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THESIS

Free-riding in Collective Agreements, a Search and Matching Model of Minimum Wages

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Abstract

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Does free riding in collective bargaining systems introduce inefficiencies in the labour market? What are the welfare losses if at all? What would the optimal bargaining protocol be? Participants of the political process of collective agreements set the institutions that fit them best, non-participants free ride on what is agreed. Building on previous research, I introduce how participants set one institution, minimum wages, in a model of two-sided heterogeneity with on-the-job search. I take the model to data using the Spanish employer-employee database 'Muestra Continua de Vidas Laborales', and the model is estimated by the Simulated Method of Moments. Workers in large firms use Minimum wages to increase the value of their jobs whereas firms avoid competitors. I show that the effects are more unemployment among low-skill workers and increase in wages for those who stay employed, reducing overall welfare. Carrying a counterfactual analysis, I have found that increasing the base of participants in the political process raises welfare and reduces unemployment, at a cost of employed earning less.

Keywords: Collective bargaining, Two-sided Heterogeneity, Minimum-Wages, Search and Matching

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1 Introduction

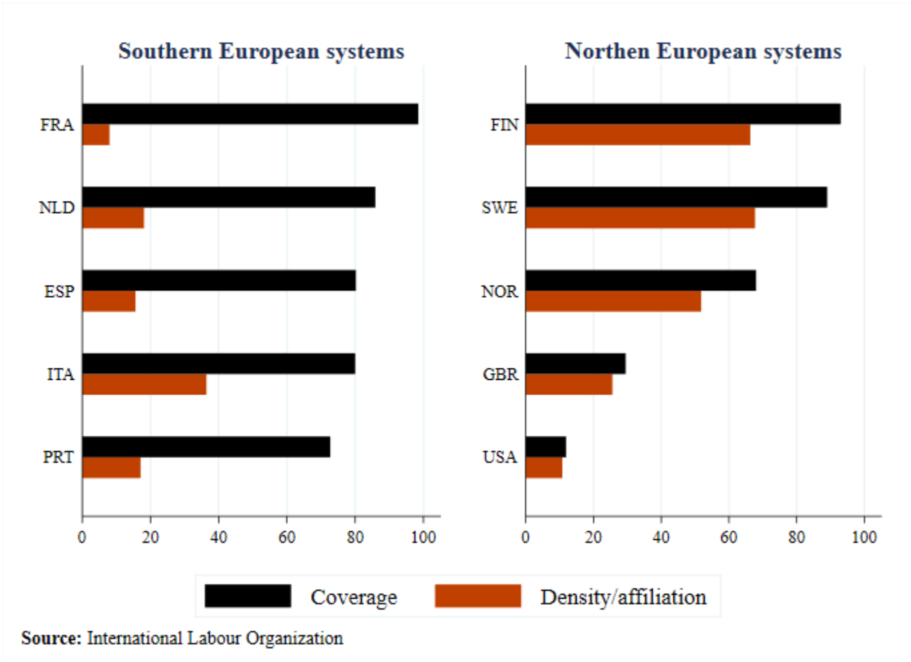
How does central and southern European systems of collective bargaining affect labour market outcomes? What are the welfare consequences? What is the optimal bargaining protocol? The enactments of these contracts cover a wide range of issues such as working conditions, fringe benefits or anti-discrimination policies. To keep the analysis simple I will focus on one of the most important aspects, namely minimum wages. Participants of the political process (insiders) might use minimum wages as a tool to gain some perks such higher wages or avoid free-entry, not internalising the effects on the outsiders. This work tries to estimate these effects and measure the impact on labour market outcomes.

Applicability of collective agreements in central and southern European countries usually extend beyond the negotiating parties. In cases as extreme as Spain, collective agreements are automatically extended to the whole labour market within a sector, regardless of the affiliation to a representative body or having participated of the political process. Therefore, collective agreements are characterised by its universal coverage. Because of legal provisions or collective action constraints, only workers in large firms are able to elect their representatives in the negotiating table. In turn, this gives them some leverage to set the institutions that are going to rule the labour market. On the other side, workers or firms who do not participate of this process might be directly affected, for good or bad, by provisions set in the contract.

From a theoretical point of view, I will try to answer these questions by constructing a search and matching model in which I include unions and employer's associations as strategical agents and not actors who take environment as given. In the literature of trade unionisation, unions bargain vis-a-vis with the employer for better wages, whereas management chooses the level of employment, see Booth (1995) for a review. This framework fits better in Anglo-Saxon and Nordic countries, but is not well suited for other systems like those of central and southern European countries. There are two chief differences, the level of negotiations and the density component.

Regarding the first difference, these models are thought to answer questions at firm-level agreements where the wage bill is actually negotiated, of course, not only payrolls are bargain over but most of union benefits that might be subject to a monetary interpretation such as health insurance, bonuses, holidays, timetables, etc. However, in systems where the coverage of the agreement is extended, the bulk of negotiations are carried out at a sectoral level. Here, wages and fringe benefits are not negotiated directly but instead the set of institutions that are going to rule the labour market at hand. Secondly, union behaviour is specified by means of a utilitarian objective function or a expected utility approach representing union preferences as a sum of utilities of members. Again, looking at figure 1 we see that this model fits well in those countries where density and coverage are very close and the free-rider problem is not an issue. Nevertheless, one might suspect that in cases as extreme as France union preferences include more people than only those affiliated. In a recent paper, Krussel and Rudanko (2016) try to fill this gap assuming the union bargains on behalf of the whole active workforce, employed or unemployed, which make sense since there is no heterogeneity in their model; yet it fails to capture the fact that there is no a priori reason why unions should worry about the whole workforce at a sector-wide level or even the unemployed.

FIGURE 1: Collective Bargaining in Europe (2017)



In order to address this vent I built on the works of Cahuc et al. (2006) (PVR) and Flinn and Mabli (2009) (FM). In the latter, authors presented an unifying framework of search and matching models with firms competing *à la Bertrand*, free-entry and minimum wages; in their model there is match specific productivity taken at random, leaving no room to model firm size. In the former, they model two-sided heterogeneity in order to account for firm fix effects, which in turn allowed to introduce a measure of firm size. In both models they account for individual wage bargaining, which I am not since as estimations of PVR show the bargaining power of the worker is close to zero for the lower ranks. The avenue that I take in this work is to account for two-sided heterogeneity in the work of FM in order to acknowledge that only workers in the largest firms will be able to elect their representatives and in this way participate the political process. On top of that, I carry on introducing unions and employers associations, letting minimum wages to be endogenised.

The empirical literature has analysed collective agreements and how their clauses affect labour market outcomes using quasi-experimental data, but missing the analytical framework with explicit channels connecting causes to effects. Some part of this literature began in the late 90's to test predictions laid out by Calmfors and Driffill (1988), who emphasised the inefficiencies brought about by intermediate levels of negotiations due to market power coupled with lack of internalisation of outcomes. A notable research in this area was done by Joop Hartog (2002) who tests this prediction using data on The Netherlands. Yet, they do not inspect how particular provisions affect labour market outcomes. The seminal paper of Cardoso and Portugal (2005) shed light on this by taking minimum wages into the analysis, they found out that adjustments are absorbed reducing the wage cushion and not laying off workers. I will follow this latter work in order to analyse how minimum wages bargained in collective agreements affect wages and unemployment, for this an exhausting database with complete working histories of sampled workers, namely *Muestra Continua de Vidas Laborales* (MCVL) is used.

The present work tries to bridge the gab between both strands of the literature by providing a behavioural framework that is confronted against data making use of the simulated method of moments. Preliminary results in the theoretical part point out that when minimum wages are considered effects on workers vary considerably depending the level of skill. High-skill workers earn higher wages in early stages of their careers whereas not having significant effects at later points. Low-skill workers start their jobs earning the minimum wage, which is higher than what otherwise would have been their entry wage, but at cost of suffering lower unemployment spells. Firms usually lose when the floor rate is imposed, they have to pay higher wages without any counterpart.

This is not so when general equilibrium effects are contemplated, higher wage floors mean relative low ability workers with respect to firm productivity not being able to match, increasing the stock of unemployed and vacancies in the economy, reducing the stock of employed and diminishing the rate at which vacancies and searchers come across. On the offer side of the market, rising vacancies coupled with a declining rate of contacts result in hump-shaped curve relating the arrival of wage offer with minimum wages. For the first stretch of the curve, relative high ability workers see that they find jobs more easily and have more dynamic tenures, whilst Low-skill workers will have more difficult to find an employer. On the demand side, The firm with lowest productivity opts out from previously profitable matches and it could not poach other workers from firms with higher productivity leaving them undoubtedly worse off. As we move on to higher productivities, firms miss less opportunities to match from those that are still possible. There will be two opposing effects; on the one hand, firms have gradually more unemployed at their disposal as we move to higher levels of productivity compared to the case without minimum wages. On the other hand, they have less employed people to poach from. The net effect depends on the search effort exerted by the employed relative to the unemployed and is subject to a quantitative estimation. Nonetheless, using reasonable parameter values simulations show that high productive firms become bigger as they have relatively more searcher to search from.

2 Base model

2.1 Setting

The Economy is populated with a continuum of workers indexed by x, representing the ability which is exogenously given and is distributed over the interval $x \in [\underline{x}, \overline{x}]$ according to a beta distribution $l(x)$ with parameters (a_x, b_x) and which quantity is normalised to L. Workers are either unemployed or employed, in both cases they search for jobs to find better alternatives, the search effort of employed is s and the unemployed effort is normalised to one. Since workers search for other jobs while employed, they have the opportunity to bring other companies into Bertrand competition with their incumbent employers in order to gain a pay rise, the process will be explained in detailed in the following sections. Let $u(x)$ be number of workers of type x among the unemployed and $U = \int u(x')dx'$ the total unemployment.

Firms are ranked according to technology y that is uniformingly distributed in $y \in \left[\underline{y}, \overline{y}\right]$. Firms exert effort $n(y)$ in hiring and retaining workers, the total effort exerted by all firms is $N = \int n(y')dy'$, which is endogenously determined due to the free-entry condition (FEC) explained below. In this setting firms may hold several vacancies and employ many workers. The number of vacancies hold by the firm with productivity y is $v(y)$ and the number of workers of type x employed in this firm is denoted by $h(x, y)$, the size of the firm is then $h_y(y) = \int h(x', y)dx'$. The total number of vacancies in the economy is $V = \int v(y')d'$ and seemingly the total number of employed people is $H = \int h_y(y')dy'$

All agents in the economy discount time at the same factor ρ. Upon matching the firm and the worker start producing a flow output $f(x, y) = xy$, the chief point is that workers are perfectly substitutable and there are no complementarities among then within the firm. Matches are exogenously terminated by a Poisson process with parameter δ.

2.2 The Matching Process

Unemployed and employed workers compete for vacancies with different search costs, the search cost of unemployed is normalised to one and for employed is denoted by s. Let k be a parameter that characterises all key rates of meeting and which definition is:

$$k = \frac{M(U + s(1-U), V)}{[U + s(1-U)]\, V}$$

From here we can define the rate at which unemployed workers meet a vacancy as $kV \cdot \frac{v(y)}{V} = kv(y)$, whereas employed workers meet vacancies at a rate $skv(y)$. On the other side of the market, vacancies meet unemployed workers at a Poisson rate $ku(x)$ and meet employed ones at a rate $skh(x, y)$

At this point it is worth noting that workers do not hold bargaining power vis-à-vis with the employer. In other words, firms take all of the surplus for themselves when a meeting is materialised in a match. The only way a worker can demand a pay rise to its employer is by dragging other firms into Bertrand competition. This setting differs from those of Dey and Flinn (2005) and Cahuc et al. (2006) where workers are assumed to hold some bargaining power, at least at a theoretical level. There are three reasons why I chose not to include it. First of all, the bargaining power of workers is close to zero as they show in their empirical extracts. Secondly, the model captures the fact that workers usually do not have bargaining power and let the agent (union) to negotiate on their behalf. Lastly, it reduces the mathematical and computational burden.

2.3 Value functions

Unemployed

The value of unemployed worker of type x is denoted by $W_0(x)$ and receives a flow bx for what produces while unemployed, notice that b is common to all workers, i.e. all have the same technology at home but the flow increases with the ability, this captures the fact that unemployed workers with high-skill have better wages and more generous unemployment benefits than those with less ability. Other rationale could be that those unemployed do some informal jobs that are going to be paid according to the ability. In any case, the unemployment insurance payment is a function of the wage earned and the time employed, at the same time the wage is a function of time and ability. Hence the mean time is captured by b which is the same for all workers and the ability by x. The wage offered to the unemployed $\phi_0(x,y)$ will be such that the firm takes all the surplus of the match for itself, so that the worker will be indifferent between taking or rejecting the offer. Notice that the offer depends on both arguments x and y. The wage is then implicitly defined as

$$W_0(x) = W_1\left(\phi_0(x,y), x, y\right),$$

with $W_1(w,x,y)$ being the value of an x employed worker earning a wage w at firm of type y. From here it can be drawn the continuation value of unemployment as:

$$(\rho + k)W_0 = bx + k\int W_1\left(\phi_0(x,y), x, y\right) v(y)dy$$

Which by the previous definition solves as:

$$\rho W_0 = bx$$

Employed

As mentioned before the value of an employed worker is $W_1(w,x,y)$, nonetheless I will introduce $W_{10}(w,x,y) = W_1(w,x,y) - W_0(x)$, which is the net surplus accounted to an x-worker earning the wage w in a y-firm, mainly to save notational burden. As workers search on the job they can bring firms into competition in order to be granted a pay rise or switch companies otherwise. Upon meeting a firm, the worker will face three situations: the alternative firm does not have enough productivity to pay the current wage and the current relation does not change; Another will result in a wage increase for the worker and a third one that will materialise in a new match (a Job-to-Job transition).

