Augmented Rectangular Load Flow Model

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***Abstract*—This paper presents some improvements to the load flow solution in rectangular coordinates. First, in an attempt to use as linear a model as possible, both the nodal equations and the bus constraints are retained. The Newton–Raphson (NR) method is then applied to the enlarged set of equations, written in terms of bus voltages and currents. This scheme, combined with a simple procedure to handle PV buses, leads to a computationally efficient algorithm, particularly advantageous in the presence of zero-injection buses. Experimental results are provided comparing the performance of the proposed approach with that of the conventional formulation.**

***Index Terms*—Current injections, load flow, rectangular coordinates.**

I. Introduction

THE LOAD flow routine is without doubt the most frequently run application in operational and planning environments.

The computational developments of Tinney *et al.* [1] in the 1960s overrode the limitations of earlier implementations of the Newton–Raphson (NR) method which became, *de facto*, the standard approach to solve the load flow problem. The second major breakthrough arose in the mid 1970s with the introduction of the fast decoupled load flow technique (FDLF) by Stott and Alsaç [2]. Nowadays, particularly when repeated solutions are required, the FDLF is the usual choice in transmission applications. However, when reliability and accuracy, rather than speed of response, is a concern, or when the decoupling principle does not hold, the NR is the preferred approach. Some cases have been reported, most of them corresponding to radial distribution systems, where even the NR encounters difficulties to reach a solution. This has motivated the development of alternative methodologies, not necessarily based on the NR iterative scheme, specifically tailored to radial cases [3]–[5].

Reference [6] constitutes an excellent review of load flow calculation methods. More recent contributions to this problem include the so-called second-order methods [7] and a modification of the FDLF intended for cases with large R/X ratios [8].

Both power-mismatch [1] and current-mismatch [9] formulations have been developed, but in all cases the unknown vector is exclusively composed of bus voltages, either in polar or rectangular coordinates [6]. An interesting feature of the current-injection rectangular formulation, recently reconsidered in [9], lies in "mutual" Jacobian terms being simply branch admittances for load buses.

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As explained in [6], the NR method's performance "is sensitive to the behavior of the functions $F(x)$ and hence to their formulation. The more linear they are, the more rapidly and reliably the Newton's method converges." This is the reason why power mismatch vector components are divided by voltage magnitudes in polar formulations.

In this paper, an attempt is made to further explore this possibility. The key idea is to solve an augmented system in which both bus voltages and current injections appear as state variables, and both power and current mismatches are zeroed. When rectangular coordinates are employed, this yields a set of linearly-coupled quadratic equations rather than the usual set of fully-coupled nonlinear equations. The same motivation guided the technique proposed in [10], exclusively applicable to radial or weekly-meshed systems.

The structure of this paper is as follows: In Section II, the proposed method is presented for networks comprising only PQ buses. This is the most common case in distribution networks. Next, Section III discusses two ways of dealing with PV buses, one of them specifically developed for the proposed augmented model. The main steps of the proposed procedure are then summarized in Section IV. Section V provides the results of applying the new method to several transmission and distribution networks. A comparison is performed with existing polar-based implementations in terms of number of iterations and computational effort. Some concluding remarks close the paper.

II. Basic Augmented Model: PQ Buses

Throughout this paper, rectangular coordinates will be used both for bus voltages and bus current injections, the notation adopted being the following:

$V_i\underline{\vert\theta_i} = e_i + jf_i$	complex voltage at bus i;
$I_i\underline{\vert\varphi_i} = I_{ai} + jI_{bi}$	complex current injected at bus i;
$G_{ij} + jB_{ij}$	ijth element of the nodal admittance matrix;
$P_i + jQ_i$	complex power injected at bus i;
sp	denotes specified magnitude;
$n+1$	number of buses.

