

ATLAS Report

I. Introduction

This report addresses recent developments on data collected in the A Toroidal Lhc Apparatus (ATLAS) project towards precision measurement of the mass of the Z boson. The Large Hadron Collider (LHC) studies proton-proton interactions to better understand fundamental forces and the Standard Model of Particle Physics. Z^0 and W^\pm bosons mediate weak interactions and carry the weak force. The Z^0 mass is measurable because 1/10 times, Z^0 bosons decay into two antiparticle leptons whose masses sum to the Z^0 mass. The total energy, particle's transverse momentum, pseudorapidity, and azimuthal beam angle give the four-momenta of the two leptons, and consequently, the Z^0 boson mass.

$$M = \sqrt{E^2 - (p_x^2 + p_y^2 + p_z^2)}$$

The ensuing sections investigate the invariant mass distribution from the ATLAS experiment's data, a physically relevant fitting routine, and a 2D parameter scan of the probability space to demonstrate the 1-sigma and 3-sigma confidence levels with respect to the best fit and quantitative efficacy of the fitting through χ^2 analysis. Data is loaded in Python's Pandas library; calculations are done in Numpy and Scipy libraries and visualized in the Matplotlib.

II. Invariant Mass Distribution and Fit

The ATLAS data is the four quantities discussed in the introduction (total energy (E), particle's transverse momentum (p_T), pseudorapidity (η), and azimuthal beam angle (ϕ)), given for each lepton. These are applied as follows:

$$p_x = p_T \cos(\phi), \quad p_y = p_T \sin(\phi), \quad p_z = p_T \sinh(\eta)$$

Then, the quantities for each lepton (E , p_x , and so forth) are summed to calculate the total Z boson mass. The errors in the boson masses are approximated as Poissonian, meaning that the uncertainty is the square root of the average count per bin. These are the error bars in Figure 2.

Using scattering theory, a Breit-Wigner distribution of decays can be constructed that for mass that depends on parameters Γ and m_0 . When performing the fit, Γ and m_0 are the fitting parameters that return the greatest agreement with the theoretical masses and the calculated data. The distribution is shown below:

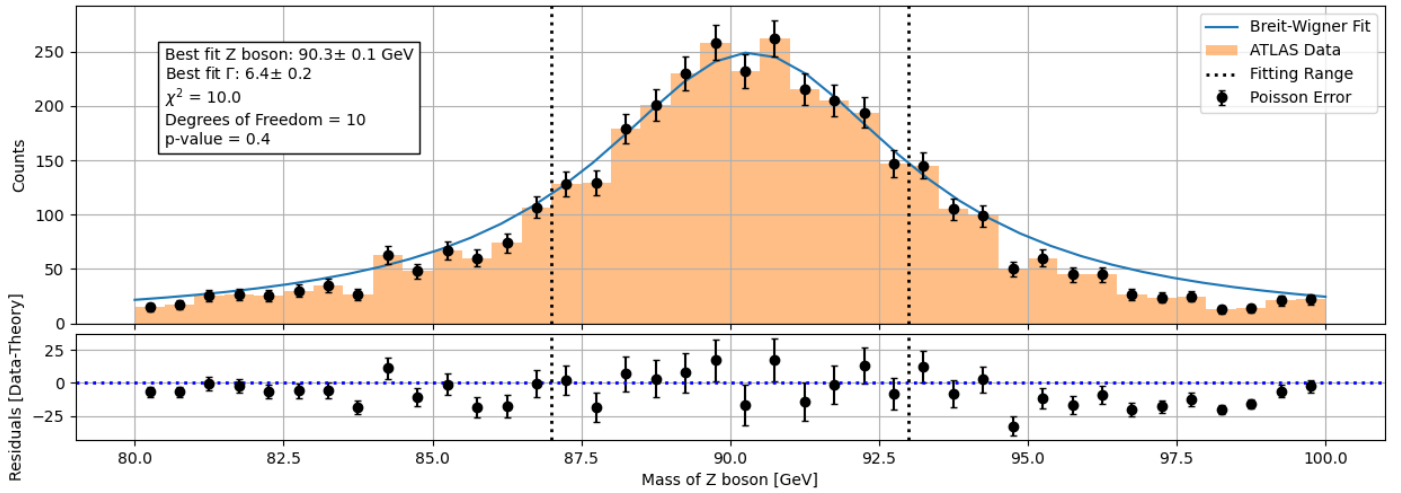
$$\mathcal{D}(m; m_0, \Gamma) = \frac{1}{\pi} \frac{\Gamma/2}{(m - m_0)^2 + (\Gamma/2)^2}$$

