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PHYS265
Lab Report 2

Lab Report 2: Mine Crafting

I. Introduction

The 4-kilometer mine shaft project proves to be one of the largest built, in history. In the below report, we present multiple feasibility calculations for the potential construction of this mine on the Earth and on the Moon. The calculations and simulations to determine the measurements of key quantities have been performed in Python using JupyterLab software. The libraries of Python employed were Numpy for large datasets, Scipy for numerical integration, and Matplotlib for visualization of results. There are four ensuing sections, fall time calculations, feasibility of depth measurements, calculation of center crossing times, and discussion and future work. As you peruse the report, the calculations below and specifically plots, were purposefully created to clarify the reference frame we establish. Rather than plot from the top of the mineshaft (4000m) to the bottom (0m), we decided to show the y-axes of most plots in km from the Earth's radius (6378.1 km) to the bottom of the shaft within our coordinate system and reference frame (6374.1 km). When not referenced, the tested object whose position and velocity in time we plot is a 1kg test mass.

II. Calculation of Fall Time

Three scenarios were calculated and tabulated for the 4km (4000m) long mineshaft. The differential equation governing the motion of this fall from the surface of the Earth (Earth's radius) is given by:

$$\frac{d^2y}{dt^2} = -g + \alpha \left|\frac{dy}{dt}\right|^\gamma$$

Where g is the gravitation acceleration, alpha is the drag coefficient, and gamma is the power of the velocity proportionality for drag.

The first calculation worked under zero drag (alpha = 0) and a constant gravitational acceleration of 9.81 m/s^2. This yielded a numerical fall time of 28.6 seconds. Because this ordinary differential equation (ODE) is analytically solvable we used Python to compare the analytical and numerical answers to get a percentage error of 2E-14%, showing our results are highly accurate for this case.

The second calculation added in a variable gravitational acceleration for a homogeneously dense Earth. This changed the g term in the ODE to a variable g(y) where y is the changing height. This is an assumption, as the Earth has a variable density as a function of radius. This will be addressed later in the report. The variable g(y) also gave a fall time of 28.6 seconds (further precision is given in the code).

Figure 2 below shows the final calculation for fall time of the test mass with variable gravity and zero drag. Notice the time it shows to hit the bottom is consistent with the reported value of 28.6 seconds.

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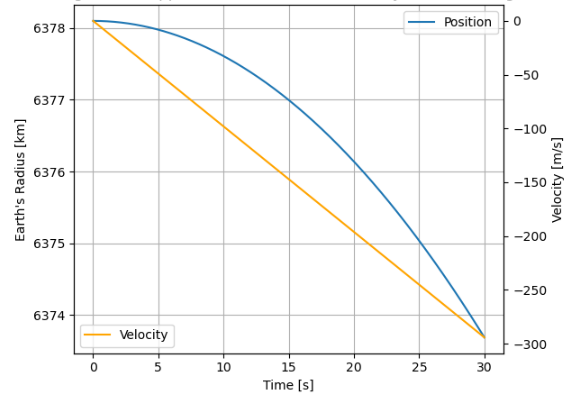
The third calculation made the drag non-zero by setting the alpha = .0039. This alpha value was chosen and calibrated by the model of a skydiver's terminal. Adding in non-zero drag greatly changed the total fall time to 83.3 seconds. This is an increase of approximately 55 seconds, which indicates that drag cannot be neglected for further investigation.

Figure 2: Dropped Mass with Variable Gravity and Zero Drag



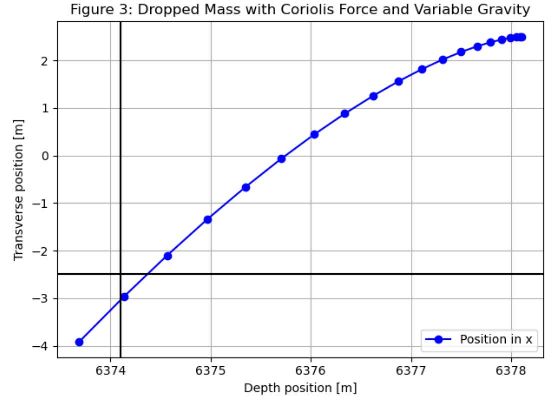
III. Feasibility of depth measurements

Depth measurements were calculated to determine the impact of variable gravity, drag, and Coriolis forces on objects descending the mine shaft. This was simulated again with a 1 kg test mass that was dropped into the shaft. The Coriolis forces change the original ODE such that:

