Firm Heterogeneity and Collective Agreements: a Structural Model of Minimum Wages in Spain

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Does widespread coverage of collective agreements introduce inefficiencies in the labour market? What would the optimal bargaining protocol be? How different sized firms are affected? Participants of the political process of collective agreements set the institutions that fit them best, not internalising the effects posed on the outsiders. I analyse how participants set minimum wages in a model of two-sided heterogeneity with on-the-job search. Then, I estimate the model using exhaustive working histories from Spanish Social Security data for the metal industry in Madrid. In a counterfactual exercise I show that the most interested in rising the wage floor are large firms because they have gains through two channels. Ones is by hampering competition of smaller firms and hiring more workers. The other channel is by slowing down wage dynamics, i.e. paying less to more experienced workers. Letting everyone participate of the political process would result in a lower wage floor of €1150 instead of €1250, with large firms reducing their profits by 4.4% and smaller firms reentering the market, increasing their profits by 6.9%.

How do central and Western European systems of collective bargaining affect labour market outcomes? What is the optimal bargaining protocol? How different sized firms are affected? These are the sort of questions that the present work answers. The most salient clause of collective contracts is minimum wages, Cardoso and Portugal (2005). Participants of the political process (insiders) might use minimum wages as a tool to gain some perks such as higher wages or reduce competition, not internalising the effects on the outsiders. This work delivers a model to rationalise these questions, to estimate the effects on welfare and to measure the impact on labour market outcomes such as unemployment, wage distributions, spillovers and job tenure.

In an economy where a subset of agents, trade unions and employers associations, have the right to set minimum wages, they also have the power to affect labour market outcomes to their advantage. In a search and matching equilibrium model similar to Flinn and Mullins (2019), I consider collective agents who decide on minimum wages. Minimum wages affect individual workers and firms through different channels. (1) Raising the lower support of wage distributions. (2) Bolstering or hindering Career dynamics depending on the concentration of firms. (3) Preventing previously viable matches from forming, and consequently rising unemployment. (4) Missing opportunities by Low productive firms due to not being able to hire low-skill workers.

Contemplating the effects of free-entry of firms in the market, higher wage floors lead to

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low ability workers relative to firm productivity not being able to match, increasing the stock of unemployed and vacancies in the economy, reducing the stock of employed and diminishing the rate at which vacancies and searchers come across. On the supply side of the market, (5) there are two opposing effects, on the one hand, there are more vacancies to match with for those who remain employed. On the other hand, the rate at which matches occur decreases. The net outcome of both effects is that increase in vacancies does not compensate for the decline in the rate of contacts, resulting in fewer wage offers and in turn less dynamic careers for workers of any skill.

On the demand side, (6) the firm with the lowest productivity opts out from previously profitable matches and it can not poach other workers from firms with higher productivity leaving it undoubtedly worse off. When we consider firms with higher productivity, there are two opposing effects: on the one hand, firms have gradually more unemployed at their disposal because they can hire workers with lower productivity and they can poach those unable to match with lower productivities due to the minimum wage; on the other hand, firms have fewer employed people to poach from. The net effect depends on the search effort exerted by the employed relative to the unemployed and is subject to a quantitative estimation. Nonetheless, using reasonable parameter values, simulations show that high productive firms become bigger as they have relatively more searchers at their disposal.

Carrying out structural estimations using the Spanish employer-employee database from administrative records for the metal industry in Madrid, it turns out that wage floors at their current level of €1,256 results in an unemployment rate of 31%. Counterfactual analysis shows that reducing the minimum wage at a sectoral level to €1,150, helps to lower unemployment to 29%, with no additional gains below that threshold, which is in line with studies that suggest that moderate increases in minimum wages have no effects on unemployment. Looking at wages, distributions become more dispersed when the minimum wage is set at €1,150; lower minimum wage incentivise firms to open more vacancies, creating a positive externality on the other side of the market, i.e. workers encounter outside offers more often, then firms pay lower entry wages but offer more vibrant wage dynamics. The demand side of the market is less concentrated and market power, the ability to hire workers from other companies, is redistributed towards the lower tail of the firm distribution, or in other words, firms have to grant pay rises more frequently.

Turning to the bargaining of the agreement, unions and employers associations keep in mind equilibrium effects when deciding the wage floor. When setting the level of wage floor, collective associations face the trade-offs of increasing the number of unemployed and vacancies balanced with higher wages and easier to fill vacancies. But, who these organisations represent? only large firms and workers of these firms can participate in the political process, they can elect their respective associations who are to negotiate. Observe that the threshold for participation in the political process is exogenously determined, a policymaker could set the cut-off point lower to allow more people to decide as a way to increase social welfare. In addition, they are the less negatively affected by the increasing minimum. Unemployed, large swathes of the employed and low productive firms are left out of this mechanism to elect representatives (outsiders). Consequently, unions and employers associations do not internalise the effects of their decisions on the outsiders.

A counterfactual derived from estimations show that workers represented by the union have a steady decrease in utility as minimum wages grow, this is because the positive effects of having more vacancies at their disposal do not compensate for the increasing competition to fill these vacancies, effectively lowering the probability to match. On the demand side, high productive employers have more unemployed at their disposal, increasing the expected value of opening a vacancy, in turn, they open more vacancies and become larger, increasing the value of their jobs. If the minimum wage is too high, employers will not be able to hire workers and will miss chances of meeting previously profitable workers. Because of this, a hump-shaped curved for the utility of the employers. As a whole, the estimates suggest that employers' associations have a larger say in setting the minimum wage, as the latter is fixed where most convenient for them, indicating that they held most of the bargaining power. Increasing the participation in the political process, negotiators would set a lower the minimum wage at €1150, with large firms reducing their profits by 4.4% and smaller firms reentering the market, and increasing their profits by 6.9%.

This work contributes to the literature in several ways. From a theoretical point of view, I answer these questions by constructing a search and matching model in which I include unions and employer's associations that bargain over the rules of the legal environment, in particular minimum wages. In the literature of trade unionisation, unions bargain vis-a-vis with the company for better wages, whereas management chooses the level of employment, see Booth (1995) for a review. This framework fits better in Anglo-Saxon and Nordic countries. The present work departs from that literature in two ways. First, negotiations are carried out at a sectoral level, not within the firms, consequently collective contracts set the minimum requirements that all agents in the market have to abide by. In a recent paper, Krusell and Rudanko (2016) try to fill this gap assuming the union bargains on behalf of the whole active workforce, employed or unemployed, which make sense since there is no heterogeneity in their model; yet it misses the fact that there is no *a priori* reason why unions should worry about the whole workforce at a sector-wide level or even the unemployed. Then the second departure of the model notices that unions and employers associations do not represent only their affiliates but a wider base of voters who can vote. This feature makes it necessary to account for the political process.

To address this vent, I built on the works of Cahuc, Postel-Vinay and Robin (2006) (CPR) and Flinn and Mabli (2009) (FM). In the latter, authors presented a unifying framework of search and matching models with firms competing *à la Bertrand*, free-entry and minimum wages; in their model there is match-specific productivity drawn at random, leaving no room to model firm size. In the former, the authors model two-sided heterogeneity to account for worker and firm fix effects, which in turn allowed to introduce a measure of firm size. Both models account for individual wage bargaining, which I do not deem right for this paper, as estimations of CPR show the bargaining power of the worker is close to zero for the lower ranks. The avenue that I take in this work is to account for two-sided heterogeneity in the work of FM to acknowledge that only workers in the largest firms will be able to elect their representatives and in this way participate the political process. On top of that, I carry on introducing unions and employers associations, letting minimum wages be endogenous.

Anther important contribution of this paper is that to the empirical literature. This strand has analysed collective agreements and how their clauses affect labour market outcomes using quasi-experimental data and a reduce form approach. The model proposed here is deemed to answer questions based on theoretical foundations that are confronted against the data. Some part of this literature began in the late nineties to test predictions laid out by Calmfors and Driffill (1988), who emphasised the inefficiencies brought about by intermediate levels of negotiations due to market power coupled with lack of internalisation of outcomes. Notable research in this area was done by Hartog, Leuven and Teulings (2002) who tests this prediction using data on The Netherlands. Yet, they do not inspect how particular provisions affect labour market outcomes. The seminal paper of Cardoso and Portugal (2005) shed light on this by taking minimum wages into the analysis, they found out that adjustments are absorbed reducing the wage cushion and not laying off workers. Another major advancement is Card and Cardoso (2021), in this work they thoroughly analyse collective agreements in Portugal looking at their outcomes such as unemployment, wages or spillovers. For this purpose, I use an exhausting database with complete working histories of sampled workers, namely *Muestra Continua de Vidas Laborales* (MCVL), that serves to test the model.

The paper is structured as follows. Section I outlines the continuous-time search model without minimum wages. Section II, minimum wages are introduced and its consequences analysed. In section III unions and employers associations are considered, they Nash-bargain to set the optimal level (for insiders) of minimum wages. Section IV describes the institutional setting. Section V presents descriptive data. Section VI structural estimation is considered. Section VII results on labour market outcomes are portrayed. Section VIII considers a welfare analysis and section IX concludes.

I. Base model

I construct a search and matching model with OTJ-search and two-sided heterogeneity where workers do not hold bargaining power and hiring is costly for firms. I built on the work of Postel-Vinay and Robin (2002) in order to draw wage and mobility dynamics and two-sided heterogeneity, this implies that person and firm fix effects can be considered and that seemingly equal pairs, matches with same productivity and ability, pay different wages since workers are subject to different histories of wage offers. From Lise, Meghir and Robin (2016), endogenous number of jobs is imported, which in their model is a measure of the number of firms. This allows to consider firms of different sizes depending on their productivity; also, notation is taken from here. Furthermore, the model pulls out from Lise and Robin (2017) the condition for the number of firms and its parameterizatiton, although in their paper is related to vacancies.

A. Setting

The Economy is populated with a continuum of workers indexed by x, representing the ability, which is exogenously given, publicly observable, and distributed over the interval $x \in [\underline{x}, \overline{x}]$ according to a beta distribution $l(x)$ with parameters (a_x, b_x) and which quantity is normalised to L. Workers are either unemployed or employed, in both cases they search for jobs to find better alternatives, the search effort of employed is s and the unemployed effort is normalised to one. Since workers search for other jobs while employed, they have the opportunity to bring other companies into Bertrand competition with their incumbent employers in order to gain a pay rise or change jobs otherwise, the process will be explained in detailed in the following sections. Let $u(x)$ be number of workers of type x among the unemployed and $U = \int u(x')dx'$ total unemployment.

Firms are ranked according to technology y that is uniformingly distributed in $y \in \left[\underline{y}, \overline{y}\right]$. Firms hold a number of jobs $n(y)$ that might be filled or vacant, the total number of jobs by held by all firms is $N = \int n(y')dy'$, which is endogenously determined due to the free-entry condition (FEC). The number of vacancies opened by the firm with productivity y is $v(y)$ and the number of workers of type x employed in this firm is denoted by $h(x, y)$, the size of the firm is then $h_y(y) = \int h(x', y)dx'$, and the distribution of workers across firms is $h_x(x) = \int h(x, y')dy'$. The total number of vacancies opened in the economy is $V = \int v(y')d'$ and seemingly the total number of employed people is $H = \int h_y(y')dy'$.

All agents in the economy discount time at the same factor ρ. The flow income as uneomployed is $f(x, y) = xb$ whereas upon matching the firm and the worker start producing a flow output $f(x, y) = xy$, the chief point is that workers are perfectly substitutable and there are no complementarities among then within the firm. Matches are exogenously terminated by a Poisson process with parameter δ or endogenously when there is a job-to-job transition.

B. The Matching Process

Search is random and undirected within the matching set, which under the case without minimum wages is $[\underline{x}, \overline{x}] \times [\underline{y}, \overline{y}]$. Notice that neither workers nor firms veto matches when they meet, since matches offer values to the unemployed or vacant jobs at least as high as their outside options. Unemployed and employed workers compete for vacancies with different search costs, the search cost of unemployed is normalised to one and for employed is denoted by s.

Let k be a parameter that characterises all key rates of meeting and which definition is:

$$k = \frac{M\left(U + s(L-U), V\right)}{\left[U + s\left(L-U\right)\right]V}.$$

Where M is a Cobb-Douglas meeting function of the searchers and vacancies with equal weights and a meeting efficiency parameter η. From here we can define the rate at which unemployed workers meet a vacancy as $kV \cdot \frac{v(y)}{V} = kv(y)$, whereas employed workers meet vacancies at a rate $skv(y)$. On the other side of the market, vacancies meet unemployed workers at a Poisson rate $ku(x)$ and meet employed ones at a rate $skh(x, y)$.

C. Value functions

At this point it is worth remembering that workers do not hold bargaining power vis-à-vis with the employer. In other words, firms take all of the surplus for themselves when a meeting is materialised in a match. The only way a worker can demand a pay rise to its employer is by dragging other firms into Bertrand competition. This setting differs from those of Dey and Flinn (2005) and Cahuc, Postel-Vinay and Robin (2006) where workers are assumed to hold some bargaining power, at least at a theoretical level. There are three reasons why I chose not to include it. First of all, the bargaining power of workers is close to zero as they show in their empirical extracts. Secondly, the model captures the fact that workers usually do not have bargaining power and let the agent (union) to negotiate on their behalf. Lastly, it reduces the mathematical and computational burden.

