Iterated Prisoner's Dilemma

1st Tournament, using the

NOVA Game Theory Interactive (NOVA GTI)

Computational Game Theory

Integrated Master (BSc. + MSc.) of
Computer Science and Engineering

Faculty of Sciences and Technology of
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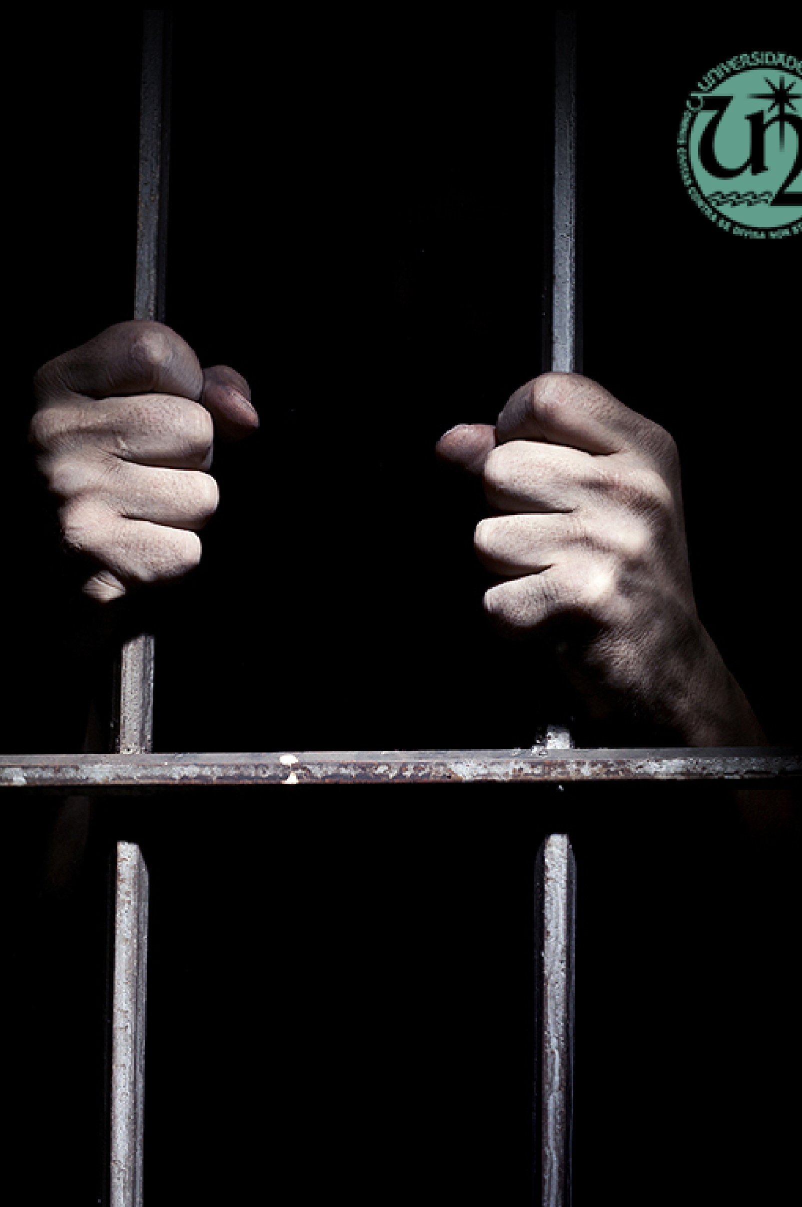

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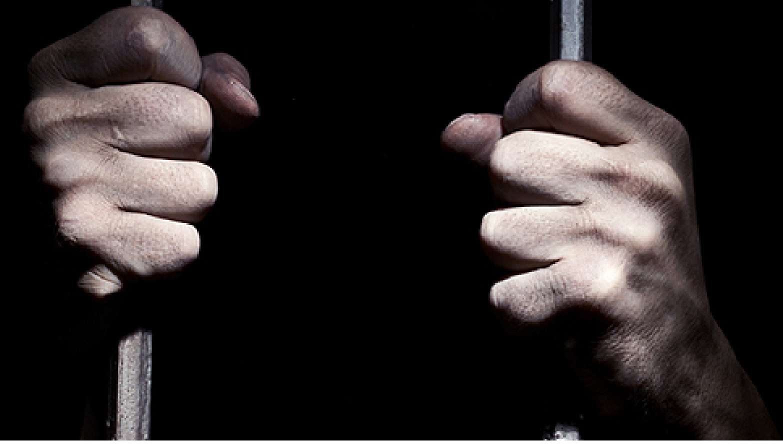
1 Introduction

ii. What is the Game Theory?

- A theoretical framework for conceiving **social situations among competing players**.
- It's considered the **science of strategy**.
- Produce the **optimal decision-making of independent and competing actors**, in a strategic setting.
- The key pioneers of this science were the mathematicians **John von Neumann** and **John Nash**, as well as economist **Oskar Morgenstern**.

iii. Basics of the Game Theory

- Models an interactive situation among rational players.
- The key aspect of this science is that one player's payoff is contingent on the strategy implemented by the other player.
- The game being played, identifies the players' identities, preferences, and available strategies, as also, how these strategies affect an outcome.
- The actions and choices of all the participants affect the outcome of each other.
- Depending on the model, other several requirements or assumptions may be necessary.



iii. Key Ingredients and Definitions

- In a situation with two or more players that involve known payouts (i.e., quantifiable consequences), we can use this science to help them to choose the most likely outcomes.

- Let’s enumerate some few **key ingredients** and their **definitions**:

  - ## Game:

    - Any set of circumstances that has a result dependent on the actions of two or more decision-makers (Players).

  - ## Players:

    - A strategic decision-maker within the context of the game.

  - ## Strategy:

    - A complete plan of action, which a player will take, given the set of circumstances decision-maker within the context of the game.

  - ## Payoff:

    - The payoff which a player receives from being at a particular outcome.

    - The payout can be in any quantifiable form, from dollars to utility.

iii. Key Ingredients and Definitions

- In a situation with two or more players that involve known payouts (i.e., quantifiable consequences), we can use this science to help them to choose the most likely outcomes.

- Let's enumerate some few key ingredients and their definitions:

  - **Information Set:**

    - The information available at a given point in the game.

    - This term is most usually applied and used, when the game it's iterated, i.e., when has a sequential component.

