plane, cut off by deformation $\mathcal{F}$ formed by Eq. (20) [58], forming the mirror-symmetrical shape in Fig. 3a. Level surfaces of Fig. 3a, d, e are mirror symmetry based on plane $c_1 = 0$, and level surfaces of Fig. 3b, c, f are mirror symmetry based on plane $c_1 = c_2$. The distance between the two separable plots becomes wider when one-norm geometric quantum discord increases. If one-norm geometric quantum discord grows greater, the surface will reach toward two vertices of $\mathcal{T}$ (see Fig. 1 in Ref. [20]) which stand for two maximally entangled Bell states. The surface also shrinks due to shrinking effect of deformation $\mathcal{F}$ [58], and the shrinking rate becomes bigger with increasing r and s. For greater r and s, the graph is moved up the plane $c_3 = 0$.

4 One-norm geometric quantum discord under correlated noise

In a realistic situation, the quantum systems have a nontrivial dynamics because of their interaction with the environment. Decoherence is a nonunitary evolution that results from the interaction between the system and the environment, which may disturb the important properties of quantum information.

Superoperators provide a way to describe nonunitarity evolution of quantum states in a noisy environment. A superoperator, Φ, is a trace-preserving and completely positive linear map, from input density operator ρ_{in} to output density operator ρ_{out} which can be expressed in operator sum or Kraus representation [60] as

$$\rho_{\text{out}} = \Phi(\rho_{\text{in}}) = \sum_i E_i \rho_{\text{in}} E_i^{\dagger}. \tag{23}$$

The trace-preserving property of Φ requires that the Kraus operators satisfy the condition

$$\sum_i E_i^{\dagger} E_i = I. \tag{24}$$

Here, we are dealing with a correlated quantum system, the correlation of which results from the memory of the channel. We analyze quantum correlations based on time-correlated noise models for two-qubit Bell-diagonal states.

The action of a noise channel with memory can be expressed as

$$\rho_{\text{out}} = (1-u) \sum_{i,j} E_{ij}^{n} \rho_{\text{in}} E_{ij}^{n\dagger} + u \sum_k E_{kk}^{c} \rho_{\text{in}} E_{kk}^{c\dagger}, \tag{25}$$

where the superscripts n and c represent the uncorrelated and correlated parts of the channel, respectively. The above relation means that with the probability u $(0 \leq u \leq 1)$ the noise is correlated and with the probability $(1-u)$ it is uncorrelated.

4.1 Amplitude-damping channel with correlated noise

First, let us consider quantum amplitude-damping channel with correlated noise. The Kraus operators for the uncorrelated part of this channel for a two-qubit system are defined as

$$E_{ij}^{n} = M_i \otimes M_j, \quad i, j \in \{0, 1\}, \tag{26}$$

where

$$M_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix},$$
$$M_1 = \begin{pmatrix} 0 & \sqrt{p} \\ 0 & 0 \end{pmatrix}. \tag{27}$$

The Kraus operators for the correlated part of the amplitude-damping channel are given by Yeo and Skeen [61] as

$$E^c_{kk} = F_k, \quad k \in \{0, 1\}, \tag{28}$$

where

$$F_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \sqrt{1-p} \end{pmatrix},$$
$$F_1 = \begin{pmatrix} 0 & 0 & 0 & \sqrt{p} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{29}$$

We consider the Bell-diagonal state with $c_1 = 1, c_2 = -\frac{3}{5}, c_3 = \frac{3}{5}$, which is an entangled state mentioned in Ref. [29]. According to Eqs. (13) and (25), the elements of the matrix ρ_{out} are given by

$$\rho_{11} = \frac{2}{5} - \frac{1}{5} p\,(2pu - 2p - u - 1)\,,$$
$$\rho_{14} = \rho_{41} = \frac{2}{5}\left[(p-1)(u-1) + u\sqrt{1-p}\right],$$
$$\rho_{22} = \rho_{33} = \frac{1}{10}\left[1 + p\,(u-1)(-3+4p)\right],$$
$$\rho_{23} = \rho_{32} = \frac{1}{10}(pu - p + 1)\,,$$
$$\rho_{44} = -\frac{2}{5}(p-1)(pu - p + 1)\,. \tag{30}$$

We can see that ρ_{out} is not a Bell-diagonal state but an X state, entropic quantum discord of which can be calculated using the method of Ref. [22]. Thus, we may say that amplitude-damping channel with correlated noise can change the type of a state.

The dynamic behavior of quantum correlations for Bell-diagonal state under amplitude-damping channel with correlated noise is depicted in Figs. 4 and 5. From Figs. 4a and 5a, we can see entropic quantum discord first decreases in a time interval of decoherence parameter p and then increases slowly. However, entanglement has a sudden death [45,46] and then has sudden birth (see Figs. 4d, 5a). Quantum discord