Unemployed

The value of unemployed worker of type x is denoted by $W_0(x)$ and receives a flow bx for what produces while unemployed, notice that b is common to all workers, i.e. all have the same technology at home but the flow increases with the ability. This captures the fact that unemployed workers with high-skill have better wages and more generous unemployment benefits than those with less ability. Other rationale could be that those unemployed do some informal jobs that are going to be paid according to the ability. In reality, the unemployment insurance payment is usually a function of the wage earned and the time employed, at the same time the wage is a function of time and ability. Hence the mean time is captured by b which is the same for all workers and the ability by x. The wage offered to the unemployed $\phi_0(x, y)$ is such that the firm takes all the surplus of the match for itself, so that the worker is indifferent between taking or rejecting the offer. Notice that the offer depends on both arguments x and y. Then, the starting wage is implicitly defined as

$$W_0(x) = W_1\left(\phi_0(x, y), x, y\right), \quad \forall y \in \left[\underline{y}, \overline{y}\right],$$

with $W_1(w, x, y)$ being the value of an x employed worker earning a wage w at firm of type y. From here it can be drawn the continuation value of unemployment as:

$$(\rho + kV)W_0 = bx + k \int W_1\left(\phi_0(x, y'), x, y'\right) v(y')dy'.$$

Which by the previous definition solves as:

$$\rho W_0 = bx$$

Employed

As mentioned before the value of an employed worker is $W_1(w, x, y)$, nonetheless I will introduce $W_{10}(w, x, y) = W_1(w, x, y) - W_0(x)$, which is the net surplus accounted to an x-worker earning the wage w in a y-firm, mainly to save notational burden. As workers search on the job they can bring firms into competition in order to be granted a pay rise or switch companies otherwise. In this way, whenever a worker comes across a wage offer from an poaching firm $y' \leq y$, she can use it as outside option to negotiate vis-a-vis with her current employer. Upon meeting a firm, the worker faces three situations: the alternative firm does not have enough productivity to pay the current wage and the current relation does not change; another situation results in a wage increase for the worker and a third one materialises in a new match (a Job-to-Job transition).

The first case might be such that the worker encounters a firm y' that does not even have enough productivity to pay for his current wage and make profits, i.e. $xy' - w \leq 0$. The set of these firms ranges from the firm with least productivity $\underline{y}$ to the threshold $q(w, x, y)$. The productivity threshold $q(w, x, y)$ leaves the worker indifferent between extracting the whole surplus of the poacher and staying in her current firm earning the same wage or in other words

$$W_{10}\left(w, x, y\right) = S(x, q).$$

In this case the worker does not swap firms nor sees her wage risen, hence the wage offer does not have any effect on her.

In the following scenario the outside firm ranks in $y' \in (q, y]$, i.e. the productivity of the incumbent company is higher that the poaching one, still the latter has enough productivity to oblige the former grant a pay rise to the worker. Let $\phi(x, y', y)$ be the offer done by firm $y' < y$ to an x-type employee working at firm y. The final job offer will leave her indifferent between staying in her current firm with a wage promotion, which is the case, or changing firms and is implicitly defined as

$$W_{10}(\phi(x, y', y), x, y) = S(x, y').$$

Notice that the poaching firm will never raise its offer above xy' since loses would materialise: $xy' - \phi(x, y', y) < 0$.

Seemingly, when the productivity of the poacher is $y' > y$ the offer granted to the worker $\phi(x, y, y')$ is such that leaves her indifferent between staying with the incumbent with a wage xy or changing jobs to a more productive firm but earning less wage. Again, the wage offer is implicitly defined as

$$W_{10}\left(\phi(x, y, y'), x, y'\right) = S(x, y).$$

With these expressions at hand the surplus continuation value of an employed worker, net of lay-off shocks and future wage offers, would be

$$\begin{aligned}\left[\rho + \delta + sk\overline{V}\left(q(w, x, y)\right)\right] W_{10}(w, x, y) &= w - \rho W_0(x) \\ &\quad + sk \int_{q(w,x,y)}^{y} W_{10}(xy', x, y')\, v(y') dy' \\ &\quad + sk \int_{y}^{\overline{y}} W_{10}(xy, x, y)\, v(y') dy' \end{aligned} \tag{1}$$

The first term in the right-hand side is wage earn at every point in time. The second is the continuation value of the loss when the match is destroyed. The third one accounts for the increase in value to the worker when promoted to a higher wage and the fourth is the continuation value that comes from a job-to-job transition. For derivations of key equations see Postel-Vinay and Robin (2002).

Vacant Jobs

Following the above discussion vacancies are filled according the search efforts of both sides of the market and the amount of each of them in the economy. Firms create jobs, filled or vacant, until the marginal cost equals the expected revenue of a filled job. Define the continuation value of holding a vacancy $\Pi_0(y)$ as

$$\rho\Pi_0(y) = -c'\left(n(y)\right) + kJ(y),$$

where

$$J(y) = \int_{\underline{x}}^{\overline{x}} S(x', y)u(x')dx' + s\int_{\underline{x}}^{\overline{x}}\int_{\underline{y}}^{y} \left(S(x', y) - S(x', y')\right)h(x', y')dy'dx'.$$

The first term in the right-hand side is the marginal cost of exerting effort $c'\left(n(y)\right) = c_0 n(y)^{c_1}$, to which I impose convexity to ensure an equilibrium exists and the second term is the expected value of filling a vacancy.

Filled Jobs

Firms discount future at the same rate as workers and have a stream flow of profits $xy - w$, when a job is exogenously destroyed, production ceases and a vacancy is immediately opened. As previously highlighted, when vacancies are open, offers accrue to both types of workers, employed and unemployed at a Poisson rates $kv(y)$ and $skv(y)$ respectively. Matches start by firms appropriating the whole surplus. As offers from less productive companies accrue to the worker, the current company has to give up the surplus that the poaching firm grants to the worker. When a worker finds a higher viable alternative the firm has no other option than let her go. Thus, the net continuation surplus $\Pi_{10}(w, x, y) = \Pi_1(w, x, y) - \Pi_0(y)$ of a filled job is

$$\begin{aligned}\left(\rho + \delta + skv\left(q(w, x, y)\right)\right)\Pi_{10}\left(w, x, y\right) = xy - w \\ + sk\int_{q}^{y} S(x, y) - S(x, y')v(y')dy'.\end{aligned}$$

The first two terms indicate the flow stream of profits made by the firm for a particular match. The second term is the continuation value of the surplus that it make out of the match minus the possible promotions that has to grant the worker when coming across wage offers.

Surplus

From previous sections it is clear that all the continuation values are defined in terms of the match surplus, whilst not being defined until now. Define the surplus in the usual way, i.e. $S(x, y) = \Pi_1(w, x, y) - \Pi_0(y) + W_1(w, x, y) - W_0(x)$. As a result summing over all these expressions in the right-hand side it can easily be proven that the equation for the surplus is

$$(\rho + \delta)\, S(x, y) = yx - bx$$

Which is quite simple expression since we have imposed $\Pi_0(y) = 0,\ \forall y$.

D. *Equilibrium*

The exogenous elements of the model are the distribution of workers $l(x)$, the support of this distribution,$[\underline{x}, \overline{x}]$, the support of this distribution,$[\underline{y}, \overline{y}]$, the discount factor ρ, the job destruction δ, the search intensity of employed workers s, the value of leisure b and the production technology $f(x, y) = xy$. Given these parameters, the equilibrium can be characterised by defining the distributions of employees, unemployed, vacancies and wages can be worked out, along with the free entry condition.

Balance Equations

In equilibrium the distribution of the unemployment rate of workers of type x, $u(x)$, and the number of vacancies of type y, $v(y)$, is determined according to the balance conditions

$$\int h(x, y)dy + u(x) = l(x)$$
$$\int h(x, y)dx + v(y) = n(y).$$

The first condition basically states that the number of employed x-type workers plus the number of unemployed of type x has to be equal to the number of people of ability x. Seemingly, the second condition says that the number of workers in firms with productivity y plus the number of vacancies of type y has to be equal to the number of firms with this productivity.

Flow Equations

The joint distribution wages and matches $G(w|x, y) \cdot h(x, y)$ follows a steady state flow equation where inflows balance the outflows. Matches of x-workers in y-firms earning w or less might arise for two reasons, either workers with ability x are hired directly from unemployment by companies with productivity y or they are poached from less productive firms than q. On the other side, matches of (x, y) pairs might be destroyed by exogenous separations that accrue at a rate δ or because outside firms, with higher productivity than q, poach the worker or make a better offer. Netting this two forces the flow equation stays as

$$\left(\delta + sk \int_q^{\overline{y}} v(y')dy'\right) G(w|x, y) \cdot h(x, y) = \left(u(x) + s \int_{\underline{y}}^{q} h(x, y')dy'\right) kv(y).$$

The same can be worked out for the number of unemployed workers and vacancies of any type, i.e.

$$kVu(x) = \delta h(x)$$
$$\left(\delta + sk\int_{y}^{\overline{y}} v(y')dy'\right) h_y(y) = \left(U + s\int_{\underline{y}}^{y} h_y(y')dy'\right) kv(y).$$

Where $h_x(x) = \int h(x,y)dy$ and $h_y(y) = \int h(x,y)dx$. Once the flow equations and the balance conditions are defined, $h(x,y)$, $u(x)$ and $v(y)$ can be derived as:

$$v(y) = \frac{\delta + sk\overline{V}(y)}{\left(\delta + sk\overline{V}(y)\right) + (kU + skH_y(y))} \cdot n(y)$$
$$u(x) = \frac{\delta}{\delta + kV} l(x)$$
$$h(x,y) = \frac{1}{H} h_x(x) h_y(y)$$
$$G(w|y) = \frac{h_y(q)}{v(q)} \cdot \frac{v(y)}{h_y(y)}$$

and $h_y(y)$ depends solely on $n(y)$, which at this point is exogenously determined. Detailed proofs of steady state equations are found in appendix A.A1. It is worth noting that the main advantage of having introduced firm heterogeneity is that we have a measure of firm size. And this firms size will be in a one-to-one correspondence with its productivity level. This turns out to be essential in the political economy part of the model. Finally, the number of jobs created by firms of type y, $n(y)$, is set by the free entry condition described below.

Free Entry Condition

Firms of type y exert increasing effort in recruiting candidates until the cost of maintaining a vacancy and retaining talent equals the expected value of filling it for every y, $\Pi_0(y) = 0$, at equilibrium:

$$c'\left(n(y)\right) = kJ(y).$$

Following the parameterization of Lise and Robin (2017), we are able to draw an expression for the effort exerted by firms of type y as

$$c_0 n(y)^{c_1} = kJ(y).$$

Equilibrium effort by firm-type is then written

$$n(y) = \left(k\frac{J(y)}{c_0}\right)^{\frac{1}{c_1}}.$$

Summing over all companies in the economy, the aggregate equilibrium number of jobs in the economy is worked out:

$$N = \int \left(k \frac{J(y')}{c_0} \right)^{\frac{1}{c_1}} dy'.$$

II. Introducing Minimum Wages

At this point minimum wages are introduced into the analysis. I add on to the works of Cahuc, Postel-Vinay and Robin (2006), Flinn and Mabli (2009) and Flinn and Mullins (2019). I consider wage and mobility dynamics based on firm heterogeneity as opposed to match quality. When considering firm heterogeneity, firms with higher productivity win the Bertrand game when competing for workers. They become larger because high productive firms hire workers from less productive ones. On top of that, larger firms will have a say in the negotiating table at a sectoral level whereas smaller firms will be left out. Furthermore, I account for a continuous measure of worker heterogeneity as a way to control for workers fix effects. At this point is important to recall that workers do no hold bargaining power, which makes sense in the present analysis as low categories of workers are considered.

Introducing minimum wages have several implications. First of all, minimum wages affect the distribution of wages beyond those directly affected, compressing the distribution for a given pair (x, y). Intuitively, when a worker is allowed to search on the job, she has to compensate the employer for the expected forgone profits when she changes companies, which might never occur. If the worker does not have bargaining power she is willing to accept less than her wage today for wage rises in the future. Upon setting a wage floor, high productive firms in the range $[t(x, y), \overline{y}]$ drag the wage down to the legal minimum, increasing the value of being employed by keeping the rate of wage offers, fixing equilibrium objects, but earning a higher wage. Low productive firms, in the range $\left[\underline{\hat{y}}(x), t(x, y)\right]$, will have to compensate the worker for this fact, raising effectively the wage earned.

Another implication is that minimum wages changes the meeting rates at which workers and firms encounter. In figure 1 the red shaded area represents the meetings that could have resulted in a match in the absence of a minimum wage but because of it now they are at the disposal of the rest, increasing the chances of meeting for those not directly affected by the minimum wage and potentially decreasing the chances for those affected. In a nutshell, upon introducing minimum wages the number of vacancies and unemployed people increase whereas the tightness, k, decreases monotonically as the minimum wage increases. Coupling both effects, it results in a hump-shaped curve of job offers; at the beginning, job offers accrue at a higher rate for relatively high skill workers and after some threshold these offers start to decrease due to fall in expected revenues of filling a vacancy and firms holding less number of jobs.