For every bus i, its nodal equations can be written as

$$\begin{bmatrix} I_{ai} \\ I_{bi} \end{bmatrix} = \sum_{j=1}^{n} \begin{bmatrix} -B_{ij} & G_{ij} \\ G_{ij} & B_{ij} \end{bmatrix} \begin{bmatrix} f_j \\ e_j \end{bmatrix} \tag{1}$$

and its boundary constraints

$$\begin{bmatrix} P_i^{sp} \\ Q_i^{sp} \end{bmatrix} = \begin{bmatrix} I_{bi} & I_{ai} \\ I_{ai} & -I_{bi} \end{bmatrix} \begin{bmatrix} f_i \\ e_i \end{bmatrix} = \begin{bmatrix} e_i & f_i \\ f_i & -e_i \end{bmatrix} \begin{bmatrix} I_{ai} \\ I_{bi} \end{bmatrix}. \tag{2}$$

Note that (1) is linear and (2) is quadratic. However, **both** equations would be nonlinear if polar coordinates were adopted.

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The usual approach to solve the load flow problem consists of using (1) to eliminate the bus current injections in (2), leading to a set of $2n$ nonlinear equations. The NR method is then employed to linearize and iteratively solve those equations.

The procedure adopted in this paper is somewhat the opposite: The NR method is first applied to the augmented model, given by (1) and (2), in which the bus current injections are retained as explicit variables. The following linearized equations result for every bus:

$$\sum_{j-1}^{n}\begin{bmatrix}-B_{ij} & G_{ij}\\ G_{ij} & B_{ij}\end{bmatrix}\begin{bmatrix}\Delta f_j\\ \Delta e_j\end{bmatrix}-\begin{bmatrix}\Delta I_{ai}\\ \Delta I_{bi}\end{bmatrix}=\begin{bmatrix}\Delta\alpha_i\\ \Delta\beta_i\end{bmatrix} \quad (3)$$

$$\begin{bmatrix}I_{bi} & I_{ai}\\ I_{ai} & -I_{bi}\end{bmatrix}\begin{bmatrix}\Delta f_i\\ \Delta e_i\end{bmatrix}+\begin{bmatrix}e_i & f_i\\ f_i & -e_i\end{bmatrix}\begin{bmatrix}\Delta I_{ai}\\ \Delta I_{bi}\end{bmatrix}=\begin{bmatrix}\Delta P_i\\ \Delta Q_i\end{bmatrix} \quad (4)$$

where the mismatch vectors are given by

$$\begin{bmatrix}\Delta\alpha_i\\ \Delta\beta_i\end{bmatrix}=\begin{bmatrix}I_{ai}\\ I_{bi}\end{bmatrix}-\sum_{j=1}^{n}\begin{bmatrix}-B_{ij} & G_{ij}\\ G_{ij} & B_{ij}\end{bmatrix}\begin{bmatrix}f_j\\ e_j\end{bmatrix} \quad (5)$$

$$\begin{bmatrix}\Delta P_i\\ \Delta Q_i\end{bmatrix}=\begin{bmatrix}P_i^{sp}\\ Q_i^{sp}\end{bmatrix}-\begin{bmatrix}e_i & f_i\\ f_i & -e_i\end{bmatrix}\begin{bmatrix}I_{ai}\\ I_{bi}\end{bmatrix}. \quad (6)$$

Excluding the slack bus, the $4n$ equations that must be solved at each iteration can be arranged as shown in (7) at the bottom of the next page, or, in compact form as

$$\begin{bmatrix}Y_B & -I\\ D_I & D_V\end{bmatrix}\begin{bmatrix}\Delta U\\ \Delta I\end{bmatrix}=\begin{bmatrix}\Delta\Lambda\\ \Delta S\end{bmatrix} \quad (8)$$

where Y_B is the 2×2-block nodal admittance matrix and D_I, D_V are 2×2-block diagonal matrices whose elements are given by (4).

Then, the above enlarged system is solved in two steps: First, (4) is used to eliminate the bus current corrections in (3), yielding

$$\left(Y_B+D_V^{-1}D_I\right)\Delta U=\Delta\Lambda+D_V^{-1}\Delta S. \quad (9)$$

The term $D_V^{-1}D_I$ only modifies the diagonal blocks of Y_B which become

$$[Y_B]_{ii}+\frac{1}{V_i^2}\begin{pmatrix}e_i & f_i\\ f_i & -e_i\end{pmatrix}\begin{pmatrix}I_{bi} & I_{ai}\\ I_{ai} & -I_{bi}\end{pmatrix}. \quad (10)$$

Next, after solving this system, the current incremental vector is computed from

$$\Delta I=D_V^{-1}\Delta S-\left(D_V^{-1}D_I\right)\Delta U \quad (11)$$

where the terms $D_V^{-1}\Delta S$, $D_V^{-1}D_I$ are already available from the former step.