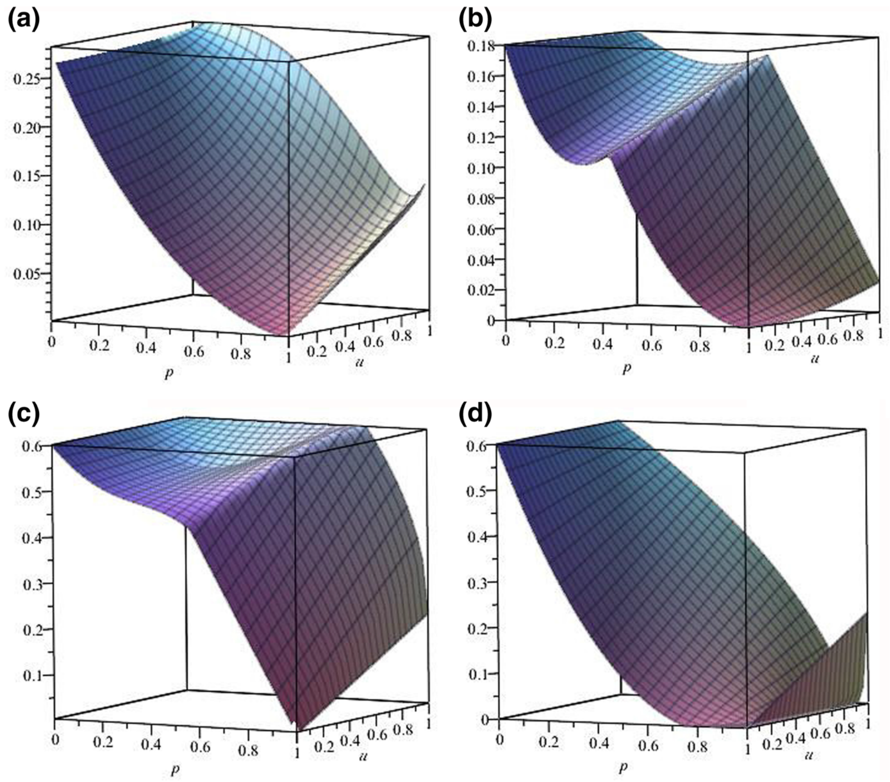

Fig. 4 **a** Entropic quantum discord, **b** two-norm geometric quantum discord, **c** one-norm geometric quantum discord, and **d** concurrence are plotted for Bell-diagonal state $\left(c_1 = 1, c_2 = -\frac{3}{5}, c_3 = \frac{3}{5}\right)$ as a function of decoherence parameter p and memory parameter u

almost does not vanish except for the case of $u = 0$ and $p = 1$ (see Fig. 4). From Figs. 4b, c and 5a, it is clear to see that geometric quantum discord has sudden transition. However, it is hard to localize the transition critical points in terms of parameters p and u for one-norm geometric quantum discord due to its complicated computing procedure for X states [see Eq. (11)]. By analyzing the evaluation procedure of two-norm geometric quantum discord, the transition critical points of two-norm geometric quantum discord can be localized through the following expression

$$\begin{aligned}
&\frac{8}{25}(up + p - 1)AB + \frac{32}{25}\left(-u^2 + u\right)(p-1)B + \frac{64}{25}\left(u^2 - 2u + 1\right)p^4 \\
&\quad + \frac{96}{25}\left(-u^2 + 2u - 1\right)p^3 + \frac{4}{5}\left(u^2 - \frac{24}{5}u + \frac{23}{5}\right)p^2 \\
&\quad + \frac{2}{25}\left(24u^2 - 15u - 1\right)p + \frac{8}{25}\left(-4u^2 + 4u - 1\right) = 0, \qquad (31)
\end{aligned}$$

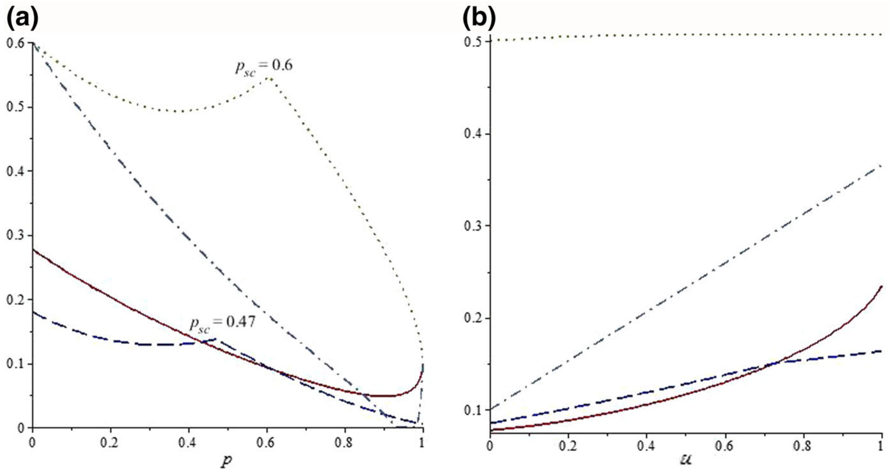

Fig. 5 Entropic quantum discord (*solid line*), two-norm geometric quantum discord (*dash line*), one-norm geometric quantum discord (*dot line*) and concurrence (*dashdot line*) are plotted as functions of decoherence parameter p with memory parameter $u = 0.5$ (**a**) and as functions of memory parameter u with decoherence parameter $p = 0.5$ (**b**). p_{sc} and u_{sc} are the critical points of sudden transition

where $A = \sqrt{(-u^2 + 2u)B + (-u^2 + 2u - 1)p + 2u^2 - 2u + 1}$ and $B = \sqrt{1 - p}$. From Figs. 4b and 5a, we can see that two-norm geometric quantum discord first monotonically decreases and then monotonically increases with decoherence parameter p until the critical points. In Fig. 4c, we see that the one-norm geometric quantum discord could still remain constant under decoherence in certain time intervals when $u = 1$. Quantum correlations grow with the increase in memory parameter u as shown in Fig. 5b. Combining with the previous discussions about geometry of one-norm geometric quantum discord for X states, we analyze that the geometric surface of the state ρ_{out} (30) will shrink with decoherence parameter p increasing or with memory parameter u decreasing. From Figs. 4 and 5, on the whole, we can say that one-norm geometric quantum discord and concurrence are greater than entropic quantum discord and two-norm geometric quantum discord.