The first case might be such that the worker encounters a firm y' that does not even have enough productivity to pay for his current wage and make profits, i.e. $xy' - w \leq 0$. The set of these firms goes from the firms with least productivity $\underline{y}'$ to the threshold $q(w,x,y)$ that leaves the worker indifferent between extracting the whole surplus of the poacher and staying in her current firm or in other words

$$W_{10}\left(w,x,y\right) = S(x,q).$$

In this case the worker will not swap firms nor will see her wage risen, hence it will not have any effect on her.

In the following scenario the outside firm ranks in $y' \in (q,y]$, i.e. the productivity of the incumbent company is higher that the poaching one, still the latter has enough productivity to oblige the former raise the wage of her worker. Let $\phi(x,y',y)$ be the offer done by $y' < y$ to an x-worker at y. The job offer will leave her indifferent between staying or changing firms and is implicitly defined as

$$W_{10}(\phi(x,y',y),x,y) = S(x,y').$$

Notice that the poaching firm will never raise its offer above xy' since loses would materialise: $xy' - \phi(x,y',y) < 0$.

Seemingly, when the productivity of the poacher is $y' \geq y$ the offer granted to the worker $\phi(x,y,y')$ will be such that leaves her indifferent between staying with the incumbent with a wage xy or changing job to a more productive firm but earning less wage. Again, it is defined as

$$W_{10}\left(\phi(x,y,y'),x,y'\right) = S(x,y).$$

With these expressions at hand the surplus continuation value of an employed worker would be

$$\begin{aligned}\left[\rho + \delta + sk\overline{V}\left(q(w,x,y)\right)\right] W_{10}(w,x,y) = w - \rho W_0(x) \\ + sk\int_{q(w,x,y)}^{y} W_{10}(xy',x,y')\, v(y')dy' \\ + sk\int_{y}^{\overline{y}} W_{10}(xy,x,y)\, v(y')dy' \qquad (1)\end{aligned}$$

For derivations of key equations see Postel-Vinay and Robin (2002).

Vacant Jobs

Following the above discussion vacancies are filled according the search efforts of both sides of the market and the amount of each of them in the economy. Firms exert effort opening vacancies and retaining talent until the marginal cost equals the expected revenue of filling a vacancy as it will be clear from the free-entry condition below. Define the continuation value of holding a vacancy as

$$\rho\Pi_0(y) = -c'(y) + kJ(y),$$

where

$$J(y) = \int_{\underline{x}}^{\overline{x}} S(x',y)u(x')dx' + s\int_{\underline{x}}^{\overline{x}}\int_{\underline{y}}^{y}\left(S(x',y) - S(x',y')\right)h(x',y')dy'dx'.$$

The first term in the right-hand side is the marginal cost of exerting effort $c'\left(n(y)\right) = c_0 n(y)^{c_1}$, to which we will impose convexity to ensure an equilibrium exists and the second term is the expected value of filling a vacancy.

Filled Jobs

Firms discount future at the same rate as workers and have a stream flow of profits $xy - w$, when a job is exogenously destroyed, production ceases and a vacancy is immediately opened. As highlighted before vacancies are open and offers accrue to both types of workers, employed and unemployed at a rates $kv(y)$ and $skv(y)$ respectively. Firms start appropriating the whole surplus of the match. As offers from less productive companies accrue to the worker, the current company has to grant a pay raise that matches the surplus of the poacher. When a worker finds a higher viable alternative the firm has no other option than let her go. With this discussion, the net continuation value of a filled job is

$$\begin{aligned}\left(\rho + \delta + skv\left(q(w,x,y)\right)\right)\Pi_{10}\left(w,x,y\right) = xy - w \\ + sk\int_{q}^{y} S(x,y) - S(x,y')v(y')dy'.\end{aligned}$$

Surplus

From previous sections it is clear that all the continuation values are defined in terms of the match surplus, whilst not being defined until now. Define the surplus in the usual way, i.e. $S(x,y) = \Pi_1(w,x,y) - \Pi_0(y) + W_1(w,x,y) - W_0(x)$. As a result summing over all these expression in the right-hand side it can easily be proven that the equation for the surplus is

$$\left(\rho + \delta\right)S(x,y) = yx - bx$$

Which is quite simple expression since we have imposed $\Pi_0(y) = 0,\ \forall y$.

2.4 Equilibrium

The exogenous elements of the model are the distribution of workers $l(x)$, the discount factor ρ, the job destruction δ, the search intensity of employed workers s, the value of leisure b and the production technology $f(x,y) = xy$.

Balance Equations

In equilibrium the distribution of the unemployment rate of workers of type x, $u(x)$, and the number of vacancies of type y, $v(y)$, is determined according to the balance conditions

$$\int h(x,y)dy + u(x) = l(x)$$
$$\int h(x,y)dx + v(y) = n(y).$$

Flow Equations

The joint distribution wages and matches $G(w|x,y) \cdot h(x,y)$ follows a steady state flow equation where inflows balance the outflows. Matches of x-workers in y-firms earning w or less might arise for two reasons, either workers with ability x are hired directly from unemployment by companies with productivity y or they are poached from less productive firms than q. On the other side, matches of (x,y) pairs might be destroyed by exogenous separations that accrue at a rate δ or because outside firms, with higher productivity than q, poach the worker or make a better offer. Netting this two forces the flow equation stays as

$$\left(\delta + sk\int_q^{\overline{y}} v(y')dy'\right) G(w|x,y)\cdot h(x,y) = \left(u(x) + s\int_{\underline{y}}^{q} h(x,y')dy'\right) kv(y).$$

The same can be worked out for the number of unemployed workers and vacancies of any type, i.e.

$$kVu(x) = \delta h(x)$$
$$\left(\delta + sk\int_y^{\overline{y}} v(y')dy'\right) h_y(y) = \left(U + s\int_{\underline{y}}^{y} h_y(y')dy'\right) kv(y).$$

Where $h_x(x) = \int h(x,y)dy$ and $h_y(y) = \int h(x,y)dx$. Once the flow equations and the balance conditions are defined, $h(x,y)$, $u(x)$ and $v(y)$ can be derived as:

$$
\begin{aligned}
v(y) &= \frac{\delta + sk\overline{V}(y)}{\left(\delta + sk\overline{V}(y)\right) + \left(kU + skH_y(y)\right)} \cdot n(y) \\
u(x) &= \frac{\delta}{\delta + kV} l(x) \\
h(x,y) &= \frac{\delta + skV}{\left(\delta + sk\int_y^{\overline{y}} v(y')dy'\right)^2} kv(y)u(x) = \frac{1}{H} h_x(x)h_y(y) \\
G(w|y) &= \frac{\left(kU + sk\int_{\underline{y}}^{q} h_y(y')dy'\right)}{\left(\delta + sk\int_q^{\overline{y}} v(y')dy'\right)} \cdot \frac{v(y)}{h_y(y)}
\end{aligned}
$$

and $h_y(y)$ depends solely on $n(y)$, which at this point is exogenously determined. Detailed proofs of steady state equations are found in appendix C. It is worth noting the main advantage of having introduced firm heterogeneity, since I will be able to set firm size in a one-to-one correspondence with its productivity level, which will be convenient in the political economy part of the model. Finally, the recruiting effort exerted firms of type y, $n(y)$, is set by the free entry condition described below.

Free Entry Condition

Firms of type y exert increasing effort in recruiting candidates until this cost of maintaining a vacancy equals the expected value of filling it for every y, $\Pi_0(y) = 0$, at equilibrium:

$$
c'\left(n(y)\right) = kJ(y).
$$

Following the parameterization of Lise and Robin (2017), we are able to draw an expression for the effort exerted by firms of type y as

$$
c_0 n(y)^{c_1} = kJ(y).
$$

Equilibrium effort by firm-type is then written

$$
n(y) = \left(k\frac{J(y)}{c_0}\right)^{\frac{1}{c_1}}.
$$

Summing over all companies in the economy, the aggregate equilibrium effort in the economy is worked out:

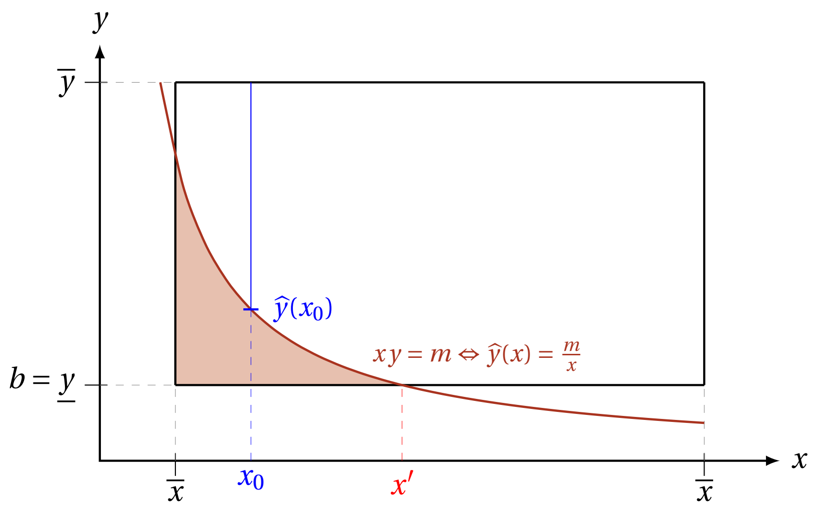
$$
N = \int \left(k\frac{J(y')}{c_0}\right)^{\frac{1}{c_1}} dy'.
$$

3 Introducing Minimum Wages

At this point minimum wages are introduced into the analysis. I add on to the work of Cahuc et al. (2006) and Flinn and Mullins (2019) (FM, hereon) without considering worker bargaining power, which makes sense in the present analysis as low categories of workers are considered. FM is the most important work of minimum wages with OTJ search in which firms compete a la Bertrand. However, Firm heterogeneity, as opposed to match quality, is introduced in this work in order to have a measure of firm size, as it will be made clear in the analysis below. Furthermore, worker heterogeneity is an advantage over PVR (online appendix), since it allows to consider differentiated effects over workers with varying abilities and firms with diverse productivities.

Introducing minimum wages will have several implications. First of all, minimum wages will affect the whole distribution of wages compressing it for a given pair (x, y) and not only those at the bottom. Intuitively, when a worker is allowed to search on the job, she has to compensate the employer for the expected forgone profits when she changes companies, which might never occur. If the worker does not have bargaining power she is willing to accept less than her wage today for wage rises in the future. Upon setting a wage floor, high productive firms in the range $\left[t(x, y), \overline{y}\right]$ drag the wage down to the legal minimum, increasing the value of being employed by keeping the rate of wage offers, fixing equilibrium objects, but earning a higher wage. Low productive firms, in the range $\left[\underline{y}*, t(x, y)\right]$, will have to compensate the worker for this fact, raising effectively the wage earned.

FIGURE 2: Minimum wages in the (x, y) space



Another implication is that minimum wages changes the meeting rates at which workers and firms encounter. In figure 2 the red shaded area represents the meetings that could have resulted in a match in the absence of a minimum wage but because of it now they will be at the disposal of the rest, increasing the chances of meeting for those not directly affected by the minimum wage and potentially decreasing the chances for those affected. In a nutshell, upon introducing minimum wages the number of vacancies and unemployed people increase whereas the tightness, k, will decrease monotonically as the minimum wage increases. Coupling both effects, it will result in a hump-shaped curve of job offers, at the beginning job offers will accrue at a higher rate for relatively high skill workers and after some threshold these offers will start to decrease due to fall in expected revenues of filling a vacancy and firms exerting less effort in contracting.

As a side effect the value of the match and the outside options of agents will change accordingly.

3.1 Value functions

In this section the lifetime utility values for the different type of workers when a minimum wage is put in place are derived. In this respect workers will be grouped in two categories, x^L-type which will be in the rank of abilities $[\underline{x}, x']$ and x^H-type employees in $(x', \overline{x}]$. So depending on their abilities employees will have a different continuation values.

Unemployed

As it is clear from figure 2 workers will only be hired when contacting a firm with productivity $y' \geq \widehat{y}(x) = min\{\frac{m}{x}, b\}$. If contacted with a firm with less productivity than $\widehat{y}(x)$ the worker will remain in unemployment enjoining a flow value $W_0(x;m)$. The flow value of the unemployed worker is increased because firms with high enough productivity cannot trade off less wages for future wage rises and the worker will enter the market when she is indifferent between being unemployed and working in $\underline{\hat{y}}(x)$ at the wage $\phi_0\left(x, \widehat{y}(x)\right)$. Also, notice that the firm with the lowest viable productivity cannot make surplus out of the match, otherwise a firm with marginally less productivity could enter the market. From these considerations the next lemma says,

LEMMA 1. *The value as unemployed is equal to the value of first employment at* $\underline{\hat{y}}(x)$. *Seemingly, the value of first employment at* $\underline{\hat{y}}(x)$ *is equal to value product of a match* $P(x, \underline{\hat{y}}(x))$. *Therefore,*

$$P\left(x, \underline{\hat{y}}(x); m\right) = W_1\left(\phi_0\left(x, \underline{\hat{y}}(x)\right), x, \underline{\hat{y}}(x); m\right) = W_0(x;m)$$

Proof. See appendix D. ■

With these considerations at hand we are ready to calculate what the minimum viable productivity of a firm to hire a worker would be, and hence the lower support of the firm distribution $\underline{\hat{y}}(x)$ for a given ability x. For some ability levels in the range $[\underline{x}, x']$, matches that were profitable without minimum wages they are not anymore and the entry wage will be the minimum wage. For those in $x \in (x', \overline{x}]$, the minimum wage only changes the values of being unemployed and employed but not their mobility decisions, therefore the minimum viable productivity of a firm to hire a worked will remain unchanged. From these considerations the following lemma states:

LEMMA 2. *Fixing equilibrium objects, the minimum viable productivity of a firm $\underline{\hat{y}}(x)$ to hire a worker under the presence of a wage floor is:*

$$\underline{\hat{y}}(x) = \begin{cases} \frac{m}{x} & if\, x < x' \\ y_{inf} = b & if\, x \geq x' \end{cases}$$

Proof. See appendix D. ■

Employed

As is common in models of OTJ-search with competition a la Bertrand, high productive firms can drag down the wage of the worker in exchange for a more dynamic tenure track, i.e. future offers that will end in wage increases. Upon introducing minimum wages, not every pair (y, y') is able to play a "wage war", more specifically when the poaching firm has very high productivity relative to the incumbent, the former will not be able to lower the wage in its full extend to the worker, because a biding minimum wage is in place. Nonetheless, it will not affect mobility decisions as they are still efficient. Workers will see the value of employment risen whenever $t(x, y) < \overline{y}$, even though they might not earn the minimum wage. Intuitively, the worker will have the opportunity to work at high productive firms earning no less than the minimum, effectively rising the value of their jobs. The continuation surplus value of an employed worker net of laid-offs would be

$$\begin{aligned}
&\left(\rho + \delta\right) W_{10}(w, x, y; m) = \\
&\qquad w - \rho W_0(x; m) + sk \int_q^y \left[S(x, y'; m) - W_{10}(w, x, y; m)\right]^+ v(y')dy' + \\
&\qquad sk \int_y^{\overline{y}} \left[max\left[S(x, y; m), W_{10}(m, x, y'; m)\right] - W_{10}(w, x, y; m)\right]^+ v(y')dy'.
\end{aligned}$$

The first object in the right-hand side is the flow income. The second one is the continuation value of being unemployed. The third term is more interesting, it represents the expected increase in surplus thanks to the fact that the worker can bring two firms into competition staying with the incumbent. The forth term, is the expected increase in surplus derived from switching to more productive firms. $max\left[S(x, y), W_{10}(m, x, y')\right]$ expresses the possibility that the worker encounters a firm with such productivity that will be able to offer no less than the minimum wage, effectively extracting more surplus for her out of the match. Theoretically, we could find a firm with enough productivity to reduce the wage rate up to the minimum. In practical terms, the firm distribution will have their productivity cap at the firm with highest productivity. In turn, it might be the case that $t(x, y) \geq \overline{y}$ and the minimum wage will have no direct impact over the worker, although she will experience general equilibrium effects inside the labour market.

