



Master's thesis

Model independent search for Dark Matter using Machine Learning

In final states with dileptons and Missing Transverse Energy with the ATLAS
detector at the LHC

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Abstract

Something something

Contents

1	Introduction	1
I	The theory behind modern particle physics / Background	3
2	The Standard Model of Particle Physics	5
2.1	Quantum Electrodynamics	6
2.2	Yang-Mills Theory	6
2.3	Weak theory	8
2.4	Quantum Chromodynamics	8
2.5	Electroweak unification and the BEH Mechanism	9
2.6	S-matrix expansion	11
2.7	Adding it all up	16
3	Dark Matter	17
3.1	Observations of existence	17
3.1.1	Cosmology	18
3.1.2	WIMP	18
3.2	Beyond Standard Model candidates	18
3.2.1	New gauge boson	18
3.2.2	Supersimmetree	18
4	Production, Detection and Analysis	19
4.1	Particle production	20
4.1.1	Particle kinematics	20
4.1.2	Proton-proton collisions	24
4.2	The ATLAS detector	27

4.2.1	Inner detector	28
4.2.2	The calorimeters	29
4.2.3	Muon spectrometer	29
4.2.4	Summary of ATLAS	30
4.3	Data analysis	32
4.3.1	Cut and count	32
4.3.2	Statistical analysis	35
4.4	Summary	39
5	Machine Learning	41
5.1	Neural Networks	43
5.1.1	Artificial neurons	43
5.1.2	Optimizers	44
5.1.3	Activation functions	46
5.1.4	Feed Forward network	46
5.1.5	Back Propagation algorithm	48
5.1.6	Summary	51
5.2	Boosted Decision Trees	52
5.2.1	Decision trees	53
5.2.2	Regression trees	53
5.2.3	Classification trees	55
5.2.4	The CART algorithm	56
5.2.5	Ensemble modeling and (extreme) gradient boosting	57
5.2.6	Summary	60
5.3	Tools and evaluation methods used for both algorithms	61
5.3.1	Cost functions	61
5.3.2	Sample weight	62
5.3.3	Area under the ROC-curve	63
5.3.4	Validation plots	64
5.3.5	Significance plots	65
5.3.6	Feature importance	65

II Implementation / Methods	67
6 Data Preparation	69
6.1 Standard Model background estimation	70
6.1.1 W and Drell Yan	70
6.1.2 TTbar	70
6.1.3 Single top	71
6.1.4 Diboson	72
6.2 Dark Matter samples	72
6.3 Object selection	72
6.4 Control region	73
6.5 Feature selection for ML	75
6.6 Transfer to ML friendly syntax	80
6.7 Problems	81
6.7.1 Missing variables	81
6.7.2 MC and data disagreement	82
7 Machine Learning	83
7.1 The datasets	84
7.1.1 Weights	85
7.1.2 Signal regions	86
7.1.3 Train and test split	86
7.2 Neural Network Training	88
7.2.1 Padding of data	88
7.2.2 Normalization of data	90
7.2.3 Balancing of signal and background	91
7.2.4 Re-weighting MC to expected events	92
7.2.5 Architecture and hyperparameter tuning	94
7.3 Boosted Desicion Tree Training	96
7.3.1 Sample weights	96
7.3.2 Architecture and hyperparameter tuning	97

III Results	99
8 Network opimization	101
8.1 Neural Network Training	101
8.1.1 Normalization of data	102
8.1.2 Balancing of signal and background	104
8.1.3 Sample weights to get expected events	107
8.1.4 Padding of data	110
8.2 Boosted Desicion Tree Training (draft)	115
8.2.1 Weights	115
8.2.2 Grid Search	117
8.3 Discussion (draft)	118
9 Model dependent approach	121
9.1 The datasets	121
9.2 Dark Higgs Heavy Dark Sector	122
9.3 Dark Higgs Light Dark Sector	125
9.4 Light Vector Heavy Dark Sector	128
9.5 Light Vector Light Dark Sector	131
9.6 Effective Field Theory Heavy Dark Sector	134
9.7 Effective Field Theory Light Dark Sector	137
9.8 Mass exclusion limits for the combined channels	139
10 Comparison to cut and count	141
11 Conclusion	147
A Dataset IDs for MC samples	149
B Kinematical variables distribution in control region	153
C Data and MC agreement jets CR	159
D Data and MC agreement jets CR	163
E BDT of depth 30	165