  - **Equilibrium:**

    - The point in a game where both players have made their decisions and an outcome is reached.

iv. Normal-Form Representation

Player #1(rows)/Player #2 (columns)	**Move #1**	...	**Move #n**
Move #1	Outcome $_{1,1}$	...	Outcome $_{1,n}$
...	...	...	...
Move #n	Outcome $_{n,1}$	...	Outcome $_{n,n}$

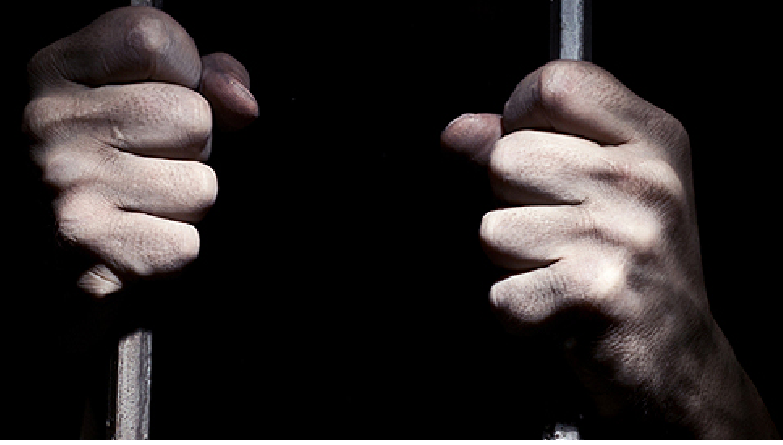
v. Pareto Optimality

- Strategy Profile S Pareto dominates a Strategy Profile S' if:
  - No Player gets a worse payoff with Strategy Profile S than with Strategy Profile S', i.e., $U_i(S) \geq U_i(S')$ for all Player i;
  - At least, one Player gets a better payoff with Strategy Profile S than with Strategy Profile S', i.e., $U_i(S) > U_i(S')$ for at least one Player i;

- A **Strategy Profile S** is **Pareto Optimal**, or **Strictly Pareto Efficient**, if there's no strategy **S'** than **Pareto Dominates S**:
  - Every game has at least one **Pareto Optimal Profile**;
  - Always at least one **Pareto Optimal Profile** in which the **Strategies** are pure;

vi. Dominant Strategy Equilibrium

- S_i **strongly dominates** S_i' if **Player *i* always gets better playoffs with** S_i **than with** S_i':
  - $\forall\, S_1, \ldots, S_{(i-1)}, S_{(i+1)}, \ldots, S_n$:
    - $U_i(S_1, \ldots, S_{(i-1)}, S_i, S_{(i+1)}, \ldots, S_n) > U_i(S_1, \ldots, S_{(i-1)}, S'_i, S_{(i+1)}, \ldots, S_n)$



vi. Dominant Strategy Equilibrium

- S_i **weakly dominates** S_i' **if Player i always gets better playoffs with** S_i **than with** S_i':
  - $\forall\ S_1, \ldots, S_{(i-1)}, S_{(i+1)}, \ldots, S_n$:
    - $U_i(S_1, \ldots, S_{(i-1)}, S_i, S_{(i+1)}, \ldots, S_n) \geq U_i(S_1, \ldots, S_{(i-1)}, S'_i, S_{(i+1)}, \ldots, S_n)$
  - $\exists\ S_1, \ldots, S_{(i-1)}, S_{(i+1)}, \ldots, S_n$:
    - $U_i(S_1, \ldots, S_{(i-1)}, S_i, S_{(i+1)}, \ldots, S_n) > U_i(S_1, \ldots, S_{(i-1)}, S'_i, S_{(i+1)}, \ldots, S_n)$

- S_i is **a Strongly Dominant Strategy** if it **strongly dominates every** $s_i' \in S_i'$.
- S_i is a **Weakly Dominant Strategy** if it **strongly dominates every** $s_i' \in S_i'$.
- A set of **Strategies** $(s_1, \ldots, s_n)$ such that each s_i is **dominant** for **Player *i***.
- Thus, **Player *i*** will get better payoffs by using s_i rather than a **different strategy**, **regardless of what strategies the other players use**.

vii. Nash Equilibrium

- Is an outcome reached that, once achieved, means no player can increase its payoff by changing unilaterally (i.e., without considering if the opponent changes its option or action).

- It can also be though as a **“no regret” situation**, in the sense that once a decision is made, the player will have no regrets, concerning decisions considering the consequences.

- Basically, it states that none of the players involved have any temptation or incentive to change their options or actions.

- The **Nash Equilibrium** is reached, over time, in most cases.

- However, once the **Nash Equilibrium** is reached, it will not be deviated from.

- Generally, there can be more than one **Nash Equilibrium** in a game.

- However, this usually occurs in games with more complex elements than two choices by two players.

- In simultaneous games that are repeated over time, one of these multiple equilibria is reached after some trial and error.

- In the following example, (e,f) is a **Nash Equilibrium**, if, and only:

    - ***e*** > ***a*** and ***f*** > ***h***;

Player #1(rows)/Player #2 (columns)	**Move #1**	**Move #2**
Move #1	(a,b)	(c,d)
Move #2	**(e,f)**	(g,h)

viii. Types of Game Theory

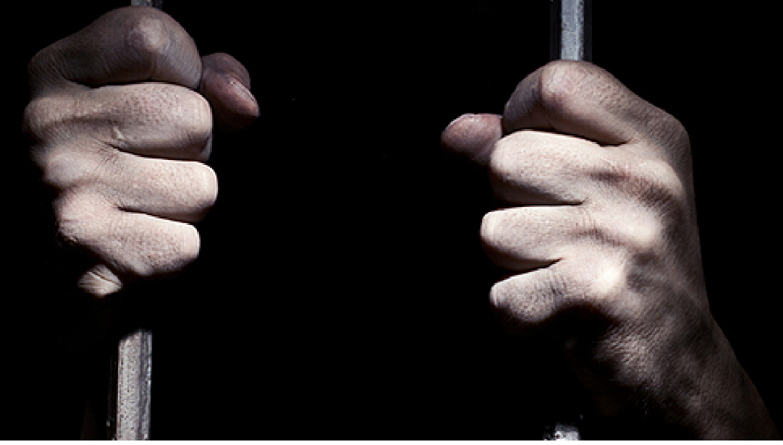
- Although there are many types (e.g., **symmetric/asymmetric**, **simultaneous/sequential**, etc.) of Game Theories:
  - **Cooperative Game Theories** and **Non-Cooperative Game Theories** are the most common;

- **Cooperative Game Theory** deals with coalitions, or cooperative groups, interact when only the payoffs are known:
  - It’s a game between coalitions of players rather than between individuals, and it questions how groups form and how they allocate the payoffs among players.

