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PHYS265

Lab 1: Apollo Mission Report

I. Introduction

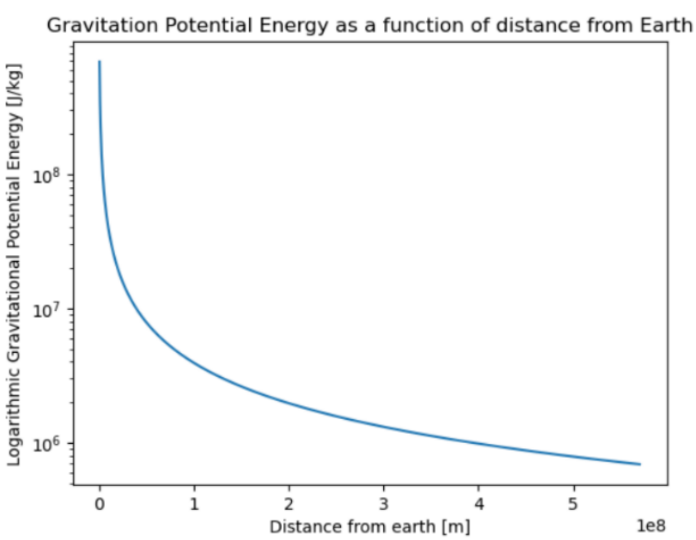
In this report, I present preliminary investigations for the Apollo program on relevant physical constraints and estimations for the Saturn V. The estimates were all made using Python programming and libraries Numpy, Matplotlib, and Scipy.

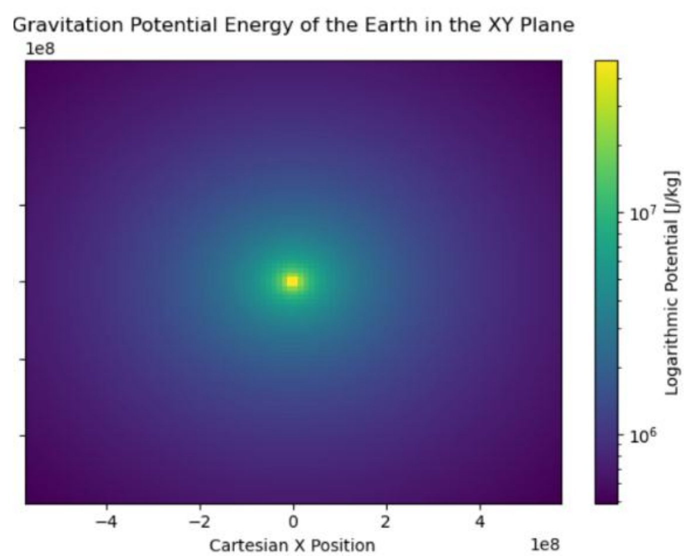
II. The gravitational potential of the Earth-Moon system

Gravitational potential measures energy per unit mass to quantify the tendency to attract another mass based on gravitational force, which has units of Joules per kilogram. I modeled the gravitation potential for the two-body Earth-Moon system using Python. The equation for gravitation potential is

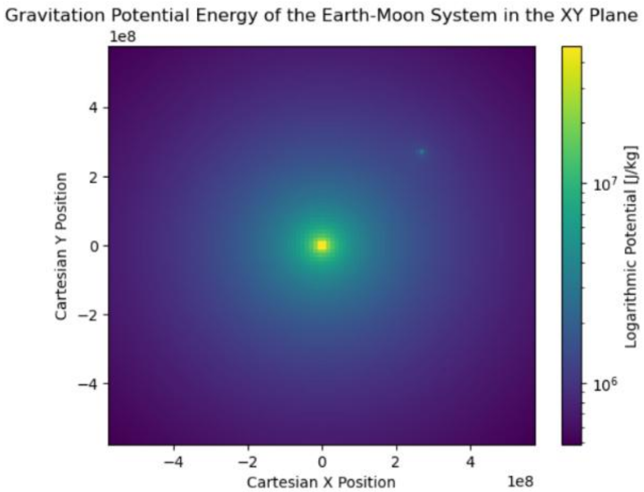
$$\phi(r) = -\frac{GM}{r}$$

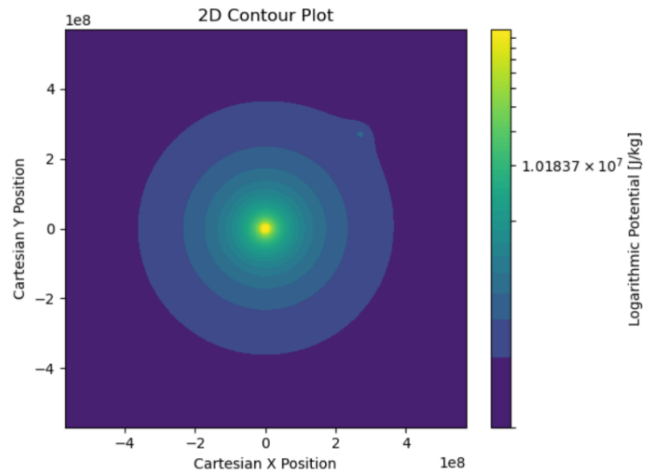
Gravitational potential is attractive and generalizable for any radius r and mass M. G is Newton's gravitational constant 6.67E-11. Using the matplotlib library on Python I first modeled the fall of gravitational potential for the Earth as a function of radius shown below.