Bibliography	172
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List of Figures

2.1	Standard Model Feynman Rules	15
2.2	Bhabha scattering Feynman diagram	15
2.3	The Standard Model	16
4.1	Feynam diagram from pp -collision	25
4.2	The ATLAS detector	27
4.3	Illustration of the ATLAS detector layers	28
4.4	The Higgs discovery on the $ZZ^{(*)}$ channel	34
5.1	Basic Neural Network Illustration	48
5.2	Basic Decision Tree illustration	60
6.1	$W/Z/\gamma + \text{jets}$ production	70
6.2	$t\bar{t}$ production	71
6.3	Single Top production	71
6.4	Diboson production	72
6.5	E_T^{miss} distribution in dilepton final state Run II	75
6.6	b- and light jet distributions in control region	75
6.7	Distribution of m_{ll} in control region.	77
6.8	Distribution of $E_T^{\text{miss},\text{sig}}$ in control region.	78
6.9	Distribution of m_{T2} in control region.	78
6.10	Distribution of $\Delta\phi(l\bar{l}, E_T^{\text{miss}})$ in control region.	79
7.1	Train-test split distribution	87
7.2	Neural Network Architecture	94
8.1	Different normalization methods for NNs	102

8.2	Comparison of best NN normalization methods and expected significance calculation	103
8.3	Difference between ADAM and SGD optimizer	104
8.4	Validation plots for different balancing methods on NN	105
8.5	ROC plots for different balancing methods on NN	106
8.6	Validation plots for re-weighting background to expected events on NNs	107
8.7	ROC plots for re-weighting background to expected events on NNs	108
8.8	Significance plots for re-weighting and balancing W dataset on NNs	109
8.9	Grid search result for pad testing on NN	110
8.10	NN parameters after 50 epochs with new features	111
8.11	NN parameters after 50 epochs when dropping features	112
8.12	ROC plots for both padding methods	113
8.13	Validation plots for both padding methods	113
8.14	Significance plots for both padding methods	114
8.15	Difference when using different weighting methods on BDTs	116
8.16	Grid search expected significance when setting $\lambda = 10^{-5}$ and $\eta = 0.1$	117
8.17	Grid search result for BDT	117
8.18	Feature importance plots of BDT	118
9.1	Validation plots for network trained on Z' DH HDS	122
9.2	ROC plots for every Z' mass point on network trained on Z' DH HDS	122
9.3	Expected significance plots for Z' mass points on network trained on Z' DH HDS	123
9.4	Mass exclusion limits of ee and $\mu\mu$ channel for all Z' DH HDS model	123
9.5	Validation plots for network trained on Z' DH LDS	125
9.6	ROC plots for every Z' mass point on network trained on Z' DH LDS	125
9.7	Expected significance plots for Z' mass points on network trained on Z' DH LDS	126
9.8	Mass exclusion limits of ee and $\mu\mu$ channel for all Z' DH LDS model	126
9.9	Validation plots for network trained on Z' LV HDS	128
9.10	ROC plots for every Z' mass point on network trained on Z' LV HDS	128
9.11	Expected significance plots for Z' mass points on network trained on Z' LV HDS	129

9.12	Mass exclusion limits of ee and $\mu\mu$ channel for all Z' LV HDS model	129
9.13	Validation plots for network trained on Z' LV LDS	131
9.14	ROC plots for every Z' mass point on network trained on Z' LV LDS	131
9.15	Expected significance plots for Z' mass points on network trained on Z' LV LDS	132
9.16	Mass exclusion limits of ee and $\mu\mu$ channel for all Z' LV LDS model	132
9.17	ROC plots for every Z' mass point on network trained on Z' EFT HDS	134
9.18	Expected significance plots for Z' mass points on network trained on Z' EFT HDS	134
9.19	Mass exclusion limits of ee and $\mu\mu$ channel for all Z' EFT HDS model	136
9.20	ROC plots for every Z' mass point on network trained on Z' EFT LDS	137
9.21	Expected significance plots for Z' mass points on network trained on Z' EFT LDS	137
9.22	Mass exclusion limits of ee and $\mu\mu$ channel for all Z' EFT LDS model	139
9.23	Mass exclusion limits of combined ee and $\mu\mu$ channel for all models	140
10.1	Expected significance of XGBoost when trained on the Full DM dataset for the DH HDS $m_{Z'} = 130$ GeV muon model.	143
10.2	Expected significance of the Neural Network when trained on the Full DM dataset for the DH HDS $m_{Z'} = 130$ GeV muon model.	144
B.1	$\Delta\phi(l_c, E_T^{miss})$, $\Delta\phi(l_l, E_T^{miss})$, $\Delta\phi(l_1, l_2)$ and m_{jj} distribution in the control region.	153
B.2	Leptons p_T, ϕ and η distribution in the control region.	154
B.3	Leading jets p_T, ϕ and η distribution in the control region.	155
B.4	p_T, ϕ and η of third leading jet and the dilepton pairs E_T , H_T and m_T distribution in the control region.	156
B.5	Number of light, b- and total jets and E_T^{miss}/H_T distribution in the control region.	157
C.1	Data and MC agreement on number of b- jets with different p_T cuts in CR.	160
C.2	Data and MC agreement on number of light jets with different p_T cuts in CR.	161