4.2 Phase-damping channel with correlated noise

In the following, we consider quantum phase-damping channel with correlated noise. The Kraus operators of the uncorrelated part of the channel are defined as

$$E^n_{ij} = M_i \otimes M_j, \quad i, j \in \{0, 1\}, \tag{32}$$

where

$$M_0 = \sqrt{1 - \frac{p}{2}} I,$$

$$M_1 = \sqrt{\frac{p}{2}}\sigma_3. \tag{33}$$

The Kraus operators of the correlated part of the phase-damping channel are given [62] as

$$E_{00}^c = \sqrt{1-\frac{p}{2}}I \otimes I,$$
$$E_{11}^c = \sqrt{\frac{p}{2}}\sigma_3 \otimes \sigma_3. \tag{34}$$

We consider the Bell-diagonal state with $c_1 = \frac{3}{50}, c_2 = -\frac{21}{50}, c_3 = \frac{3}{10}$. This is an example of an initially separable state. The elements of the matrix ρ_{out} are given by

$$\rho_{11} = \rho_{44} = \frac{13}{40},$$
$$\rho_{14} = \rho_{41} = -\frac{9}{100}[1 - p(u-1)(p-2)],$$
$$\rho_{22} = \rho_{33} = \frac{7}{40},$$
$$\rho_{23} = \rho_{32} = \frac{3}{25}[1 - p(pu - 2u - p - 2)]. \tag{35}$$

It can be easily verified that ρ_{out} is a Bell-diagonal state with

$$c_1' = \frac{3}{50}[1 - p(u-1)(p-2)],$$
$$c_2' = \frac{21}{50}[1 - p(u-1)(p-2)],$$
$$c_3' = \frac{3}{10}. \tag{36}$$

Its entropic quantum discord can be calculated using the approach of Ref. [21]. As a result, we can see that phase-damping channel with correlated noise does not change the type of a state.

Figures 6 and 7 display the dynamics of quantum correlations for the Bell-diagonal state under phase-damping channel with correlated noise. From Figs. 6 and 7a, we can see quantum discord does not disappear completely and vanishes only when $p = 1$ and $u = 0$, although the state is a separable state since its concurrence is 0 (see Fig. 6d). We observe that quantum discord has sudden transition and the critical transition points are the same (see Fig. 7). Geometric quantum discord remains constant during a time interval (see Figs. 6b, c, 7), where quantum discord is unaffected by the noise. This may be important for quantum information processing. Sudden transition is related to maximization or minimization procedure in computing quantum discords, as analyzed in Ref. [29]. Interestingly, we find that the transition critical points of three measures of quantum discord are determined by the following function

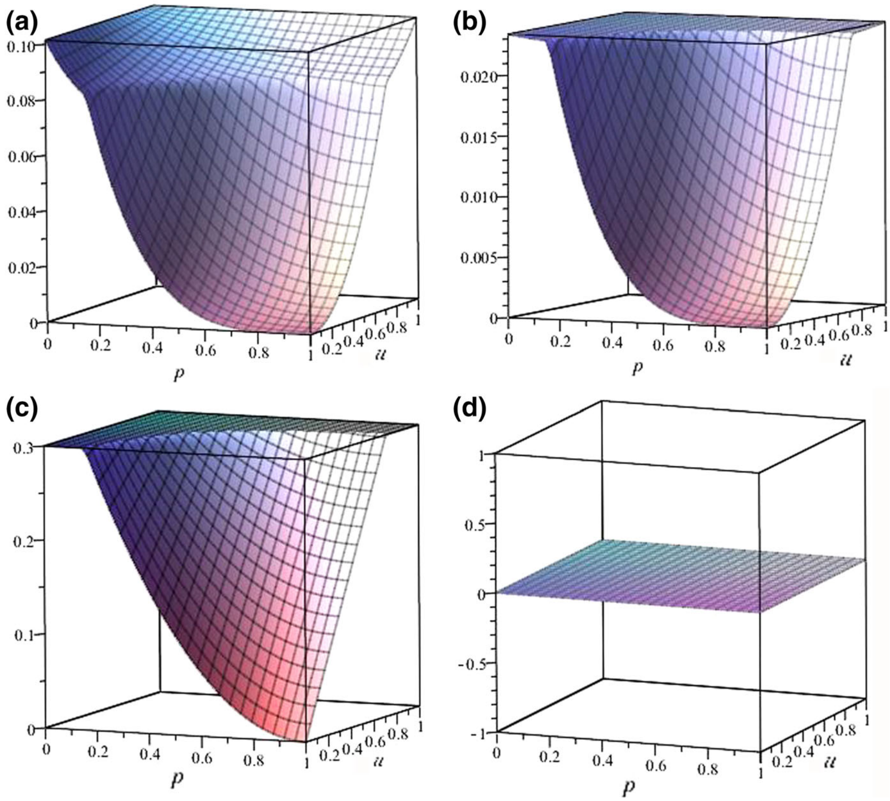

Fig. 6 **a** Entropic quantum discord, **b** two-norm geometric quantum discord, **c** one-norm geometric quantum discord, and **d** concurrence are plotted for Bell-diagonal state $\left(c_1 = \frac{3}{50}, c_2 = -\frac{21}{50}, c_3 = \frac{3}{10}\right)$ as functions of decoherence parameter p and memory parameter u