Physically, m_0 is the true rest mass of the Z boson, and Γ is a linewidth of the decay transition. The fitting is only performed over a central range, 87-93 GeV. Natively, the units of mass are in GeV/c² but the speed of light is set to 1 in the current unit system. Shown in Figure 1 below, the results of this section are:

- (1) Best fit mass of Z boson: 90.3 GeV
- (2) Uncertainty of the best fit mass of Z boson: .1 GeV
- (3) χ^2 of 10.0
- (4) Degrees of freedom = # of Comparisons – Fitting parameters = 10
- (5) P-value of .4

Additionally, Figure 2 gives a best fit for Γ and its uncertainty. The P-value of .4 indicates that the data and theory (Breit-Wigner fit) are in reasonable agreement with each other. A value below .05 would indicate significant disagreement and above .95 would indicate overfitting. This allows the conclusion that the Z boson mass value of $90.3 \pm .1$ GeV is statistically significant for our fitting range. Because the fitting range was done only between 87 and 93 GeV, the results are not applicable to outside of that range.

Figure 1: Breit-Wigner Fit for Z^0



Beneath the histogram and fitted curve is a residuals plot, the difference between data and theory, which visualizes the fit's accuracy. A model in perfect agreement with the data has 0 for its residuals shown as the blue line on Figure 2.

III. 2D Parameter Scan

This fit is 2-dimensional because the mass distribution has been fitted with both the rest mass and width parameters. Visiting every point in the fitting range of Γ and m_0 through a double for-loop and evaluating a $\Delta\chi^2$ there (represented along the z-axis), we scan the parameter space.

$\Delta\chi^2$ is the minimum chi-squared value subtracted from the chi-squared value at that (mass, gamma) grid-point. Overall, Figure 2 is a map of these $\Delta\chi^2$ values. The best fit value

grid-point is a minimum of this plot. Additionally, the 1-sigma and 3-sigma confidence levels are plotted as contours on this plot. These values should contain 68% and 99.7% of the data respectively. The confidence levels were calculated using Python's `scipy.stats.chi2.ppf`. Under the hood, this is calculated by computing a critical value for the inverse of the cumulative distribution function which is the confidence level boundary.

IV. Discussion and Future Work

Using four-momenta, scattering theory, and numerical computations, we established

- (1) Best fit Z boson mass
(and uncertainty): 90.3 ± 1 GeV
- (2) Minimum chi-squared value of 10, and therefore a P-value of .4, indicating reasonable agreement between the data and theory for the Breit-Wigner fit.
- (3) A $\Delta\chi^2$ map of the probability space and the 1-sigma and 3-sigma confidence levels.

While the calculations presented here provide a good start for the fitting and mass estimation of the Z boson from the ATLAS data, further work must be done to flush out results. One simplification applied here is a limited fitting range of 87-93 GeV. Because we know the literature value to be between this range, this simplification is generally semi-appropriate. Fitting a larger range could also generally improve the best fit mass. Second, the quantities established in the introduction used to calculate the four-momenta are here assumed to have zero uncertainty. A more rigorous calculation would include experimental uncertainties in those quantities and the resolution of the ATLAS detector. Currently the Particle Data Group (PDG) provides the literature value for the Z boson at $91.1880 \pm 0.0020 \frac{\text{GeV}}{c^2}$. Comparing our fit value to this literature value we find a percentage error of .9%. When considering the PDG's uncertainty, the ratio of $\frac{\text{diff}}{\sigma_{\text{diff}}}$ is 423.6 which indicates significant disagreement between our value and the PDG's value.

Signed,

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