E.1	Grid search expected significance going to a depth of up to 30	165
E.2	Grid search AUC going to a depth of up to 30	166
E.3	Feature importance of depth 30 network trained on FULL Z' DM data set when testing it on DH HDS $m_{Z'} = 130$ GeV model.	167
E.4	Comparison of the network performance when having a depth of 6 and 30. Figure a) and b) show the validation data of both cases, c) and d) show the expected significance of the validation plots when making a cut on the output.	168

List of Tables

6.1	Electron selection criteria	73
6.2	Muon selection criteria	73
6.3	Jet selection criteria	73
6.4	Control region for model-independent search	76
6.5	Kinematic variables used as features	80
7.1	Dataset used for ML	85
7.2	New kinematic variables that need no padding	90
9.1	Inputs for the $Z'h_D \rightarrow l^+l^-\chi\bar{\chi}$ HDS σB calculations	124
9.2	Inputs for the $Z'h_D \rightarrow l^+l^-\chi\bar{\chi}$ LDS σB calculations	127
9.3	Inputs for the $Z'V \rightarrow l^+l^-\chi\bar{\chi}$ HDS σB calculations	130
9.4	Inputs for the $Z'V \rightarrow l^+l^-\chi\bar{\chi}$ LDS σB calculations	133
9.5	Inputs for the EFT $\rightarrow Z'\chi\bar{\chi}$ HDS σB calculations	135
9.6	Inputs for the EFT $\rightarrow Z'\chi\bar{\chi}$ LDS σB calculations	138
10.1	Cut and count cuts	141
10.2	Cut and count significance ee	142
10.3	Cut and count significance uu	142
10.4	Cut and count significance ee	146
10.5	Unbalanced DM training dataset	146
A.1	Drell Yan background MC samples	149
A.2	Single top and TTbar background MC samples	150
A.3	Diboson background MC samples	150
A.4	W background MC samples	151

List of Acronyms

SM Standard Model

DM Dark Matter

LHC Large Hadron Collider

MET Missing Transverse Energy

MC Monte Carlo

ML Machine Learning

NN Neural Network

BDT Boosted Decision Tree

Chapter 1

Introduction

After the discovery of the Higgs boson ...

Part I

The theory behind modern particle
physics / Background

Chapter 2

The Standard Model of Particle Physics

The standard model of particle physics

$$\mathcal{L}_{SM} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\not{D}\psi + \psi_i y_{ij}\psi_j\phi + h.c. + |D_\mu\phi|^2 - V(\phi) \quad (2.1)$$

The elegant equation that can explain phenomena in nature with great precision. It consists of three *symmetry groups*. The group explaining electromagnetism $U(1)$, the group describing the weak force $SU(2)_L$ and the group describing the strong force $SU(3)_C$. In this chapter we will delve into each of these groups, the spontaneous symmetry breaking phenomena that gives mass to particles, as well as how to calculate the cross section of processes.

The theory of this section is mainly based of Peskin's and Schroeder's "An Introduction to Quantum Field Theory" [1] and Thomson's "Modern Particle Physics" [2].

2.1 Quantum Electrodynamics

In the begining there was nothing; *then God said, “Let there be light,” and there was light.* This lead us to the first part of the Standard Model, Quantum Electrodynamics, or QED for short,

$$\mathcal{L}_{QED} = \bar{\psi} (i\cancel{D} - m) \psi - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} \quad (2.2)$$

using the notation $\cancel{D} \equiv \gamma^\mu D_\mu$ where $iD_\mu = i\partial_\mu - eB_\mu$ is the covariant derivative and $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ is the electromagnetic field tensor¹. B_μ is the vector potential of the Lagrangian.