  - Some examples are:
    - **Groups’ behaviors against dictatorships**;
    - **Groups’ behaviors against pandemics**;

- **Non-Cooperative Game Theory** deals with how rational and self-interested players deal with each other to achieve their own goals:
  - The most known common **Non-Cooperative Games** are the **Strategic** ones, in which only the available strategies and the outcomes that result from a combination of choices are known;

  - Some examples are:
    - **Prisoner’s Dilemma**;
    - **Rock-Paper-Scissors**;



ix. Impact/Usage of Game Theory

- Has a wide range of applications, including **psychology**, **evolutionary biology**, **war**, **politics**, **economics**, and **business**.
- Brought about a revolution in **economics** by addressing crucial problems in prior **mathematical economic models**.
- For instance, **neoclassical economics** struggled to understand entrepreneurial anticipation and could not handle the imperfect competition.
- Turned attention away from steady-state equilibrium toward the **market process**.
- Is beneficial for modeling **competing behaviors** between **economic agents**.
- Can be used by **businesses companies** which may face dilemmas such as whether to retire existing products or develop new ones, set **lower prices relative to the competition**, or **employ new marketing strategies**.
- Can be used to understand **oligopoly firm behavior**.
- Helps to **predict** likely outcomes when firms **engage in certain behaviors**, such as **price-fixing** and **collusion**.
- It's very applicable to the business world, when **two** (**or more**) **firms** are **determining prices for highly interchangeable products**, such as **airfare**, **soft drinks**, among many others.
- It can be used to study behaviors during election or voting processes, in **cooperative organizations** and **politics**.

ix. Impact/Usage of Game Theory

- It's very useful to study the **behaviors of society and its global welfare**, like in situations of **dictatorship**, **rebellion**, **pandemics**, per example.

- For example, the current **pandemic COVID-19 situation worldwide**, could be represented by something like this:

$$U_{ruben}(\text{action}) \begin{cases} 100, \# \{j: a_j = stay_{home}\} \geq 6{,}000{,}000{,}000 \wedge action_{ruben} = stay_{home} \\ 50, \quad \# \{j: a_j = stay_{home}\} < 6{,}000{,}000{,}000 \wedge action_{ruben} = stay_{home} \\ -200, \quad action_{ruben} = go_{out} \end{cases}$$

- Following this example, it's more reasonable to argue that the best-response to the **COVID-19 pandemic** (and the most benefit) for the **global social welfare of the worldwide population**, should be $stay_{home}$.

- It can be also used in **consensus proofs**, where it's necessary to choose a leader or some entity responsible for some action or property, like per example, **Cryptocurrencies**, **Blockchains**, **Decentralized Distributed Systems** or **Byzantine Agreements**, which need to take in account, some trust aspects.

- Despite its many advances, **Game Theory** is still a **young** and **developing science**.

2 Prisoner's Dilemma

i. What is the Prisoner' Dilemma?

- The **Prisoner's Dilemma** is the most well-known example of **Game Theory**.
- Consider the example of **two criminals**, partners in crime, **Rúben** e **João**, **who were arrested**.
- **Prosecutors have no hard evidence to convict them**.
- However, **to gain a confession**, **officials remove the prisoners from their solitary cells** and **question each one in separate chambers**.
- **Neither prisoner has the means to communicate with each other**.

ii. Outcomes of Prisoner's Dilemma

- The rules and respective outcomes for the Prisoner's Dilemma, in this case, are the following:

  1. If both Rúben and João confess, they will each receive a payoff of 3.

  2. If Rúben confesses, but João doesn't, Rúben and João will receive, payoffs of 0 and 4, respectively.

  3. If João confesses, but Rúben doesn't, João and Rúben will receive, payoffs of 0 and 4, respectively.

  4. If both Rúben and João don't confess, they will each receive a payoff of 1.

iii. Normal-Form Representation in Prisoner's Dilemma

- The **Prisoner's Dilemma** can be seen, as a whole category of games, with the following **Normal-Form Representation** format:

Player #1(rows)/Player #2 (columns)	Move #1	Move #2
Move #1	(a,a)	(b,c)
Move #2	(c,b)	(d,d)

with $b < d < a < c$, which can be translated to $0 < 1 < 3 < 4$.

- The **Prisoner's Dilemm**a, in the previously defined example, with assigned values to the variables **a, b, c** and **d**, and considering the possible actions of **Cooperate** and **Defect**, for each Player (**Rúben** and **João**), should have the following **Normal-Form Representation** format:

Rúben (rows)/João (columns)	Cooperate	Defect
Cooperate	(3,3)	(0,4)
Defect	(4,0)	(1,1)

iv. Pareto Optimality in Prisoner's Dilemma

- Following again the example of the **Prisoner's Dilemma**:

Rúben (rows)/João (columns)	Cooperate	Defect
Cooperate	(3,3)	(0,4)
Defect	(4,0)	(1,1)

- **(Cooperate, Cooperate) is Pareto Optimal**:
  - No Strategy Profile gives both players a higher payoff;
- **(Defect, Cooperate) is Pareto Optimal**:
  - No Strategy Profile gives Rúben a higher payoff;
- **(Defect, Cooperate) is Pareto Optimal:**
  - No Strategy Profile gives João a higher payoff;
- **(Defect, Defect) is not a Pareto Optimal:**
  - (Defect, Defect) is Pareto dominated by (Cooperate, Cooperate);
  - (Defect, Defect) is not Pareto dominated by (Cooperate, Defect) neither (Defect, Cooperate);
  - But ironically, (Defect, Defect) is the Dominant Strategy Equilibrium;

v. Dominant Strategies and Iterated Removal of Dominated Strategies in Prisoner's Dilemma

- Following, once more, the previously described example of the **Prisoner's Dilemma**:

Rúben (rows)/**João** (columns)	**Cooperate**	**Defect**
Cooperate	(3,3)	(0,4)
Defect	(4,0)	(1,1)

- **Rúben's action of Defect Strictly Dominates his action of Cooperate:**
  - Rúben's (Defect, Cooperate) payoff is higher than (Cooperate, Cooperate) and (Cooperate, Defect):
    - $4 > 3 \wedge 1 > 0$;
- Thus, **Rúben should have no incentive to play the action of Cooperate, instead of playing the action of Defect**:

Rúben (rows)/**João** (columns)	**Cooperate**	**Defect**
Defect	(4,0)	(1,1)

v. Dominant Strategies and Iterated Removal of Dominated Strategies in Prisoner's Dilemma

- Following, once more, the previously described example of the **Prisoner's Dilemma**:

Rúben (rows)/João (columns)	Cooperate	Defect
Defect	(4,0)	(1,1)

- But the same idea it's applied to **João**'s thoughts:
  - João's (Defect, Defect) payoff is higher than (Defect, Cooperate):
    - $1 > 0$;
- Thus, **João** also should have no incentive to play the action of **Cooperate**, instead of playing the action of **Defect**:

Rúben (rows)/João (columns)	Defect
Defect	(1,1)

- So, once again, we conclude that **(Defect, Defect)** it's the **Nash Equilibrium** in this game.

vi. Nash Equilibrium in Prisoner's Dilemma

- The **most favorable** (and **safest**) **strategy** is to play a **Defect** action.