As a side effect the value of the match and the outside options of agents change accordingly. The whole surplus is still seized by the employer, workers benefit from the raising wage floor through the higher value of unemployment.

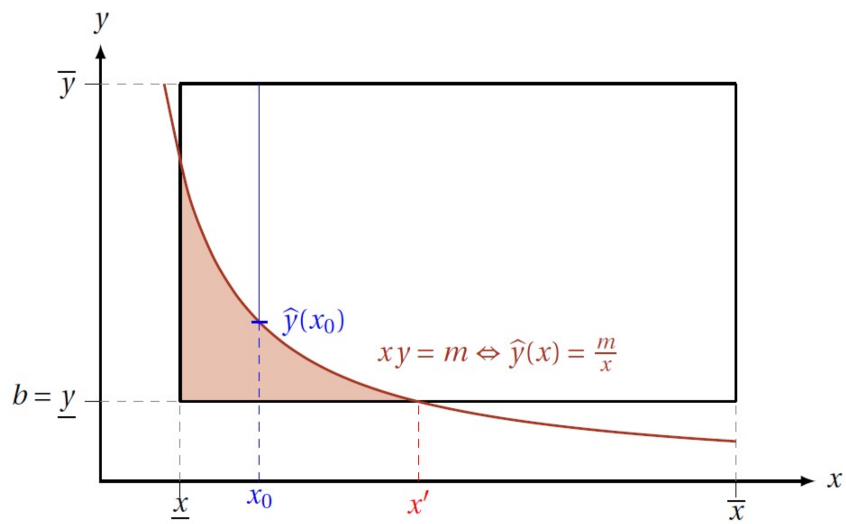


Figure 1. : Matching Space

A. Value functions

In this section the lifetime utility values for the different type of workers are derived when a minimum wage is put in place. For the ease of exposition, workers are grouped in two categories, x^L-type which is in the rank of abilities $[\underline{x}, x']$ and x^H-type employees in $(x', \overline{x}]$. So, employees have different continuation values depending on their abilities.

Unemployed

As it is clear from figure 1 workers are only hired when contacting a firm with productivity $y' \geq \widehat{\underline{y}}(x) = min\left\{\frac{m}{x}, b\right\}$. Firms with lower productivity than $\widehat{\underline{y}}(x)$ are out of the scope of an x-type unemployed worker and never contacted. The flow value of the unemployed worker $W_0(x;m)$ is increased because firms with high enough productivity cannot trade off less wages today for pay rises tomorrow. I assume that individuals are ex-ante heterogeneous in their valuations before they enter the labour market. Subsequently, they participate of the labour force whenever the value of staying out is strictly lower than the unemployment value $W_0(x;m)$, which under the minimum wage is equivalent to working in $\hat{\underline{y}}(x)$ at the wage $\phi_0\left(x, \widehat{\underline{y}}(x)\right)$. Furthermore, notice that the firm with the lowest viable productivity cannot make surplus out of the match, otherwise a firm with marginally less productivity could enter the market. From these considerations the next lemma says,

LEMMA 1. *The value as unemployed is equal to the value of first employment at $\hat{\underline{y}}(x)$. Seemingly, the value of first employment at $\hat{\underline{y}}(x)$ is equal to value product of a match $P(x, \hat{\underline{y}}(x))$.*

Therefore,

$$P\left(x, \underline{\hat{y}}(x); m\right) = W_1\left(\phi_0\left(x, \underline{\hat{y}}(x)\right), x, \underline{\hat{y}}(x); m\right) = W_0(x; m)$$

Proof. *See appendix B.B1.*

With these considerations at hand we are ready to calculate what the minimum viable productivity of a firm to hire a worker would be, and hence the lower support of the firm distribution $\underline{\hat{y}}(x)$ for a given ability x. For some ability levels in the range $[\underline{x}, x']$, matches that were profitable without minimum wages they are not anymore and the entry wage is the minimum wage. For those in $x \in (x', \overline{x}]$, the minimum wage only changes the values of being unemployed and employed but not their mobility decisions, therefore the minimum viable productivity of a firm to hire a worker remains unchanged. From these considerations the following lemma states:

LEMMA 2. *Fixing equilibrium objects, the minimum viable productivity of a firm $\underline{\hat{y}}(x)$ to hire a worker under the presence of a wage floor is:*

$$\underline{\hat{y}}(x) = \begin{cases} \frac{m}{x} & \text{if } x < x' \\ y_{inf} = b & \text{if } x \geq x' \end{cases}$$

Proof. *See appendix B.B2.*

EMPLOYED

As is common in models of OTJ-search with competition a la Bertrand, high productive firms can drag down the wage of the worker in exchange for a more dynamic tenure track, i.e. future offers that will end in wage increases. Upon introducing minimum wages, not every pair (y, y') is able to play a "wage war", more specifically when the poaching firm has very high productivity relative to the incumbent, the former will not be able to lower the wage in its full extend to the worker, because a biding minimum wage is in place. Nonetheless, it does not affect mobility decisions as they are still efficient. Workers see the value of employment risen whenever $t(x, y) < \overline{y}$, even though they might not earn the minimum wage. Intuitively, the worker has the opportunity to work at high productive firms earning no less than the minimum, effectively rising the value of their jobs. The continuation surplus value of an employed worker net of laid-offs would be

$$\begin{aligned}
&(\rho + \delta) W_{10}(w, x, y; m) = \\
&\qquad w - \rho W_0(x; m) + sk \int_q^y \left[S(x, y'; m) - W_{10}(w, x, y; m)\right]^+ v(y') dy' + \\
&\qquad sk \int_y^{\overline{y}} \left[max\left[S(x, y; m), W_{10}(m, x, y'; m)\right] - W_{10}(w, x, y; m)\right]^+ v(y') dy'.
\end{aligned}$$

Where $[a]^+ = max[a, 0]$. The first object in the right-hand side is the flow income. The second one is the continuation value of being unemployed. The third term is more interesting, it represents the expected increase in surplus thanks to the fact that the worker can bring two firms into competition staying with the incumbent. The forth term, is the expected increase in surplus derived from switching to more productive firms. $max\left[S(x,y), W_{10}(m,x,y')\right]$ expresses the possibility that the worker encounters a firm with such productivity that will be able to offer no less than the minimum wage, effectively extracting more surplus for her out of the match. Theoretically, we could find a firm with enough productivity to reduce the wage rate up to the minimum. In practical terms, the firm distribution has its productivity cap at the firm with highest productivity. In turn, it might be the case that $t(x,y) \geq \overline{y}$ and the minimum wage has no direct impact over the worker, although she experiences general equilibrium effects inside the labour market.

As it is obvious, the notation has slightly changed. The reason why is because there is no analytical expression for the threshold $t(x,y)$. This threshold is worked out by iterating the value function to achieve a fix point and finding the cut point where $S(x,y;m) = W_{10}(m,x,t(x,y);m)$, at which point $t(x,y)$ is implicitly defined.

Employed earning the minimum wage

Because we have introduced the minimum wage we have to consider what is the value for an employed worker earning the minimum wage either because she is been hired directly from unemployment or because she has received and offer for a high productive firm. At the minimum wages the value function is

$$
\begin{aligned}
&(\rho+\delta)W_{10}(m,x,y;m) = \\
&\qquad m - \rho W_0(x;m) + sk\int_{\underline{\hat{y}}(x)}^{y} \left[S(x,y';m) - W_{10}(m,x,y;m)\right]^+ v(y')dy' + \\
&\qquad sk\int_{y}^{\overline{y}} \left[max\left[S(x,y;m), W_{10}(m,x,y';m)\right] - W_{10}(m,x,y;m)\right]^+ v(y')dy'.
\end{aligned}
$$

The only thing that is likely to change is the lower limit of the integral in the third term. I seems that the new limit restricts the matching space of the worker to meet another firm. However, this is not the case, once employed and earning the minimum wage, the worker experience the same restriction as if the minimum were not in place.

Surplus

The new surplus takes into account the increased in value as an unemployed worker, effectively reducing the surplus, together with the increase in the value for an employed worker, leaving the expression

$$
(\rho+\delta)\,S(x,y;m) = yx - \rho W_0(x;m) + sk\int \left[W_{10}(m,x,y';m) - S(x,y;m)\right]^+ v(y')dy'
$$

The change of surplus with respect to the case without minimum wages depends on the interplay of the mentioned objects. Still, the whole surplus is appropriated by the employer and the legal minimum affects the employee positively in two ways. First, it increases the value as unemployed conditioned on participating in the labour market, although participation is not a case of study in the present work, now the worker is indifferent between being unemployed or working at a firm with the least viable productivity higher than before, leaving her better off. The other mechanism at her disposal is again the competition a la Bertrand between firms. Nonetheless, in this case the worker receives a wage offer potentially higher than the one that she would have been given without the presence of a minimum wage, even if she were not earning the statuary minimum. The intuition is that the worker has the opportunity to work at high productive firms earning no less that the minimum, if a poacher did not compensate the employee for this fact, the worker would find it profitable to wait for the next offer come. This is not optimal for the poacher who loses the value of the filled job. Consequently, the poacher offers the employee a wage that leaves her indifferent between working with them at relatively higher wage rate or waiting another period of time.

COROLLARY 3. *The surplus generated at the firm with minimum viable productivity is* 0
Proof. Trivial from LEMMA 1.

B. *Equilibrium Under minimum wages*

In this section I concentrate in the labour market equilibrium effects of establishing a minimum wage. On the one hand, the set of possible matches is reduced, as $\{x, y\}$ pairs that fall short of m do not form a match anymore. On the other hand, there are more vacancies at the disposal of the rest of workers and more unemployed at the disposal of high productive firms, having ambiguous effects over different firm productivities.

Low productive firms are in general worse off as they find harder to find workers of relatively low ability. However as we move up through the productivity distribution, firms can match with workers with lower abilities. At the same time these firms will have a new stock of unemployed at their disposal, in particular, from those firms with lower productivity that are unable to hire low ability workers. These points are made clear in the following sections.

Balance Equations

These balance conditions have to be rearranged to account for the fact that some meetings do not come true anymore, instead these pairs of workers and vacancies are at the disposal of the rest. More precisely they are

$$l(x) = \underbrace{u(x) + \int_{\underline{y}}^{\hat{y}(x)} h(x, y')dy'}_{\tilde{u}(x)} + \underbrace{\int_{\hat{y}(x)}^{\overline{y}} h(x, y')dy'}_{\tilde{h}_x(x)}$$

$$n(y) = \underbrace{v(y) + \int_{\underline{x}}^{\hat{x}(y)} h(x', y)dx'}_{\tilde{v}(y)} + \underbrace{\int_{\hat{x}(y)}^{\overline{x}} h(x', y)dx'}_{\tilde{h}_y(y)}.$$

Where $\int_{\underline{y}}^{\hat{y}(x)} h(x,y')dy'$ is the new stock of unemployed and $\int_{\hat{y}(x)}^{\overline{y}} h(x,y')dy'$ are are workers that remain employed with ability x that are able to keep their jobs. Exactly the same argument applies for the second and third terms in the firm balance equation. In the first equation, the number of unemployed and employed depend just on x, now affecting the limits of the integral. In the second, the number of vacancies and employed depend just on y.

Flow equations

Remember that no functional forms where assumed on $h(x,y)$, $u(x)$ and $v(y)$; and the new $\tilde{h}(x,y)$, $\tilde{u}(x)$ and $\tilde{v}(y)$ still depend only on their respective variables. Then, the number of vacancies and unemployed people are substituted by their minimum wage counterparts, i.e. $\tilde{v}(y)$ and $\tilde{u}(y)$, which can be readily worked out from the stocks. whereas the flow equation $\tilde{G}_t(w|x,y) \cdot \tilde{h}(x,y)$ is rewritten following exactly the same derivations as the case without minimum wages, showing the same functional form

$$\tilde{h}(x,y) = \begin{cases} \frac{1}{\tilde{H}}\tilde{h}_x(x)\tilde{h}_y(y) & \textit{if} \quad xy \geq m \\ 0 & \textit{if} \quad xy < m \end{cases}$$

$$\tilde{G}(w|x,y) = \begin{cases} \frac{\tilde{h}_y(q)}{\tilde{v}(q)}\frac{\tilde{v}(y)}{\tilde{h}_y(y)} \cdot \frac{\tilde{H}}{\tilde{h}_x(x)} & \textit{if} \quad xy \geq m \\ 0 & \textit{if} \quad xy < m \end{cases}$$

Again, these objects are pinned down by the firm recruiting effort which is determined by the FEC.

Free-Entry Condition

The free-entry condition in which $\Pi_0 = 0$ still holds, as do all the derivations to arrived to the expression for $n(y)$, the only object that changes in this case is the expected value of filling a vacancy which is

$$\begin{aligned} kJ(y;m) = k &\int_{\underline{\hat{x}}(y)}^{\overline{x}} S(x',y;m)\tilde{u}(x')dx' \\ &+ sk \int_{\underline{y}}^{y} \int_{\underline{\hat{x}}(y)}^{\overline{x}} \left(S(x',y;m) - S(x',y';m)\right) \tilde{h}(x',y')dy'dx'. \end{aligned}$$

As it seems clear from the above equation, and figure 1, the firm with lowest productivity is undoubtedly worse off because it has less workers to fish from. As we consider higher productivities, firms still lose from those they cannot make profits any more, however they have at their disposal the unemployed not poached by less productive firms, leaving them gradually better off.