$$p = \frac{7u - 7 + \sqrt{49u^2 - 84u + 35}}{7(u-1)}, \tag{37}$$

which results from the optimization procedures depending on c_2' and c_3' [see Eq. (36)] in calculating the three measures of quantum discord.

Quantum correlation monotonously decreases with the decoherence parameter p and increases with the memory parameter u as shown in Fig. 7. Combining Fig. 6c with the previous discussions about geometry of one-norm geometric quantum discord for Bell-diagonal states, we can deduce that the geometric surface of the state ρ_{out} (Eq. 35) will shrink with increasing decoherence parameter p or decreasing memory parameter u. From Figs. 6 and 7, it can be seen that one-norm geometric quantum discord is greater than entropic quantum discord, and entropic quantum discord is greater than two-norm geometric quantum discord.

Next, we consider the Bell-diagonal state with $c_1 = \frac{3}{5}, c_2 = -\frac{2}{5}, c_3 = \frac{3}{10}$, which is an initially entangled state meeting conditions of double sudden changes of one-norm geometric quantum discord [39]. Note that it does not meet the universal freezing

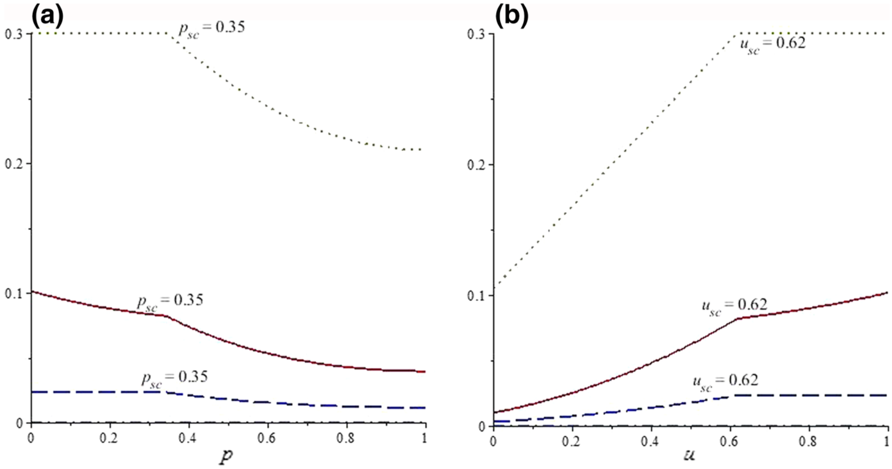

Fig. 7 Entropic quantum discord (*solid line*), two-norm geometric quantum discord (*dash line*) and one-norm geometric quantum discord (*dot line*) are plotted as functions of decoherence parameter p and memory parameter $u = 0.5$ (**a**) and as functions of memory parameter u and decoherence parameter $p = 0.5$ (**b**). p_{sc} and u_{sc} are the critical points of sudden transition

conditions in Refs. [63–66]. The elements of the matrix ρ_{out} are

$$\begin{aligned} \rho_{11} &= \rho_{44} = \frac{13}{40}, \\ \rho_{14} &= \rho_{41} = -\frac{1}{4}[1 - p(u-1)(p-2)], \\ \rho_{22} &= \rho_{33} = \frac{7}{40}, \\ \rho_{23} &= \rho_{32} = -\frac{1}{20}[1 - p(u-1)(p-2)]. \end{aligned} \tag{38}$$

It can be easily verified that ρ_{out} is a Bell-diagonal state with

$$\begin{aligned} c'_1 &= \frac{3}{5}[1 - p(u-1)(p-2)], \\ c'_2 &= -\frac{2}{5}[1 - p(u-1)(p-2)], \\ c'_3 &= \frac{3}{10}. \end{aligned} \tag{39}$$

From Figs. 8 and 9a, it can be seen that quantum discord has sudden change and disappears only when $p = 1$ and $u = 0$. However entanglement has sudden death (see Figs. 8d, 9). From Figs. 8c and 9, we observe that one-norm geometric quantum discord shows the phenomenon of double sudden change, which coincides with the results of Ref. [39] when memory parameter $u = 0$. The first transition point of one-norm geometric quantum discord is localized by the following function