Employed earning the minimum wage

Because we have introduced the minimum wage we have to consider what is the value for an employed worker earning the minimum wage either because she is been hired directly from unemployment or because she has received and offer for a high productive firm. At the minimum wages the value function is

$$
\begin{aligned}
&(\rho+\delta)W_{10}(m,x,y;m) = \\
&\qquad m-\rho W_0(x;m)+sk\int_{\underline{\hat{y}}(x)}^{y}\left[S(x,y';m)-W_{10}(m,x,y;m)\right]^{+}v(y')dy'+ \\
&\qquad sk\int_{y}^{\overline{y}}\left[max\left[S(x,y;m),W_{10}(m,x,y';m)\right]-W_{10}(m,x,y;m)\right]^{+}v(y')dy'.
\end{aligned}
$$

The only thing that is likely to change is the lower limit of the integral in the third term. I seems that the new limit restricts the matching space of the worker to meet another firm. However, this is not the case, once employed and earning the minimum wage, the worker experience the same restriction as if the minimum were not in place.

Surplus

The new surplus takes into account the increased in value as an unemployed worker, effectively reducing the surplus, together with the increase in the value for an employed worker, leaving the expression

$$
(r+\delta)\,S(x,y;m)=yx-\rho W_0(x;m)+sk\int\left[W_{10}(m,x,y';m)-S(x,y;m)\right]^{+}v(y')dy'
$$

The change of surplus with respect to the case without minimum wages will depend on the interplay of the mentioned objects. Still, the whole surplus is appropriated by the employer and the legal minimum will affect the employee positively in two ways. First, it increases the value as unemployed conditioned on participating in the labour market, although participation is not a case of study in the present work, now the worker will be indifferent between being unemployed or working at a firm with the least viable productivity, leaving her better off. The other mechanism at her disposal will again be the competition a la Bertrand between firms. Nonetheless, in this case the worker will receive a wage offer potentially higher than the one that she would have been given without the presence of a minimum wage, even if she were not earning the statuary minimum. The intuition is that the worker has the opportunity to work at high productive firms earning no less that the minimum, if a poacher would not compensate the employee for this fact, the worker would find it profitable to wait for the next offer come. This is not optimal for the poacher who will lose the value of the filled job. Consequently, the poacher will offer the employee a wage that leaves her indifferent between working with them at relatively higher wage rate or waiting another period time.

COROLLARY 3. *The surplus generated at the firm with minimum viable productivity is* 0

Proof. Trivial from LEMMA 1. ∎

3.2 Equilibrium Under minimum wages

In this section I will concentrate in the labour market equilibrium effects of establishing a minimum wage. On the one hand, the set of possible matches will be reduced, as $\{x, y\}$ pairs that fall short of m will not form a match anymore. On the other hand, there will be more vacancies at the disposal of the rest of workers and more unemployed at the disposal of high productive firms, having ambiguous effects over different firm productivities. Low productive firms will in general be worse off, however as we move up through the productivity distribution, they will find harder to find workers of low ability but will filled vacancies from relatively high ability workers affected by the minimum wage more quickly. These points will be made clear in the following sections.

Balance Equations

These pair of conditions will have to be rearranged to account for the fact that some meetings will not come true anymore, instead these pairs of workers and vacancies will be at the disposal of the rest. More precisely they will be

$$l(x) = \underbrace{u(x) + \int_{\underline{y}}^{\hat{y}(x)} h(x, y')dy'}_{\tilde{u}(x)} + \underbrace{\int_{\hat{y}(x)}^{\overline{y}} h(x, y')dy'}_{\tilde{h}_x(x)}$$

$$n(y) = \underbrace{v(y) + \int_{\underline{x}}^{\hat{x}(y)} h(x', y)dx'}_{\tilde{v}(y)} + \underbrace{\int_{\hat{x}(y)}^{\overline{x}} h(x', y)dx'}_{\tilde{h}_y(y)}.$$

Where $\int_{\underline{y}}^{\hat{y}(x)} h(x, y')dy'$ is the new stock of unemployed and $\int_{\hat{y}(x)}^{\overline{y}} h(x, y')dy'$ are the remaining employed with ability x that are able to keep their jobs. Exactly the same argument applies for the second and third terms in the firm balance equation. Still both equations just depend on the ability of the worker in the first and the productivity in the second.

Flow equations

Remember that no functional forms where made on $h(x, y)$, $u(x)$ and $v(y)$; and the new $\tilde{h}(x, y)$, $\tilde{u}(x)$ and $\tilde{v}(y)$ still depend only on their respective variables. Then, the number of vacancies and unemployed people will be substituted by their minimum wage counterparts, i.e. $\tilde{v}(y)$ and $\tilde{u}(y)$, which can be readily worked out from the stocks. whereas the flow equation $\tilde{G}_t(w|x, y) \cdot \tilde{h}(x, y)$ is rewritten following exactly the same derivations as the case without minimum wages, showing the same functional form

$$\tilde{h}(x,y) = \begin{cases} \frac{\delta + sk\tilde{V}}{\left(\delta + sk\overline{\tilde{V}}(y)\right)^2} \cdot k\tilde{u}(x)\tilde{v}(y) & \textit{if} \quad xy \geq m \\ 0 & \textit{if} \quad xy < m \end{cases}$$

$$\tilde{G}(w|x,y) = \begin{cases} \frac{\left(k\tilde{v}(y)\tilde{u}(x) + sk\int_{\underline{y}}^{q}\tilde{h}(x,y')dy'\tilde{v}(y)\right)}{\left(\delta + sk\int_{q}^{\overline{y}}\tilde{v}(y')dy'\right)} \cdot \frac{1}{\tilde{h}(x,y)} & \textit{if} \quad xy \geq m \\ 0 & \textit{if} \quad xy < m \end{cases}$$

Again these objects are pinned down by the firm recruiting effort which is pinned down by the FEC.

Free-Entry Condition

The free-entry condition in which $\Pi_0 = 0$ still holds, as do all the derivations to arrived to the expression for $n(y)$, the only object that changes in this case is the expected value of filling a vacancy which is

$$\begin{aligned} kJ(y;m) = k &\int_{\underline{\hat{x}}(y)}^{\overline{x}} S(x',y;m)\tilde{u}(x')dx' \\ &+ sk\int_{\underline{y}}^{y}\int_{\underline{\hat{x}}(y)}^{\overline{x}} \left(S(x',y;m) - S(x',y';m)\right)\tilde{h}(x',y')dy'dx'. \end{aligned}$$

As it seems clear from the above equation, and figure 2, the firm with lowest productivity will be undoubtedly worse off because it has less workers to fish from. As we consider higher productivities, firms will still lose from those they cannot make profits any more, however they will have at their disposal the unemployed not poached by less productive firms, leaving them gradually better off.

4 Political Economy

Once the basic framework of the labour market with minimum wages has already been deployed, it is time to consider the bargaining protocol between working unions and employers associations. One of the chief contributions of this work is to endogenise the decision to set minimum wages by reckoning the role of unions and employers. As it is common in collective bargaining systems where the bulk of negotiations are carried out at a sectoral level, what is agreed between unions and employers is usually extended to other participants in the labour market. I focus on the extreme case where collective agreements are applied to the whole labour market, regardless of workers and firms being affiliated to their representative associations or having participated in the political process.

There is much to say about union and employer preferences, what triggers the decision to vote and who can actually vote (there is usually no universal suffrage), representation at the negotiating table, who is affected and accountability. However, I will abstract from most of these concerns to keep the model simple and tractable. Nonetheless, two important factors within the political economy sphere are considered. First, who is allowed or able to vote? Either because legal clauses or collective action constraints, participation in the negotiating table is subject to firms reaching a certain size, which in turn means that only the voice of workers in these firms will be heard. In terms of my model this requires the following condition

$$h(y) \geq \overline{h}.$$

Another concern is about union and employers preferences. Traditionally, unions preferences have been model to take into account the fact that they may care in one way or another about wages, unemployment, and income distribution. On the other side of the market, firms have been assumed to maximise profits. At this point is when the complexity of the model starts to pay off. In the present work I deviate from the assumption that unions represent their affiliates and instead they will consider the utility of those who actually vote. All the concerns about wages, unemployment and income distribution are directly or indirectly considered through the values that employees assigned to them. The functional form consider for the union is that of an utilitarian objective function like

$$T = \int_{\underline{y}}^{\overline{y}} \int_{\underline{x}}^{\overline{x}} \int_{w_{min}}^{\overline{w}} \left(W_1\left(w, x, y\right) - W_0\left(x\right)\right) G(w|x, y) h(x, y) dw dx dy.$$

And same will be applicable in the firm side

$$E = \int_{\underline{y}}^{\overline{y}} \int_{\underline{x}}^{\overline{x}} \int_{w_{min}}^{\overline{w}} \left(\Pi_1\left(w, x, y\right) - \Pi_0\left(x\right)\right) G(w|x, y) h(x, y) dw dx dy.$$

Once we know the preferences of unions and employers; and who can vote, we are ready to introduce them into the analysis. In this case, unions and employers will not bargain for wages, employment levels and income distribution directly but they will set the level of minimum wages that maximises the Nash-bargaining solution to their respective utilities, or in other words

$$\begin{aligned} m^* = argmax \quad & E^{1-\alpha} \cdot T^{\alpha} \\ s.t. \quad & h(y) \geq \overline{h}. \end{aligned}$$

On the offered side of the market, unions face the typical trade off, higher wages despite higher unemployment for their represented, assuming there is no general equilibrium effects; in the firm side, principals will be worse off if just because they will have to pay higher wages, in addition low productive firms will not be able to hire low productive workers. On the other hand, both coalitions will face an additional channel due to the congestion externalities that they exert on each other. Less employment means vacancies are easier to fill, especially for those firms that do not have to lay off workers; on the other side of the market, high skill workers will encounter wage offers more frequently. The net effect is ambiguous and structural estimation is carried out to discern what effect is stronger.

5 The Institutional Setting

Most of the theoretical literature about trade unions revolves around their affiliates. However, labour market in Spain is framed within the class of collective negotiating systems where bargaining is a public good, typical of Southern Europe. These systems are characterised by a low density but high coverage of the labour force, regardless the employee is affiliated or not.

Nonetheless the regulatory framework in Spain has some peculiarities worth mentioning. First of all, the important agent in the industrial relations is not the trade union but the work council or committee (comité) instead, which is the collegiate organism within companies of more than 50 workers in charge of representing the staff. This council is composed of 5 to 75 members, depending on the size of the firm, and is directly elected among the workforce. Members of the committee, also delegates, can be either union affiliates or independent workers. This 'elections feature' is what justifies the 'public good' facet of the settlement. Then, the number of elected committee members in each firm are recorded by the Ministry of Labour to assign representation at higher levels of negotiations.

Another relevant trait is the way sector-wide negotiations are carried out. The key institution here is the 'Bargaining Commission' which is the body in charge of reaching an understanding. This body is composed of employers associations and trade unions, not independent workers in this case. As noted before, the number of unionised delegates at firm-level elections are recorded and taking into account to assign the representation at higher levels, therefore each union is represented in accordance to their popularity. For example, if there is Union 1 and Union 2 in Firm 1 obtaining 3 and 1 delegates respectively and there is Union 1 and Union 2 in Firm 2 obtaining 1 delegate each, then the bargaining commission will be composed of two thirds of Union 1 and one third of Union 2.

The result of these negotiations is the 'Collective Agreement', whether carried out at firm-level by the committee or at sector-wide by the bargaining commission. This contract rules over any possible matter related to labour, being the most prominent: wage increases, minimum wages, hours and employment. The agreement is published in the Official Bulletin of the State (BOE), has rank of law and Judges might use it to solve potential disputes between workers and employers. Then, this collective contract serves as the minimum standard individual contracts must have. What is more, firm-level agreements override to sector-wide ones, within the latter a narrower scope cannot confront those with a wider one. For example, a province-level agreement for metalurgy cannot set lower standards than a national-level one, I will discuss in more detail in the following section.