The above Lagrangian is part of a $U(1)$ symmetry group, meaning that there is only one unique gauge freedom, this is the rotation of phase angle of the field θ . The m is the mass of the *Dirac* particle described by the Dirac field ψ .

2.2 Yang-Mills Theory

While QED is expressed by the Lagrangian in Eq. 2.2, which is part of a $U(1)$ symmetry group, it can still be described by a more general Lagrangian using *Yang-Mills* theory

$$\mathcal{L}_{YM} = \bar{f} (i\cancel{D} - m) f - \frac{1}{4} (F_{\mu\nu}^a)^2 \quad (2.3)$$

where we use the notation $f \equiv \begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{pmatrix}$ where f_i is a field. The Yang-Mills Lagrangian looks

almost identical to the QED Lagrangian, the differences being that the gauge freedom changes from a rotation of the phase angle θ to a more general

$$\psi \rightarrow (1 + i\alpha^a t^a + \mathcal{O}(\alpha^2)) \psi$$

¹Taking $\partial_\mu B^{\mu\nu} = 0$ gives all of Maxwells equations.

This also changes the vector fields A_μ^a such that it transforms

$$A_\mu^a \rightarrow A_\mu^a + \frac{1}{g} \partial_\mu \alpha^a + f^{abc} A_\mu^b \alpha^c$$

where e is replaced for a more general scale factor g , and f^{abc} is a set of numbers called the *structure constants*. From these changes follow that

$$iD_\mu = i\partial_\mu - eA_\mu \rightarrow iD_\mu = i\partial_\mu - gA_\mu^a t^a$$

The important thing to note about the Yang-Mills is that the covariant derivative can be *non Abelian*, meaning that

$$[D_\mu, D_\nu] = -igF_{\mu\nu}^a t^a$$

this new t^a factor is a Lie algrbra generator, which for our purposes represent a local gauge symmetry we have on the field. The structure constant has to fullfull the criteria that

$$[t^a, t^b] = if^{abc}t^c \quad (2.4)$$

The difference between the Yang-Mills Lagrangian is most noticeable when fully writing

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$$

Giving that equatin of motion

$$\partial^\mu F_{\mu\nu}^a + g f^{abc} A^{b\mu} F_{\mu\nu}^c = -g \bar{\psi} \gamma_\nu t^a \psi \quad (2.5)$$

Using Noether's thoere on the Langrangian gives us the conserved quantity of the symmetry group. There is also a theorem that states that for a $SU(n)$ symmetry group there are $n^2 - 1$ vector fields.

If we were to choose $U(1)$ as our local symmetry group the generator $t^a = \mathbb{I}$ and all the structure constants to be $f^{abc} = 0$, meaning that we get the QED back. Noether's theorem also states that the conserved quantity of the Lagrangian is the electric charge e .

2.3 Weak theory

Using the Yang-Mills Lagrangian from Eq. (2.3) on a $SU(2)$, choosing $t^a = \frac{\sigma^i}{2}$, where σ^i are the Pauli matrices, and the structure constants to be the Levi-Civita $f^{abc} = \epsilon^{ijk}$. Where $i, j, k \in [1, 2, 3]$. The Weak theory gives us the gauge freedoms of

$$\psi \rightarrow (1 + i\alpha^i \sigma^i + \mathcal{O}(\alpha^2)) \psi$$

Using the notation of $A_\mu \rightarrow W_\mu$ we get that there are $2^2 - 1$, meaning three vector fields

$$W_\mu^i \rightarrow W_\mu^i + \frac{1}{g} \partial_\mu \alpha^i + \epsilon^{ijk} W_\mu^j \alpha^k$$

If one were to apply Noether's theorem on this Lagrangian we would get that two quantities are conserved, the *hypercharge* and *isospin*.

