

This exam has 7 problems on 6 pages. There are no calculators, phones, or other electronic devices allowed during this exam. Be sure to show all your work.

Name: _____

Score:

id-number:

Problem 1. Solve the following limits when possible. (*Hints: try to add and subtract, multiply and divide by the same number; make use of the complement i.e. complement of $(a+b)$ is $(a-b)$ and write numbers as quotients if possible*)

(a) $\lim_{x \rightarrow 0} \frac{5}{x-1}$

(b) $\lim_{x \rightarrow \infty} \frac{162000x^2 + 7800x + 10}{1 + x + x^2 + x^3}$

(c) $\lim_{x \rightarrow 2} \frac{x^2 - 7x + 10}{x^2 - 5x + 6}$

(d) $\lim_{x \rightarrow 0} \frac{x}{|x|}$

(e) $\lim_{x \rightarrow \infty} (\sqrt{x+1} - \sqrt{x})$

(f) $\lim_{x \rightarrow \infty} \frac{2^x - 2^{-x}}{2^x + 2^{-x}}$

Problem 2. Find the $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ and $\frac{\partial^2 z}{\partial x \partial y}$ of the following functions:

(a) $z = x^{\ln 4} e^{x^2 + y^2}$

(b) $z = \frac{x^{\frac{1}{4}} y^{\frac{1}{3}}}{e^x}$

(c) $z = \ln x^3 y^2$

Problem 3. Solve the following integrals by direct methods, substitution and parts respectively:

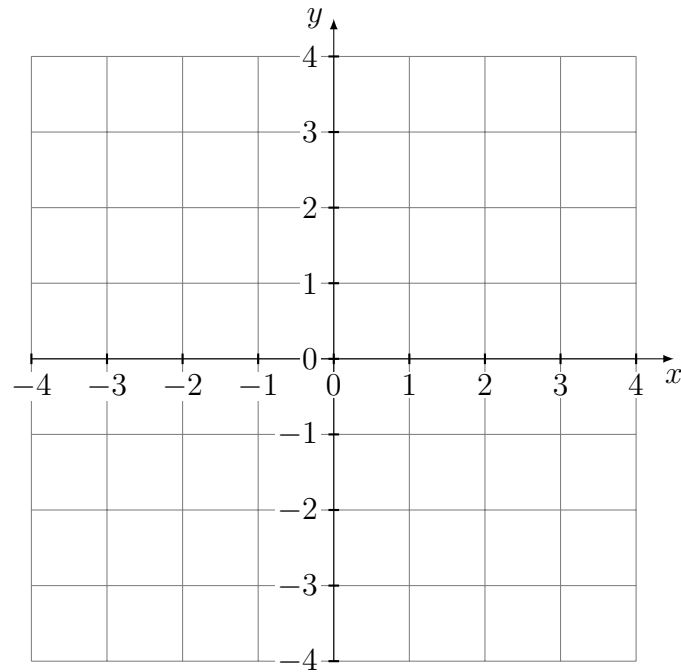
(a) $\int_0^1 x^{-4} dx$

(b) $\int_0^{\frac{3}{2}} \frac{x dx}{\sqrt{9-4x^2}}$

(c) $\int x \ln x dx$

Problem 4. Find the derivative of w with respect to z of the following function $\ln(x^4 y^3 z^6 t^2 w)$ by making use of the implicit function theorem.

Problem 5. Sketch the following function e^{-x^2} by showing analytically over what intervals the functions is increasing and decreasing, over what intervals the function is convex or concave and where are the maxima and minima of the function if at all.



Problem 6. Solve the following problem using the Khun-Tucker conditions:

$$\begin{array}{ll}\max_{x,y} & f(x,y) = 3xy - x^3 \\ \text{s.t.} & 2x - y = -5 \\ & 5x + 2y \geq 37 \\ & x, y \geq 0\end{array}$$

Problem 7. Solve the following system of equations using the inverse matrix.

$$x_1 + 2x_2 + x_3 = 4$$

$$x_1 - x_2 + x_3 = 5$$

$$2x_1 + 3x_2 - x_3 = 1$$