III. Political Economy

Once the basic framework of the labour market with minimum wages has already been deployed, it is time to consider the bargaining protocol between working unions and employers associations. One of the chief contributions of this work is to endogenise the decision to set minimum wages by reckoning the role of unions and employers. As it is common in collective bargaining systems where the bulk of negotiations are carried out at a sectoral level, what is agreed between unions and employers is usually extended to other participants in the labour market. I focus on the extreme case where collective agreements are applied to the whole labour market, regardless of workers and firms being affiliated to their representative associations or having participated in the political process.

There is much to say about union and employer preferences, what triggers the decision to vote and who can actually vote (there is usually no universal suffrage), representation at the negotiating table, who is affected and accountability. However, I abstract from most of these concerns to keep the model simple and tractable. Nonetheless, two important factors within the political economy sphere are considered. First, who is allowed or able to vote? Either because legal clauses or collective action constraints, participation in the negotiating table is subject to firms reaching a certain size, which in turn means that only the voice of workers in these firms are heard. In terms of my model this requires the following condition

$$\tilde{h}_y(y) \geq \overline{h}_y.$$

Where $\overline{h}_y$ is the minimum size of a firm to participate of the political process, and the workers within it.

Another concern is about union and employer preferences. Traditionally, unions preferences have been modelled to take into account the fact that they may care in one way or another about wages, unemployment, or income distribution. On the other side of the market, firms have been assumed to maximise profits. At this point is when the complexity of the model starts to pay off. In the present work I deviate from the assumption that unions represent their affiliates and instead they consider the utility of those who actually vote. All the concerns about wages, unemployment and income distribution are directly or indirectly considered through the values that employees assign to them. The functional form reckoned for the union is that of an utilitarian objective function like

$$T(m) = \int_{\underline{y}}^{\overline{y}} \int_{\underline{x}}^{\overline{x}} \int_{w_{min}}^{\overline{w}} \left(W_1\left(w, x, y; m\right) - W_0\left(x; m\right)\right) \tilde{G}(w|x, y)\tilde{h}(x, y) dw dx dy.$$

And same is applicable in the firm side

$$E(m) = \int_{\underline{y}}^{\overline{y}} \int_{\underline{x}}^{\overline{x}} \int_{w_{min}}^{\overline{w}} \left(\Pi_1\left(w, x, y; m\right) - \Pi_0\left(y; m\right)\right) \tilde{G}(w|x, y)\tilde{h}(x, y) dw dx dy.$$

Once we know the preferences of unions and employers; and who can vote, we are ready to introduce them into the analysis. In this case, unions and employers do not bargain for wages, employment levels and income distribution directly but they set the level of minimum wages that maximises the Nash-bargaining solution of their respective utilities, or in other words

$$m^* = \arg\max_m \quad E^{1-\alpha}(m) \cdot T^{\alpha}(m)$$
$$s.t. \quad \tilde{h}_y(y) \geq \overline{h}_y.$$

On the offered side of the market, unions face the typical trade off, higher wages despite higher unemployment for their represented, assuming there is no general equilibrium effects; in the firm side, principals are worse off if just because they have to pay higher wages, in addition low productive firms are not able to hire low productive workers. On the other hand, both coalitions face an additional channel due to the congestion externalities that they exert on each other. Less employment means vacancies are easier to fill, especially for those firms that do not have to lay off workers; on the other side of the market, high skill workers encounter wage offers more frequently. The net effect is ambiguous and structural estimation is carried out to discern what effect is stronger.

A. *Preliminary Theoretical Results*

The estimation of the model is a work in progress at the point of writing these lines. therefore I show the results for particular set of parameters. The exercise consist on considering the value of jobs in firms that meet a certain threshold, specifically I consider the matches in the largest firms that add up to the 10% of the working force. Unions and employer's associations bargain on behalf these workers and firms. As pointed out previously, they choose the level of minimum wages that maximise their weighted utilities, or in other words a their social welfare function which is $E^{1-\alpha} \cdot T^{\alpha}$. Then, when pushing up the wage floor this social welfare function displays a hump-shape form as shown in figure 2, where the red line is a fit of a third degree polynomial. It seems clear from the picture that the level of minimum wages that these negotiator would choose would be around 1.5

However, other thresholds might be consider as well. It is fair to ask, what would the legal minimum had been if we had increased the threshold to allow the top 20% to participate in negotiations? And what about 30%, 40%, etc. till we allow the who workforce to participate? The next graph in figure 3 shows the social welfare functions for the different thresholds and their respective firm size cuts. Higher thresholds means lower skill workers and low productivity firms taking part in negotiations. Since they are the most negatively affected by the increase, they are the most interested in blocking any upward update of the minimum.

As it is clear from the picture, higher thresholds mean less minimum wages being lower. The next graph makes the relation between the participation threshold and minimum wage somewhat more transparent

All in all, these preliminary results show that rising the legal threshold for participation in the negotiating table or encouraging participation, has positive results in the welfare of participants in the labour market as a whole, for this look at the black curve in figure 3 representing the social welfare function when every one is allowed to vote, this curve attains its maximum when there is no minimum wage in place.

Figure 2. : Top 10% SWF

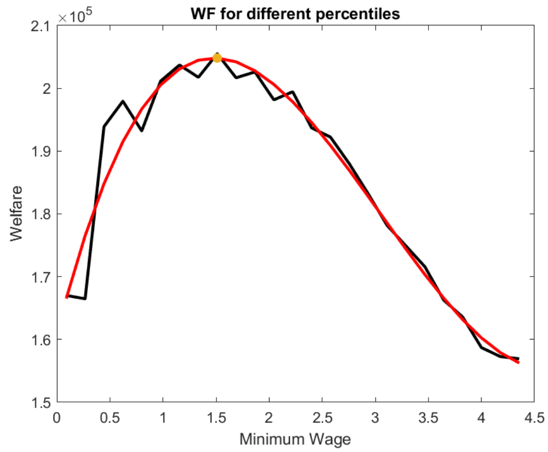
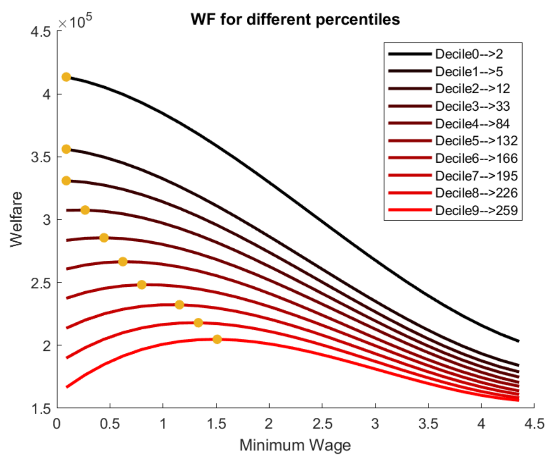


Figure 3. : Different Percentiles

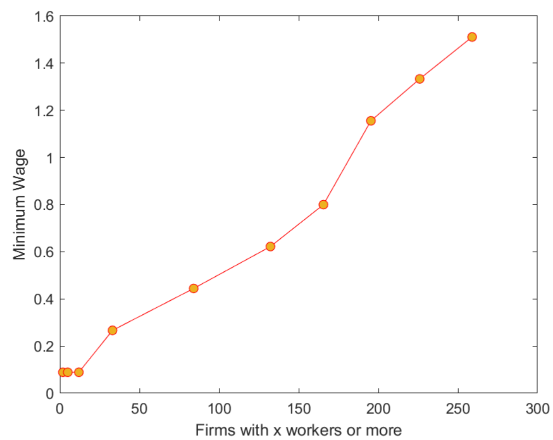


IV. The Institutional Setting

Most of the theoretical literature about trade unions revolves around their affiliates. However, labour market in Spain is framed within the class of collective negotiating systems where bargaining is a public good, typical of Southern Europe. These systems are characterised by a low density but high coverage of the labour force, regardless the employee is affiliated or not.

Nonetheless the regulatory framework in Spain has some peculiarities worth mentioning. First of all, the important agent in the industrial relations is not the trade union but the

Figure 4. : Minimum Wage as Function of Threshold



work council or committee (comité) instead, which is the collegiate organism within companies of more than 50 workers in charge of representing the staff. This council is composed of 5 to 75 members, depending on the size of the firm, and is directly elected among the workforce. Members of the committee, also delegates, can be either union affiliates or independent workers. This 'elections feature' is what justifies the 'public good' facet of the settlement. Then, the number of elected committee members in each firm are recorded by the Ministry of Labour to assign representation at higher levels of negotiations.

Another relevant trait is the way sector-wide negotiations are carried out. The key institution here is the 'Bargaining Commission' which is the body in charge of reaching an understanding. This body is composed of employers associations and trade unions, not independent workers in this case. As noted before, the number of unionised delegates at firm-level elections are recorded and taking into account to assign the representation at higher levels, therefore each union is represented in accordance to their popularity. For example, if there is Union 1 and Union 2 in Firm 1 obtaining 3 and 1 delegates respectively and there is Union 1 and Union 2 in Firm 2 obtaining 1 delegate each, then the bargaining commission will be composed of two thirds of Union 1 and one third of Union 2.

The result of these negotiations is the 'Collective Agreement', whether carried out at firm-level by the committee or at sector-wide by the bargaining commission. This contract rules over any possible matter related to labour, being the most prominent: wage increases, minimum wages, hours and employment. The agreement is published in the Official Bulletin of the State (BOE), has rank of law and Judges might use it to solve potential disputes between workers and employers. Then, this collective contract serves as the minimum standard individual contracts must have. What is more, firm-level agreements override to sector-wide ones, within the latter a narrower scope cannot confront those with a wider one. For example, a province-level agreement for metalurgy cannot set lower standards than a national-level one, I will discuss in more detail in the following section.

V. Data

A. MCVL

To test the model described in previous sections, I make use the *Muestra Continua de Vidas Laborales* (MCVL), a database with complete working histories of employees. The MCVL is a 4% sample of population having a relation with the *Tesorería General de la Seguridad Social* (TGSS) in the year of reference (2015-2017). The TGSS is the institution in charge of the social security finances and releases this database on a yearly basis.

This database has exhaustive information of complete working histories of workers. Relevant for this work are variables relating personal characteristics of the worker such as age, nationality, genre, etc.; Information about firms like id, type of employer, sector and size; large set of features regarding the job relation e.g. type of contract, start and end dates of the relationship, skill, etc. and the database provides information about the monthly base rates of contributions.

Due to information and time constraints explained below the sub-sample of workers selected for the present study are those in the metal sector in the region of Madrid between years 2015 and 2017. As pointed out previously all of them are to be covered by a collective agreement, in particular the agreement *'CONVENIO COLECTIVO DE LA INDUSTRIA, SERVICIOS E INSTALACIONES DEL METAL DE LA COMUNIDAD DE MADRID'* that appeared in the Spanish Official Bulletin on 2 January 2016.

Workers in this subsample are grouped in four categories according to wage floors: unskilled manual workers, administrative stuff, skilled workers, technical supervisors and engineers and graduates. Table 3 shows descriptive statistics of the mean wage, wage floor, wage cushion and the percentage of people for which the wage floor binds.

Table 1—: Number of observations and means for selected variables

Categories	Number of observations	Wage, (euros)	Wage floor (euros)	Wage cushion (euros)	Workers earning the wage floor (%)
Unskilled manual workers	1500	1915.1	1273.6	644.1	4.00
Administrative stuff	160	1612.4	1372.7	235.6	16.25
Skilled workers	235	2407.3	1471.6	950.3	4.68
Technical supervisors	137	2776.9	1646.4	1141.0	8.76
Engineers and graduates	67	2816.9	1813.4	1009.7	11.94
Managers	174	3395.2	2161.3	1265.8	0.57

As expected, higher categories earn higher wages and higher wage floors. What is interesting to notice is the fact that higher categories have also higher wage cushions, meaning that the excess of salary above the wage floor is also lager. As one can see, the wage cushion is not monotonically increasing with the category hold, this is because the skill level of the employed in this classification is taken into account two dimensions, responsibility and level of education. In this respect, if we consider two skilled workers, one with a level of education vocational training that is supervisor and the other a graduate but that has just been hired in the company, they will end in two different categories, namely technical supervisors and engineers respectively, the latter will be in a higher category, with a higher wage floor, just because she is a graduate.

It is worth noticing that managers have been left out of the analysis, the reason why they have not been taken into the analysis is twofold. There is no clear sign in the data that this category are affected by its corresponding wage floor. One rationale is that these high level categories have some bargaining power to set their salaries well above and beyond the wage floor. Which leads to the second point, all throughout my analysis I have imposed that wages are not bargained; then had I taken managers, it would have distorted the results, see Cahuc, Postel-Vinay and Robin (2006) for a seminal paper on this matter.