2.4 Quantum Chromodynamics

Using the Yang-Mills Lagrangian from Eq. (2.3) on a $SU(3)$, choosing $t^a = \frac{\lambda^a}{2}$, where λ^a , the *Gell-Mann matrices*, are the equivalent of the $SU(2)$ Pauli matrices:

$$\begin{aligned} \lambda^1 &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, & \lambda^2 &= \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, & \lambda^3 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\ \lambda^4 &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, & \lambda^5 &= \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, & \lambda^6 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \\ \lambda^7 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, & \lambda^8 &= \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \end{aligned} \tag{2.6}$$

as the structure constant has to obey Eq. (2.4) we get

$$\begin{aligned} f^{123} &= 1 \\ f^{147} = -f^{156} = f^{246} = f^{257} = f^{345} = -f^{367} &= \frac{1}{2} \\ f^{458} = f^{678} &= \frac{\sqrt{3}}{2} \end{aligned}$$

Using the notation of $A_\mu \rightarrow G_\mu$ we get that there are $3^2 - 1$, meaning *eight* vector fields

$$G_\mu^a \rightarrow G_\mu^a + \frac{1}{g} \partial_\mu \alpha^a + \epsilon^{abc} G_\mu^b \alpha^c$$

If one were to use Noether's theorem on the $SU(3)$ Yang-Mills Lagrangian, we would get that there are three conserved quantities for this symmetry group. These are the colors, r, g and b . With all the information above we can fully describe Quantum Chrono Dynamics, which describe quark interactions. The eight vector fields are the eight different gluons that are in the theory.

In this theory we have color triplets

$$f = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix}$$

where f_i is the color state of the particle we are studying.

2.5 Electroweak unification and the BEH Mechanism

We have so far explained the three symmetry groups that form the SM, but this is not the full picture yet. To get the full SM we need to talk about the electroweak $U(1)_Y \otimes SU(2)_L$ unification that can be explained when studying the spontaneous symmetry breaking (SSB) phenomena, which also includes the *Higgs* boson.

If we introduce a new complex scalar field ϕ that transforms in the $SU(2)$ representation,

we can write the Lagrangian

$$\mathcal{L} = -\frac{1}{2} \text{Tr}(W_{\mu\nu}W^{\mu\nu}) - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + (D_\mu\phi)^\dagger(D^\mu\phi) - \lambda \left(\phi^\dagger\phi - \frac{v^2}{2} \right)^2 \quad (2.7)$$

with

$$D_\mu\phi = \partial_\mu\phi - igW_\mu^a\sigma^a - i\frac{e}{2}B_\mu\phi \quad (2.8)$$

where W_μ and B_μ are vector fields of the $SU(2)$ and $U(1)$ group respectively. And where $-\lambda \left(\phi^\dagger\phi - \frac{v^2}{2} \right)^2 = V(\phi)$ is the potential of the new complex scalar field. Writing

$$\phi = \begin{pmatrix} o \\ v + H(x) \end{pmatrix} \quad (2.9)$$

where H is real scalar field, expands the potential to be

$$V(H) = \frac{1}{2}m_H^2H^2 + \frac{1}{2v}m_H^2H^3 + \frac{1}{8v^2}m_H^2H^4 \quad (2.10)$$

where we have used that $m_H = \sqrt{2\lambda}v$. This potential is the so-called *Sombrero potential* and we can reveal that this scalar field H is the Higgs boson. The consequences of writing our complex scalar field using Eq. 2.9 comes when from the shape of the potential, this gives rise to the mass of particles. The SSB comes from the fact that the fields have two minima on the potential but can move around in a higher dimensional space (around the sombrero edge) to get to the other minima. To explicitly show how this SSB phenomena gives mass to particles one can do a unitary gauge transformation of the scalar field, and this would ass mass terms to every particle that interacts with the Higgs field on the Lagrangian.

Another thing that follows from Eq. 2.9 is that expanding on Eq. (2.8) gives rise (after a lot of math) to the following relations

$$W_\mu^\pm = \frac{W_\mu^1 \mp iW_\mu^2}{\sqrt{2}} \quad (2.11)$$

$$Z_\mu = \frac{gW_\mu^3 - eB_\mu}{\sqrt{g^2 + e^2}} \quad (2.12)$$

Should
I add a
figure?