6 Data

In order to carry out my future analysis I will dispose of two databases. On the one hand I will make use of new-brand database released under demand by the Ministry of Labour: *Registro de Convenios* (REGCON). This database contains the outcomes of CA signed from 2010 to 2018. Data is collated for each contract by means of the 'Statistical Sheet' which negotiating parts and the administration fill.

It owns three types of data, firstly it holds administrative data necessary for the Ministry of Labour to follow the timing of negotiations and the scope of the agreement. Another sort of data is related to the contends and clauses of the collective contract itself, good examples are increment in wages, wage complements, working hours in a year, working holidays, etc. The last sort of data connects to the number of people covered by the collective agreement.

Data related to administrative records and the clauses of the understanding is accurate and reliable as negotiating parties are in charge of filling it out. Nevertheless, data related to coverage is more questionable. In this respect, we must differentiate between firm-level agreements and sector-wide ones. I regard the former ones as trustworthy, because people filling the Statistical Sheet usually have the census of the company. It is not the same though for sector-wide agreements, parties write down the number of companies affected within the scope of the agreement insofar they dispose of that information, which is rarely the case, needles to say about the workers. Thus, estimates of coverage released by the Ministry of Labour should be taken with a pinch of salt, actual these estimates will differ from my preliminary statistics.

Yet, this database encounters two key challenges. First of all, it does not contain the hierarchy of collective agreements, consequently it might be the case that a worker is under several collective agreements, not identifying which one to apply, matter that I should come back to latter. The second issue to tackle is the lack of information about individual contracts to check to what extend CA are being rendered. To tackle this point I will use the well-known *Muestra Continua de Vidas Laborales* (MCVL): a database with working histories of employees. The MCVL is a 4% sample of population having a relation with the *Tesorería General de la Seguridad Social* (TGSS) in the year of reference (2004-2017) and provides a vast amount of data about individual contracts like the duration, base wage, worker and person characteristics, and firm attributes among others. Its main flaw is the lack of information about the CA being applied to each contract.

Now, the fact that it is not possible to identify uniquely what CA is being applied to each worker, i.e. there potentially be many CA covering the same employee, poses a challenge. Therefore, I will tackle this issue by assigning just one CA from REGCON to an employee in the MCVL. The common variables that I will use to allocate a collective contract to an employee are Province, Economic Activity (CNAE) and Date. Province is the regional territory where the worker is located and the collective contract applies. CNAE is the economic activity that the firm and the worker are inscribed in according to the Statistical Classification of Economic Activities. And Date is the month and year when the collective contract is valid and the worker is recorded as being occupied. Whenever these three variables coincide the CA is allocated, as mentioned it could be that several collective agreements meet the same criteria, hence the question is what arrangement should be imposed upon.

So, if the worker is under two or more CA I proceed in two stages: in the first one I make a hierarchy among CA according to a law-base rule which is applied in a lexicographic manner. The first rule is about the regional level, lower levels in the regional scope prevail over higher levels. If there are still more than two CA concurring for one worker, I implement the next rule: the collective contract that applies to fewest economic activities is prioritised over the ones that rule over more. The third rule is the date of the signature, according to the law, sooner signed CA predominate over later ones. The last few rules are wage increases, the settlement of a minimum wage and hours worked per year, I will apply the ones more favourable to the worker.

In the second stage, it is just left to cross both databases and integrate information of the individual and collective contracts following the method specified above. Table 1 shows the result of giving priority to contracts, as shown the amount of contracts actually in place reduces by more than half from 2,653 contracts to 1,265, this occurs because contracts finer in scope override those that are at higher levels. Interestingly the orders of magnitude are kept at a regional level. As seen, Province and Nation-wide agreements gain representation by increasing 2.4 and 1.6 percentage points respectively. Percentages that are lost by Autonomous Community collective agreements decreasing by 4 pp.

	Total CA		**Prioritised CA**	
	N	%	N	%
Province	1.968	74.2	969	76.6
Autom. Com.	451	17.0	165	13.0
Nation-wide	234	8.8	131	10.4
	2.653		1.265	

TABLE 1: Collective Agreements before and after prioritisation

Table 2 shows the outcome of merging both databases, in order to make comparisons easier and give some orders of magnitude I have included non-finished contracts of employees. As seen in the table 67.8% of workers are covered by a collective contract, whereas 32.2% are not. This figure is in line with official statistics offered by the OCDE, who sets the collective bargaining coverage at 73%, furthermore these estimates are similar to those announced by the Ministry of Labour which oscillate from 69%-70%. Nonetheless, these figures demand some careful examination.

As noted earlier, people filling the Statistical Sheet, where data comes from, do not know the exact number of firms or workers that are covered, as a result statistics shown in table 2 are quite different from the official ones. If we normalised the percentage of people covered by CA to 100% it would be straight forward to see that according to table 2 Province, Autonomous Community and Nation-wide agreements represent 17%, 11% and 73% (not shown) of covered employees, whereas these percentages for the Ministry of labour dramatically change to 40%, 11% and 37% respectively. In my future research I will keep using the resulting figures from merging both databases since it is what several researchers have suggested as being the correct way to measure collective bargaining coverage (Infante 2017).

	Total	No CA	With CA
Region			
Province			11.4
Auto. Com.			7.3
National			49.0
Total	100	32.2	67.8
Gender			
Female	45.9	29.4	70.6
male	54.1	34.5	65.5
Nationality			
Spanish	89.8	33.4	66.6
Foreigner	10.2	17	83
Sector			
Manufacturing	17.6	51	49
Construction	5.6	25.5	74.5
Services	76.8	28.4	71.6
Contract			
Permanent	82.1	90.5	78.1
Fix-term	17.9	9.5	21.9
Number of contracts	38,056	12,248	25,708

TABLE 2: Individual Contracts: covered and not covered by a CA and by worker's characteristics

Appendix A

Value Functions

A.1 Derivations of The Value Functions Basic model

Unemployed

In this appendix expressions for the value functions of unemployed, employed, match and surplus are derived for the basic model. I will closely follow the work of PV-R in deriving these analytical forms. First I will start setting the Value function of an unemployed worker in discrete time.

$$\begin{aligned} W_0(x) &= bx\Delta + e^{-r\Delta}\left\{e^{-kV\Delta}W_0(x) + \left(1-e^{-kV\Delta}\right)E\left[W_1\left(\phi_0(x,y),x,y\right)\right]\right\} \\ &= bx\Delta + e^{-r\Delta}\left\{e^{-kV\Delta}W_0(x) + \left(1-e^{-kV\Delta}\right)\int_{\underline{y}}^{\overline{y}} W_1\left(\phi_0(x,y'),x,y'\right)\frac{v(y')}{V}dy'\right\} \end{aligned}$$

rearranging

$$\left(1-e^{-(r+kV)\Delta}\right)W_0(x) = bx\Delta + e^{-r\Delta}\left(1-e^{-kV\Delta}\right)\int_{\underline{y}}^{\overline{y}} W_1\left(\phi_0(x,y'),x,y'\right)\frac{v(y')}{V}dy'$$

Dividing into Δ and taking the limit as Δ tends to zero, i.e. time is continuous, the expressions have the indeterminate form $\frac{0}{0}$. Then applying L'Hospital Rule

$$(r+\kappa V)\,W_0(x) = bx + \kappa\int_{\underline{y}}^{\overline{y}} W_1\left(\phi_0(x,y'),x,y'\right)v(y')dy'.$$

Taking into account that the worker has not bargaining power, in other words $W_0(x) = W_1\left(\phi_0(x,y'),x,y'\right)$, then the above expression is left as

$$rW_0(x) = bx.$$

Employed

Turning to the employed workers of any type, the value function of an x employee working on a firm of type y and earning the wage $w \le xy$ is derived in the same fashion. Starting from the value of the an employed worker in discrete time

$$
\begin{aligned}
&W_1\left(w,x,y\right)\\
&\quad = w\Delta + e^{-r\Delta}\left\{\left(1-e^{-\delta\Delta}\right)W_0(x)\right.\\
&\quad + e^{-\delta\Delta}\left[e^{-skV\Delta}W_1\left(w,x,y\right)+\left(1-e^{-skV\Delta}\right)\left(\int_{\underline{y}}^{q(w,x,y)}W_1(w,x,y)\frac{v(y')}{V}dy'\right.\right.\\
&\quad + \left.\left.\left.\int_{q(w,x,y)}^{y}W_1(xy',x,y)\frac{v(y')}{V}dy'+\int_{y}^{\overline{y}}W_1(xy,x,y')\frac{v(y')}{V}dy'\right)\right]\right\}
\end{aligned}
$$

rearranging

$$
\begin{aligned}
&\left(1-e^{-(r+\delta+skV)\Delta}\right)W_1\left(w,x,y\right)\\
&\qquad = w\Delta + e^{-r\Delta}\left(1-e^{-\delta\Delta}\right)W_0(x)\\
&\qquad + e^{-r\Delta}e^{-\delta\Delta}\left(1-e^{-skV\Delta}\right)\left(\int_{\underline{y}}^{q(w,x,y)}W_1(w,x,y)\frac{v(y')}{V}dy'\right.\\
&\qquad + \left.\int_{q(w,x,y)}^{y}W_1(xy',x,y)\frac{v(y')}{V}dy'+\int_{y}^{\overline{y}}W_1(xy,x,y')\frac{v(y')}{V}dy'\right)
\end{aligned}
$$

Dividing into Δ and taking the limit as Δ tends to zero, i.e. time is continuous, the expressions have the indeterminate form $\frac{0}{0}$. Then applying L'Hospital Rule

$$
\begin{aligned}
&(r+\delta+skV)\,W_1\left(w,x,y\right)\\
&\qquad = w+\delta W_0(x)\\
&\qquad + sk\int_{\underline{y}}^{q(w,x,y)}W_1(w,x,y)v(y')dy'\\
&\qquad + sk\int_{q(w,x,y)}^{y}W_1(xy',x,y)v(y')dy'\\
&\qquad + sk\int_{y}^{\overline{y}}W_1(xy,x,y')v(y')dy'
\end{aligned}
$$

rearranging

$$
\begin{aligned}
&\left(r+\delta+sk\overline{V}(q)\right)W_1\left(w,x,y\right)\\
&\qquad = w+\delta W_0(x)\\
&\qquad + sk\int_{q(w,x,y)}^{y}W_1(xy',x,y)v(y')dy'\\
&\qquad + sk\int_{y}^{\overline{y}}W_1(xy,x,y')v(y')dy'
\end{aligned}
$$

Now we can subtract $\left(r+\delta+sk\overline{V}(q)\right)W_0(x)$ to both sides of the equation and noticing that $W_1(xy,x,y)-W_0(x)=S(x,y)$, we can obtain the continuation value of the surplus accounted to the worker as a function of the whole surplus

$$\begin{aligned}\left(r+\delta+sk\overline{V}(q)\right)W_{10}\left(w,x,y\right)\\ &= w - rW_0(x)\\ &+ sk\int_{q(w,x,y)}^{y} S(x,y')\,v(y')dy'\\ &+ sk\int_{y}^{\overline{y}} S(x,y)\,v(y')dy'\end{aligned}$$

Or rewritting for computational purposes

$$\begin{aligned}(r+\delta)\,W_{10}\left(w,x,y\right)\\ &= w - rW_0(x)\\ &+ sk\int\left[min\left(S(x,y'),S(x,y)\right)-W_{10}\left(w,x,y\right)\right]^{+} v(y')dy'\end{aligned}$$

Where $[a]^{+}$ is equivalent to $max\,[a,0]$

Value of a Match and Surplus

Define the value of a Match and Surplus as $P(x,y)=\Pi_1(w,x,y)+W_1(w,x,y)$ and $S(x,y)=P(x,y)-W_0(x)$ respectively, again we start setting the value of a match as

$$\begin{aligned}&P(x,y)\\ &= yx\Delta + e^{-r\Delta}\left\{\left(1-e^{-\delta\Delta}\right)W_0(x)\right.\\ &+ e^{-\delta\Delta}\left[e^{-skV\Delta}P(x,y)+\left(1-e^{-skV\Delta}\right)\left(\int_{\underline{y}}^{q} P(x,y)\frac{v(y')}{V}dy'\right.\right.\\ &+ \left.\left.\left.\int_{q}^{y} P(x,y)\frac{v(y')}{V}dy'+\int_{y}^{\overline{y}} P(x,y)\frac{v(y')}{V}dy'\right)\right]\right\}\end{aligned}$$

rearranging

$$\begin{aligned}\left(1-e^{-(r+\delta+skV)\Delta}\right)P(x,y)\\ &= yx\Delta + e^{-r\Delta}\left\{\left(1-e^{-\delta\Delta}\right)W_0(x)\right.\\ &+ \left.e^{-\delta\Delta}\left[\left(1-e^{-skV\Delta}\right)\int_{\underline{y}}^{\overline{y}} P(x,y)\frac{v(y')}{V}dy'\right]\right\}\end{aligned}$$

Dividing into Δ and taking the limit as Δ tends to zero, i.e. time is continuous, the expressions have the indeterminate form $\frac{0}{0}$. Then applying L'Hospital Rule

$$\begin{aligned}(r+\delta+skV)\,P(x,y)\\ &= yx+\delta W_0(x)\\ &+ sk\int_{\underline{y}}^{\overline{y}} P(x,y)\,v(y')dy'\end{aligned}$$

Cancelling terms

$$(r+\delta)\,P(x,y) = yx + \delta W_0(x)$$

We just need to subtract $(r+\delta)W_0(x)$ to both sites to have a close expression for the surplus

$$(r+\delta)\,S(x,y) = yx - rW_0(x)$$

Remember that I am assuming that $\Pi_0(y) = 0,\ \forall y$

A.2 Derivations of The Value Functions under Minimum Wages

Unemployed

In this appendix expressions for equilibrium wages are determined for the basic model. I will closely follow the work of PV-R in deriving these analytical forms. First I will start setting the Value function of an unemployed worker in discrete time.

$$W_0(x) = bx\Delta + e^{-r\Delta}\Big\{e^{-kV\Delta}W_0(x) + \left(1-e^{-kV\Delta}\right)\int max\left[W_1\left(\phi_0(x,y),x,y\right), W_1\left(m,x,y'\right)\right]\frac{v(y')}{V}dy'\Big\}$$