- Thus, it's reasonable to conclude that the **Prisoner's Dilemma** have only one **Nash Equilibrium**:

Rúben (rows)/João (columns)	Cooperate	Defect
Cooperate	(3,3)	(0,4)
Defect	(4,0)	**(1,1)**

- If both **Rúben** and **João** play **Defect**, **none of them**, **have any incentive to unilaterally deviate from it**, because will get no payoff, at all.

- However, neither is aware of the other's strategy and without certainty that one will **Defect**, both will likely **Cooperate** and **receive a better payoff**.

- The **Nash Equilibrium** suggests that in a **Prisoner's Dilemma**, both players will make the move that is best for them individually, but worse for them collectively.

- And this, is one of the main reasons for the huge study and research interest in this game, because it's considered a **very complex** and **hard paradox to solve**.

3 Analysis of Iterated Prisoner's Dilemma

i. Iterated Prisoner's Dilemma

- The **Iterated Prisoner's Dilemma** is a variant of **repeated rounds** (or **game stages**) successively in **Prisoner's Dilemma**.
- In this variant, **each player doesn't know how the other player will act** (**the rounds are simultaneous for each player**) but **know how the other player acted previously** (**in the previous rounds played**).

ii. Types of Strategies for Iterated Prisoner's Dilemma

- There are an infinity of types of possible strategies and approaches for Iterated Prisoner's Dilemma, where the commonly known are:
  - Deterministic Strategies:
    - Basic Strategies;
    - Periodic Strategies;
    - Triggering Strategies;
    - Handshakes/Group Strategies;
  - Probabilistic Strategies:
    - Random Strategies;
    - Equalizer Strategies;
    - Extortion Strategies;

iii. Brief Description of some of the most known Strategies for Iterated Prisoner’s Dilemma

- Let’s enumerate some few commonly known **Strategies** for **Iterated Prisoner’s Dilemma**:
  - **Deterministic Strategies:**
    - **Basic Strategies:**
      - ***All Cooperate:***
        - The ***Player*** always play a ***Cooperate*** action, independently of the other ***Player***’s action;
      - ***All Defect:***
        - The ***Player*** always play a ***Defect*** action, independently of the other ***Player***’s action;
    - **Periodic Strategies:**
      - ***Periodic CD Strategy:***
        - The ***Player*** always play a periodic sequence of ***Cooperate***, ***Defect*** actions, independently of the other ***Player***’s action;

iii. Brief Description of some of the most known Strategies for Iterated Prisoner's Dilemma

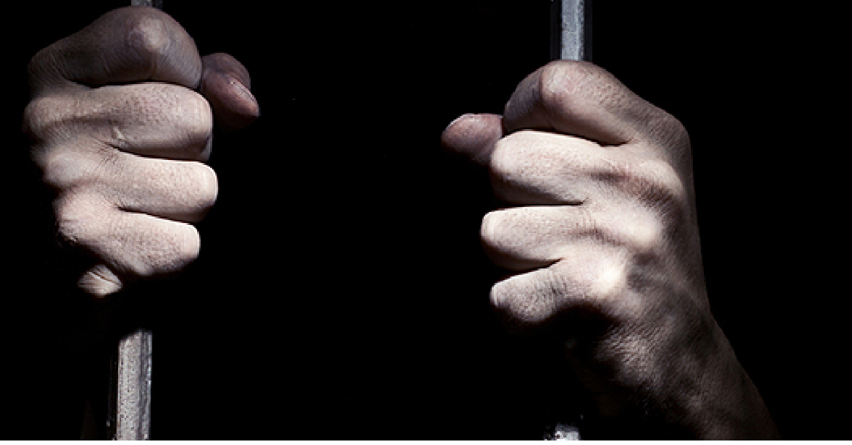
- Let's enumerate some few commonly known **Strategies** for **Iterated Prisoner's Dilemma**:
  - **Deterministic Strategies:**
    - **Periodic Strategies:**
      - ***Periodic CCD Strategy:***
        - The ***Player*** always play a periodic sequence of ***Cooperate***, ***Cooperate***, ***Defect*** actions, independently of the other ***Player***'s action;
      - ***Periodic DDC Strategy:***
        - The ***Player*** always play a periodic sequence of ***Defect***, ***Defect***, ***Cooperate*** actions, independently of the other ***Player***'s action;
    - **Triggering Strategies:**
      - ***Tit For Tat Strategy:***
        - The ***Player*** starts to play a ***Cooperate*** action, and then, plays the action that the other ***Player*** played in the previous ***Game Stage/Round***;
        - It's also known as a ***Mimic Strategy***;

iii. Brief Description of some of the most known Strategies for Iterated Prisoner's Dilemma

- Let's enumerate some few commonly most known **Strategies** for **Iterated Prisoner's Dilemma**:
  - **Deterministic Strategies:**
    - **Triggering Strategies:**
      - ***Grim Trigger Strategy:***
        - The ***Player*** starts to play a ***Cooperate*** action, and keep playing ***Cooperate*** actions until the other Player played a ***Defect*** action, in the previous ***Game Stage/Round***, after that, plays always a ***Defect*** action as ***punishment***;
        - It's also known as a ***Spiteful Strategy***;
      - ***Gradual Strategy:***
        - The ***Player*** starts to play a ***Cooperate*** action, then play a ***Defect*** action ***n times*** after the ***n^{th} Defect*** action played by the other ***Player***;

iii. Brief Description of some of the most known Strategies for Iterated Prisoner’s Dilemma