$$A_\mu = \frac{gB_\mu - eW_\mu^3}{\sqrt{g^2 + e^2}} \quad (2.13)$$

These are the W and Z boson as well as the photon, respectively. It also adds an extra term on the Lagrangian called the *Yukawa* Lagrangian

$$\mathcal{L}_{YU} = \psi_i y_{ij} \psi_j \phi = \bar{U}_L G_u U_R \phi^0 + \bar{D}_L G_u U_R \phi^- \bar{U}_L G_D D_R \phi^+ \bar{D}_L G_u D_R \phi^0$$

where $G_{u,d}$ is the Yukawa coupling and as we are looking at a complex scalar field in the $SU(2)$ representation we have from the Yang-Mills theory that we can write

$$f = \begin{pmatrix} U \\ D \end{pmatrix}$$

where U, D is the field *up and down* spinor. For example this can be the quark two component spinor $Q = \begin{pmatrix} u \\ d \end{pmatrix}$ consisting of the *up* and *down* quark. The standard for weak interactions is to define *right* and *left* handed helicity using the operators

$$L = \frac{1}{2}(\mathbb{I} - \gamma_5), \quad \text{and} \quad R = \frac{1}{2}(\mathbb{I} + \gamma_5)$$

where $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. Such that we can write

$$U_L = L \begin{pmatrix} U \\ D \end{pmatrix} = \begin{pmatrix} U \\ 0 \end{pmatrix}, \quad \text{and} \quad D_R = R \begin{pmatrix} U \\ D \end{pmatrix} = \begin{pmatrix} 0 \\ D \end{pmatrix}$$

as well as their hermitian conjugates.

2.6 S-matrix expansion

But how do we calculate processes? To explain this generally, we can conduct an S-matrix expansion. To be more precise and use terms from quantum field theory rather than quantum mechanics, we can use the scattering operator \mathcal{S} on the interaction term of

the SM Lagrangian \mathcal{L}_{int} . Starting by defining the general operator

$$\begin{aligned} \mathcal{S} &= \sum_{i=vertices} \mathcal{S}^{(i)}, \quad \text{where } \mathcal{S}^{(0)} = \mathbb{I} \quad \text{and} \\ \mathcal{S}^{(n)} &= (-i)^n \mathcal{T} \int_{-\infty}^{\infty} d^4 \mathbf{x}_1 \cdots \int_{-\infty}^{\infty} d^4 \mathbf{x}_n [\mathcal{H}_{int}(x_1) \cdots \mathcal{H}_{int}(x_n)] \end{aligned} \quad (2.14)$$

where \mathcal{T} is the *time operator* ordering the terms such that we calculate our desired process, $\mathcal{H}_{int} = -\mathcal{L}_{int}$ is the interaction Lagrangian, and \mathbf{x}_i are coordinates in spacetime. The \mathcal{S} operator sums over all *vertices* of a *Feynman diagram*. To calculate the probability amplitude we can start with an easy example using QED, the Bhabha scattering process $e^- e^+ \rightarrow e^- e^+$. The interaction Lagrangian in that gives us this scattering process is $\mathcal{L}_{int} = e \bar{\psi}(x) \gamma^\mu \psi(x) A_\mu$. To calculate the probability amplitude we use the scattering operator in the following way

$$\begin{aligned} \langle f | \mathcal{S}^{(n)} | i \rangle &\rightarrow \langle e^- e^+ | S^{(2)} | e^- e^+ \rangle \\ &= \langle e^- e^+ | \left(-e^2 \mathcal{T} \int_{-\infty}^{\infty} d^4 \mathbf{x} \int_{-\infty}^{\infty} d^4 \mathbf{y} \bar{\psi}(x) \gamma^\mu \psi(x) A_\mu(x) \bar{\psi}(y) \gamma^\nu \psi(y) A_\nu(y) \right) | e^- e^+ \rangle \end{aligned}$$

Starting by using the time operator \mathcal{T} on the integrals, which gives us the *normal order* of the operations, we can start by using *Wicks theorem* to "connect" the two points in spacetime \mathbf{x} and \mathbf{y} . As we know there are no photons on the initial or final state, we can connect the photons as *propagators*, meaning

$$S^{(2)} = -\frac{e^2}{2} \int_{-\infty}^{\infty} d^4 \mathbf{x} \int_{-\infty}^{\infty} d^4 \mathbf{y} \mathcal{N} \left(\overline{\psi}(x) \gamma^\mu \overline{A_\mu(x) \psi(x)} \overline{\psi}(y) \gamma^\nu A_\nu(y) \psi(y) \right)$$

where

$$\overline{A_\mu(x) A_\nu(y)} = \langle 0 | \mathcal{T}(A_\mu(x) A_\nu(y)) | 0 \rangle = \cdots = D_{\mu\nu}^F(x - y) = \int_{-\infty}^{\infty} \frac{d^4 q}{(2\pi)^4} D_{\mu\nu}^F(q) e^{-iq(x-y)}$$

with

$$D_{\mu\nu}^F(k) = \frac{-g_{\mu\nu} + r_{\mu\nu}}{(q^2 + i\epsilon)} \quad (2.15)$$