Since $W_1(w,x,y)$ is monotonically increasing in y there will be a threshold in which $W_1\left(m,x,y'\right) \geq W_1\left(\phi_1(x,y),x,y\right)\ \forall y' \geq t_0$ or as implied by the lack of bargaining power of the worker $W_{10}\left(m,x,y'\right) \geq 0$, hence the the threshold is implicitly defined as $W_{10}\left(m,x,t_0(x,y)\right) = 0$

$$W_0(x) = bx\Delta + e^{-r\Delta}\Big\{e^{-kV\Delta}W_0(x) + \left(1-e^{-kV\Delta}\right)\left(\int_{\underline{y}}^{t_0(x,y)} W_1\left(\phi_0(x,y),x,y\right)\frac{v(y')}{V}dy' + \int_{t_0(x,y)}^{\overline{y}} W_1\left(m,x,y'\right)\frac{v(y')}{V}dy'\right)\Big\}$$

Because the lack of bargaining power of the worker $W_1\left(\phi_0(x,y),x,y\right) = W_0(x)$ we can rewrite

$$W_0(x) = bx\Delta + e^{-r\Delta}\Big\{e^{-kV\Delta}W_0(x) + \left(1-e^{-kV\Delta}\right)\left(\int_{\underline{y}}^{t_0(x,y)} W_0(x)\frac{v(y')}{V}dy' + \int_{t_0(x,y)}^{\overline{y}} W_1\left(m,x,y'\right)\frac{v(y')}{V}dy'\right)\Big\}$$

rearranging

$$\left(1-e^{-(r+kV)\Delta}\right)W_0(x) = bx\Delta + e^{-r\Delta}\left(1-e^{-kV\Delta}\right)\left(\int_{\underline{y}}^{t_0(x,y)} W_0(x)\frac{v(y')}{V}dy' + \int_{t_0(x,y)}^{\overline{y}} W_1\left(m,x,y'\right)\frac{v(y')}{V}dy'\right)$$

Dividing into Δ and taking the limit as Δ tends to zero, i.e. time is continuous, the expressions have the indeterminate form $\frac{0}{0}$. Then applying L'Hospital Rule

$$(r+\kappa V)W_0(x) = bx + \kappa\left(\int_{\underline{y}}^{t_0(x,y)} W_0(x)v(y')dy' + \int_{t_0(x,y)}^{\overline{y}} W_1\left(m,x,y'\right)v(y')dy'\right)$$

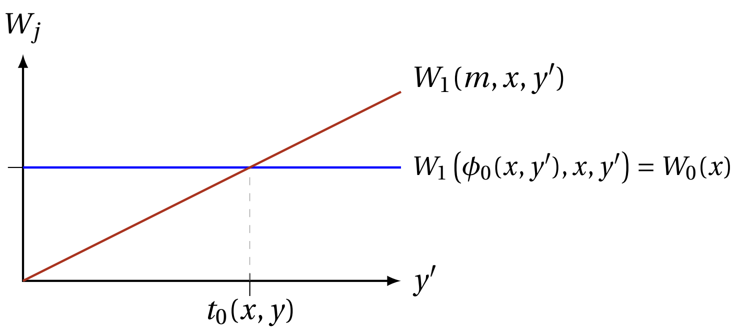
rearranging

$$\left(r+\kappa\overline{V}\left(t_0(x,y)\right)\right)W_0(x) = bx + \kappa\int_{t_0(x,y)}^{\overline{y}} W_1\left(m,x,y'\right)v(y')dy'$$

$$rW_0(x) = bx + \kappa\int_{t_0(x,y)}^{\overline{y}} W_{10}\left(m,x,y'\right)v(y')dy'$$

For computational purposes it is more convenient to write the equation as

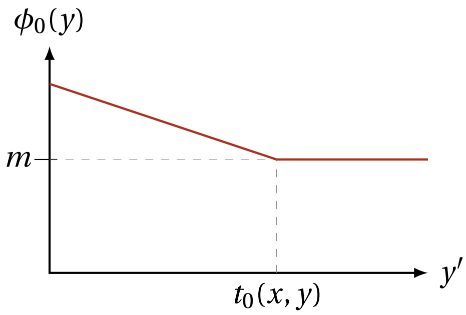
$$rW_0(x) = bx + \kappa\int max\left[W_{10}\left(m,x,y'\right),0\right]v(y')dy'$$



Threshold $t_0(x,y)$

Employed

Turning to the employed workers of any type, the value function of an x employee working on a firm of type y and earning the wage $w \le xy$ is derived in the same fashion. Because there is a minimum wage in place, for a particular x, the lower bound of firms might change being $\underline{y}^* = min[\underline{y},\hat{y}(x)]$, where $\hat{y}(x)$ is minimum viable productivity of a firm to hire a worker of type x when the minimum wage is



Wage offer $\phi_0(x, y)$

binding.

$$
\begin{aligned}
&W_1\left(w, x, y\right)\\
&\quad = w\Delta + e^{-r\Delta}\left\{\left(1 - e^{-\delta\Delta}\right) W_0(x) + e^{-\delta\Delta}\left[e^{-skV\Delta} W_1\left(w, x, y\right)\right.\right.\\
&\quad + \left(1 - e^{-skV\Delta}\right)\left(\int_{\underline{y}^*}^{q(w,x,y)} W_1(w, x, y)\frac{v(y')}{V}dy' + \int_{q(w,x,y)}^{y} W_1(xy', x, y)\frac{v(y')}{V}dy'\right.\\
&\quad + \left.\left.\left. + \int_{y}^{t_1(x,y)} W_1(xy, x, y')\frac{v(y')}{V}dy' + \int_{t_1(x,y)}^{\overline{y}} W_1(m, x, y')\frac{v(y')}{V}dy'\right)\right]\right\}
\end{aligned}
$$

Rearranging

$$
\begin{aligned}
&\left(1 - e^{-(r+\delta+skV)\Delta}\right) W_1\left(w, x, y\right)\\
&\quad = w\Delta + e^{-r\Delta}\left\{\left(1 - e^{-\delta\Delta}\right) W_0(x)\right.\\
&\quad + \left(1 - e^{-skV\Delta}\right)\left(\int_{\underline{y}^*}^{q(w,x,y)} W_1(w, x, y)\frac{v(y')}{V}dy' + \int_{q(w,x,y)}^{y} W_1(xy', x, y)\frac{v(y')}{V}dy'\right.\\
&\quad + \left.\left. + \int_{y}^{t_1(x,y)} W_1(xy, x, y')\frac{v(y')}{V}dy' + \int_{t_1(x,y)}^{\overline{y}} W_1(m, x, y')\frac{v(y')}{V}dy'\right)\right\}
\end{aligned}
$$

Dividing into Δ and taking the limit as Δ tends to zero, i.e. time is continuous, the expressions have the indeterminate form $\frac{0}{0}$. Then applying L'Hospital Rule

$$
\begin{aligned}
(r + \delta + skV)\, W_1\left(w, x, y\right)&\\
&= w + \delta W_0(x)\\
&+ sk\int_{\underline{y}^*}^{q(w,x,y)} W_1(w, x, y)\, v(y')dy'\\
&+ sk\int_{q(w,x,y)}^{y} W_1(xy', x, y)\, v(y')dy'\\
&+ sk\int_{y}^{t_1(x,y)} W_1(xy, x, y')\, v(y')dy'\\
&+ sk\int_{t_1(x,y)}^{\overline{y}} W_1(m, x, y')\, v(y')dy'
\end{aligned}
$$

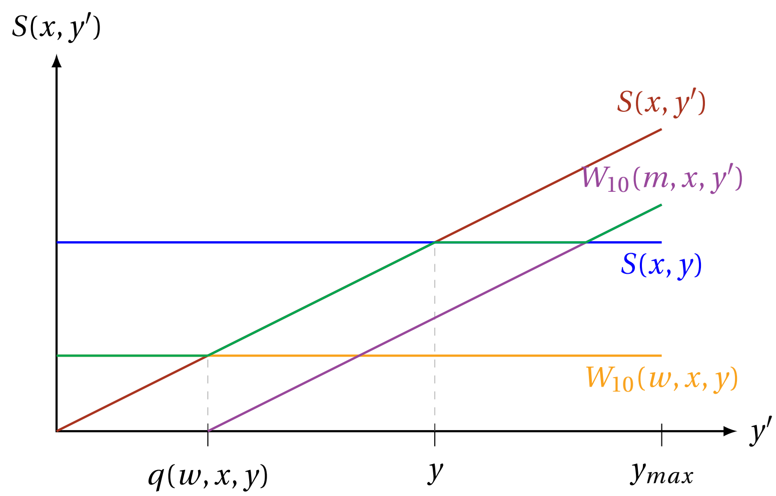
Now, because $\Pi_0(y) = 0,\ \forall y$ then $W_1(xy, x, y) = P(x, y)$, substituting this into the previous equation

$$
\begin{aligned}
&(r+\delta+skV)\,W_1\left(w,x,y\right)\\
&\qquad = w + \delta W_0(x)\\
&\qquad + sk\int_{\underline{y}^*}^{q(w,x,y)} W_1(w,x,y)v(y')dy'\\
&\qquad + sk\int_{q(w,x,y)}^{y} P(x,y')v(y')dy'\\
&\qquad + sk\int_{y}^{t_1(x,y)} P(x,y)v(y')dy'\\
&\qquad + sk\int_{t_1(x,y)}^{\overline{y}} W_1(m,x,y')v(y')dy'
\end{aligned}
$$

Subtracting $(r+\delta+skV)\,W_0(x)$ from both sites and rearranging

$$
\begin{aligned}
&\left(r+\delta+sk\overline{V}(q)\right)W_{10}\left(w,x,y\right)\\
&\qquad = w - rW_0(x)\\
&\qquad + sk\int_{q(w,x,y)}^{y} S(x,y')v(y')dy'\\
&\qquad + sk\int_{y}^{t_1(x,y)} S(x,y)v(y')dy'\\
&\qquad + sk\int_{t_1(x,y)}^{\overline{y}} W_{10}(m,x,y')v(y')dy'
\end{aligned}
$$

Following the picture we can rewrite the expression for computational purposes



Threshold $t_1(x,y)$

and thus avoid calculating $q(w,x,y)$ for every wage

$$
\begin{aligned}
(r & +\delta + skV)\, W_1\left(w,x,y\right) \\
&= w + \delta W_0(x) \\
&+ sk \int max\left\{min\left[max\left(W_{10}(w,x,y), S(x,y')\right), S(x,y)\right], W_{10}(m,x,y')\right\} v(y')dy'
\end{aligned}
$$

And the threshold $t_1(x,y)$ is defined as

$$
\begin{aligned}
W_1\left(xy,x,y\right) &= W_1\left(m,x,t_1(x,y)\right)\ or \\
S\left(x,y\right) &= W_{10}\left(m,x,t_1(x,y)\right)
\end{aligned}
$$

value of a Match and Surplus

Define the value of a Match and Surplus as $P(x,y) = \Pi_1(w,x,y) + W_1(w,x,y)$ and $S(x,y) = P(x,y) - W_0(x)$ respectively. Because there is a minimum wage in place, for a particular x, the lower bound of firms might change being $\underline{y}^* = min[\underline{y}, \hat{y}(x)]$, where $\hat{y}(x)$ is minimum viable productivity of a firm to hire a worker of type x when the minimum wage is unemployment binding. again we start setting the value of a match as

$$
\begin{aligned}
& P(x,y) \\
&= yx\Delta + e^{-r\Delta}\left\{\left(1-e^{-\delta\Delta}\right)W_0(x)\right. \\
&+ e^{-\delta\Delta}\left[e^{-skV\Delta}P(x,y) + \left(1-e^{-skV\Delta}\right)\left(\int_{\underline{y}^*}^{q} P(x,y)\frac{v(y')}{V}dy'\right.\right. \\
&+ \left.\left.\left.\int_{q}^{y} P(x,y)\frac{v(y')}{V}dy' + \int_{y}^{t_1(x,y)} P(x,y)\frac{v(y')}{V}dy' + \int_{t_1(x,y)}^{\overline{y}} W_1(m,x,y')\frac{v(y')}{V}dy'\right)\right]\right\}
\end{aligned}
$$

rearranging

$$
\begin{aligned}
& \left(1-e^{-(r+\delta+skV)\Delta}\right)P(x,y) \\
&= yx\Delta + e^{-r\Delta}\left\{\left(1-e^{-\delta\Delta}\right)W_0(x)\right. \\
&+ \left. e^{-\delta\Delta}\left[\left(1-e^{-skV\Delta}\right)\left(\int_{\underline{y}^*}^{t_1(x,y)} P(x,y)\frac{v(y')}{V}dy' + \int_{t_1(x,y)}^{\overline{y}} W_1(m,x,y')\frac{v(y')}{V}dy'\right)\right]\right\}
\end{aligned}
$$

Dividing into Δ and taking the limit as Δ tends to zero, i.e. time is continuous, the expressions have the indeterminate form $\frac{0}{0}$. Then applying L'Hospital Rule

$$
\begin{aligned}
(r+\delta+skV)\,P(x,y) & \\
&= yx + \delta W_0(x) \\
&+ sk\int_{\underline{y}^*}^{t_1(x,y)} P(x,y)v(y')dy' + sk\int_{t_1(x,y)}^{\overline{y}} W_1(m,x,y')v(y')dy'
\end{aligned}
$$

Rearranging

$$\left(r+\delta+sk\overline{V}(t_1)\right)P(x,y)$$
$$= yx+\delta W_0(x)$$
$$+sk\int_{t_1(x,y)}^{\overline{y}} W_1(m,x,y')v(y')dy'$$

We just need to subtract $\left(r+\delta+sk\overline{V}(t_1)\right)W_0(x)$ to both sites to have a close expression for the surplus

$$\left(r+\delta+sk\overline{V}(t_1)\right)S(x,y) = yx - rW_0(x) + sk\int_{t_1(x,y)}^{\overline{y}} W_{10}(m,x,y')v(y')dy'$$