Remarks:

1) Equation (9) is structurally identical to that solved in [9]. Except for the diagonal blocks, the coefficient matrix is also constant. However, the numerical values of diagonal blocks differ and, hence, different convergence rates are expected.
2) Since (1) is linear, the current mismatch vector in (3) remains null after the first iteration ($\Delta\Lambda=0$). Furthermore, computation of the power mismatch vector ΔS is extremely fast as a consequence of current injections being updated at every iteration. In fact, each element of $D_V^{-1}\Delta S$

$$\left[D_V^{-1}\Delta S\right]_i=\frac{1}{V_i^2}\begin{bmatrix}e_1 & f_i\\ f_i & -e_1\end{bmatrix}\begin{bmatrix}P_i^{sp}\\ Q_i^{sp}\end{bmatrix}-\begin{bmatrix}I_{ai}\\ I_{bi}\end{bmatrix} \quad (12)$$

can be computed by means of just five adds and six mults (only four mult/adds are required if ΔP_i, ΔQ_i are sought).
3) Unlike in earlier formulations of the load flow problem, advantage can be taken of zero-injection buses, which usually represent a nonnegligible fraction of the total. In the same way as voltage magnitudes are omitted for PV buses when polar coordinates are used, the current components are removed in this formulation from the set of variables when they equal zero (or any other value). Therefore, the respective bus constraint (2) is not enforced, and there is no need to compute its power mismatch components. Note that this way of handling zero-injection buses may affect the convergence, because the more buses of this type the more linear the resultant equation system.

There are at least two ways of initializing the iterative process.

A) Assign initial values so that the linear nodal equations are satisfied ($\Delta\Lambda^0=0$)

$$e_i^0=0,\quad f_i^0=0$$
$$a_i^0=\sum_{j=1}^{n}G_{ij}=0,\quad b_i^0=\sum_{j=1}^{n}B_{ij}.$$

With these initial values, the power mismatches become

$$\Delta P_i^0=P_i^{sp},\qquad \Delta Q_i^{sp}=Q_i^{sp}+b_i^0.$$

B) Assign initial values so that the quadratic equations (bus constraints) are satisfied ($\Delta S^0=0$)

$$e_i^0=1,\qquad f_i^0=0$$
$$a_i^0=P_i^{sp},\qquad b_i^0=-Q_i^{sp}.$$

With these initial values, the current mismatches become

$$\Delta\alpha_i^0=a_i^0,\qquad \Delta\beta_i^0=b_i^0-\sum_{j=1}^{n}B_{ij}.$$

III. Inclusion of PV Buses

The natural way of including PV buses in the enlarged model is by replacing the reactive component of (2)

$$Q_i^{sp}=I_{ai}f_i-I_{bi}e_i$$

with the respective voltage magnitude constraint

$$\left(V_i^{sp}\right)^2=e_i^2+f_i^2 \quad (13)$$

yielding

$$\begin{bmatrix} P_i^{sp} \\ (V_i^{sp})^2 \end{bmatrix} = \begin{bmatrix} I_{bi} & I_{ai} \\ f_i & e_i \end{bmatrix} \begin{bmatrix} f_i \\ e_i \end{bmatrix}. \tag{14}$$

Equation (4) then becomes

$$\begin{bmatrix} I_{bi} & I_{ai} \\ 2f_i & 2e_i \end{bmatrix} \begin{bmatrix} \Delta f_i \\ \Delta e_i \end{bmatrix} + \begin{bmatrix} e_i & f_i \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta I_{ai} \\ \Delta I_{bi} \end{bmatrix} = \begin{bmatrix} \Delta P_i \\ \Delta V_i^2 \end{bmatrix}. \tag{15}$$

The major difference between (4) and (15) lies in the fact that the block elements of D_V corresponding to PV buses are singular. Therefore, (9) cannot be used. A simple modification to that equation

$$(D_V Y_B + D_I)\Delta U = D_V \Delta\Lambda + \Delta S \tag{16}$$

would circumvent the problem, but then all elements of the Jacobian would have to be recomputed.