Here we have omitted the mathematical details, but to summarize it with words. We calculated the photon propagator which connects two spacetime points \mathbf{x} and \mathbf{y} by calculating the Greens function of the Minkowski space using among other the residue theorem

from complex analysis. If we were now to define $|e^- e^+\rangle, \langle e^- e^+|, \bar{\psi}$ and ψ we could directly calculate the scattering amplitude by setting in the values, as this usually involves solving integrals that use Dirac delta function². To complete our example we define

$$|e^- e^+\rangle = (\sqrt{2E_p}a_p^\dagger)(\sqrt{2E_k}b_k) |0\rangle, \quad \text{and} \quad \langle e^- e^+| = (2\sqrt{E_p E_k}) \langle 0| a_p b_k^\dagger$$

where E_i is the energy of the particle and a_i, b_i and a_i^\dagger, b_i^\dagger are creation and annihilation operators, respectively. In general fermion fields have

$$\psi \equiv \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left(\sum_s a_{\mathbf{p}}^s u^s(p) e^{-ip \cdot x} + b_{\mathbf{p}}^{s\dagger} v^s(p) e^{ip \cdot x} \right) = \psi_p^+ + \psi_p^-$$

where depending ψ^+ (for $u(p)$) and ψ^- (for $v(p)$) are the particle and anti-particle fields, respectively. It is standard convention to say that ingoing particles and anti-particles on an interaction are written as ψ^+ and $\bar{\psi}^-$, respectively, and their hermitian conjugate when they are outgoing. With this information we can connect the initial and final particles to their fields, solving the bra we have

$$\langle e^- e^+ | \bar{\psi}(x) \psi(x) = (2\sqrt{E_p E_k}) \langle 0| a_p b_k^\dagger \bar{\psi}_{k'}^- \psi_{p'}^+,$$

Giving

$$\langle e^- e^+ | \bar{\psi}(x) \psi(x) = \langle 0| \int \frac{d^3 p'}{(2\pi)^3} \int \frac{d^3 k'}{(2\pi)^3} \sqrt{\frac{E_p E_k}{E_{p'} E_{k'}}} \left(\sum_{s,t} a_{\mathbf{p}'}^{\dagger,s} u^s(p') e^{-ip' \cdot x} + b_{\mathbf{k}'}^t \bar{v}^t(k') e^{-ik' \cdot x} \right) a_p b_k^\dagger$$

using the relation $a_{p's} a_{pt}^\dagger = (2\pi)^2 \delta^{(3)}(p' - p) - a_{p's}^\dagger a_{pt}$ where the latter part kills vacuum, we get

$$\langle e^- e^+ | \bar{\psi}(x) \psi(x) = u(x) \bar{v}(x) \langle 0|$$

Doing the same for the ket, and setting all of this into the intial equation we get

$$\langle f | \mathcal{S}^{(n)} | i \rangle = -\frac{e^2}{2} \langle 0 | \int_{-\infty}^{\infty} d^4 \mathbf{x} \int_{-\infty}^{\infty} d^4 \mathbf{y} u(x) \gamma^\mu \bar{v}(x) D_{\mu\nu}^F(x-y) \bar{u}(y) \gamma^\nu v(y) | 0 \rangle$$

Giving us

$$(2\pi)^4 \delta^{(4)}(p_1 + p_2 - p'_1 - p'_2) \bar{v}(p_1) \gamma^\mu u(p_2) \frac{g_{\mu\nu}}{q^2} \bar{u}(p'_1) \gamma^\nu v(p'_2)$$

²At least while omitting loops, which we will do in this thesis

check if
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real name

is this the
real name

we can relate the scattering amplitude to a variable called the *matrix amplitude*, \mathcal{M}_{fi} by

using

$$\langle f | \mathcal{S} | i \rangle = i(2\pi)^4 \delta^{(4)} \left(\sum_i p_i - \sum_f p_f \right) \mathcal{M}_{fi}$$

Meaning that we have

$$\mathcal{M}_{bhbabha} = \bar{v}(p_1) \gamma^\mu u(p_2) \frac{g_{\mu\nu}}{q^2} \bar{u}(p'_1) \gamma^\nu v(p'_2)$$