Rewriting for computational purposes

$$(r+\delta+skV)\,S(x,y) = yx - rW_0(x) + sk\int max\left[S(x,y), W_{10}(m,x,y')\right]v(y')dy'$$

Remember that I am assuming that $\Pi_0(y)=0,\ \forall y$

Appendix B

Wage Offer

B.1 Equilibrium Wage Determination

Employed

In this section I will work out specific expressions for value functions and wages

$$\left[r+\delta+s\kappa\overline{V}\left(q(w,x,y)\right)\right]W_1(w,x,y)=w+\delta W_0(x) + s\kappa\int_{q(w,x,y)}^{y} W_1(xy',x,y')\;v(y')dy' + s\kappa\int_{y}^{\overline{y}} W_1(xy,x,y)\;v(y')dy' \tag{B.1}$$

Now, imposing $w=xy$ implies that $q(xy,x,y)=y$, introducing these concerns into the previous equation:

$$[r+\delta]\,W_1(xy,x,y)=xy+\delta W_0(x)$$

This last expression can be derived with respect to y to have a specific expression of the derivative, then differentiating implicitly and solving for $W_1'(xy,x,y)$

$$W_1'(xy,x,y)=\frac{x}{[r+\delta]}$$

Integrating (B.1) by parts, we have

$$\left[r+\delta+s\kappa\overline{V}\left(q(w,x,y)\right)\right]W_1(w,x,y)=w+\delta W_0(x) + s\kappa W_1\left(xy,x,y\right)V(y)-s\kappa W_1\left(w,x,y\right)V\left(q\right)-s\kappa\int_q^y W_1'\left(xy',x,y'\right)V(y')\,dy' + s\kappa W_1(xy,x,y)\left[V(\overline{y})-V(y)\right]$$

Noticing that $W_1(w,x,y)=W_1(xq,x,q)$ and after cancelling terms

$$[r+\delta]W_1(w,x,y)=w+\delta W_0(x) + s\kappa W_1(xy,x,y)V-s\kappa W_1\left(xq,x,q\right)V-s\kappa\int_q^y W_1'\left(xy',x,y'\right)V(y')\,dy'$$

And by the FTC

$$\begin{aligned}[r+\delta]\, W_1(w,x,y) = w + \delta W_0(x) \\ + s\kappa \int_q^y W_1'\left(xy',x,y'\right)\overline{V}(y')\, dy'\end{aligned}$$

With this general expression at hand we are ready to compute a particular expression of the wages. Start with the fact that an outside offer coming from a firm $\tilde{y} < y$ should comply with the equality $W_1(x\tilde{y},x,x\tilde{y}) = W_1(\phi(x,\tilde{y},y),x,y)$, notice that the threshold $q(x\tilde{y},x,\tilde{y}) = \tilde{y}$

$$\begin{aligned}x\tilde{y} + \delta W_0(x) + s\kappa \int_{\tilde{y}}^{\tilde{y}} W_1'\left(xy',x,y'\right)\overline{V}(y')\, dy' \\ = \phi(x,\tilde{y},y) + \delta W_0(x) + s\kappa \int_{\tilde{y}}^{y} W_1'\left(xy',x,y'\right)\overline{V}(y')\, dy'\end{aligned}$$

Cancelling terms and rearranging

$$\phi(x,\tilde{y},y) = x\tilde{y} - s\kappa \int_{\tilde{y}}^{y} W_1'\left(xy',x,y'\right)\overline{V}(y')\, dy'$$

Unemployed

For the unemployed x workers, entry wages at y firms are $\phi_0(x,y) = \phi(x,y_{inf},y)$.

$$\begin{aligned}\phi_0(x,y) &= xy_{inf} - s\kappa \int_{y_{inf}}^{y} W_1'\left(xy',x,y'\right)\overline{V}(y')dy' \\ &= \phi(x,y_{inf},y)\end{aligned}$$

Where y_{inf} is the minimum viable productivity of a firm to hire a worker and make no profits or in other words $W_1(xy_{inf},x,y_{inf}) = W_0(x)$, multiplying both sides by $(r+\delta)$ and replacing the LHS by its expression

$$xy_{inf} + \delta W_0(x) + s\kappa \int_{y_{inf}}^{y_{inf}} W_1'\left(xy',x,y'\right)\overline{V}(y')dy' = (r+\delta)W_0(x)$$

Which after cancelling terms becomes

$$y_{inf} = b$$

B.2 Equilibrium Wage Determination under Minimum wages

Employed

Turning to the employed workers of any type, the value function of an x employee working on a firm of type y and earning the wage $w \leq xy$ is set as to equal the wage, plus the value as unemployed with a laid-off rate of δ and offers from outside firms, accruing at a rate $s\kappa V$. When a worker is poached by a firm with productivity $\tilde{y} \le q$ then $\phi(x, \tilde{y}, y) \le w$, the poacher does not even reach the necessary productivity to hire the worker and make profits. At this point the minimum viable productivity that the firm has to have in order to provoke a wage increase is defined implicitly in the usual way as $\phi\left(x, q(x, w, y), y\right) = w$. If the offering firm has productivity $[q, y]$, the worker will receive a wage increase due to the Bertrand competition. In the case where the firm would have productivity higher than y, up to a threshold $t_1(x, y)$, the worker will switch jobs, the commonly known interplay between the wage offer and future wages increases plays its role, leaving the discounted future value of wealth fixed in this interval, i.e. the value function reaches a plateau since the firm is able to extract all the value from the match, thus higher productivity (and more likely future wage increases) is offset by a reduction in the wage offered. Now, offers from firms with higher productivity than $t_1(x, y)$ cannot reduce the offer made to the worker to extract all the match value, since the minimum wage acts as a lower bound for wages, in other words the minimum wage is binding. From the onset I will assume that this threshold exists and is unique as will be shown later. The value function of the worker is left as

$$\begin{aligned}\left[r + \delta + s\kappa\overline{V}\left(q(w, x, y)\right)\right] W_1(w, x, y) = w + \delta W_0(x) \\ + s\kappa \int_{q(w,x,y)}^{y} W_1(xy', x, y')\, v(y') dy' \\ + s\kappa \int_{y}^{t_1(x,y)} W_1(xy, x, y)\, v(y') dy' \\ + s\kappa \int_{t_1(x,y)}^{\overline{y}} W_1(m, x, y')\, v(y') dy' \end{aligned} \tag{B.2}$$

Where the threshold is defined as $W_1(xy, x, y) = W_1\left(m, x, t_1(x, y)\right)$

Adding and subtracting $W_1(xy, x, y)$ in the range $[t_1(x, y), \overline{y}]$ and rearranging terms we have

$$\begin{aligned}\left[r + \delta + s\kappa\overline{V}\left(q(w, x, y)\right)\right] W_1(w, x, y) = w + \delta W_0(x) \\ + s\kappa \int_{q(w,x,y)}^{y} W_1(xy', x, y')\, v(y') dy' \\ + s\kappa \int_{y}^{\overline{y}} W_1(xy, x, y)\, v(y') dy' \\ + s\kappa \int_{t_1(x,y)}^{\overline{y}} W_1(m, x, y') - W_1(xy, x, y)\, v(y') dy' \end{aligned}$$

The last term in the equation is the increase in utility granted by the minimum wage. Now, imposing $w = xy$ implies that $q(xy, x, y) = y$, introducing these concerns into the previous equation:

$$\begin{aligned}\left[r+\delta+s\kappa\overline{V}\left(y\right)\right]W_1(xy,x,y) &= xy+\delta W_0(x)\\ &\quad+\underbrace{s\kappa\int_{y}^{\overline{y}}W_1(xy,x,y)v(y')dy'}_{s\kappa\overline{V}(y))W_1(xy,x,y)}\\ &\quad+s\kappa\int_{t_1(x,y)}^{\overline{y}}W_1(m,x,y')-\underbrace{W_1(xy,x,y)}_{W_1(xy,x,y)\,\overline{V}(t_1(x,y))}\quad v(y')dy'\end{aligned}$$

Which results in

$$\left[r+\delta+s\kappa\overline{V}\left(t_1(x,y)\right)\right]W_1(xy,x,y)=xy+\delta W_0(x)+s\kappa\int_{t_1(x,y)}^{\overline{y}}W_1(m,x,y')\,v(y')dy'$$

This last expression can be derived with respect to y to have a specific expression of the derivative, then differentiating implicitly

$$\begin{aligned}-s\kappa v\left(t_1(x,y)\right)t_1'(x,y)W_1(xy,x,y)+\left[r+\delta+s\kappa\overline{V}\left(t_1(x,y)\right)\right]W_1'(xy,x,y)\\ =x-s\kappa\underbrace{W_1(m,x,t_1(x,y))}_{W_1(xy,x,y)}v\left(t_1(x,y)\right)t_1'(x,y)\end{aligned}$$

Solve for $W_1'(xy,x,y)$

$$W_1'(xy,x,y)=\frac{x}{\left[r+\delta+s\kappa\overline{V}\left(t_1(x,y)\right)\right]} \tag{B.3}$$

Integrating (B.2) by parts, we have

$$\begin{aligned}&\left[r+\delta+s\kappa\overline{V}\left(q(w,x,y)\right)\right]W_1(w,x,y)=w+\delta W_0(x)\\ &\quad+s\kappa W_1\left(xy,x,y\right)V(y)-s\kappa W_1\left(w,x,y\right)V\left(q\right)-s\kappa\int_{q}^{y}W_1'\left(xy',x,y'\right)V(y')\,dy'\\ &\quad+s\kappa W_1(xy,x,y)\left[V\left(t_1(x,y)\right)-V(y)\right]\\ &\quad+s\kappa W_1\left(m,x,\overline{y}\right)V-s\kappa\underbrace{W_1\left(m,x,t_1(x,y)\right)}_{W_1(xy,x,y)}V\left(t_1(x,y)\right)-\int_{t_1(x,y)}^{\overline{y}}W_1'\left(m,x,y'\right)\,V(y')dy'\end{aligned}$$

Which after cancelling terms and rearranging, results in

$$\begin{aligned}
[r+\delta]W_1(w,x,y) &= w+\delta W_0(x) \\
&+ s\kappa\left[W_1(xy,x,y)V - W_1(w,x,y)V - \int_q^y W_1'(xy',x,y')V(y')\,dy'\right] \\
&+ s\kappa\left[W_1(m,x,\overline{y})V - \underbrace{W_1(xy,x,y)V}_{W_1(m,x,t_1(x,y))} - \int_{t_1(x,y)}^{\overline{y}} W_1'(m,x,y')V(y')\,dy'\right]
\end{aligned}$$

Now, thanks to the fundamental theorem of calculus we get to the usual expression

$$\begin{aligned}
[r+\delta]W_1(w,x,y) = w+\delta W_0(x) + s\kappa\int_q^y W_1'(xy',x,y')\overline{V}(y')\,dy' \\
+ s\kappa\int_{t_1(x,y)}^{\overline{y}} W_1'(m,x,y')\overline{V}(y')\,dy' \qquad \text{(B.4)}
\end{aligned}$$

However this expression is not very intuitive, instead it would be better to have the value function defined in the whole support of firms, for which the following change of variables can be performed

$$\left.\begin{array}{r} y' = t_1(x,z) \\ dy' = t_1'(x,z)dz \end{array}\right\} W_1(m,x,y') = W_1(xz,x,z) \Rightarrow W_1(m,x,t_1(x,z)) = W_1(xz,x,z)$$

Deriving with respect to z

$$W_1'(m,x,t_1(x,z))\, t_1'(x,z)dz = W_1'(xz,x,z)$$

Making use of (B.3), (B.4) and the previous expression the final form follows

$$\begin{aligned}
[r+\delta]W_1(w,x,y) = w+\delta W_0(x) + s\kappa x\int_q^y \frac{\overline{V}(y')}{\left[r+\delta+s\kappa\overline{V}\left(t_1(x,y)\right)\right]}\,dy' \\
+ s\kappa x\int_y^{\overline{y}} \frac{\overline{V}\left(t_1(x,y')\right)}{\left[r+\delta+s\kappa\overline{V}\left(t_1(x,y)\right)\right]}\,dy' \qquad \text{(B.5)}
\end{aligned}$$

With this general expression at hand we are ready to compute a particular expression of the wages. Start with the fact that an outside offer coming from a firm $\tilde{y} < y$ should comply with the equality

$$W_1\left(\phi(x,\tilde{y},y),x,y\right) = max\left[W_1\left(x\tilde{y},x,\tilde{y}\right), W_1\left(m,x,y\right)\right]$$

Notice that because $\tilde{y} < y$, m is not going to be binding and the maximum function will result in $W_1\left(x\tilde{y},x,\tilde{y}\right)$. Then, taking the specific forms of the value functions derived in (B.5) at the particular wages, we can write

$$
\begin{aligned}
\phi(x,\tilde{y},y)+\delta W_0(x)+s\kappa x&\left\{\int_{\tilde{y}}^{y}\frac{\overline{V}(y')dy'}{\left[r+\delta+s\kappa\overline{V}\left(t_1(x,y)\right)\right]}+\int_{y}^{\overline{y}}\frac{\overline{V}\left(t_1(x,y')dy'\right)}{\left[r+\delta+s\kappa\overline{V}\left(t_1(x,y)\right)\right]}\right\}\\
&=x\tilde{y}+\delta W_0(x)+s\kappa x\left\{\int_{\tilde{y}}^{\tilde{y}}\frac{\overline{V}(y')dy'}{\left[r+\delta+s\kappa\overline{V}\left(t_1(x,y)\right)\right]}+\int_{\tilde{y}}^{\overline{y}}\frac{\overline{V}\left(t_1(x,y')dy'\right)}{\left[r+\delta+s\kappa\overline{V}\left(t_1(x,y)\right)\right]}\right\}
\end{aligned}
$$