Apart from that, workers with a salary more than 5000 euros have been removed from the sample for two reasons. First of all, the maximum base of contribution that is taken into account is around 4000 euros for each job relation. However, the social security does not cap the base if the worker happens to have more than one employer or if there has been a job-to-job transition with a substantial wage increase. A second ground is more practical, wages over 5000 euros distorts the wage distribution unreliably. Last, it is unlikely that a worker of a low category earns such a salary. Although considering wages with measurement error is a possibility, it is not within the scope of the current study.

Last but not least workers with a tenure of more than 12 months have been taken into account since lower thresholds apply for those with a lesser duration.

B. *Collective Agreements*

As I only focus in one sector and province, the collective agreement that applies is clearly identified. Nonetheless, the skill categories that the social security assigns to workers does not have to coincide with the ones negotiated in collective agreements. In this respect, I have followed the work of Adamopoulou and Villanueva (2020) where they match skill categories for the MCVL. So one challenge is to assign the levels of minimum wages to the correct skill which in the case of the present work has been done by visual exploration, i.e. identifying where is the mass point in the wage level data for each level in the MCVL and assign it to the corresponding level in the collective contract.

Although Card and Cardoso (2021) have pointed the way to go with a vast linkage of collective agreements to worker level data, this process turns out to be a daunting task in the case of Spain since the TGSS does not deliver such information, even though they dispose of it for their inner purposes. Despite of that, the MCVL has two advantages with the respect to *Quadros de Pesoal* which is the Portuguese counterpart. As is common in collective agreements they are usually negotiated for long periods of time that entered the periods over which the clauses are being negotiated, consequently retroactive measures have to be taken. For example, negotiations of collective agreements end up after the period covered. So, a collective agreement signed in 2016, could well rule in 2015. Thus, if wages are updated at a posterior date, one should take into account what part of the increase corresponds to what period. The advantage of MCVL is that it writes a correction in the previous months that are affected and then we do not need to worry of taking backdating into account.

Another concerned could have been the statuary minimum wage which potentially could overlap with wage floors considered. This is certainly not the case of this work, as the sector considered is well above the national minimum wage at the time considered (2017). Even so, minimum wages are unlikely to interfere with wage floors as in the time considered minimum wages were so low that it is unlikely that they overrule sectoral ones.

VI. Estimation

A. Method

Using the data described in the previous section I take the model to data using the simulated minimum distance estimator (SMD). As is common in the literature, see McFadden (1989), moments are taken from data and stored in a vector in which the k^{th}-element is $m_k = \frac{1}{N}\sum_{i=1}^{N} m_{ki}$, where m_k might be any statistic of interest like mean duration of unemployment. Then, with the model at hand, the same information is simulated given a set of parameters θ, as such the k^{th}-simulated moment is defined as $\widehat{m_k}(\theta) = \frac{1}{N}\sum_{i=1}^{N} \widehat{m_{ki}}(\theta)$. Then the objective is to make these simulated moments as close as possible as their data counterparts. Then, the problem lays on retrieving the parameters that make the loss function as close to zero as possible, or in other words

$$\widehat{\theta}_N = argmin\Big\{ \big(m_k - \widehat{m_k}\,(\theta)\,\big) W \big(m_k - \widehat{m_k}\,(\theta)\,\big) \Big\}.$$

Where W is a variance weighting matrix, although in the econometric estimation it is just a diagonal matrix with the inverse of variance of their respective moments, which is the proper method to account for different in orders of magnitude.

Once the programme has been set, it is only to choose the right moments to match in order to retrieve the underlying parameters. Since my model introduces free entry of firms to the canonical Postel-Vinay and Robin (2002), it is difficult to argue what moments identify what parameters since moments are affected by all parameters of the model in some way. The approach taken here is the one of Lise and Robin (2017), where they use an heuristic approach to identify all the parameters at once, making sense of the sensitivity of some moments with respect to the parameters that they mean to identify.

Another possible approach, it is the one taken by Flinn and Mullins (2019), where they first solve for a partial equilibrium, i.e. keeping fix the rates at which searchers encounter, which in the present work would be $kv(y), skv(y), u(x)$ and $skh(x, y)$, and take them from data, then they estimate the offer side parameters like the measurement error of wages and worker heterogeneity. Last, they impose some values on the elasticity of number of matches with respect to the number of vacancies and the parameter of the cost function, which they use to back out the rest of demand side parameters. So, even though all parameters are identified not all of them are treated as general equilibrium objects, which is the reason why the work of Lise and Robin (2017).

This work shows a heuristic representation of identification as in the works of Lise and Robin (2017) and Lise, Meghir and Robin (2016) measuring the sensitivity of some moments as parameters change. I use of the observed distribution of wages, actually 9 deciles, and the mass point at the minimum wage in order to identify parameters related to worker ability $\underline{x}, \overline{x}, a_x, b_x$ and firm productivity $\underline{y}$ and $\overline{y}$. The higher the upper support of worker ability and firm productivity the larger the upper support of the wage distribution, and upper deciles will be more affected as a result. Lower deciles are more sensitive to the lower support of $\underline{x}$ and $\underline{y}$. Depending on how concentrated are wages along deciles, a_x and b_x, skew wages towards the left or right tail depending on the relative strength of the two parameters. Special mention deserves the difference between $\underline{y}$ and $\overline{y}$,

Moments related to durations serve to identify the relative search intensity s, the more effort employed workers exert with respect to to those unemployed the shorter the tenures and the shorter the duration of unemployment. Deciles of the size distribution of firms have also been considered to identify parameters related to costs of opening vacancies c_0 and c_1, which are linked to the free entry condition. The lesser the costs the more vacancies each company opens, there will be more matches and consequently k will rise, in turn durations will be affected accordingly by this.

B. *Estimation of Parameters*

Table of the full set of parameters to be estimated is presented in table 2.

Table 2—: Parameters

Abilities support	$\overline{x}$	11.67	Productivity support	$\overline{y}$	245.14
	$\underline{x}$	1.33		$\underline{y}$	285.41
Worker heterogeneity	a_x	15.63	Vacancy costs	c_0	1,448.17
	b_x	19.61		c_1	0.09
Search intensity	s	0.18			

Search effort exerted by employed workers is in line with with works of Lise, Meghir and Robin (2016), Lise and Robin (2017) or Flinn and Mullins (2019), the estimate of $s = 0.18$ is a little bit smaller than in those works, which is coherent with the fact of higher unemployment seen in the Spanish labour market. Parameters that govern the number of jobs, c_0 and c_1, resemble also those seen in the mentioned literature, it actually lies in between of those estimated by Flinn and Mullins (2019). Every opening is more expensive than the previous one, i.e. $1 + c_1 = 1.09$, being the function convex and guaranteeing the equilibrium exists. Also each new opening is going to increase by a factor of $c_0 = 1,448.17$, which seems high but explains the rationale of low firm competition in Spain. Turning to firm related parameters $\underline{y} = 245.14$ and $\overline{y} = 285.41$, it seems that they are relatively close with respect to other estimations carried out, actually they need not to be far apart since it is known that these models give too much market power to large firms, if the difference between these two parameter were too high, wage distributions would be unreasonably skewed towards the lef-tail, which is at odds to a mere visual inspection of the data. Little more can be said about these parameters since they are just a way of ranking firms.

Figure 5. : Distribution of workers abilities

Parameters related to workers do not have a direct economic interpretation, as stated before $\underline{x} = 1.33$ and $\overline{x} = 11.57$ pin down the support of wage distribution together with parameters connected to firm productivity. As seen in figure 5 the distribution of abilities in the population is slightly skewed towards the right tail, meaning that there are relatively more workers of low ability.

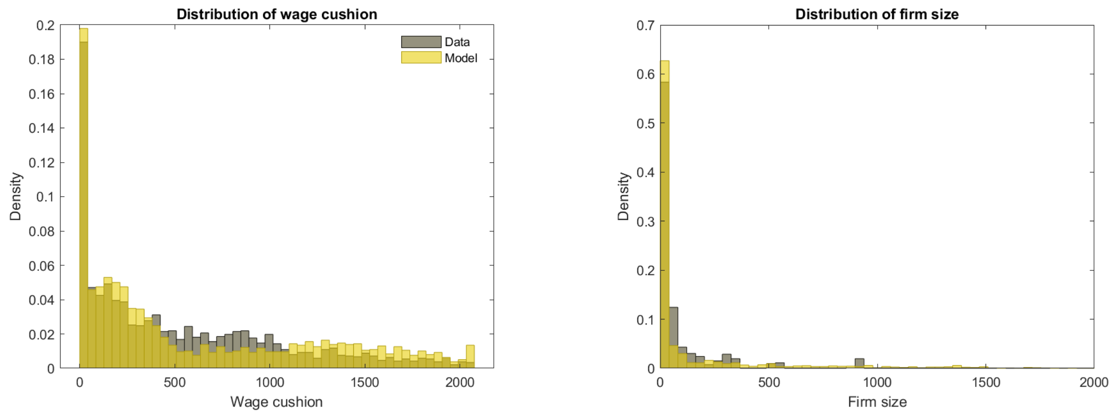
C. Fit

This section presents empirical and simulated moments in graphs 6a and 6b as well as table 3.

As it seems clear from figure 6a the model fit the distribution of wage cushions closely. Nonetheless, the model tends to over concentrate the distribution over right-end, this is because there are too many large firms in the model. Large firms find it easier to poach workers form low productive ones, because workers are able to bring these same firms into competition, they are granted wage increases more often of what we see in the data, and as a result higher wages. Now, turning to distribution of firms across workers we see that is not perfect, the model underestimates the number of workers in low productive firms, whereas it has a thick long right-tail. High productive firms always win the sequential auction model of offers and counter offers, whenever they come across an unemployed or an employed from a firm with lower productivity they sum one worker to their workforce. As such, in order to generate a wage distribution with a big mass point at the minimum, unreliably large firms are encounter.

Looking at moments associated to durations, we see that the model does a good job fitting the $\frac{H}{U}$-ratio, meaning that there are three times more employed workers than unemployed

Figure 6. : Fit of data distributions: wage cushion and firms



(a) Distribution of the wage cushion (b) Distribution of firm size across workers

which results in about 25% of unemployment ratio. As a side effect, unemployment duration will be inevitably high, being the mean duration of unemployment of about 2 years and a half, as unreasonably high as it might, I am focusing at non-employment, and I am not correcting for the fact that people might have leaved the labour force. The largest deviation is from the moments of job duration where my model predicts 7 years of a duration of a job, instead of 5 as we see in the data.

Table 3—: Duration moments

Moments	Data	Model
Unemployment duration	30.1	34.4
Job duration	61.2	84.3
$\frac{H}{U}$-ratio	2.98	2.98

VII. Labour Market Outcomes

Minimum wages affect both sides of the market through different channels. Minimum wages raise the unemployment value of the worker creating incentives to become part of the labour force. Workers that previously earned less that the minimum see their wages rise and simultaneously it is been documented that minimum wages have spill-overs through the wage distribution, so workers that have wages close to the minimum also experience wage increases, Autor, Manning and Smith (2016). This comes at a cost of reducing employment, however, little evidence has been found in this respect, see Card and Krueger (2015) book for a review. Nonetheless, Neumark, Schweitzer and Wascher (2004) have pointed that low skilled workers suffer the most through reduction of hours and employability. On the demand side, firms might adjust through several channels, the most obvious one being employment as commented before. Another possible channel is by hiring higher skill workers, indeed as minimum wage increases, before profitable matches are not anymore and firms have to hire workers with higher ability to cover the same vacancy. Less attention has been drawn to hours effects and the probabilities of part-time or full-time unemployment, for example Katz and Krueger (1992) find that after a wage increase causes firms to substitute part-time jobs by full-time employment. Out of the labour market firms might be able to pass-through higher costs to prices in their products, which depends on the level of competition in the product market.

In the following sections variables like unemployment, employability, wage dynamics, spill-overs and wage inequality will be considered. Because of how the model has been constructed or because data constraints I will not look at other channels through which minimum wages affect labour market outcomes. Hours will be left out of the analysis since the model is not well suited for this purpose, upon dealing with data wages have been considered in full-time equivalent units (8 hours). Labour market participation is not considered due to the nature of data, as opposed to surveys like the Current Population Survey where people are asked about their searching status, administrative data only records the periods that workers has been on formal employment and because of this limitation, non-employment is regarded instead. Channelling hiking costs via prices is also out the scope of the present work since the data is not well suited for this purpose nor it is the theoretical model.

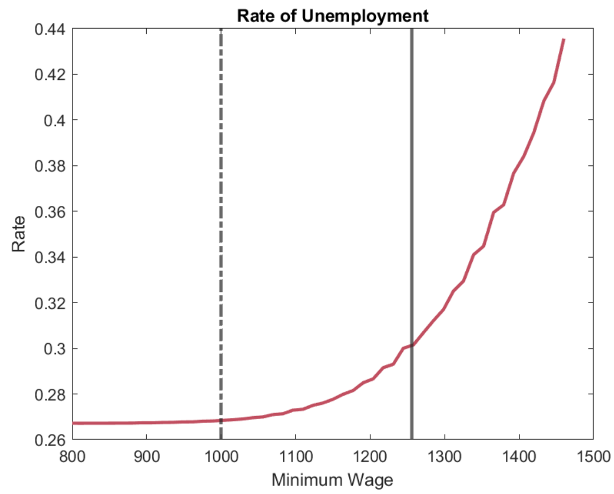
In the following sections I will discus the effect of minimum wages on unemployment, employability, and wage inequality.