- Let’s enumerate some few commonly known **Strategies** for **Iterated Prisoner’s Dilemma**:
  - **Deterministic Strategies:**
    - **Handshakes/Group Strategies:**
      - ***Prober Strategy***:
        - The ***Player*** starts to play a sequence of ***Defect***, ***Cooperate***, ***Cooperate*** actions, then:
          - Play an ***Always Defect Strategy*** if the other ***Player*** played a ***Cooperate*** action in the ***2nd and 3rd Game Stage/Rounds***;
          - Play a ***Tit For tat Strategy***, otherwise;
  - **Probabilistic Strategies:**
    - **Random Strategies:**
      - ½ Random Strategy:
        - The ***Player*** always play ***randomly***, with ½ of play each ***Cooperate*** or ***Defect*** actions;



iv. Some concerns about the Iterated Prisoner's Dilemma

- Let's clarify some key aspects to take in account in this game version of **Prisoner's Dilemma**:
  - In a **Prisoner's Dilemma** game with only one **Game Stage/Round**, the best approach Strategy and respectively best response to the other Player's behavior it's very discussable:
    - A **Player** will never know how the other **Player** will act:
      - From an altruist view, a Player can believe that the other Player doesn't have any reason to try to harm it, and may play a **Cooperate** action, and, in response to that, should play also a **Cooperate** action, sharing the same payoff with it;
      - But any reasonable **Player** will always think in play a **Defect** action, in order to try to maximize its payoff and, in that case it's very likely that the other **Player** will play a **Defect** action, and in that case, the best response that a **Player** can give it's also a **Defect** action, receiving both, in that case, a lower payoff than if the both played a **Cooperate** action;
      - This it's the main reason for calling this game, a **Dilemma** or a **Paradox**;

iv. Some concerns about the Iterated Prisoner’s Dilemma

- Let’s clarify some key aspects to take in account in this game version of **Prisoner’s Dilemma**:
  - But in an **Iterated Prisoner’s Dilemma** game with various **sequential Game Stages/Rounds**, the best approach **Strategy** should be to try to **avoid starting to play a Defect action**, **delaying it to later as possible**.

  - And there are some reasons for that:
    - ***Mistrust:***
      - If a **Player** choose to play a **Defect** action, in some moment of the game, **there is no guarantee that it won’t do it again, in a near future**;
      - This leads to the other rational **Player** to **lose the trust on it**;

    - ***Vengeance:***
      - If a **Player** harm other **Player**, in some moment of the game, by playing a **Defect** move, this will lead to some **vengeance thoughts** and the **Player would probably like to avenge it, sooner or later**;
      - This leads to the other **rational Player** to want to harm it, in a **revenge action**;

iv. Some concerns about the Iterated Prisoner's Dilemma

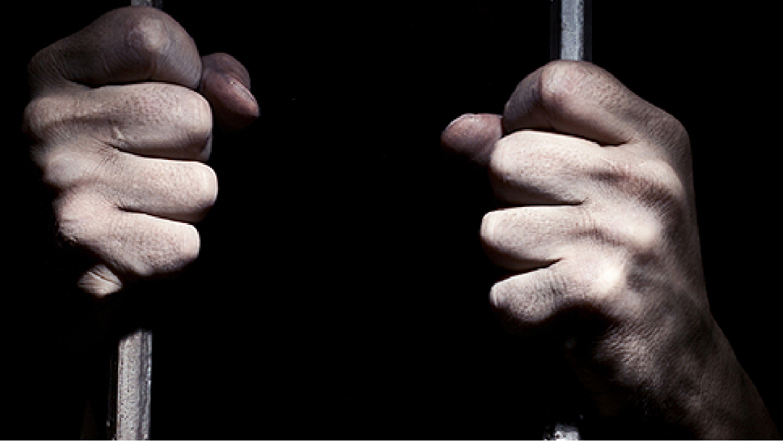
- Let's clarify some key aspects to take in account in this game version of **Prisoner's Dilemma**:
  - But in an **Iterated Prisoner's Dilemma** game with various **sequential Game Stages/Rounds**, the best approach Strategy should be to try to **avoid starting to play a Defect action, delaying it to later as possible**.
  - And there are some reasons for that:
    - ***Envy:***
      - If a **Player** choose to play a **Defect** action, in some moment of the game, probably it will **awake some envy feelings** on the other **Player**, by the **Player** who played a **Defect** action gained some payoff points on that other **Player**;
      - This leads to the other **Player** desire the same gain of payoff points, regarding to what it gained before;

iv. Some concerns about the Iterated Prisoner’s Dilemma

- Let’s clarify some key aspects to take in account in this game version of **Prisoner’s Dilemma**:
  - From a **deep analysis** from **Robert Axelrod**, it’s reasonable to state some conditions necessary for a **Strategy** to be successful:
    - ***Nice***:
      - A successful **Strategy** must be **“nice”**, that is, it will **not** play **Defect** actions before the other **Player** does it;
      - A **purely selfish strategy** will **not “cheat”** on its opponent, for purely self-interested reasons first;
    - ***Retailing***:
      - A **successful Strategy must not be blind optimist**, it must retaliate sometimes;
      - A **very optimist** or **altruist Strategy** it’s also a **bad choice**, because **“nasty” Strategies** will ruthlessly exploit such **Players**;

iv. Some concerns about the Iterated Prisoner’s Dilemma

- Let’s clarify some key aspects to take in account in this game version of **Prisoner’s Dilemma**:
    - From a **deep analysis** from **Robert Axelrod**, it’s reasonable to state some conditions necessary for a **Strategy** to be successful:
        - ***Forgiving***:
            - A successful **Strategy** must also be **forgiving**, because if the other **Players will retaliate**, but they will be very likely to fall back in **Cooperate** actions, if the **Player** doesn’t continue to play **Defect** actions;
            - This kind of **Strategies will stop long runs of revenge and counter revenge** and **counter-revenge**, **maximizing the payoff points**;
        - ***Non-Envious***:
            - A successful **Strategy should be non-envious**;
            - This kind of **Strategies** will choose very likely to **maximize the current score of payoff points** (which it’s the main goal of this type of game), **instead of score more than the other Player**;
    - Thus, a **Successful Top-Scoring Strategy** should try to take advantage of all these four important conditions.
    - For that reason, **Strategies** like **Tit For Tat**, **Grim Trigger** and **Gradual should work fine for this kind of games**.