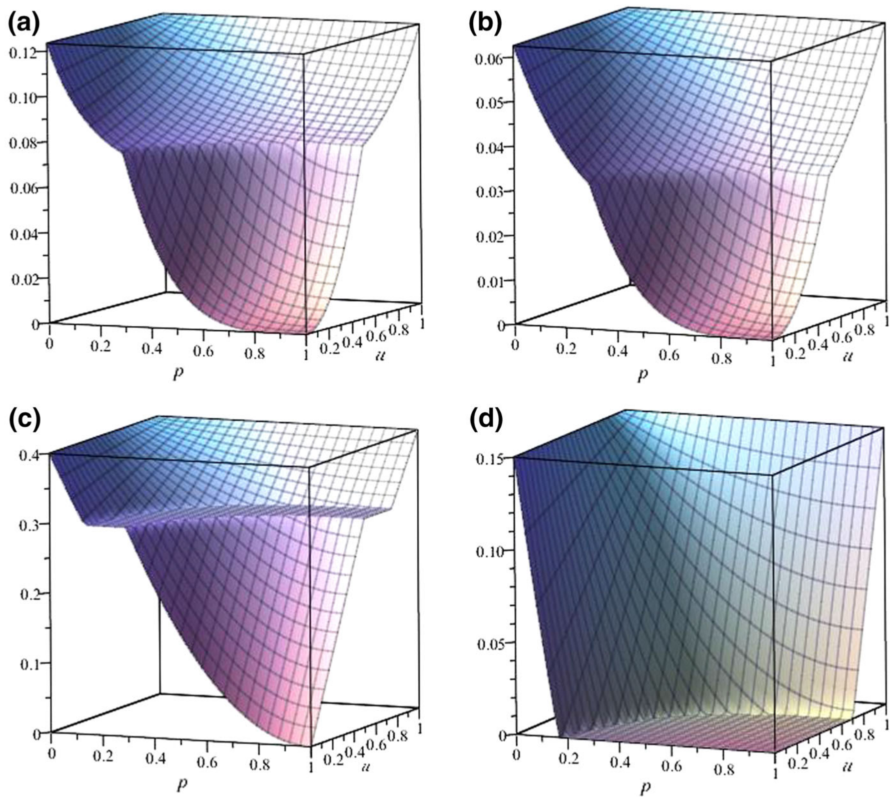

Fig. 8 **a** Entropic quantum discord, **b** two-norm geometric quantum discord, **c** one-norm geometric quantum discord, and **d** concurrence are plotted for Bell-diagonal state $\left(c_1 = \frac{3}{5}, c_2 = -\frac{2}{5}, c_3 = \frac{3}{10}\right)$ as functions of decoherence parameter p and memory parameter u

$$p = \frac{2u - 2 + \sqrt{4u^2 - 7u + 3}}{2(u-1)}, \tag{40}$$

which is determined by c_2' and c_3' of Eq. (39). Crossing the critical point, one-norm geometric quantum discord only depends on the constant component $c_3' = \frac{3}{10}$, which result in freezing for a certain interval (robust to decoherence in this interval). And then a second critical point will occur involving c_1' and c_3', determined by the following function

$$p = \frac{2u - 2 + \sqrt{4u^2 - 6u + 2}}{2(u-1)}. \tag{41}$$

Entropic quantum discord and two-norm geometric quantum discord have the same critical point with the second critical point.

Interestingly, we find that the shape of entropic quantum discord is very similar to the shape of two-norm geometric quantum discord. Combining Fig. 8c with the previous

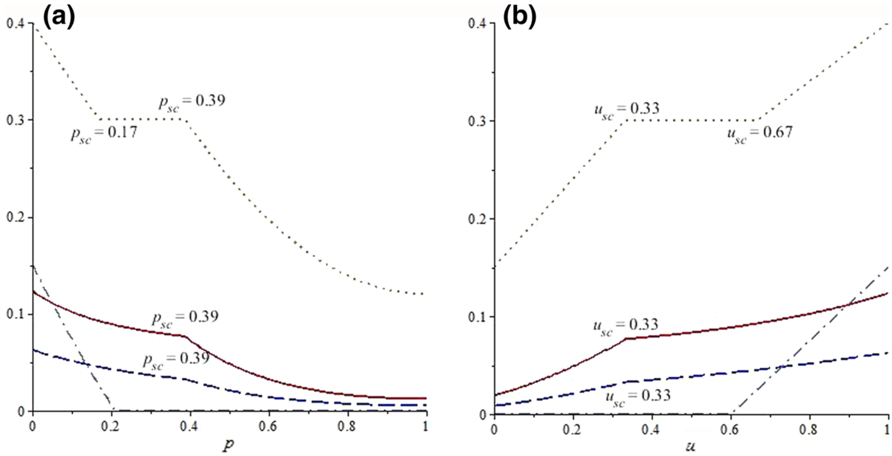

Fig. 9 Entropic quantum discord (*solid line*), two-norm geometric quantum discord (*dash line*), one-norm geometric quantum discord (*dot line*) and concurrence (*dashdot line*) are plotted as functions of decoherence parameter p and memory parameter $u = 0.2$ (**a**) and as functions of memory parameter u and decoherence parameter $p = 0.5$ (**b**). p_{sc} and u_{sc} are the critical points of sudden transition

discussions about geometry of one-norm geometric quantum discord for Bell-diagonal states, we can obtain that the geometric surface of the state ρ_{out} (38) will shrink with increasing decoherence parameter p and enlarge with increasing memory parameter u. From Figs. 8 and 9, it can be seen that one-norm geometric quantum discord is greater than entropic quantum discord and entropic quantum discord is greater than two-norm geometric quantum discord. Overall, quantum correlation decreases with the increase in decoherence parameter p and increases with the growth of memory parameter u.