In this paper, the following two alternatives have been considered.

A) The first approach consists of using (9) for PQ buses and (16) for PV buses. Denoting PQ buses by "l," and PV buses by "g," the resulting equation is

$$\left[\begin{array}{c|c} Y_{ll} + D_{V_l}^{-1} D_{I_l} & Y_{lg} \\ \hline D_{V_g} Y_{gl} & D_{I_g} + D_{V_g} Y_{gg} \end{array}\right] \begin{bmatrix} \Delta U_l \\ \Delta U_g \end{bmatrix} = \begin{bmatrix} \Delta\Lambda_l + D_{V_l}^{-1} \Delta S_l \\ D_{V_g} \Delta\Lambda_g + \Delta S_g \end{bmatrix}. \tag{17}$$

Now, only the rows of the Jacobian corresponding to PV buses must be recomputed (besides the diagonal

$$\left[\begin{array}{cc|cc|c|cc||cc|cc|c|cc}
-B_{11} & G_{11} & -B_{12} & G_{12} & \cdots & -B_{1n} & G_{1n} & -1 & 0 & 0 & 0 & \cdots & 0 & 0 \\
G_{11} & B_{11} & G_{12} & B_{12} & & G_{1n} & B_{1n} & 0 & -1 & 0 & 0 & & 0 & 0 \\ \hline
-B_{21} & G_{21} & -B_{22} & G_{22} & \cdots & -B_{2n} & G_{2n} & 0 & 0 & -1 & 0 & \cdots & 0 & 0 \\
G_{21} & B_{21} & G_{22} & B_{22} & & G_{2n} & B_{2n} & 0 & 0 & 0 & -1 & & 0 & 0 \\ \hline
\vdots & & \vdots & & \ddots & \vdots & & \vdots & & \vdots & & \ddots & \vdots & \\ \hline
-B_{n1} & G_{n1} & -B_{2n} & G_{2n} & \cdots & -B_{nn} & G_{nn} & 0 & 0 & 0 & 0 & \cdots & -1 & 0 \\
G_{n1} & B_{n1} & G_{2n} & B_{2n} & & G_{nn} & B_{nn} & 0 & 0 & 0 & 0 & & 0 & -1 \\ \hline\hline
I_{b1} & I_{a1} & 0 & 0 & \cdots & 0 & 0 & e_1 & f_1 & 0 & 0 & \cdots & 0 & 0 \\
I_{a1} & -I_{b1} & 0 & 0 & & 0 & 0 & f_1 & -e_1 & 0 & 0 & & 0 & 0 \\ \hline
0 & 0 & I_{b2} & I_{a2} & \cdots & 0 & 0 & 0 & 0 & e_2 & f_2 & \cdots & 0 & 0 \\
0 & 0 & I_{b2} & I_{a2} & & 0 & 0 & 0 & 0 & f_2 & -e_2 & & 0 & 0 \\ \hline
\vdots & & \vdots & & \ddots & \vdots & & \vdots & & \vdots & & \ddots & \vdots & \\ \hline
0 & 0 & 0 & 0 & \cdots & I_{bn} & I_{an} & 0 & 0 & 0 & 0 & \cdots & e_n & f_n \\
0 & 0 & 0 & 0 & & I_{an} & -I_{bn} & 0 & 0 & 0 & 0 & & f_n & -e_n
\end{array}\right]
\cdot
\left[\begin{array}{c}
\Delta f_1 \\ \Delta e_1 \\ \hline \Delta f_2 \\ \Delta e_2 \\ \hline \vdots \\ \hline \Delta f_n \\ \Delta e_n \\ \hline\hline \Delta I_{a1} \\ \Delta I_{b1} \\ \hline \Delta I_{a2} \\ \Delta I_{b2} \\ \hline \vdots \\ \hline \Delta I_{an} \\ \Delta I_{bn}
\end{array}\right]
=
\left[\begin{array}{c}
\Delta\alpha_1 \\ \Delta\beta_1 \\ \hline \Delta\alpha_2 \\ \Delta\beta_2 \\ \hline \vdots \\ \hline \Delta\alpha_n \\ \Delta\beta_n \\ \hline\hline \Delta P_1 \\ \Delta Q_1 \\ \hline \Delta P_2 \\ \Delta Q_2 \\ \hline \vdots \\ \hline \Delta P_n \\ \Delta Q_n
\end{array}\right] \tag{7}$$

blocks). The current incremental vector is computed from (11) for PQ buses and from the following expression:

$$\Delta I_g = Y_{gl}\Delta U_l + Y_{gg}\Delta U_g - \Delta\Lambda_g \tag{18}$$

for PV buses.