v. Explanation of the developed Strategies

- Taking in account the previously mentioned important conditions for a **Successful Strategies** for the **Iterated Prisoner's Dilemma**, it was decided, initially to develop some initial known base **Strategies**, in order to perform some **Experimental Tests**:
  - **Tit For Tat Strategy**;
  - **Omega Tit For Tat Strategy**:
    - A variant of **Tit For Tat**, **leading with random behaviors** from the other **Player** and **acts more severely, in that case**;
  - **Grim Trigger Strategy**;
- Then, it was considered the more relevant aspects of the three games of the **proposed Tournament** of the **Iterated Prisoner's Dilemma**:
  - **The prior known fixed number of iterations, (Game Stages/Rounds);**
  - **The probability of continue to play the next Game Stages/Rounds;**
- Following that relevant aspects, it was developed some additional **Strategies**, considering that:
  - **Omega Tit For Tat Strategy, for Fixed Number of Rounds**;
  - **Omega Tit For Tat Strategy**, **for Fixed Number of Rounds with Probability of continue to the next Game Stage/Round**;
  - **Omega Tit For Tat Strategy, for Unknown Number of Rounds with Probability of continue to the next Game Stage/Round**;

v. Explanation of the developed Strategies

- Following that relevant aspects, it was developed some additional **Strategies**, considering that:
    - **Grim Trigger Strategy, for Fixed Number of Rounds**;
    - **Grim Trigger Strategy, for Fixed Number of Rounds with Probability of continue to the next Game Stage/Round**;
    - **Grim Trigger Strategy, for Unknown Number of Rounds with Probability of continue to the next Game Stage/Round**;
- This **Strategies** was developed, like will be explained following:
    - **Tit For Tat**:
        - Starts playing a **Cooperate** action;
        - Plays the same action that the other **Player** played in the previous **Game Stage/Round**, for the next **Game Stage/Rounds**;
    - **Omega Tit For Tat Strategy**:
        - Starts playing a **Cooperate** action;
        - Implements and uses a **randomness threshold**, in a form of **Boolean flag**, **which will be activated, in the case of being detected a random behavior in the actions of the other Player**:
            - If $\text{TOTAL\_NUMBER\_OF\_TIMES\_MY\_OPPONENT\_MOVES\_CHANGED} \geq \text{FOR\_ALLOWING\_MY\_OPPONENT\_MOVES\_CHANGES}$, play an All Defect Strategy from then;
            - Otherwise, play the well-known **Tit For Tat Strategy**, as described previously;

v. Explanation of the developed Strategies

- This **Strategies** was developed, like will be explained following:
  - **Grim Trigger Strategy**:
    - Starts playing a **Cooperate** action;
    - Keeps playing a Cooperate action, until the other **Player** play a **Defect** action, after that, it will play an **All Defect Strategy** action, from then;
  - **Tit For Tat Strategy/Omega Tit For Tat Strategy/Grim Trigger Strategy, for Fixed Number of Rounds**:
    - Starts playing a **Cooperate** action;
    - Plays very similar to the general **Tit For Tat Strategy/Omega Tit For Tat Strategy/Grim Trigger Strategy**;
    - But, if $(\text{GAIN\_DEFECT} > \text{DET\_LOSS\_ DEFECT})$, play a Defect action:
      - $\text{GAIN\_DEFECT} = (\text{I\_DEFECT\_OPPONENT\_COOPERATE - BOTH\_COOPERATE})$
      - $\text{DET\_LOSS\_DEFECT} = (\text{BOTH\_COOPERATE - BOTH\_DEFECT}) \times (\#\text{ROUND\_LEFT} - 1)$
      - $\text{I\_DEFECT\_OPPONENT\_COOPERATE} = 4$
      - $\text{BOTH\_COOPERATE} = 3$
      - $\text{BOTH\_DEFECT} = 1$

v. Explanation of the developed Strategies

- This **Strategies** was developed, like will be explained following:
  - **Tit For Tat Strategy/Omega Tit For Tat Strategy/Grim Trigger Strategy, for Fixed Number of Rounds with Probability of continue to the next Game Stage/Round**:
    - Starts playing a **Cooperate** action;
    - Plays very similar to the general **Tit For Tat Strategy/Omega Tit For Tat Strategy/Grim Trigger Strategy**;
    - But, if $(\text{GAIN\_DEFECT} > \text{DET\_LOSS\_ DEFECT})$ or $(\text{GAIN\_DEFECT} > \text{PROB\_LOSS\_DEFECT})$, play a Defect action:
      - $\text{GAIN\_DEFECT} = (\text{I\_DEFECT\_OPPONENT\_COOPERATE - BOTH\_COOPERATE})$
      - $\text{DET\_LOSS\_DEFECT} = (\text{BOTH\_COOPERATE - BOTH\_DEFECT}) \times (\text{\#ROUND\_LEFT} - 1)$
      - $\text{PROB\_LOSS\_DEFECT} = (\,(\text{BOTH\_COOPERATE - BOTH\_DEFECT}) \times \text{P(NEXT\_ROUND)}\,) \div (1 - \text{P(NEXT\_ROUND)})$
      - $\text{I\_DEFECT\_OPPONENT\_COOPERATE} = 4$
      - $\text{BOTH\_COOPERATE} = 3$
      - $\text{BOTH\_DEFECT} = 1$

v. Explanation of the developed Strategies

- This **Strategies** was developed, like will be explained following:
  - **Tit For Tat Strategy/Omega Tit For Tat Strategy/Grim Trigger Strategy, for Unknown Number of Rounds with Probability of continue to the next Game Stage/Round**:
    - Starts playing a **Cooperate** action;
    - Plays very similar to the general **Tit For Tat Strategy/Omega Tit For Tat Strategy/Grim Trigger Strategy**;
    - But, if $(\text{GAIN\_DEFECT} > \text{PROB\_LOSS\_ DEFECT})$, play a Defect action:
      - $\text{GAIN\_DEFECT} = (\text{I\_DEFECT\_OPPONENT\_COOPERATE - BOTH\_COOPERATE})$
      - $\text{PROB\_LOSS\_DEFECT} = (\,(\text{BOTH\_COOPERATE - BOTH\_DEFECT}) \times \text{P(NEXT\_ROUND)}\,) \div (1 - \text{P(NEXT\_ROUND)})$
      - $\text{I\_DEFECT\_OPPONENT\_COOPERATE} = 4$
      - $\text{BOTH\_COOPERATE} = 3$
      - $\text{BOTH\_DEFECT} = 1$