4.3 Depolarizing channel with correlated noise

In the subsection, we consider quantum depolarizing channel with correlated noise. The Kraus operators of the uncorrelated part of this channel are defined as

$$E_{ij}^{n} = M_i \otimes M_j, \quad i, j \in \{0, 1, 2, 3\}, \tag{42}$$

where

$$\begin{aligned} M_0 &= \frac{1}{2}\sqrt{4-3p}I, \\ M_1 &= \frac{1}{2}\sqrt{p}\sigma_1, \\ M_2 &= \frac{1}{2}\sqrt{p}\sigma_2, \\ M_3 &= \frac{1}{2}\sqrt{p}\sigma_3. \end{aligned} \tag{43}$$

The Kraus operators of the correlated part of the depolarizing channel are given [62] as

$$
\begin{aligned}
E_{00}^c &= \frac{1}{2}\sqrt{4-3p}I \otimes I,\\
E_{11}^c &= \frac{1}{2}\sqrt{p}\sigma_1 \otimes \sigma_1,\\
E_{22}^c &= \frac{1}{2}\sqrt{p}\sigma_2 \otimes \sigma_2,\\
E_{33}^c &= \frac{1}{2}\sqrt{p}\sigma_3 \otimes \sigma_3.
\end{aligned}
\tag{44}
$$

We consider the initially entangled Bell-diagonal state with $c_1 = 1$, $c_2 = -\frac{3}{5}$, $c_3 = \frac{3}{5}$. The elements of the matrix ρ_{out} are

$$
\begin{aligned}
\rho_{11} = \rho_{44} &= \frac{2}{5} - \frac{3}{20}p(u-1)(p-2),\\
\rho_{14} = \rho_{41} &= \frac{2}{5}[1 - p(u-1)(p-2)],\\
\rho_{22} = \rho_{33} &= \frac{1}{10} + \frac{3}{20}p(u-1)(p-2),\\
\rho_{23} = \rho_{32} &= \frac{1}{10}[1 - p(u-1)(p-2)].
\end{aligned}
\tag{45}
$$

It is obvious that ρ_{out} is a Bell-diagonal state with

$$
\begin{aligned}
c_1' &= 1 - p(u-1)(p-2),\\
c_2' &= -\frac{3}{5}[1 - p(u-1)(p-2)],\\
c_3' &= \frac{3}{5}[1 - p(u-1)(p-2)].
\end{aligned}
\tag{46}
$$

Thus we can say that depolarizing channel with correlated noise does not change the type of a state as well.

Figures 10 and 11 show the dynamics of quantum correlation for the Bell-diagonal state under depolarizing channel with correlated noise. From Fig. 10, it is clear that quantum discord does not suddenly vanish except for $p = 1$ and $u = 0$, in contrast to sudden death of entanglement (see Figs. 10d, 11). However, we see that quantum discord does not have sudden transition in the noisy channel and the shapes of three measures of quantum discord are very similar, which results from the optimization procedures of calculating the three measures only depending on one of c_1', c_2' and c_3' in Eq. (46). Combining Fig. 10c with the previous analysis about geometry of one-norm geometric quantum discord for Bell-diagonal states, we can obtain that the geometric surface of the state ρ_{out} [Eq. (45)] will shrink with increasing decoherence parameter p and enlarge with increasing memory parameter u. From Figs. 10 and 11, it can be seen that one-norm geometric quantum discord is greater than entropic quantum

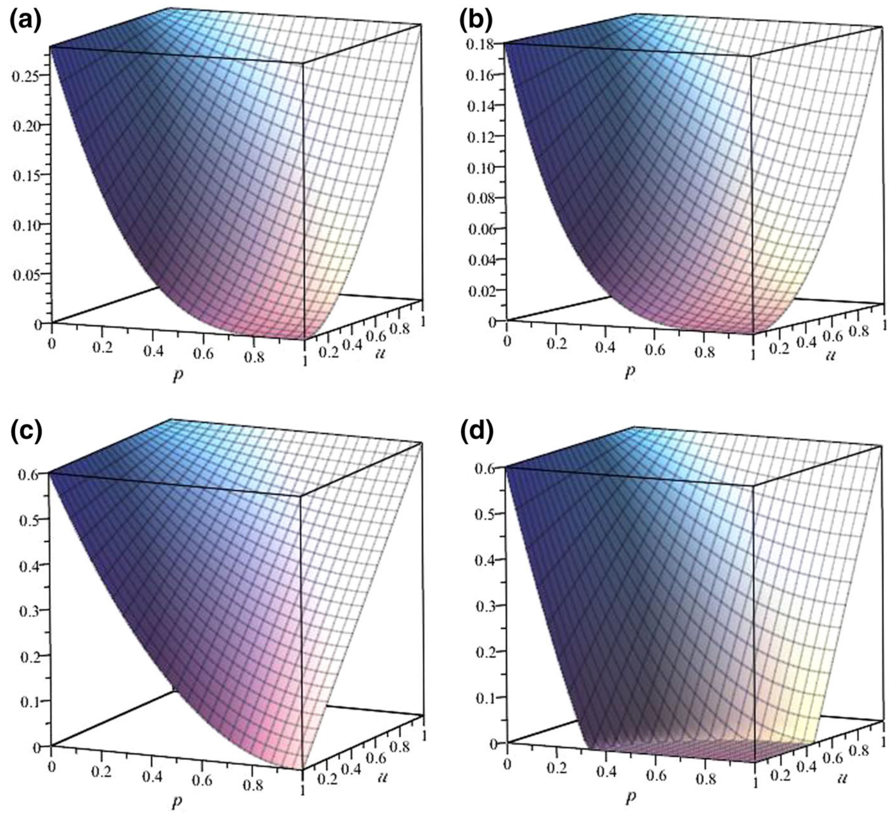

Fig. 10 **a** Entropic quantum discord, **b** two-norm geometric quantum discord, **c** one-norm geometric quantum discord, and **d** concurrence are plotted for Bell-diagonal state $\left(c_1 = 1, c_2 = -\frac{3}{5}, c_3 = \frac{3}{5}\right)$ as functions of decoherence parameter p and memory parameter u