B) A popular way of handling PV buses in the FDLF method, intended to retain the structure of the B'' matrix, consists of switching the respective diagonal between a large number (when the required reactive power is within bounds) and its actual value (when a limit is reached). In this manner, PV buses are "grounded" in the reactive subproblem, leading to $\Delta V \approx 0$. Experience has shown that the risk of numerical instability is minimal on modern extended-wordlength computers.

A similar idea can be applied to the proposed augmented model. Let us add the reactive component of (4) times ε and the voltage component of (15) times $(1-\varepsilon)$. This yields

$$\left[\begin{array}{c|c} I_{bi} & I_{ai} \\ \hline \varepsilon I_{ai} + 2(1-\varepsilon)f_i & -\varepsilon I_{bi} + 2(1-\varepsilon)e_i \end{array}\right]\begin{bmatrix}\Delta f_i \\ \Delta e_i\end{bmatrix} + \left[\begin{array}{c|c} e_i & f_i \\ \hline \varepsilon f_i & -\varepsilon e_i \end{array}\right]\begin{bmatrix}\Delta I_{ai} \\ \Delta I_{bi}\end{bmatrix} = \begin{bmatrix}\Delta P_i \\ \varepsilon\Delta Q_i + (1-\varepsilon)\Delta V_i^2\end{bmatrix}. \tag{19}$$

Clearly, when $\varepsilon = 1$ the above equation reduces to that of PQ buses, while a sufficiently small ε (e.g., 10^{-4}) will model the behavior of PV buses [there is no problem using $\varepsilon = 0$ in the right-hand side (RHS)]. With this scheme, a large number $(1/\varepsilon)$ will show up in certain diagonal blocks of (9), but all off-diagonal blocks remain constant. Therefore, this approach is computationally more attractive provided there is no numerical deterioration. Switching between PV and PQ bus types is as simple as changing the value of ε when updating the respective diagonal block.

IV. Solution Methodology

The main steps of the proposed procedure can be summarized as follows (the bus-type switching logic is omitted for simplicity).

Step 1) Initialize the state variables. In practice, the scheme *B)* described above has proved to be superior. Build Y_B and perform the minimum degree and symbolic factorization on its block structure.

Step 2) Compute the mismatch vectors. During the first iteration $\Delta S = 0$ but $\Delta\Lambda^0 \neq 0$ if scheme B is adopted. At subsequent iterations the opposite happens, i.e., $\Delta\Lambda = 0$ and only ΔS must be computed. Note that zero-injection nodes are skipped from the computations. If all mismatch vector components are smaller than a threshold, STOP.

Step 3) Compute the reduced mismatch vector [RHS of (9) or (17)]. Modify Y_B according to the way PV buses are handled: If (17) is adopted, the diagonal blocks plus the rows corresponding to PV buses must be updated. If the "grounding" ε-based technique is chosen, update only the diagonal blocks. In any case, no updating is necessary for zero-injection buses.

Step 4) Solve (17) or (9) by LU factorization (and simultaneous elimination) followed by back substitution (both scalar or block arithmetic can be adopted). Update bus voltages.

Step 5) Obtain incremental and new values for bus current injections. If the maximum number of iterations is not exceeded go to Step 2.

Major computational differences with existing current injection methodologies are:

- Bus injected currents, being used as explicit variables, are updated at the end of each iteration [Step 5]. In [9], injected currents are instead computed at the beginning from bus voltages ($I = Y_B U$) which is more time consuming.
- Updating the diagonal blocks by means of (10) is also less expensive than using the expressions provided in [9].
- It is possible to deal with PV buses without having to modify whole columns or rows.
- Rectangular coordinates are used throughout, both for voltages and currents. The way state variables are updated in [9] requires that polar coordinates be used at intermediate steps.