Subsequently, I added in the potential of the moon. The combined potential of the Earth-Moon system is the sum of the Earth's and Moon's potential at every point in space. Below, the Earth is located at (0,0) and the moon at the distance of the earth to the moon divided by the square root of 2 for both the x and y coordinates. Note that the Moon is small compared to the Earth. This is because potential depends on mass and the mass of the Earth is 2 orders of magnitude greater than the Moon. I have also added a 2D contour plot which displays potential at discrete levels.



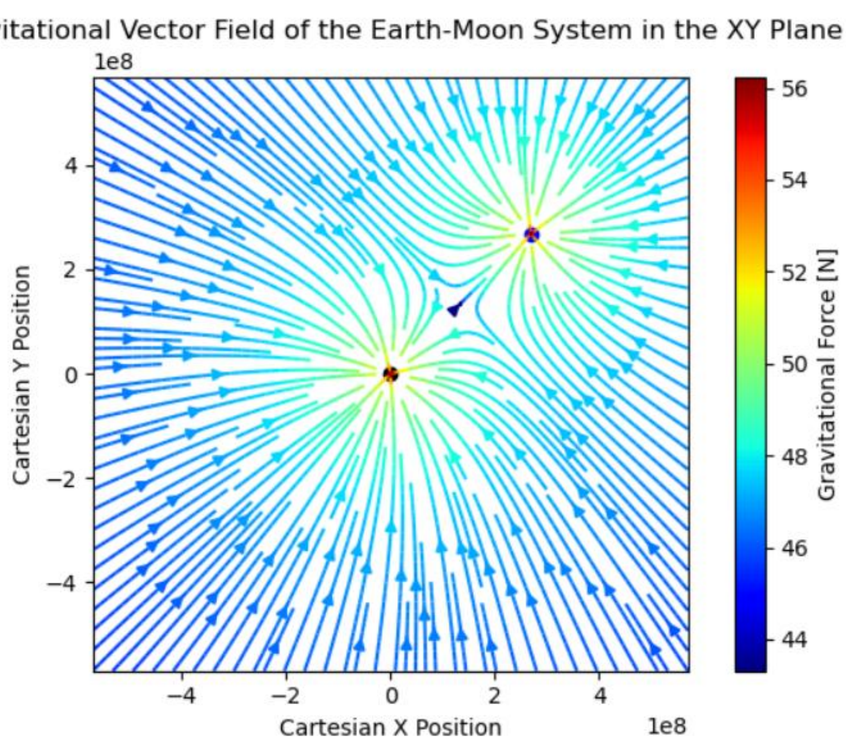


III. The gravitation force of the Earth-Moon system

The gravitation force is a vector field as each point in space has a magnitude (a measure of how strong the force is) and direction. The direction always points inward in the radial direction of the mass. The equation for gravitation force is

$$\vec{F}_{21} = -G\frac{M_1 m_2}{|\vec{r}_{21}|^2}\hat{r}_{21}$$

The same Cartesian positions taken previously are used again here. The Earth is a black point and the Moon is a blue point. You can visualize that there is a point where the Earth and Moon's gravitational force cancel each other out.



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IV. Projected performance of the Saturn V Stage 1

Using the Tsiolkovsky rocket equation for an ideal rocket I calculated the maximum altitude of Saturn V and the maximum burn time to be:

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This is the total burn time for the first stage of the Saturn V: 157.69230769230768 seconds
This is the projected altitude for the Saturn V Rocket: 74093.98013366401 meters
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The rocket equation works under the principle that the fuel expelled is directly proportional to the forward momentum of the rocket. This is given by:

$$\Delta v(t) = v_e \ln\left(\frac{m_0}{m(t)}\right) - gt$$

I used the built-in scipy.integrate.quad Python function to perform numerical integration of the equation from time 0 to the burnout time to get the maximum altitude. The quad function originally comes from the Fortran language.

V. Discussion and Future Work

In the calculations and plots presented today, I used the simplifying assumption that the earth behaves as a point mass. This is a valid assumption outside of the Earth's radius however, Saturn V will launch and experience a strong gravitation pull that is so close to the Earth's surface that you must account for potential when the radius is less than that of the Earth's. Practically, I dealt with this assumption by returning Not A Number (np.nan) values when the radius was equal to zero.

The calculation for burn time and maximum altitude of Saturn V neglects a drag force or air resistance that is usually proportional to the velocity. After Saturn V goes beyond the atmosphere this approximation is valid but not inside the atmosphere. Clearly, the test results show that the true altitude of 70 km is less than the calculated 74 km and 160 seconds is longer than the calculated 157 seconds.