A. *Unemployment*

Among the different channels that firms use to accommodate an increase in minimum wages, the most controversial one that has been in the limelight since Card and Krueger (1994) paper is unemployment. Here, I show a counterfactual analysis for the particular market under study showing that moderate increases of minimum wages do not have an effect on unemployment. However, when minimum wages are hiked too much they start having detrimental effects.

The current level of wage floor in this market, metal industry in Madrid 2015-2017, is negotiated at €$1,255.91$, with the rest of wage floors normalised to this level. At this level the rate of unemployment is 30.3%, which is the one we actually see in the data. After parameters of the model have been estimated for this level of wage floor, the counterfactual analysis has been carried out taking these parameters as fixed and shifting the wage from the actual level in a range that spans between $m \in [800, 1450]$ euros, as we can see from figure 7 moderate increases up to €$1,000$ in the minimum wage do not interfere with firm productivity and the level of unemployment rate would be 26.9%, this percentage can be taken as the unemployment rate in the present of frictions. Once this threshold is surpassed, the wage floor adds to these frictions and unemployment increases. In the end the wage floor add 3.49p.p. to unemployment from what it would have been without them at their current level.

Figure 7. : Rate of unemployment as a function of the minimum wage



Note: The solid vertical line represents the current level of wage floor at €$1,255.91$ The dash-dotted line indicates the level at €$1,000$

B. Employability

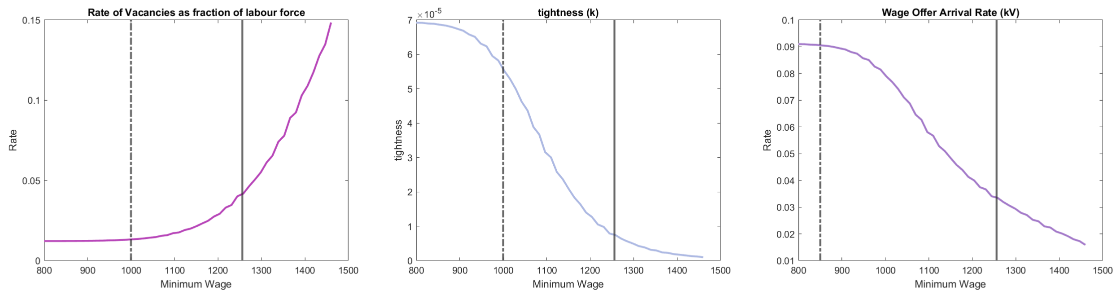
When imposing a wage floor there are two effects that impact in employability. The first one is mechanical, from the supply side, as the wage floor is increased the set of possible firms that are available for pairing is reduced, as shown in the theoretical part, only firms in the range $y \in [\underline{\hat{y}}(x), \overline{y}]$ will be available for workers with ability x, and the same mechanics are present in the other side of the market, where relatively low productive firms are not able to match workers with low capabilities to produce enough to pay for the minimum wage.

The second factor takes into account general equilibrium effects inside the labour market, the discussion goes along the same lines as with the unemployment rate. At the current wage floor of €$1,255.91$ the vacancy rate is 5% of the population, had the wage floor been set at €$1,000$, the vacancy rate would have been about 1%. At the same time the measure of the tightness falls sharply as the increase in the number of matches does not offset the increasing numbers in the stocks of unemployed and vacancies in the economy. when coupling both effects the outcome is the offer arrival rate to unemployed worker kV, the offer arrival rate is just a vertical stretching of size s. The result could be either increasing or decreasing, but for reasonable estimates of model parameters we see that the offer arrival rate is decreasing in the whole domain of wage floors. The probability of encountering a valid job offer is three times smaller than if there was not a wage floor in place, see figure 8c.

C. Wage Inequality

I now turn to the analysis of wage inequality. Dinardo, Fortin and Lemieux (1996) show how the diminishing of real minimum wage is a major reason for increasing inequality. There,

Figure 8. : Fit of data distributions: wage cushion and firms



(a) Vacancy rate $\frac{v}{L}$ (b) Tightness: k (c) Offer arrival rate: kV

they assume there are no spill overs and no disemployment effects, which is not the case of the present paper and I take both causes, and the direct effect, in turn. As in the cited paper, minimum wages affect directly the distribution of wages by just mechanically cutting off all the wages below it, some workers will go to unemployment and others will experience a wage increase.

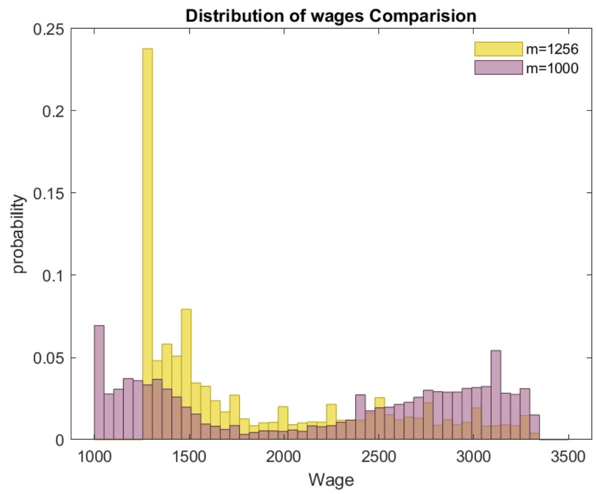
Another channel is the spillover effect. As we have seen in the theoretical part, when minimum wages are increased, workers experience higher values as employed just because they now have the opportunity to work in high productivity firms earning no less than the minimum wage. Higher values of employment result in wages above what they would have had if the minimum wage had not been in place, even if the minimum wage is not biding.

Now, in the previous section we have seen that raising the minimum wage has negative effects on the probability of arrival of wage offers and therefore U-E, E-E transitions and wage increases will be reduced accordingly. Looking at the case of an unemployed worker, she receives less offers when employed, meaning that she will lose fewer opportunities in the future and will start with a higher entry wage. So, raising the minimum wage does not only have the direct effect of compressing the wage distribution by rising the lower support or through spillovers but also by increasing entry wages.

Turning to the employed, they encounter less wage offers that end up in a wage increase, also they receive less wage offers from high productive firms, implying in fewer job-to-job transitions. Thus, careers begin with a higher starting wage but at a cost of being stagnated for longer.

In figure 9 the effects of previous mechanisms are at play for two levels of the wage floor. As we can see, when the level of wage floor is at the current degree of €$1,255.91$, the wage distribution has a large mass point in the lower support and wage are concentrated right above this threshold for the reason explained before: Higher lower support, spillovers and higher entry wages. As we move along the distribution to the right tail, wages are being less concentrated because workers do not have as many opportunities. If the minimum wage were lowered to €$1,000$ all of this channels would be at work in the reverse order. Lower minimum means the lower support is going to be inferior, also poaching firms will be able to drag wages

Figure 9. : Comparison of Wage distributions under Two Wage Floors



down further, which results in spillovers being attenuated. Higher rate of wage offer arrivals turns out in reduced entry wages. On the other end of the distribution, these higher rate of wage offers subsequently end in more vibrant careers with more job-to-job transitions and pay rises more often.

VIII. Welfare and Bargaining

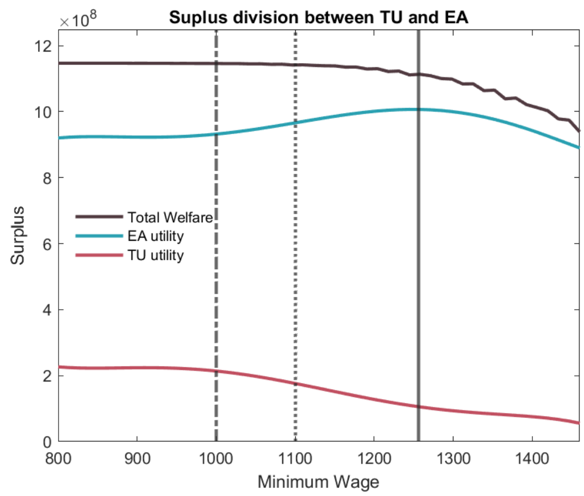
Once the effects and channels through which minimum wages affect labour market outcomes have already been scrutinised, it is just left to know how these minimum wages come about. In Spain, only firms with with 50 workers of more are able to set working councils and not all of them actually set one, large firms and their workers might impose the level of minimum wages to their advantage. Also the level of minimum wages will depend on the bargaining power of both parties. As suggested in the theory section, unions and employers associations negotiate to impose the minimum wage subject to the firms being big enough, the approach taken there is axiomatic and no assumption is done about what the bargaining power α means. Nonetheless, it might be reinterpreted as the relative discounting factors, Ariel Rubinstein (1982).

Since unions and employers associations have different bargaining power and utilities as a function of the minimum wage, it might well be that they reach an agreement where neither of them attain maximum utility. The key question is which party will be able to drag the level of wage floor closer to their interests. This question can be answered properly after estimating the model parameters and retrieving the values of agents in this market which are subsequently aggregated by their representatives giving equal weights to all individuals.

From figure 10 we can see that minimum wages below €1,000 do not interfere with neither the total welfare nor the utilities of representative agents. Surprisingly, from ranges above

€1,000 the utility of the employers associations starts increasing up to €1,260, the reason why this behaviour is due to the increment of unemployed stock, who in turn are at the disposal of these firms. Because it is easier to poach an unemployed worker, firms derive more expected value of filling a vacancy. When this level of minimum wages is surpassed, the level of unemployment becomes too high and hiring becomes more and more difficult, reducing effectively the utility of the employers associations. On the other side of the market, workers are negatively affected by the increase in minimum wages in the whole domain. Regarding total welfare, it is worth realising that as utilities of both organisations move in opposite directions up to €1,100, in this way they offset each other when considering Total welfare in the labour market and starts diminishing as wage floors are again too high.

Figure 10. : Welfare division



All in all, the fact that the wage floor is set at €1,255.91 suggests that employers associations have the most bargaining power when deciding the level. Back of the envelope estimations of this bargaining power have been estimated to be $\alpha = .77$.

IX. Discussion and Conclusions

The most important contribution of this paper is a theoretical framework that accommodates the European collective bargaining system, so as to rationalise how negotiating parties use minimum wages to affect labour market outcomes to their advantage. Using this framework, I have carried out the estimation that allows me to measure the different channels through which wage floors spread their impact. Another important trait is that minimum wages are endogeneised, leading to results that partially contradict the view that minimum wages do not have an effect on employment. Nonetheless, there are two features that should be taken into account for future research. First, several wage floors should be considered, so as to capture that different categories are subject to higher wage cushions, research in this area has recently been done and easily to conform. Another feature to be addressed is the large market power of firms, which remains unchallenged by the present work, however this problem could be tackled introducing multiworker firms and cost specific functions. The main advantage of this work is that it can be readily be used in other environments where the access to administrative data and collective agreements is easily accessible.

REFERENCES

Adamopoulou, Effrosyni, and Ernesto Villanueva. 2020. "Wage Determination and the Bite of Collective Contracts in Italy and Spain: Evidence from the Metal Working Industry."

Ariel Rubinstein. 1982. "Perfect Equilibrium in a Bargaining Model." *The Econometric Society*, 50(1): 97–109.

Autor, By David H, Alan Manning, and Christopher L Smith. 2016. "The Contribution of the Minimum Wage to US Wage Inequality over Three Decades: A Reassessment." *American Economic Journal: Applied Economics*, 8(1): 58–99.

Booth, Alison. 1995. *The economics of the Trade Union.* Cambridge:Cambridge University Press.

Cahuc, Pierre, Fabien Postel-Vinay, and Jean Marc Robin. 2006. "Wage bargaining with on-the-job search: Theory and evidence." *Econometrica*, 74: 323–364.

Calmfors, Lars, and John Driffill. 1988. "Centralization of wage bargaining and macroeconomic performance." *Economic Policy*, , (131): 1–60.

Card, David, and Alan B. Krueger. 1994. "Minimum Wages and Employment: A Case Study of the Fast-Food Industry in New Jersey and Pennsylvania." *The American Economic Review*, 84(4): 772–793.

Card, David, and Alan B Krueger. 2015. *Myth and Measurement: The New Economics of the Minimum Wage-Twentieth-Anniversary Edition.* Princeton University Press.

Card, David, and Ana Rute Cardoso. 2021. "Wage Flexibility under Sectoral Bargaining Wage Flexibility under Sectoral Bargaining."

Cardoso, Ana Rute, and Pedro Portugal. 2005. "Contractual Wages and the Wage Cushion under Different Bargaining Settings." *Journal of Labor Economics*, 23(4): 875–902.

Dey, Matthew S., and Christopher J. Flinn. 2005. "An equilibrium model of health insurance provision and wage determination." *Econometrica*, 73: 571–627.

Dinardo, B Y John, Nicole M Fortin, and Thomas Lemieux. 1996. "Labor Market Institutions and the Distribution of Wages , 1973-1992 : A Semiparametric Approach." *Econometrica*, 64(5): 1001–1044.