discord and entropic quantum discord is greater than two-norm geometric quantum discord. In general, quantum correlation decreases with the increase in decoherence parameter p and increases with the growth of memory parameter u.

5 Dynamical behavior of renormalized one-norm geometric quantum discord in XXZ model

The Hamiltonian of the XXZ model on a periodic chain of N site is written as

$$H = \frac{J}{4} \sum_{i}^{N} \left(\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \Delta \sigma_i^z \sigma_{i+1}^z \right), \tag{47}$$

where $J, \Delta > 0$, J is the exchange coupling constant, Δ is the anisotropy parameter, and σ^x, σ^y and σ^z are Pauli matrices. The model is solvable by means of Bethe Ansatz and critical for $0 \leq \Delta \leq 1$.

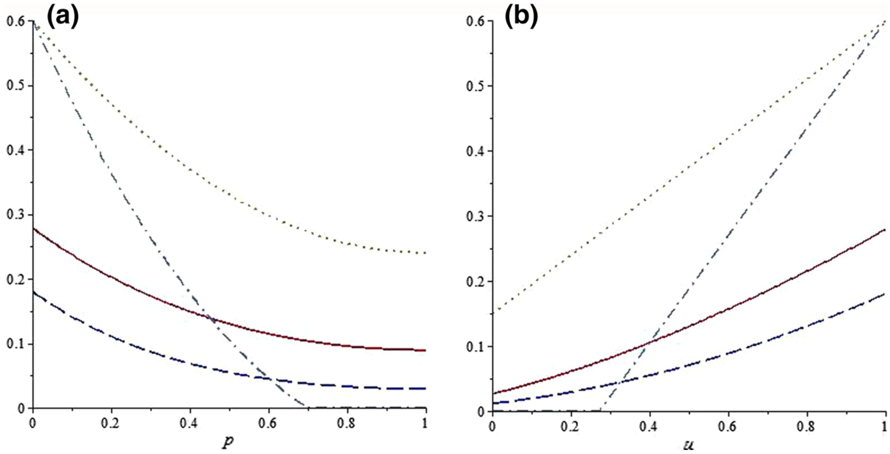

Fig. 11 Entropic quantum discord (*solid line*), two-norm geometric quantum discord (*dash line*), one-norm geometric quantum discord (*dot line*) and concurrence (*dashdot line*) are plotted as functions of decoherence parameter p and memory parameter $u = 0.4$ (**a**) and as functions of memory parameter u and decoherence parameter $p = 0.5$ (**b**)

The main idea of the quantum renormalization group method is the elimination of freedom degrees followed by an iteration which reduces the number of variables step by step until a manageable situation. Following the Kadanoff's approach, the lattice is split into blocks. Each block is treated independently to obtain the lower energy renormalized Hilbert subspace. The full Hamiltonian is projected onto the renormalized space to achieve an effective Hamiltonian H^{eff} [52,53].

We implement this approach to calculate the one-norm geometric quantum discord of the XXZ spin chain. A decomposition of three-site blocks is chosen to obtain a self-similar Hamiltonian for the Hamiltonian [Eq. (47)], which is necessary for guaranteeing of self-similarity after each iterative step [52,53]. The degenerate ground states of the block Hamiltonian are

$$
\begin{aligned}
|\varphi_0\rangle &= \frac{1}{\sqrt{2+q^2}}\left(|001\rangle + q|010\rangle + |100\rangle\right), \\
|\varphi_0'\rangle &= \frac{1}{\sqrt{2+q^2}}\left(|100\rangle + q|101\rangle + |110\rangle\right),
\end{aligned} \tag{48}
$$

where $|0\rangle$, $|1\rangle$ being the eigenstates of Pauli matrix, σ^z stands for the spin-up state and spin-down state of particles in spin-1/2 Heisenberg XXZ spin chain, and

$$
q = -\frac{1}{2}\left(\Delta + \sqrt{\Delta^2 + 8}\right). \tag{49}
$$

The effective Hamiltonian of the renormalized chain is turned into the form

$$H^{\text{eff}} = \frac{J'}{4}\sum_{i}^{N/3}\left(\sigma_i^x\sigma_{i+1}^x + \sigma_i^y\sigma_{i+1}^y + \Delta'\sigma_i^z\sigma_{i+1}^z\right), \tag{50}$$

where the iterative relationship is

$$J' = J\left(\frac{2q}{2+q}\right)^2, \quad \Delta' = \Delta\frac{q^2}{4}. \tag{51}$$

By solving $\Delta = \Delta'$, we can obtain stable fixed point at $\Delta = 0$ and unstable fixed point at $\Delta = 1$.