$$F_{c_x} = +2m\Omega v_y$$
$$F_{c_y} = -2m\Omega v_x$$
$$F_{c_z} = 0$$

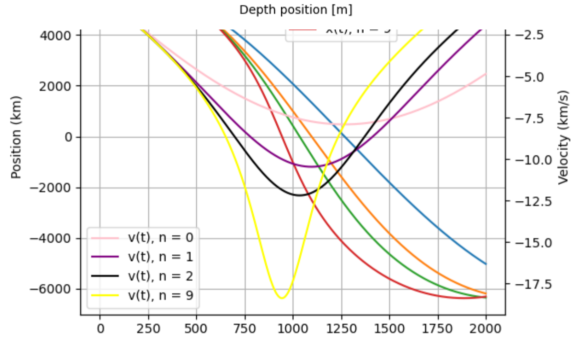
These components change the simulated accelerations and hence, the positions and velocities as functions of time. After simulation under such forces, the test mass hits the wall of the mine shaft before the bottom at t = 21.9 seconds. Including drag force also affects this measurement, changing the time it hits the side of the wall to t = 29.5 seconds. The test mass's original transverse position is set to 2.5 meters to simulate dropping it from the middle of the shaft. Figure 3 shows the plot of the simulations. Our recommendation is then not to proceed with such a depth measurement technique because it will hit the wall first.

Figure 3: Dropped Mass with Coriolis Force and Variable Gravity



IV. Calculation of crossing times

Further calculations were done to simulate motion under homogenous and inhomogeneous earth density distributions. We have incremented values of n to demonstrate how the position and velocity change for different approximations of a variable density. Figure 9 shows such calculations. The n=0 case describes a constant density and the n=9 case describes a highly non-constant density given by the equation below.



The crossing over times for inhomogeneous Earth, and their speeds during the crossover are tabulated below:

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Time to reach center for n=0: 1267.3427996212383 [s]. Speed at center:7905.277114448316 [m/s]
Time to reach center for n=1: 1096.512985261829 [s]. Speed at center:10457.698647710951 [m/s]
Time to reach center for n=2: 1035.075646309803 [s]. Speed at center:12182.850249245026 [m/s]
Time to reach center for n=9: 943.7775309879296 [s]. Speed at center:18370.673737782512 [m/s]
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The lunar surface was also studied as a potential for a mine shaft. On the Earth, under a homogenous density of 5494.9 kg/m^3, the crossing over time is 1266.6 seconds. On the Moon, under a homogeneous density of 3341.8, the crossing over time is 1624.9 seconds. We also show that the orbital period is approximately 4 times the crossing over time for the mass dropped through an infinitely long mineshaft. These values and the behavior observed in the report when working under a homogenous density (oscillations occur), produce the conclusion that crossing time is inversely proportional to the square root of density. This is demonstrated as:

$$T \sim \frac{1}{\sqrt{\rho}}$$

V. Discussion and future work

In this report, we have produced many calculations presented here and mathematically done in a Jupyter notebook to find fall times under different approximations, depth measurement feasibility and calculation of center crossing times for the Earth and the Moon. Some notable results we present are that when testing the depth, the test mass will hit the wall before it hits the bottom without drag, and 8 seconds later with drag. Since we have done the calculations for the orientation of the Coriolis force, a future project could determine if there was an optimal position to drop the test mass from such that it hits the bottom first and accurately measures the depth. The simulation above used a transverse drop position of 2.5 meters, 5 meters may outcompete the effects of the Coriolis force. Another notable result is the established proportionality of orbit time to the reciprocal square root of density. However, our report uses certain simplifications that negatively impact the accuracy of our results. One major assumption made is that the Earth is purely spherical and has a circular orbit. The spherical assumption greatly enhances the ease of computing the volume integrals when calculating both homogenous and inhomogeneous density. To enhance realism and accuracy, implementation of a non-spherical Earth, and an elliptical orbit that uses Keppler's laws and more recent advances, would be a large improvement.