Which after rearranging becomes

$$
\begin{aligned}
\underbrace{\phi(x,\tilde{y},y)}_{\text{wage offer}}&=\underbrace{x\tilde{y}}_{\text{max. productivity of the match}}\underbrace{-s\kappa x\int_{\tilde{y}}^{y}\frac{\overline{V}(y')dy'}{\left[r+\delta+s\kappa\overline{V}\left(t_1(x,y)\right)\right]}}_{\text{Trade off of lower wages for future increases}}\underbrace{+s\kappa x\int_{\tilde{y}}^{y}\frac{\overline{V}\left(t_1(x,y')\right)dy'}{\left[r+\delta+s\kappa\overline{V}\left(t_1(x,y)\right)\right]}}_{\text{Extra rent granted by imposing a min. wage}}\\
&=x\left\{\tilde{y}+s\kappa\int_{\tilde{y}}^{y}\frac{\overline{V}\left(t_1(x,y')\right)-\overline{V}(y')}{\left[r+\delta+s\kappa\overline{V}\left(t_1(x,y)\right)\right]}dy'\right\}
\end{aligned}
$$

Unemployed

As in the case without minimum wages we just need to define the minimum viable productivity of a firm $\hat{y}(x)$ as the value that leaves the worker indifferent between looking for a job and working; and at the same time makes the surplus of the match equal to zero, if the surplus of the match were not zero at the firm with the lowest viable productivity, any other firm with marginally lower productivity could make an offer to the worker and make profits at the same time. Since, there is no restriction on how low the marginal productivity of a firm can be, this is a contradiction, and thus:

$$P(x,\hat{y}(x))=W_1(m,x,\hat{y}(x))=W_0(x;m)$$

Where $\hat{y}(x)=\frac{m}{x}$

Proof:

$$
\begin{aligned}
P(x,\hat{y}(x))&=x\hat{y}(x)+s\kappa\int_{\hat{y}(x)}^{t_1}P(x,y)v(y')dy'+sk\int_{t_1}^{\overline{y}}W_1(m,x,y')v(y')dy'\\
W_1(m,x,\hat{y}(x))&=m+s\kappa\int_{\hat{y}(x)}^{t_1}P(x,y)v(y')dy'+sk\int_{t_1}^{\overline{y}}W_1(m,x,y')v(y')dy'
\end{aligned}
$$

Equating terms we arrive at $x\hat{y}(x)=m$, hence the result $\hat{y}(x)=\frac{m}{x}$.

And the entry wage will be m in any case, $\phi_0(x,\tilde{y})=m$.

High-skill workers

High-skill workers will show similar expressions as those worked out of low-skill ones. For employed workers the expression for salaries will remain unchanged, since the process for poaching workers is basically the same. Also, since the effective minimum wage $\hat{y}(x)$ is below the minimum viable productivity of a firm, $\hat{y}(x) \leq y_{inf}$, the support of the distribution will remain unchanged. What is likely to change is the support of the distribution of wages since now the entry wage will be the largest between $\phi_0(x, y; m) = [\phi_0(x, y), m]$, depending on the productivity of the initial poacher. Then the expression for entry wages will be a piece-wise function of the form

$$\phi_0(\epsilon, y; m) = \begin{cases} x\left\{b + s\kappa \int_{y_{inf}}^{y} \frac{\overline{V}(t_1(x,y')) - \overline{V}(y')}{\left[\rho+\delta+\lambda_1 \overline{F}(t_1(x,y))\right]} dy'\right\} & \text{if} \quad y' < t_1(x, y)) \\ m & \text{if} \quad y' \geq t_1(x, y)) \end{cases}$$

Appendix C

Steady State distributions

C.1 Base Model

In this appendix detailed derivations for $v(y)$, $u(x)$ and $h(x,y)$ are worked out. We start with the balance conditions, which are no more that accounting identities, that are met in every point in time

$$\int h_t(x,y')dy' + u_t(x) = l_t(x)$$
$$\int h_t(x',y)dx' + v_t(y) = n_t(y)$$

and flow equations in discrete time.

$$\begin{aligned}
h_{t+1}(x,y) = h_t(x,y) \quad & +kv_t(y)u_t(x)\Delta + sk\int_{\underline{y}}^{y} h_t(x,y')dy'v_t(y)\Delta \quad -\delta h_t(x,y)\Delta - sk\int_{y}^{\overline{y}} v_t(y')dy'h_t(x,y)\Delta \\
u_{t+1}(x) = u_t(x) \quad & -kV_t u_t(x)\Delta \quad +\delta h_{x,t}(x)\Delta \\
v_{t+1}(y) = v_t(y) \quad & -kv(y)U_t\Delta - sk\int_{\underline{y}}^{y} h_{y,t}(y')dy'v_t(y)\Delta \quad +\delta h_{y,t}(y)\Delta + sk\int_{y}^{\overline{y}} v_t(y')dy'h_{y,t}(y)\Delta
\end{aligned}$$

Where $h_{x,t}(x) = \int_{\underline{y}}^{\overline{y}} h_t(x,y')dy'$ and $h_{y,t}(y) = \int_{\underline{x}}^{\overline{x}} h_t(x',y)dx'$. The expressions are easier to work with in continuous time so I rearrange stocks to the LHS and flows to the RHS, Divide by Δ and take the limit as $\Delta \to 0$ to have

$$\begin{aligned}
\dot{h}_t(x,y) = kv_t(y)u_t(x) \quad & +sk\int_{\underline{y}}^{y} h_t(x,y')dy'v_t(y) - \delta h_t(x,y) \quad -sk\int_{y}^{\overline{y}} v_t(y')dy'h_t(x,y) \\
\dot{u}_t(x) = -kV_t u_t(x) \quad & +\delta h_{x,t}(x) \\
\dot{v}_t(y) = -kv_t(y)U_t \quad & -sk\int_{\underline{y}}^{y} h_{y,t}(y')dy'v_t(y) + \delta h_{y,t}(y) \quad +sk\int_{y}^{\overline{y}} v_t(y')dy'h_{y,t}(y)
\end{aligned}$$

Before working out specific expressions for every type of firm and worker, it will be useful to calculate aggregate balance conditions, just aggregate over the set of firm productivities and worker abilities to have

$$H_t + U_t = L_t$$
$$H_t + V_t = N_t$$

Where $H_t = \int\int h_t(x', y')dx'dy'$. L_t is exogenous and given N_t, V_t is pinned down, at this point both conditions are knotted by H_t, so we can write

$$L_t - U_t = N_t - V_t$$

And deriving with respect to time we have that $\dot{U}_t = \dot{V}_t$. Also, we need to have the expressions for the aggregate flows. Integrate $v(y)$, $u(x)$ and $h(x, y)$ over the variables that they depend on. Furthermore, in the aggregate all the workers that quit are the same as those who are poached, i.e. $\int_{\underline{y}}^{y} h_t(x, y')dy'v_t(y) = \int_{y}^{\overline{y}} v_t(y')dy'h_t(x, y)$, hence

$$\left.\begin{aligned} \dot{H}_t &= kV_tU_t - \delta H_t \\ \dot{U}_t &= -kV_tU_t + \delta H_t \\ \dot{V}_t &= -kV_tU_t + \delta H_t \end{aligned}\right\} \overset{SS}{\Longrightarrow} \delta H_t = kV_tU_t$$

Since we are in the S.S., time dependence can be dropped from the notation. Now, Plug the aggregate balance conditions to have H as a function of N, k (endogenous objects) and δ, L (parameters), $\delta H = k(N - H)(L - H)$, this is a quadratic equation on H

$$kH^2 - (\delta + kL + kN)\,H + kLN = 0$$

Which solves as

$$H = \frac{1}{2}\left\{\left(\frac{\delta}{k} + L + N\right) - \sqrt{\left(\frac{\delta}{k} + L + N\right)^2 - 4LN}\right\}$$

Below, it is the proof of why only the negative part is taken. First consider the positive part, i.e.

$$H(N) = \frac{1}{2}\left\{\left(\frac{\delta}{k} + L + N\right) + \sqrt{\left(\frac{\delta}{k} + L + N\right)^2 - 4LN}\right\} > \frac{1}{2}\left\{\left(\frac{\delta}{k} + L + N\right) + \sqrt{\left(\frac{\delta}{k} + L + N\right)^2}\right\} \Leftrightarrow$$
$$\frac{1}{2}\left\{\left(\frac{\delta}{k} + L + N\right) + \sqrt{\left(\frac{\delta}{k} + L + N\right)^2 - 4LN}\right\} > \left(\frac{\delta}{k} + L + N\right)$$

Which means that the number of employed is higher than the number of people in the economy, an absurdity. Turning to the negative part, I would like to work out the maximum and minimum values as a function of N. The minimum value can be easily worked out as

$$H(0) = \frac{1}{2}\left\{\left(\frac{\delta}{k}+L\right) - \sqrt{\left(\frac{\delta}{k}+L\right)^2}\right\} = 0$$

And the maximum

$$\lim_{N\to\infty} H(N) = \lim_{N\to\infty} \frac{1}{2}\left\{\left(\frac{\delta}{k}+L+N\right) - \sqrt{\left(\frac{\delta}{k}+L+N\right)^2 - 4LN}\right\}$$

Which is undetermined, dividing and multiplying by the complement and later on dividing by N in the numerator and denominator we have

$$\begin{aligned}
\lim_{N\to\infty} H(N) &= \lim_{N\to\infty} \frac{1}{2}\left\{\frac{4LN}{\left(\frac{\delta}{k}+L+N\right) + \sqrt{\left(\frac{\delta}{k}+L+N\right)^2 - 4LN}}\right\} \\
&= \lim_{N\to\infty} \frac{1}{2}\left\{\frac{4L}{\left(\frac{\delta}{Nk}+\frac{L}{N}+1\right) + \sqrt{\left(\frac{\delta}{Nk}+\frac{L}{N}+1\right)^2 + \frac{4L}{N}}}\right\} \\
&= \frac{1}{2}\left\{\frac{4L}{1+\sqrt{1}}\right\} = L
\end{aligned}$$

Which means that as the number of firms tends to infinity the number of employed workers tends to the number of people in the economy. Also, it would be convenient to check if the function is increasing in the whole domain

$$\frac{\partial H}{\partial N} = \frac{1}{2}\left\{1 - \frac{2\left(\frac{\delta}{k}+L+N\right) - 4L}{\sqrt{\left(\frac{\delta}{k}+L+N\right)^2 - 4LN}}\right\} > 0$$

Using the balance conditions U and V can easily be derived. With this expressions at hand we can work out their desegregated counterparts. First, consider the S.S. and drop the time dependence,so

$$\begin{aligned}
0 &= kv(y)u(x) && +sk\int_{\underline{y}}^{y} h(x,y')dy'v(y) - \delta h(x,y) && -sk\int_{y}^{\overline{y}} v(y')dy'h(x,y) \\
0 &= -kVu(x) && \qquad\qquad\qquad\qquad +\delta h_x(x) && \\
0 &= -kv(y)U && -sk\int_{\underline{y}}^{y} h_y(y')dy'v(y) + \delta h_y(y) && +sk\int_{y}^{\overline{y}} v(y')dy'h(_y y)
\end{aligned}$$

Now, consider the last expression and rearrange such that U-to-H and H-to-U flows are on the LHS and H-to-H flows are in the RHS. Also, for notational convenience let's write $F_z(z) = \int_{\underline{z}}^{z} f_z(z')dz'$ for any function over the z-characteristic and denote its complement counterpart as $\overline{F}_z(z) = F_z(\overline{z}) - F_z(z) = \int_z^{\overline{z}} f_z(z')dz'$

$$kv(y)U - \delta h_y(y) = -skH_y(y)v(y) + sk\overline{V}(y)h_y(y)$$

Integrate both sites from $\underline{y}$ to y to have

$$kV(y)U - \delta H_y(y) = skH_y(y)\overline{V}(y) \tag{C.1}$$

And Integrate the balance condition from $\underline{y}$ to y to have

$$H_y(y) + V(y) = N(y)$$

Then solve last expression for V(y) and plug it into the aggregate flow equation for vacancies to arrive at

$$\begin{aligned} k\left(N(y) - H_y(y)\right)U - \delta H_y(y) &= skH_y(y)\left(V - N(y) + H_y(y)\right) \\ kUN(y) - kUH_y(y) - \delta H_y(y) &= skVH_y(y) - skN(y)H_y(y) + skH^2(y) \\ skH^2(y) + \left(\delta + kU + skV - skN(y)\right)H_y(y) - kUN(y) &= 0 \end{aligned}$$

Again, this is a quadratic equation in $H_y(y)$, which depends solely on k and $N(y)$ and the rest of variables have been previously worked out, as we can see below

$$\underbrace{sk}_{A}H_y^2(y) + \underbrace{\left(\delta + kU + skV - skN(y)\right)}_{B(N(y))}H_y(y) - \underbrace{kUN(y)}_{C(N(y))} = 0$$

Then

$$H_y(y) = \frac{-B\left(N(y)\right) + \sqrt{B^2\left(N(y)\right) + 4AC\left(N(y)\right)}}{2A}$$

The negative part can safely be discarded as

$$H_y(y) = \frac{-B\left(N(y)\right) - \sqrt{B^2\left(N(y)\right) + 4AC\left(N(y)\right)}}{2A} < \frac{-B\left(N(y)\right) - \sqrt{B^2\left(N(y)\right)}}{2A} = -\frac{2B\left(N(y)\right)}{2A} < 0$$

Whereas the positive part is always greater than zero

$$H_y(y) = \frac{-B\left(N(y)\right) + \sqrt{B^2\left(N(y)\right) + 4AC\left(N(y)\right)}}{2A} > \frac{-B\left(N(y)\right) + \sqrt{B^2\left(N(y)\right)}}{2A} = 0$$

Now, derive the quadratic equation implicitly with respect to to y to find $h_y(y)$

$$2skH_y(y)h_y(y) + \left(\delta + kU + skV - skN(y)\right)h_y(y) - skn(y)H_y(y) - kUn(y) = 0$$

Solving for $h_y(y)$

$$h_y(y) = \frac{kU + skH_y(y)}{\left(\delta + skV + skH_y(y) - skN(y)\right) + \left(kU + skH_y(y)\right)} \cdot n(y)$$

Where

- $\left(\delta + skV + skH_y(y) - skN(y)\right) = \left(\delta + sk\overline{V}(y)\right)$: are flows out of $H_y(y)$ and
- $kU + skH_y(y)$: are flows into $H_y(y)$

It's worth noting that $H_y(y)$ depends on $N(y)$ and so does $h_y(y)$.