Then we consider one of the degenerate ground states, because $|\varphi_0'\rangle$ will yield the same results. Defining $\rho = |\varphi_0\rangle\langle\varphi_0|$, by tracing over site 1, we obtain the reduced density matrix

$$\rho_{23} = \frac{1}{q^2+2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & q & 0 \\ 0 & q & q^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{52}$$

Through Eq. (12), we can get the following equation:

$$\gamma_1 = \gamma_2 = \frac{2q}{q^2+2}, \quad \gamma_3 = -\frac{q^2}{q^2+2}, \quad x_3 = -\frac{q^2-2}{q^2+2}. \tag{53}$$

So it is straightforward to compute one-norm geometric quantum discord of ρ_{23}

$$\mathcal{D}_1(\rho_{23}) = \frac{2|q|}{q^2+2}, \tag{54}$$

which is exactly equal to concurrence.

The variation of one-norm geometric quantum discord versus Δ for different renormalization group iteration steps is displayed in Fig. 12. After enough renormalization group steps, $\mathcal{D}_1(\rho_{23})$ develops two saturated values, for $0 \leq \Delta \leq 1$, $\mathcal{D}_1(\rho_{23}) \to 0$ for $\Delta > 1$, which is resulted from the different initial phase (the former is gapless and the latter is gap). The values of $\mathcal{D}_1(\rho_{23})$ are equal when $\Delta = 0$ and $\Delta = 1$ in each iteration, which is related to the solution of iterative relationship [Eq. (51)]. Thus, we can observe that all curves of one-norm geometric quantum discord cross each other at $\Delta = 1$. From Fig. 12, it can been seen that after several renormalization group iteration steps, the one-norm geometric quantum discord at critical point $\Delta = 1$ appears sudden change, revealing quantum phase transition. The two saturated values are associated with two different phases, for $0 \leq \Delta < 1$, $\mathcal{D}_1(\rho_{23}) = 0.7$, representing spin-fluid phase, while for $\Delta > 1$, $\mathcal{D}_1(\rho_{23}) \to 0$, representing Néel phase. Spin-fluid phase ($\mathcal{D}_1(\rho_{23}) = 0.7$) contains the quantum correlation, while Néel phase ($\mathcal{D}_1(\rho_{23}) \to 0$) lacks quantum correlation. In comparison with the results of Ref. [53], one-norm geometric quantum discord develops two saturated values more quickly, because about four iteration steps are needed for developing two saturated values.

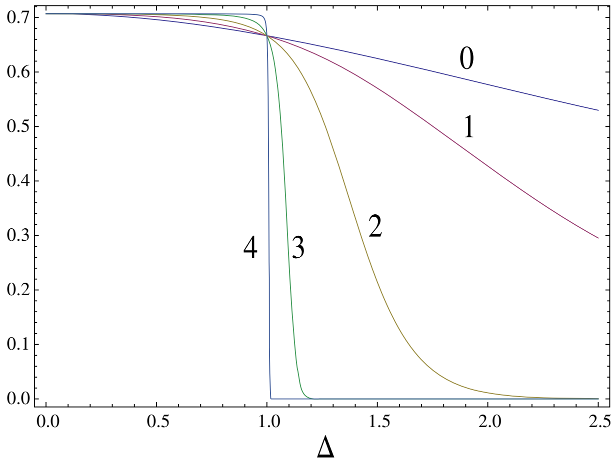

Fig. 12 The evolution of the one-norm geometric quantum discord versus Δ in terms of renormalization group iteration steps in the XXZ model

6 Conclusions

In this work, we have studied the one-norm geometric quantum discord for a class of X states. The level surfaces of one-norm geometric quantum discord have been depicted. It has been shown that the level surfaces of one-norm geometric quantum discord have different geometry compared to that in Refs. [20,25,43]. The pictorial approach provides a complete understanding to geometric change of one-norm geometric quantum discord and quantum correlation.

We have also investigated the dynamics of one-norm geometric quantum discord and other quantum correlation under correlated noise for Bell-diagonal states. It has been demonstrated that quantum discord does not suddenly disappear in contrast to entanglement sudden death. For some initial states, quantum discord has the sharp transition and one-norm geometric quantum discord has double sudden change, and quantum discord remains constant in a certain interval of time for certain initial states. It is possible to utilize such a property to perform quantum computation or communication tasks without any disturbance from the noisy environment. Otherwise, amplitude-damping channel with correlated noise may change the type of a state. Overall, quantum correlation decays with the increase in decoherence parameter or the decrease in memory parameter, and geometric surface of one-norm geometric quantum discord shrinks with increasing decoherence parameter and enlarges with increasing memory parameter. Almost one-norm geometric quantum discord is greater than entropic quantum discord and entropic quantum discord is greater than two-norm geometric quantum discord.

According to the analysis of geometry and decoherent dynamics of one-norm geometric quantum discord, because of the robust feature of quantum discord, it may be more practical than entanglement in some tasks of quantum information processing. There are some tasks of quantum computing and information processing in which quantum discord has advantages over entanglement [5–9].

Finally, we have demonstrated the dynamical behavior of one-norm geometric quantum discord. One-norm geometric quantum discord can effectively detect quantum phase transition employing the quantum renormalization group method in anisotropic XXZ model. The results show that one-norm geometric quantum discord develops two saturated values which are associated with two different phase, spin-liquid and Néel phases after some renormalization steps.

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