Hence, although we are handling and solving an enlarged equation system, it turns out that the computational effort per iteration is lower than that of [9] and any other polar-based coupled formulation.

An immediate modification of the proposed procedure is obtained by keeping the Jacobian constant after the first iteration.

A detailed treatment of voltage regulating transformers is out of the scope of the paper, but they can be included in the proposed model essentially as in existing formulations [11]. Assume a transformer tap t is regulating the voltage of a PQ bus i. Then, the state vector is augmented with the variable t, and the constraint (13) is added to compensate for the extra unknown. New Jacobian entries corresponding to the column $\partial/\partial t$ must be computed, and the bus-related block structure is lost. Furthermore, some care must be exercised when solving the resulting equation system as a zero pivot may arise if Δt is eliminated prematurely.

V. Experimental Results

The polar-based NR and the augmented rectangular-based model have been coded in Fortran using the same sparse matrix package and programming techniques. Scalar rather than 2×2-block arithmetic has proved to be a less expensive scheme.

Both transmission and radial distribution networks have been tested. Table I shows the main figures corresponding to transmission networks.

Table II compares the number of iterations required by the proposed method with those of the polar-based NR (unless otherwise noticed, the convergence threshold is 10^{-4} p.u.). Although both ways of handling PV buses have been tested, only the results corresponding to the ε-based technique have been reported, as the computational effort is smaller and the number of iterations seldom differs. Relative computation times per iteration are also shown (NR is 100%). Actually, these are average

TABLE I
MAIN DATA OF THE TRANSMISSION NETWORKS TESTED

Number of Buses	Number of Branches	% PV Buses	% Zero-inj. Buses
57 [a]	78	12.3	26.3
118 [a]	178	45.7	8.4
127	219	19.7	26.8
298 [a]	407	23.1	22.14
731 [b]	951	22.3	39.8
1467 [b]	2087	22.6	33.3
4376 [c]	5216	4.2	32.24

TABLE II
NUMBER OF ITERATIONS AND RELATIVE COMPUTATION TIMES PER ITERATION FOR TRANSMISSION SYSTEMS

Number of Buses	NR Method	Proposed Method	Relative times (%)
57	3	3	78.68
118	4	4	97.33
127	4	3	85.79
298	5	6	85.7
731	5	6	81.49
1467	9	8	89.6
4376	6	6	90.1

TABLE III
MAIN DATA OF THE RADIAL DISTRIBUTION NETWORKS TESTED

Buses	Zero-inj. Buses (%)
30	30
43	51
69	29
85	30.6
90	47.7
690	44.7

values, as the execution time for a particular iteration is slightly affected by the bus-type switching logic.[1] [2] [3]

A similar convergence pattern, but a smaller computational cost can be noticed in all cases. The worst performance for the proposed model takes place on the 118-bus system, as a consequence of the abnormally high proportion of PV buses (46%) and low number of zero-injection buses (8%).

A noteworthy experiment consists of reducing the number of PV buses for the IEEE 300-bus system from 69 to 22. In this situation, the conventional NR requires ten iterations while the proposed scheme needs only five. Furthermore, each iteration is 21% rather than 14% faster. This shows the advantage of polar-based formulations over rectangular ones when the number of PV buses is excessive, and vice versa.

Tables III and IV are the counterpart of Tables I and II for radial balanced distribution networks, most of which have been regarded in the literature as very ill-conditioned [5], [12]–[15]. Except for the slack bus, no PV buses are present in those networks. Note also that the number of zero-injections buses (30–50%) is in average larger than that of transmission networks. Both facts probably explain why the augmented model is this time significantly more efficient than the polar-based formulation, usually requiring less iterations which, in average, are 45% faster.

[1]IEEE test cases.

[2]Spanish transmission networks.

[3]PowerWorld test case.