  - But how the base version of **Grim Trigger Strategy**, it's a very severe Strategy and can originate some situations of **deadlock** (i.e., **Situations of runs of revenge and constant mutual Defect actions played**), it was developed one more last **Strategy** based on the **Grim Trigger**, but with a little modification, but only considering a **Fixed Number of Rounds**:
    - **Forgiving Grim Trigger, for a Fixed Number of Rounds**;

v. Explanation of the developed Strategies

- This **Strategies** was developed, like will be explained following:
  - **Forgiving Grim Trigger Strategy, for Fixed Number of Rounds:**
    - Starts playing a **Cooperate** action;
    - Plays very similar to the general **Grim Trigger Strategy**, with some little differences:
      - In order to **avoid deadlock situations**, mentioned before, it's considered the following scenario and the respective response to that:
        - If the **Player** it's in a current situation of triggering the other **Player** with **punishments**, through an **All Defect Strategy** and by some reason, the other **Player play two consecutive Cooperate actions**, the **Player** will forgive it and suspend the current **punishment**, starting to play **Cooperate** actions from then, until the other **Player** play a **Defect** action again;
        - **This forgiving behavior it will be allowed only two times**;
        - If the other **Player** have this behavior **more than two times**, then, the punishment it will last until the current **Iterated Game** end, even if it will originate a **deadlock situation**;

v. Explanation of the developed Strategies

- This **Strategies** was developed, like will be explained following:
  - **Forgiving Grim Trigger Strategy, for Fixed Number of Rounds**:
    - But, once again, if $(\text{GAIN\_DEFECT} > \text{PROB\_LOSS\_ DEFECT})$, play a Defect action:
      - $\text{GAIN\_DEFECT} = (\text{I\_DEFECT\_OPPONENT\_COOPERATE} - \text{BOTH\_COOPERATE})$
      - $\text{DET\_LOSS\_DEFECT} = (\text{BOTH\_COOPERATE} - \text{BOTH\_DEFECT}) \times (\#\text{ROUND\_LEFT} - 1)$
      - $\text{I\_DEFECT\_OPPONENT\_COOPERATE} = 4$
      - $\text{BOTH\_COOPERATE} = 3$
      - $\text{BOTH\_DEFECT} = 1$

vi. Some Experimental Tests

- There was realized some **Experimental Tests**, regarding the three type of games proposed to the **Tournament**:
  - Game #1 (20 Iterations – 100% of Probability of Continue to the next Game Stage/Round):

ForgivingGrimTriggerForFixedNumRounds	Random
93	57

vi. Some Experimental Tests

- There was realized some **Experimental Tests**, regarding the three type of games proposed to the **Tournament**:
  - Game #1 (20 Iterations – 100% of Probability of Continue to the next Game Stage/Round):

ForgivingGrimTriggerForFixedNumRounds	AllCooperate
122	114

ForgivingGrimTriggerForFixedNumRounds	AllDefect
38	46

ForgivingGrimTriggerForFixedNumRounds	GrimTrigger
122	114

ForgivingGrimTriggerForFixedNumRounds	GrimTriggerForFixedNumRounds
116	116

ForgivingGrimTriggerForFixedNumRounds	OmegaTitForTat
122	114

ForgivingGrimTriggerForFixedNumRounds	OmegaTitForTatForFixedNumRounds
116	116

ForgivingGrimTriggerForFixedNumRounds	TitForTat
122	114

vi. Some Experimental Tests

- There was realized some **Experimental Tests**, regarding the three type of games proposed to the **Tournament**:

  - Game #1 (20 Iterations – 100% of Probability of Continue to the next Game Stage/Round):

ForgivingGrimTriggerForFixedNumRounds	TitForTatForFixedNumRounds
116	116

  - Game #2 (1000 Iterations – 60% of Probability of Continue to the next Game Stage/Round):

GrimTriggerForFixedNumRoundsWithProbability	Random
7	11

GrimTriggerForFixedNumRoundsWithProbability	AllCooperate
18	18

GrimTriggerForFixedNumRoundsWithProbability	AllDefect
4	12

GrimTriggerForFixedNumRoundsWithProbability	GrimTrigger
30	30

GrimTriggerForFixedNumRoundsWithProbability	OmegaTitForTat
6	6

vi. Some Experimental Tests

- There was realized some **Experimental Tests**, regarding the three type of games proposed to the **Tournament**:

  - Game #2 (1000 Iterations – 60% of Probability of Continue to the next Game Stage/Round):

GrimTriggerForFixedNumRoundsWithProbability	OmegaTFTForFixedRoundsWProbability
6	6

GrimTriggerForFixedNumRoundsWithProbability	TitForTat
24	24

GrimTriggerForFixedNumRoundsWithProbability	TFTForFixedRoundsWProbability
12	12

  - Game #3 (1000 Iterations, which are not considered – 40% of Probability of Continue to the next Game Stage/Round):

GrimTriggerForUnknownNumRoundsWithProbability	Random
3	7

GrimTriggerForUnknownNumRoundsWithProbability	AllCooperate
24	24

GrimTriggerForUnknownNumRoundsWithProbability	AllDefect
4	12

vi. Some Experimental Tests

- There was realized some **Experimental Tests**, regarding the three type of games proposed to the **Tournament**:
  - Game #3 (1000 Iterations, which are not considered – 40% of Probability of Continue to the next Game Stage/Round):

GrimTriggerForUnknownNumRoundsWithProbability	GrimTrigger
6	6

GrimTriggerForUnknownNumRoundsWithProbability	OmegaTitForTat
6	6

GTriggerUnknownRoundsWProbability	OTFTUnknownRoundsWProbability
12	12

GrimTriggerForUnknownNumRoundsWithProbability	TitForTat
6	6

GTriggerUnknownRoundsWProbability	TFTUnknownRoundsWProbability
6	6

vii. Strategies Used in the Tournament

- **Game #1 (20 Iterations – 100% of Probability of Continue to the next Game Stage/Round):**
    - **Forgiving Grim Trigger For Fixed Number of Rounds Strategy:**
        - Total Score Points (4th Place): 56.400
        - Highest Score Points: 57.200
- **Game #2 (∞ Iterations – Unknown Probability of Continue to the next Game Stage/Round, less than 100%):**
    - **Grim Trigger For Fixed Number of Rounds With Probability Strategy:**
        - Total Score Points (10th Place): 62.355
        - Highest Score Points: 66.000
- **Game #3 (Unknown Iterations, higher than 1 – Unknown Probability of Continue to the next Game Stage/Round, less than 100%):**
    - **Grim Trigger For Unknown Number of Rounds With Probability Strategy:**
        - Total Score Points (6th Place): 39.000
        - Highest Score Points: 44.500