Flinn, Christopher J., and James Mabli. 2009. "On-the-Job Search, Minimum Wages, and Labor Market Outcomes in an Equilibrium Bargaining Framework." *mimeo*.

Flinn, Christopher J., and Joseph Mullins. 2019. "Firms' Choices of Wage-Setting Protocols in the Presence of Minimum Wages." *Mimeo*, 1–71.

Hartog, Joop, Edwin Leuven, and Coen Teulings. 2002. "Wages and the bargaining regime in a corporatist setting: The Netherlands." *European Journal of Political Economy*, 18: 317–331.

Katz, Lawrence F, and Alan B Krueger. 1992. "The effect of the minimum wage on the fast-food industry." *ILR Review*, 46(1): 6–21.

Krusell, Per, and Leena Rudanko. 2016. "Unions in a frictional labor market." *Journal of Monetary Economics*, 80: 35–50.

Lise, Jeremy, and Jean Marc Robin. 2017. "The macrodynamics of sorting between workers and firms." *American Economic Review*, 107: 1104–1135.

Lise, Jeremy, Costas Meghir, and Jean Marc Robin. 2016. "Matching, sorting and wages." *Review of Economic Dynamics*, 19: 63–87.

McFadden, Daniel. 1989. "A Method of Simulated Moments for Estimation of Discrete Response Models Without Numerical Integration." *Econometrica*, 57(5): 995–1026.

Neumark, David, Mark Schweitzer, and William Wascher. 2004. "Minimum Wage Effects throughout." *The Journal of Human Resources*, 39(2): 425–450.

Postel-Vinay, Fabien, and Jean Marc Robin. 2002. "Equilibrium wage dispersion with worker and employer heterogeneity." *Econometrica*, 70: 2295–2350.

Mathematical Appendix

A1. Distributions

In this appendix detailed derivations for $v(y)$, $u(x)$ and $h(x,y)$ are worked out. We start with the balance conditions, which are no more that accounting identities, that are met in every point in time

$$\int h_t(x,y')dy' + u_t(x) = l_t(x)$$
$$\int h_t(x',y)dx' + v_t(y) = n_t(y)$$

and flow equations in discrete time.

$$
\begin{aligned}
h_{t+1}(x,y) &= h_t(x,y) && +kv_t(y)u_t(x)\Delta + sk\int_{\underline{y}}^{y} h_t(x,y')dy'v_t(y)\Delta && -\delta h_t(x,y)\Delta - sk\int_{y}^{\overline{y}} v_t(y')dy'h_t(x,y)\Delta \\
u_{t+1}(x) &= u_t(x) && -kV_t u_t(x)\Delta && +\delta h_{x,t}(x)\Delta \\
v_{t+1}(y) &= v_t(y) && -kv(y)U_t\Delta - sk\int_{\underline{y}}^{y} h_{y,t}(y')dy'v_t(y)\Delta && +\delta h_{y,t}(y)\Delta + sk\int_{y}^{\overline{y}} v_t(y')dy'h_{y,t}(y)\Delta
\end{aligned}
$$

Where $h_{x,t}(x) = \int_{\underline{y}}^{\overline{y}} h_t(x,y')dy'$ and $h_{y,t}(y) = \int_{\underline{x}}^{\overline{x}} h_t(x',y)dx'$. The expressions are easier to work with in continuous time so I rearrange stocks to the LHS and flows to the RHS, Divide by Δ and take the limit as $\Delta \to 0$ to have

$$
\begin{aligned}
\dot{h}_t(x,y) &= kv_t(y)u_t(x) && +sk\int_{\underline{y}}^{y} h_t(x,y')dy'v_t(y) - \delta h_t(x,y) && -sk\int_{y}^{\overline{y}} v_t(y')dy'h_t(x,y) \\
\dot{u}_t(x) &= -kV_t u_t(x) && +\delta h_{x,t}(x) && \\
\dot{v}_t(y) &= -kv_t(y)U_t && -sk\int_{\underline{y}}^{y} h_{y,t}(y')dy'v_t(y) + \delta h_{y,t}(y) && +sk\int_{y}^{\overline{y}} v_t(y')dy'h_{y,t}(y)
\end{aligned}
$$

Before working out specific expressions for every type of firm and worker, it will be useful to calculate aggregate balance conditions, just aggregate over the set of firm productivities and worker abilities to have

$$
\begin{aligned}
H_t + U_t &= L_t \\
H_t + V_t &= N_t
\end{aligned}
$$

Where $H_t = \int\int h_t(x',y')dx'dy'$. L_t is exogenous and given N_t, V_t is pinned down, at this point both conditions are knotted by H_t, so we can write

$$
L_t - U_t = N_t - V_t
$$

And deriving with respect to time we have that $\dot{U}_t = \dot{V}_t$. Also, we need to have the expressions for the aggregate flows. Integrate $v(y)$, $u(x)$ and $h(x,y)$ over the variables that they depend on. Furthermore, in the aggregate all the workers that quit are the same as those who are poached, i.e. $\int_{\underline{y}}^{y} h_t(x,y')dy'v_t(y) = \int_{y}^{\overline{y}} v_t(y')dy'h_t(x,y)$, hence

$$
\left.
\begin{aligned}
\dot{H}_t &= kV_tU_t - \delta H_t \\
\dot{U}_t &= -kV_tU_t + \delta H_t \\
\dot{V}_t &= -kV_tU_t + \delta H_t
\end{aligned}
\right\} \overset{SS}{\Longrightarrow} \delta H_t = kV_tU_t
$$

Since we are in the S.S., time dependence can be dropped from the notation. Now, Plug the aggregate balance conditions to have H as a function of N, k (endogenous objects) and δ, L (parameters), $\delta H = k(N-H)(L-H)$, this is a quadratic equation on H

$$
kH^2 - (\delta + kL + kN)H + kLN = 0
$$

Which solves as

$$H = \frac{1}{2}\left\{\left(\frac{\delta}{k} + L + N\right) - \sqrt{\left(\frac{\delta}{k} + L + N\right)^2 - 4LN}\right\}$$

Below, it is the proof of why only the negative part is taken. First consider the positive part, i.e.

$$H(N) = \frac{1}{2}\left\{\left(\frac{\delta}{k} + L + N\right) + \sqrt{\left(\frac{\delta}{k} + L + N\right)^2 - 4LN}\right\} > \frac{1}{2}\left\{\left(\frac{\delta}{k} + L + N\right) + \sqrt{\left(\frac{\delta}{k} + L + N\right)^2}\right\} \Leftrightarrow$$
$$\frac{1}{2}\left\{\left(\frac{\delta}{k} + L + N\right) + \sqrt{\left(\frac{\delta}{k} + L + N\right)^2 - 4LN}\right\} > \left(\frac{\delta}{k} + L + N\right)$$

Which means that the number of employed is higher than the number of people in the economy, an absurdity. Turning to the negative part, I would like to work out the maximum and minimum values as a function of N. The minimum value can be easily worked out as

$$H(0) = \frac{1}{2}\left\{\left(\frac{\delta}{k} + L\right) - \sqrt{\left(\frac{\delta}{k} + L\right)^2}\right\} = 0$$

And the maximum

$$\lim_{N\to\infty} H(N) = \lim_{N\to\infty} \frac{1}{2}\left\{\left(\frac{\delta}{k} + L + N\right) - \sqrt{\left(\frac{\delta}{k} + L + N\right)^2 - 4LN}\right\}$$

Which is undetermined, dividing and multiplying by the complement and later on dividing by N in the numerator and denominator we have

$$\begin{aligned}
\lim_{N\to\infty} H(N) &= \lim_{N\to\infty} \frac{1}{2}\left\{\frac{4LN}{\left(\frac{\delta}{k} + L + N\right) + \sqrt{\left(\frac{\delta}{k} + L + N\right)^2 - 4LN}}\right\} \\
&= \lim_{N\to\infty} \frac{1}{2}\left\{\frac{4L}{\left(\frac{\delta}{Nk} + \frac{L}{N} + 1\right) + \sqrt{\left(\frac{\delta}{Nk} + \frac{L}{N} + 1\right)^2 + \frac{4L}{N}}}\right\} \\
&= \frac{1}{2}\left\{\frac{4L}{1 + \sqrt{1}}\right\} = L
\end{aligned}$$

Which means that as the number of firms tends to infinity the number of employed workers tends to the number of people in the economy. Also, it would be convenient to check if the function is increasing in the whole domain

$$\frac{\partial H}{\partial N} = \frac{1}{2}\left\{1 - \frac{2\left(\frac{\delta}{k} + L + N\right) - 4L}{\sqrt{\left(\frac{\delta}{k} + L + N\right)^2 - 4LN}}\right\} > 0$$

Using the balance conditions U and V can easily be derived. With this expressions at hand we can work out their desegregated counterparts. First, consider the S.S. and drop the time dependence,so

$$\begin{aligned}
0 &= kv(y)u(x) && +sk\int_{\underline{y}}^{y} h(x,y')dy'v(y) - \delta h(x,y) && -sk\int_{y}^{\overline{y}} v(y')dy'h(x,y)\\
0 &= -kVu(x) && \qquad\qquad\qquad\qquad + \delta h_x(x) && \\
0 &= -kv(y)U && -sk\int_{\underline{y}}^{y} h_y(y')dy'v(y) + \delta h_y(y) && +sk\int_{y}^{\overline{y}} v(y')dy'h_y(y)
\end{aligned}$$

Now, consider the last expression and rearrange such that U-to-H and H-to-U flows are on the LHS and H-to-H flows are in the RHS. Also, for notational convenience let's write $F_z(z) = \int_{\underline{z}}^{z} f_z(z')dz'$ for any function over the z-characteristic and denote its complement counterpart as $\overline{F}_z(z) = F_z(\overline{z}) - F_z(z) = \int_{z}^{\overline{z}} f_z(z')dz'$

$$kv(y)U - \delta h_y(y) = -skH_y(y)v(y) + sk\overline{V}(y)h_y(y)$$

Integrate both sites from $\underline{y}$ to y to have

$$kV(y)U - \delta H_y(y) = skH_y(y)\overline{V}(y) \tag{A1}$$

And Integrate the balance condition from $\underline{y}$ to y to have

$$H_y(y) + V(y) = N(y)$$

Then solve last expression for V(y) and plug it into the aggregate flow equation for vacancies to arrive at

$$\begin{aligned}
k\left(N(y) - H_y(y)\right)U - \delta H_y(y) &= skH_y(y)\left(V - N(y) + H_y(y)\right)\\
kUN(y) - kUH_y(y) - \delta H_y(y) &= skVH_y(y) - skN(y)H_y(y) + skH^2(y)\\
skH^2(y) + \left(\delta + kU + skV - skN(y)\right)H_y(y) - kUN(y) &= 0
\end{aligned}$$

Again, this is a quadratic equation in $H_y(y)$, which depends solely on k and $N(y)$ and the rest of variables have been previously worked out, as we can see below

$$\underbrace{sk}_{A} H_y^2(y) + \underbrace{(\delta + kU + skV - skN(y))}_{B(N(y))} H_y(y) - \underbrace{kUN(y)}_{C(N(y))} = 0$$

Then

$$H_y(y) = \frac{-B\left(N(y)\right) + \sqrt{B^2\left(N(y)\right) + 4AC\left(N(y)\right)}}{2A}$$

The negative part can safely be discarded as

$$H_y(y) = \frac{-B\left(N(y)\right) - \sqrt{B^2\left(N(y)\right) + 4AC\left(N(y)\right)}}{2A} < \frac{-B\left(N(y)\right) - \sqrt{B^2\left(N(y)\right)}}{2A} = -\frac{2B\left(N(y)\right)}{2A} < 0$$

Whereas the positive part is always greater than zero

$$H_y(y) = \frac{-B\left(N(y)\right) + \sqrt{B^2\left(N(y)\right) + 4AC\left(N(y)\right)}}{2A} > \frac{-B\left(N(y)\right) + \sqrt{B^2\left(N(y)\right)}}{2A} = 0$$

Now, derive the quadratic equation implicitly with respect to to y to find $h_y(y)$

$$2skH_y(y)h_y(y) + \left(\delta + kU + skV - skN(y)\right) h_y(y) - skn(y)H_y(y) - kUn(y) = 0$$

Solving for $h_y(y)$

$$h_y(y) = \frac{kU + skH_y(y)}{\left(\delta + skV + skH_y(y) - skN(y)\right) + \left(kU + skH_y(y)\right)} \cdot n(y)$$

Where

- $\left(\delta + skV + skH_y(y) - skN(y)\right) = \left(\delta + sk\overline{V}(y)\right)$: are flows out of $H_y(y)$ and
- $kU + skH_y(y)$: are flows into $H_y(y)$

It's worth noting that $H_y(y)$ depends on $N(y)$ and so does $h_y(y)$.