TABLE IV
NUMBER OF ITERATIONS AND RELATIVE COMPUTATION TIMES PER ITERATION FOR DISTRIBUTION SYSTEMS

Number of Buses	NR Method	Proposed Method	Relative times (%)
30	3	2	55.72
43	6	5	51.72
69	3	2	50.45
85	3	2	53.18
90	2	2	47.26
690	3	2	59.67

TABLE V
MAX. $|\Delta P_i|$, $|\Delta Q_i|$ FOR THE 690-BUS SYSTEM

Iteration	NR Method	Proposed Method
0	1620.57	1.20
1	12.23	$1.57 \cdot 10^{-2}$
2	$2.52 \cdot 10^{-2}$	$1.62 \cdot 10^{-5}$
3	$5.31 \cdot 10^{-6}$	$2.36 \cdot 10^{-11}$
4	$1.01 \cdot 10^{-9}$	$2.22 \cdot 10^{-16}$

TABLE VI
NUMBER OF ITERATIONS FOR THE 690-BUS SYSTEM WHEN THE RATIO R/X IS INCREASED

$R = k \cdot R_o$	NR Method	Proposed Method
$k = 1$	3	2
$k = 2$	3	2
$k = 3$	4	3
$k = 6.5$	5	4

TABLE VII
NUMBER OF ITERATIONS FOR THE 690-BUS SYSTEM WHEN LOADS ARE INCREASED

$k \cdot P_L$, $k \cdot Q_L$	NR Method	Proposed Method
$k = 1$	3	2
$k = 1.3$	3	2
$k = 1.75$	4	3
$k = 2.5$	5	4

Table V compares the largest mismatch vector component at every iteration for the 690-bus network. The good convergence rate shown for the proposed implementation is representative of its behavior on most distribution networks tested. In fact, the results provided by the proposed methodology are better than those obtained with radial load flow techniques (see, for instance, the comparison performed in [10]). Although this paper addresses only the single-phase load flow problem, the explicit inclusion of current injections in the state vector makes the augmented approach an interesting choice to model the typical three-phase load arrangements of three-phase load flows.

In order to test more stressed situations, the same experiments are repeated for the 690-bus radial network when the R/X ratio is augmented for all branches (see Table VI) or the load is proportionally increased at all buses (see Table VII). In all cases, the proposed rectangular version attains convergence in less iterations.

TABLE VIII
OPERATIONS COUNT PER ITERATION FOR THE LARGEST T&D NETWORKS

Network	690 buses		4376 buses	
Method	NR	Prop.	NR	Prop.
Jacobian & mismatch vector computation				
-Trigon. functions	2756	0	20852	0
- Products	13094	6858	94494	54633
-Additions	11026	4953	77833	39264
Solution & Updating				
-Products	15831	16666	187211	214154
-Additions	18583	20869	492057	637357
Total Equivalent Operations	86094	49346 (57.3%)	1060115	945408 (89.17%)

Finally, Table VIII presents the number of arithmetic operations per iteration corresponding to the largest T&D networks. Data corresponding to Steps 2 and 3 of the proposed procedure (formation of Jacobian and mismatch vector) and Steps 4 and 5 (solution and updating of the state vector) are separately collected. The last row shows the total number of equivalent operations, taking into account that, in double-precision arithmetic, a product and a sum are approximately equivalent and that a sine (cosine) amounts to ten sums, on a Pentium computer. As discussed above, the proposed approach is more efficient when computing the Jacobian and mismatch vector, but it has to solve a larger system in the presence of PV buses and it has to update, additionally, the current injection vector. Differences between the relative execution times of Tables II and IV, and the percentages shown in Table VIII are due to the nonarithmetic overhead.

VI. CONCLUSIONS

This paper presents an alternative to the way the load flow equations are currently solved. Instead of combining the nodal equations and the bus constraints into a single set of $2n$ nonlinear equations, the NR method is applied to the two primitive sets of equations. The enlarged model, in which current injections are retained in the state vector, leads to a very simple solution methodology if rectangular coordinates are adopted. A straightforward approach to dealing with PV buses is also proposed.

Experiments confirm that, depending on the number of PV buses, the computational effort per iteration ranges between 50 and 80% of that required by polar formulations. Not only comes this saving from the simplicity of the Jacobian terms, as in other rectangular-based methods, but from the mismatch vector computation as well, particularly when many zero-injection buses are present.

While the convergence rate of the proposed method, when transmission networks are solved, is similar to that of existing implementations, a noticeable improvement is obtained when dealing with distribution networks.

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