At this point we are ready to come with an expression for $v(y)$. Consider again the expression coming from the integrated flow of vacancies in the S.S.

$$kV(y)U - \delta H_y(y) = skH_y(y)\overline{V}(y)$$

It is just left to solve for $V(y)$ and derive to reach the desire result

$$V(y) = \frac{\delta + skV}{kU + skH_y(y)} H_y(y)$$

And deriving

$$v(y) = \frac{\delta + skV}{\left(kU + skH_y(y)\right)^2} kUh_y(y)$$

Which is not very intuitive. In order to have an expression in terms of flows, change $h_y(y)$ by its last derived expression; plug the definition of $V(y)$ and use the integrated flow of vacancies in the S.S. to arrive to the desired result

$$v(y) = \frac{\delta + sk\overline{V}(y)}{\left(\delta + sk\overline{V}(y)\right) + \left(kU + skH_y(y)\right)} \cdot n(y)$$

Once we have worked out a close form expression for the number of vacancies and the number of workers in y-type firms, we can deal with the number of unemployed and the number of workers with x-characteristic. From the differential equation for unemployed, substitute the balance condition for $h_x(x)$, such that

$$kVu(x) = \delta h_x(x) \Leftrightarrow kVu(x) = \delta\,(l(x) - u(x)) \Leftrightarrow (\delta + kV)\,u(x) = \delta l(x)$$
$$u(x) = \frac{\delta}{(\delta + kV)} l(x)$$

With this expression and basic algebra we work out $h_x(x)$

$$h_x(x) = \frac{kV}{(\delta + kV)} l(x)$$

Finally, we are ready to calculate $h(x, y)$. The steps to arrive at the solution are basically the same as those to compute $h_y(y)$. THen, consider the last expression and rearrange such that U-to-H and H-to-U flows are on the LHS and H-to-H flows are in the RHS.

$$\delta h(x,y) - kv(y)u(x) = sk\int_{\underline{y}}^{y} h(x,y')dy'v(y) - sk\int_{y}^{\overline{y}} v(y')dy'h(x,y)$$

Integrate both sites from $\underline{y}$ to y to have

$$\delta\int_{\underline{y}}^{y} h(x,y')dy' - ku(x)V(y) = -sk\int_{\underline{y}}^{y} h(x,y')dy'\overline{V}(y)$$

rearranging

$$\left(\delta + sk\overline{V}(y)\right)\int_{\underline{y}}^{y} h(x,y')dy' = ku(x)V(y)$$

$$\int_{\underline{y}}^{y} h(x,y')dy' = \frac{ku(x)V(y)}{\left(\delta + sk\overline{V}(y)\right)}$$

And deriving with respect to to y we arrive at the final form

$$h(x,y) = \frac{\delta + skV}{\left(\delta + sk\overline{V}(y)\right)^2}\cdot ku(x)v(y)$$

Which is difficult to interpret. However, we can prove that the following interesting result holds, $h(x,y) = \frac{1}{H}h_x(x)h_y(y)$. Start by plugging in $h(x,y)$ the expressions for $ku(x) = \frac{\delta}{V}h_x(x)$, $v(y)$, and solve for $\left(\delta + sk\overline{V}(y)\right)$ in equation C.1, so that we get

$$h(x,y) = \frac{(\delta + skV)}{\left(\frac{kV(y)U}{H_y(y)}\right)^2}\cdot\frac{(\delta + skV)}{\left(kU + skH_y(y)\right)^2}\cdot kUh_y(y)\frac{\delta}{V}h_x(x)$$

Solve for $\left(kU + skH_y(y)\right)$ in C.1 and use the fact that $kU = \frac{\delta}{V}H$

$$h(x,y) = \frac{(\delta + skV)}{\left(\frac{kV(y)U}{H_y(y)}\right)^2}\cdot\frac{(\delta + skV)}{\left(\frac{(\delta+skV)H_y(y)}{V(y)}\right)^2}\cdot\left(\frac{\delta}{V}\right)^2 Hh_x(x)h_y(y)$$

Cancelling out terms and substituting

$$h(x,y) = \left(\frac{\delta}{kUV}\right)^2 Hh_x(x)h_y(y) = \frac{1}{H^2}Hh_x(x)h_y(y) = \frac{1}{H}h_x(x)h_y(y)$$

Now, we are ready to calculate the distribution of wages from the flow equation

$$\frac{dG_t(w|x,y)\cdot h_t(x,y)}{dt} = kv_t(y)u_t(x) + sk\int_{\underline{y}}^{q} h(x,y')dy'\cdot v(y) - \delta G_t(w|x,y)\cdot h_t(x,y) - sk\int_{q}^{\overline{y}} v(y')dy'G_t(w|x,y)\cdot h_t(x,y) = 0$$

rearranging

$$\left(\delta + sk\int_q^{\overline{y}} v(y')dy'\right) G(w|x,y)\cdot h(x,y) = kv(y)u(x) + sk\int_{\underline{y}}^{q} h(x,y')dy'\,v(y)$$

solving for $G_t(w|x,y)$ we have

$$G(w|x,y) = \frac{\left(kv(y)u(x) + sk\int_{\underline{y}}^{q} h(x,y')dy'\,v(y)\right)}{\left(\delta + sk\int_q^{\overline{y}} v(y')dy'\right)}\cdot\frac{1}{h(x,y)}$$

Substitute $h(x,y)$ by the product of the marginals $\frac{1}{H}h_x(x)h_y(y)$. Also use the flow equation for the unemployed $kVu(x) = h_x(x)$ and the aggregate flow equation $kVU = \delta H$ to arrive at $u(x) = \frac{U}{H}h_x(x)$, then after cancelling terms

$$G(w|y) = \frac{\left(kU + sk\int_{\underline{y}}^{q} h_y(y')dy'\right)}{\left(\delta + sk\int_q^{\overline{y}} v(y')dy'\right)}\cdot\frac{v(y)}{h_y(y)}$$

Which shows what intuition could have told us in advance, namely that the distribution of wages does not depend on x.

C.2 Distributions with Minimum Wages

To find out the distributions of matches, vacancies and unemployed under the minimum wage, $\tilde{h}(x,y)$, $\tilde{v}(y)$ and $\tilde{u}(x)$ respectively, it will just suffice to rewrite them in terms of the old ones. Under the minimum wage some meetings that could have ended in a match are not going to be possible, as the flow revenue of the match it is not enough to pay the minimum wage. Then the balance conditions can be rewritten as

$$l(x) = \underbrace{u(x) + \int_{\underline{y}}^{\hat{y}(x)} h(x,y')dy'}_{\tilde{u}(x)} + \underbrace{\int_{\hat{y}(x)}^{\overline{y}} h(x,y')dy'}_{\tilde{h}_x(x)}$$

$$n(y) = \underbrace{v(y) + \int_{\underline{x}}^{\hat{x}(y)} h(x',y)dx'}_{\tilde{v}(y)} + \underbrace{\int_{\hat{x}(y)}^{\overline{x}} h(x',y)dx'}_{\tilde{h}_y(y)}$$

Because the previous analysis without minimum wages, we know that under random search there is no sorting under the model assumptions. The implication being to express $h(x,y)$ as the product of two functions that describe abilities and productivities independently, namely $h(x,y) = \frac{1}{H}h_y(y)h_x(x)$, then the balance conditions can be rewritten as

$$l(x) = \underbrace{u(x) + h_x(x)\int_{\underline{y}}^{\hat{y}(x)} \frac{h_y(y')}{H}dy'}_{\tilde{u}(x)} + \underbrace{h_x(x)\int_{\hat{y}(x)}^{\overline{y}} \frac{h_y(y')}{H}dy'}_{\tilde{h}_x(x)}$$

$$n(y) = \underbrace{v(y) + h_y(y)\int_{\underline{x}}^{\hat{x}(y)} \frac{h_x(x')}{H}dx'}_{\tilde{v}(y)} + \underbrace{h_y(y)\int_{\hat{x}(y)}^{\overline{x}} \frac{h_x(x')}{H}dx'}_{\tilde{h}_y(y)}$$

And then as

$$l(x) = \underbrace{u(x) + h_x(x)F_y\left(\hat{y}(x)\right)}_{\tilde{u}(x)} + \underbrace{h_x(x)\overline{F}_y\left(\hat{y}(x)\right)}_{\tilde{h}_x(x)}$$

$$n(y) = \underbrace{v(y) + h_y(y)F_x\left(\hat{x}(y)\right)}_{\tilde{v}(y)} + \underbrace{h_y(y)\overline{F}_x\left(\hat{x}(y)\right)}_{\tilde{h}_y(y)}$$

Integrating over abilities in the first condition and over productivities in the second we work out the aggregate balance conditions

$$L = \underbrace{U + \int_{\underline{x}}^{\overline{x}} h_x(x')F_y\left(\hat{y}(x')\right)dx'}_{\tilde{U}} + \underbrace{\int_{\underline{x}}^{\overline{x}} h_x(x')\overline{F}_y\left(\hat{y}(x')\right)dx'}_{\tilde{H}}$$

$$N = \underbrace{V + \int_{\underline{y}}^{\overline{y}} h_y(y')F_x\left(\hat{x}(y')\right)dy'}_{\tilde{V}} + \underbrace{\int_{\underline{y}}^{\overline{y}} h_y(y')\overline{F}_x\left(\hat{x}(y')\right)dy'}_{\tilde{H}}$$

For the join distribution of jobs under the minimum wage it will suffice to solve the follow equation for jobs:

$$\begin{aligned} 0 = k\tilde{v}(y)\tilde{u}(x) \qquad\qquad & +sk\int_{\underline{y}}^{y} \tilde{h}(x,y')dy'\,\tilde{v}(y) \\ -\delta\tilde{G}_t(w|x,y)\cdot\tilde{h}(x,y) \qquad\qquad & -sk\int_{y}^{\overline{y}} \tilde{v}(y')dy'\,\tilde{G}_t(w|x,y)\cdot\tilde{h}(x,y) \end{aligned}$$

Now the number of vacancies and unemployed people will be substituted by their minimum wage counterparts, i.e. $\tilde{v}(y)$ and $\tilde{u}(y)$, since those vacancies lost by the unemployed or workers with low ability will be at the disposal of the rest, and seemingly the same argument applies for those unemployed that will not be able to cover vacancies in low productive firms. Hence, following the same procedure as before we will arrive at

$$\tilde{h}(x,y) = \frac{\delta + sk\tilde{V}}{\left(\delta + sk\overline{\tilde{V}}(y)\right)^2} \cdot k\tilde{u}(x)\tilde{v}(y)$$

$$\tilde{G}(w|x,y) = \frac{\left(k\tilde{v}(y)\tilde{u}(x) + sk\int_{\underline{y}}^{q}\tilde{h}(x,y')dy'\tilde{v}(y)\right)}{\left(\delta + sk\int_{q}^{\overline{y}}\tilde{v}(y')dy'\right)} \cdot \frac{1}{\tilde{h}(x,y)}$$

Lack of assortative matching will not be the case upon introducing minimum wages, now the minimum viable productivity of a firm (or worker) to form a match will be a function of the worker ability (firm productivity). In this respect minimum wages will introduce negative sorting in our analysis.

Appendix D

Proofs

LEMMA.1

The firm with the minimum viable productivity to hire a worker cannot make any surplus out of the match, i.e. $\Pi_1\left(x, \underline{\hat{y}}(x)\right) = 0$, otherwise a firm with marginally less productivity could enter the market, hire a worker and make profits, being a contradiction; then $P\left(x, \underline{\hat{y}}(x)\right) = W_1\left(\phi_0\left(x, \underline{\hat{y}}(x)\right), x, \underline{\hat{y}}(x)\right)$.

With respect to $W_1\left(\phi_0\left(x, \underline{\hat{y}}(x)\right), x, \underline{\hat{y}}(x)\right) = W_0(x, m)$, the same argument along the above lines can be devised. If $W_1\left(\phi_0\left(x, \underline{\hat{y}}(x)\right), x, \underline{\hat{y}}(x)\right) > W_0(x, m)$ then another firm with marginally less productivity could enter and make profits, once again a contradiction.

LEMMA.2

Making use of LEMMA.1 we can equate $P\left(x, \underline{\hat{y}}(x)\right) = W_1\left(\phi_0\left(x, \underline{\hat{y}}(x)\right), x, \underline{\hat{y}}(x)\right)$, which means that

$$\underline{\hat{y}}(x)x + \delta W_0(x) + sk\int_{\underline{\hat{y}}(x)}^{t(x,y)} P(x,y)v(y')dy' + sk\int_{t(x,y)}^{\overline{y}} W_1(m,x,y')v(y')dy'$$
$$= \phi_0\left(x, \underline{\hat{y}}(x)\right) + \delta W_0(x) + sk\int_{\underline{\hat{y}}(x)}^{t(x,y)} P(x,y)v(y')dy' + sk\int_{t(x,y)}^{\overline{y}} W_1(m,x,y')v(y')dy'.$$

And after cancelling terms we arrive at:

$$\underline{\hat{y}}(x)x = \phi_0\left(x, \underline{\hat{y}}(x)\right).$$

For convenience define the threshold x' such that $\phi_0\left(x', y_{inf}\right) = m$. There are two cases of interest:

CASE.1: $x < x'$

$$\phi_0\left(x', \underline{\hat{y}}(x)\right) = m \Leftrightarrow \underline{\hat{y}}(x) = \frac{m}{x}.$$

CASE.2: $x \geq x'$

$$\phi_0\left(x', y_{inf}\right) > m \Leftrightarrow \underline{\hat{y}}(x) = y_{inf}.$$

$\blacksquare$

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