viii. Explanation about the chosen Strategies for the Tournament

- **Game #1 (20 Iterations – 100% of Probability of Continue to the next Game Stage/Round):**
  - **Forgiving Grim Trigger For Fixed Number of Rounds Strategy:**
    - It was thought in an implementation of an initial **Handshake Process** with a colleague, in order to, if in the case of be **detected**, **maximize each other payoff gain**. This process to be reliable would take a **couple of rounds (e.g., 5)** and some **Defect** actions as sacrifice or prober, and, as the number of iterations of the game it was **only 20**, **it was decided to be not implemented**.
    - The **Strategy** starts by playing a **Cooperate** action.
    - The **Strategy**, globally, implements the behaviour of the well-known **Grim Trigger**, **but in a more forgiving sense**.
    - It's **conceded two opportunities of forgiveness**, in the case of the opponent plays **two consecutives Cooperate** actions, after played some **Defect** action(s), in order to not play in a very severe way and try to be **"nice"**, sometimes.
    - If the opponent **waste** that **two opportunities of forgiveness**, the strategy will perform completely as a pure **Grim Trigger Strategy**, until the last **Game Stages/Rounds**.
    - The **Strategy** will play a **Defect** action, when the possible **Gain From Defect** was worthy, i.e., when the **Gain From Defect** would be greater than the **Deterministic Loss From Defect**, in order to, **avoid earlier unnecessary punishments**.

viii. Explanation about the chosen Strategies for the Tournament

- **Game #2 (∞ Iterations – Unknown Probability of Continue to the next Game Stage/Round, less than 100%):**
  - **Grim Trigger For Fixed Number of Rounds With Probability Strategy:**
    - The **Strategy** starts by playing a **Cooperate** action.
    - The **Strategy**, globally, implements the behaviour of the well-known **Grim Trigger**.
    - The **Strategy** will play a **Defect** action, when the possible **Gain From Defect** was worthy, i.e., when the **Gain From Defect** would be greater than the **Deterministic** or the **Probabilistic Loss From Defect**, in order to, **avoid earlier unnecessary punishments**.
- **Game #3 (Unknown Iterations, higher than 1 – Unknown Probability of Continue to the next Game Stage/Round, less than 100%):**
  - **Grim Trigger For Unknown Number of Rounds With Probability Strategy:**
    - The **Strategy** starts by playing a **Cooperate** action.
    - The **Strategy**, globally, implements the behaviour of the well-known **Grim Trigger**.
    - The **Strategy** will play a **Defect** action, when the possible **Gain From Defect** was worthy, i.e., when the **Gain From Defect** would be greater than the **Probabilistic Loss From Defect**, in order to, **avoid earlier unnecessary punishments**.

ix. Conclusions

- First, the **Prisoner's Dilemma** it's a **very complex** and **difficult problem** to solve.

- The most known and **commonly used Strategies** for this problem are:

    - **Tit For Tat;**

    - **Grim Trigger;**

    - **Gradual;**

- But, considering some few little modifications, in order to make a **Defect** action, **in the last round(s)**, it's always a good idea, like per example:

    - Consider the number of iterations (Game Stage/Rounds);

    - Consider the Probability of Continue to the next iteration (Game Stage/Round);

- Follow the recommendations of **Robert Axelrod**, for successful **Top-Scoring Strategies**, it's also a good idea:

    - Be "nice".

    - Retailing sometimes.

    - Forgiving sometimes.

    - Don't be envious.

- It's also, always, a good idea to don't underestimate too much our opponents:

    - Assume with 100% of certainty how our opponents are going to play, it's a very bad idea.

ix. Conclusions

- For that reason, the opposite idea it's also a good thought:
  - Think "outside the box", to take our opponents by surprise, in the last round(s), but also, not too early.
- But, after all, it's always very hard to deal with some aspects:
  - Predict with accuracy the behaviours of our opponents.
  - Deal with randomness behaviours of our opponents.
- Other approaches can be used in order to lead with **randomness** and **unpredictability**:
  - Using the emergent **Quantum Computing**, per example, to lead with much better with randomness and unpredictability should be a good choice.
  - Use some properties of **Quantum Physics/Mechanics** together with Informatics, such as:
    - **Quantum Superposition:**
      - In order to test all the possible outcome's payoffs of several actions, at once, and choose always the best one for us;
    - **Quantum Entanglement:**
      - In order to don't allow some "cheat" behaviours from our opponents and punish them, in a very severe way;
      - Or even, to build some unbreakable coordination between the **Players**, in some kind of "blood-contracts", which no one will want to deviate from it;

ix. Conclusions

- And this different point of views, lead to a new topic of research, called **Quantum Game Theory**:

  - Using the topic of **Design of Mechanisms** in **Game Theory**, it's possible to build a new version of **Prisoner's Dilemma**, using **Quantum Rules** and **adding a third possible move**, called something like **"Q-Move",** in order to **punish severely** our opponents, **in the case of "cheating"**, resulting in a new **Normal-Form** for this game, like the following, per example:

Rúben (rows)/João (columns)	Cooperate	Defect	Q-Move
Cooperate	(3,3)	(0,4)	(1,1)
Defect	(4,0)	(1,1)	(0,4)
Q-Move	(1,1)	(4,0)	(3,3)

- In this new version of **Prisoner's Dilemma**, which we can call **Quantum Prisoner's Dilemma**, the **Nash Equilibrium** it's no more the **(Defect, Defect)**, thus, we can state the following:

  - In a situation of **(Defect, Defect)**, both the **Players** have now an incentive to change unilaterally their choices.

  - Both **Players** will be punished severely, in the case of "cheating".

  - Now, the game has a new **Nash Equilibrium**, the **(Q-Move, Q-Move)**.

- **But it's this unpredictability and uncertainty, offered by its "Classical Version", that makes the Prisoner's Dilemma so funny!!!**

x. Bibliography/Suggested Literature

- The **Bibliography** and **Suggested Literature** are the following:
  - https://en.wikipedia.org/wiki/Prisoner%27s_dilemma
  - https://www.investopedia.com/terms/g/gametheory.asp
  - https://medium.com/thinking-is-hard/a-prisoners-dilemma-cheat-sheet-4d85fe289d87
  - http://jasss.soc.surrey.ac.uk/20/4/12.html
  - http://www.prisoners-dilemma.com/strategies.html

xi. GitHub Repository for the Code

- The **GitHub Repository** hosting the **Java Code implementation** of the **Strategies** and, the report it's the following:
  - https://github.com/rubenandrebarreiro/nova-game-theory-interactive-tournaments

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- And never forget… **You should never betray a trusted comrade**, because:
  - *"Snitches get stitches!!!"*