At this point we are ready to come with an expression for $v(y)$. Consider again the expression coming from the integrated flow of vacancies in the S.S.

$$kV(y)U - \delta H_y(y) = skH_y(y)\overline{V}(y)$$

It is just left to solve for $V(y)$ and derive to reach the desire result

$$V(y) = \frac{\delta + skV}{kU + skH_y(y)} H_y(y)$$

And deriving

$$v(y) = \frac{\delta + skV}{(kU + skH_y(y))^2} kUh_y(y)$$

Which is not very intuitive. In order to have an expression in terms of flows, change $h_y(y)$ by its last derived expression; plug the definition of $V(y)$ and use the integrated flow of vacancies in the S.S. to arrive to the desired result

$$v(y) = \frac{\delta + sk\overline{V}(y)}{\left(\delta + sk\overline{V}(y)\right) + (kU + skH_y(y))} \cdot n(y)$$

Once we have worked out a close form expression for the number of vacancies and the number of workers in y-type firms, we can deal with the number of unemployed and the number of workers with x-characteristic. From the differential equation for unemployed, substitute the balance condition for $h_x(x)$, such that

$$kVu(x) = \delta h_x(x) \Leftrightarrow kVu(x) = \delta\left(l(x) - u(x)\right) \Leftrightarrow \left(\delta + kV\right)u(x) = \delta l(x)$$
$$u(x) = \frac{\delta}{(\delta + kV)} l(x)$$

With this expression and basic algebra we work out $h_x(x)$

$$h_x(x) = \frac{kV}{(\delta + kV)} l(x)$$

Finally, we are ready to calculate $h(x, y)$. The steps to arrive at the solution are basically the same as those to compute $h_y(y)$. THen, consider the last expression and rearrange such that U-to-H and H-to-U flows are on the LHS and H-to-H flows are in the RHS.

$$\delta h(x, y) - kv(y)u(x) = sk \int_{\underline{y}}^{y} h(x, y')dy'v(y) - sk \int_{y}^{\overline{y}} v(y')dy'h(x, y)$$

Integrate both sites from $\underline{y}$ to y to have

$$\delta \int_{\underline{y}}^{y} h(x, y')dy' - ku(x)V(y) = -sk \int_{\underline{y}}^{y} h(x, y')dy'\overline{V}(y)$$

rearranging

$$\left(\delta + sk\overline{V}(y)\right) \int_{\underline{y}}^{y} h(x, y')dy' = ku(x)V(y)$$
$$\int_{\underline{y}}^{y} h(x, y')dy' = \frac{ku(x)V(y)}{\left(\delta + sk\overline{V}(y)\right)}$$

And deriving with respect to to y we arrive at the final form

$$h(x,y) = \frac{\delta + skV}{\left(\delta + sk\overline{V}(y)\right)^2} \cdot ku(x)v(y)$$

Which is difficult to interpret. However, we can prove that the following interesting result holds, $h(x,y) = \frac{1}{H}h_x(x)h_y(y)$. Start by plugging in $h(x,y)$ the expressions for $ku(x) = \frac{\delta}{V}h_x(x)$, $v(y)$, and solve for $\left(\delta + sk\overline{V}(y)\right)$ in equation A1, so that we get

$$h(x,y) = \frac{(\delta + skV)}{\left(\frac{kV(y)U}{H_y(y)}\right)^2} \cdot \frac{(\delta + skV)}{(kU + skH_y(y))^2} \cdot kUh_y(y)\frac{\delta}{V}h_x(x)$$

Solve for $(kU + skH_y(y))$ in A1 and use the fact that $kU = \frac{\delta}{V}H$

$$h(x,y) = \frac{(\delta + skV)}{\left(\frac{kV(y)U}{H_y(y)}\right)^2} \cdot \frac{(\delta + skV)}{\left(\frac{(\delta + skV)H_y(y)}{V(y)}\right)^2} \cdot \left(\frac{\delta}{V}\right)^2 Hh_x(x)h_y(y)$$

Cancelling out terms and substituting

$$h(x,y) = \left(\frac{\delta}{kUV}\right)^2 Hh_x(x)h_y(y) = \frac{1}{H^2}Hh_x(x)h_y(y) = \frac{1}{H}h_x(x)h_y(y)$$

Now, we are ready to calculate the distribution of wages from the flow equation

$$\begin{aligned}\frac{dG_t(w|x,y)\cdot h_t(x,y)}{dt} &= kv_t(y)u_t(x) + sk\int_{\underline{y}}^{q} h(x,y')dy' \cdot v(y) \\ &- \delta G_t(w|x,y)\cdot h_t(x,y) - sk\int_{q}^{\overline{y}} v(y')dy' G_t(w|x,y)\cdot h_t(x,y) = 0\end{aligned}$$

rearranging

$$\left(\delta + sk\int_{q}^{\overline{y}} v(y')dy'\right) G(w|x,y)\cdot h(x,y) = kv(y)u(x) + sk\int_{\underline{y}}^{q} h(x,y')dy' v(y)$$

solving for $G_t(w|x,y)$ we have

$$G(w|x,y) = \frac{\left(kv(y)u(x) + sk\int_{\underline{y}}^{q} h(x,y')dy'v(y)\right)}{\left(\delta + sk\int_{q}^{\overline{y}} v(y')dy'\right)} \cdot \frac{1}{h(x,y)}$$

Substitute $h(x,y)$ by the product of the marginals $\frac{1}{H}h_x(x)h_y(y)$. Also use the flow equation for the unemployed $kVu(x) = h_x(x)$ and the aggregate flow equation $kVU = \delta H$ to arrive at

$u(x) = \frac{U}{H} h_x(x)$, then after cancelling terms

$$G(w|y) = \frac{\left(kU + sk\int_{\underline{y}}^{q} h_y(y')dy'\right)}{\left(\delta + sk\int_{q}^{\overline{y}} v(y')dy'\right)} \cdot \frac{v(y)}{h_y(y)}$$

Which shows what intuition could have told us in advance, namely that the distribution of wages does not depend on x.

A2. Distributions with Minimum Wages

To find out the distributions of matches, vacancies and unemployed under the minimum wage, $\tilde{h}(x, y)$, $\tilde{v}(y)$ and $\tilde{u}(x)$ respectively, it will just suffice to rewrite them in terms of the old ones. Under the minimum wage some meetings that could have ended in a match are not going to be possible, as the flow revenue of the match it is not enough to pay the minimum wage. Then the balance conditions can be rewritten as

$$l(x) = \underbrace{u(x) + \int_{\underline{y}}^{\hat{y}(x)} h(x, y')dy'}_{\tilde{u}(x)} + \underbrace{\int_{\hat{y}(x)}^{\overline{y}} h(x, y')dy'}_{\tilde{h}_x(x)}$$

$$n(y) = \underbrace{v(y) + \int_{\underline{x}}^{\hat{x}(y)} h(x', y)dx'}_{\tilde{v}(y)} + \underbrace{\int_{\hat{x}(y)}^{\overline{x}} h(x', y)dx'}_{\tilde{h}_y(y)}.$$

Because the previous analysis without minimum wages, we know that under random search there is no sorting under the model assumptions. The implication being to express $h(x, y)$ as the product of two functions that describe abilities and productivities independently, namely $h(x, y) = \frac{1}{H} h_y(y) h_x(x)$, then the balance conditions can be rewritten as

$$l(x) = \underbrace{u(x) + h_x(x)\int_{\underline{y}}^{\hat{y}(x)} \frac{h_y(y')}{H}dy'}_{\tilde{u}(x)} + \underbrace{h_x(x)\int_{\hat{y}(x)}^{\overline{y}} \frac{h_y(y')}{H}dy'}_{\tilde{h}_x(x)}$$

$$n(y) = \underbrace{v(y) + h_y(y)\int_{\underline{x}}^{\hat{x}(y)} \frac{h_x(x')}{H}dx'}_{\tilde{v}(y)} + \underbrace{h_y(y)\int_{\hat{x}(y)}^{\overline{x}} \frac{h_x(x')}{H}dx'}_{\tilde{h}_y(y)}$$

.

And then as

$$l(x) = \underbrace{u(x) + h_x(x)F_y\left(\hat{y}(x)\right)}_{\tilde{u}(x)} + \underbrace{h_x(x)\overline{F}_y\left(\hat{y}(x)\right)}_{\tilde{h}_x(x)}$$
$$n(y) = \underbrace{v(y) + h_y(y)F_x\left(\hat{x}(y)\right)}_{\tilde{v}(y)} + \underbrace{h_y(y)\overline{F}_x\left(\hat{x}(y)\right)}_{\tilde{h}_y(y)}.$$

Integrating over abilities in the first condition and over productivities in the second we work out the aggregate balance conditions

$$L = \underbrace{U + \int_{\underline{x}}^{\overline{x}} h_x(x')F_y\left(\hat{y}(x')\right)dx'}_{\tilde{U}} + \underbrace{\int_{\underline{x}}^{\overline{x}} h_x(x')\overline{F}_y\left(\hat{y}(x')\right)dx'}_{\tilde{H}}$$
$$N = \underbrace{V + \int_{\underline{y}}^{\overline{y}} h_y(y')F_x\left(\hat{x}(y')\right)dy'}_{\tilde{V}} + \underbrace{\int_{\underline{y}}^{\overline{y}} h_y(y')\overline{F}_x\left(\hat{x}(y')\right)dy'}_{\tilde{H}}.$$

For the join distribution of jobs under the minimum wage it will suffice to solve the follow equation for jobs:

$$0 = k\tilde{v}(y)\tilde{u}(x) \qquad + sk\int_{\underline{y}}^{q} \tilde{h}(x,y')dy'\tilde{v}(y)$$
$$- \delta\tilde{G}_t(w|x,y)\cdot\tilde{h}(x,y) \qquad - sk\int_{q}^{\overline{y}} \tilde{v}(y')dy'\tilde{G}_t(w|x,y)\cdot\tilde{h}(x,y).$$

Now the number of vacancies and unemployed people will be substituted by their minimum wage counterparts, i.e. $\tilde{v}(y)$ and $\tilde{u}(y)$, since those vacancies lost by the unemployed or workers with low ability will be at the disposal of the rest, and seemingly the same argument applies for those unemployed that will not be able to cover vacancies in low productive firms. Hence, following the same procedure as before we will arrive at

$$\tilde{h}(x,y) = \frac{\delta + sk\tilde{V}}{\left(\delta + sk\overline{\tilde{V}}(y)\right)^2}\cdot k\tilde{u}(x)\tilde{v}(y)$$
$$\tilde{G}(w|x,y) = \frac{\left(k\tilde{v}(y)\tilde{u}(x) + sk\int_{\underline{y}}^{q}\tilde{h}(x,y')dy'\tilde{v}(y)\right)}{\left(\delta + sk\int_{q}^{\overline{y}}\tilde{v}(y')dy'\right)}\cdot\frac{1}{\tilde{h}(x,y)}$$

Lack of assortative matching will not be the case upon introducing minimum wages, now the minimum viable productivity of a firm (or worker) to form a match will be a function of the worker ability (firm productivity). In this respect minimum wages will introduce negative sorting in our analysis.

Mathematical Proofs

B1. Lemma 1

The firm with the minimum viable productivity to hire a worker cannot make any surplus out of the match, i.e. $\Pi_1\left(x,\underline{\hat{y}}(x)\right)=0$, otherwise a firm with marginally less productivity could enter the market, hire a worker and make profits, being a contradiction; then $P\left(x,\underline{\hat{y}}(x)\right)=W_1\left(\phi_0\left(x,\underline{\hat{y}}(x)\right),x,\underline{\hat{y}}(x)\right)$.

With respect to $W_1\left(\phi_0\left(x,\underline{\hat{y}}(x)\right),x,\underline{\hat{y}}(x)\right)=W_0(x,m)$, the same argument along the above lines can be devised. If $W_1\left(\phi_0\left(x,\underline{\hat{y}}(x)\right),x,\underline{\hat{y}}(x)\right)>W_0(x,m)$ then another firm with marginally less productivity could enter and make profits, once again a contradiction.

B2. Lemma 2

Making use of LEMMA.1 we can equate $P\left(x,\underline{\hat{y}}(x)\right)=W_1\left(\phi_0\left(x,\underline{\hat{y}}(x)\right),x,\underline{\hat{y}}(x)\right)$, which means that

$$\begin{aligned}
&\underline{\hat{y}}(x)x+\delta W_0(x)+sk\int_{\underline{\hat{y}}(x)}^{t(x,y)}P(x,y)v(y')dy'+sk\int_{t(x,y)}^{\overline{y}}W_1(m,x,y')v(y')dy'\\
&\quad=\phi_0\left(x,\underline{\hat{y}}(x)\right)+\delta W_0(x)+sk\int_{\underline{\hat{y}}(x)}^{t(x,y)}P(x,y)v(y')dy'+sk\int_{t(x,y)}^{\overline{y}}W_1(m,x,y')v(y')dy'.
\end{aligned}$$

And after cancelling terms we arrive at:

$$\underline{\hat{y}}(x)x=\phi_0\left(x,\underline{\hat{y}}(x)\right).$$

For convenience define the threshold x' such that $\phi_0\left(x',y_{inf}\right)=m$. There are two cases of interest:

CASE.1: $x<x'$

$$\phi_0\left(x',\underline{\hat{y}}(x)\right)=m\Leftrightarrow\underline{\hat{y}}(x)=\frac{m}{x}.$$

CASE.2: $x\geq x'$

$$\phi_0\left(x',y_{inf}\right)>m\Leftrightarrow\underline{\hat{y}}(x)=y_{inf}.$$