The reason we did all this is because we can calculate the *differential cross section* (in the centre of mass frame) from this, using

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{p_f^*}{p_i^*} |\mathcal{M}_{fi}|^2 \quad (2.16)$$

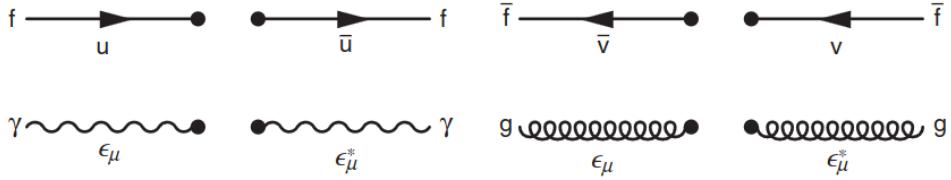
where p_f^* and p_i^* are the magnitudes of the final- and initial-state momenta, respectively. From the differential cross section we can also calculate the cross section by integrating over $d\Omega$.

This method is a general way to calculate the cross section of an event we want to study, but usually one just constructs the matrix element \mathcal{M}_{fi} from *Feynman diagrams*. In this calculation we already introduced the photon propagator in Eq. (2.15), and we implicitly simplified it to be $\frac{g_{\mu\nu}}{q^2}$, which is valid for our purposes, and can be used in any event where the photon is the propagator. Feynman realized this and made Feynman rules to construct diagrams. The rules are shown in Figure 2.1.

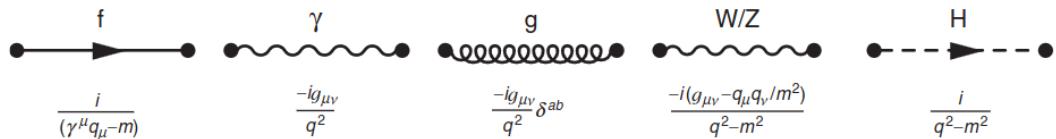
Meaning that following these rules we just constructed the Feynman diagram for Bhabha scattering shown in Figure 2.2

Lowest-order Feynman rules

External particles:



Propagators:



Three-point vertices:

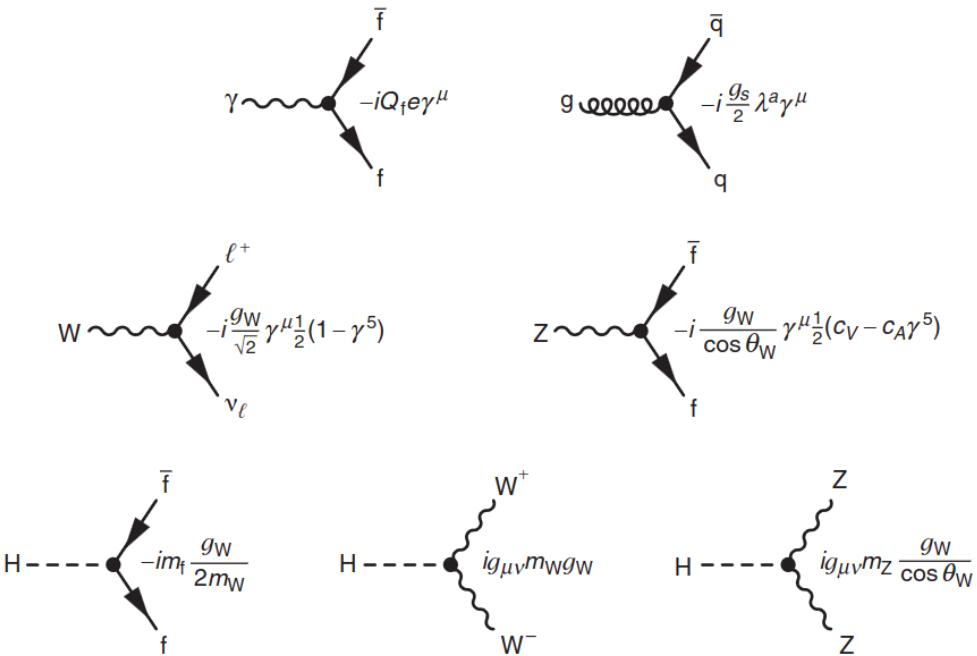


Figure 2.1: The Standard Model Feynman rules. Image taken from Thomson's "Modern Particle Physics" [2]

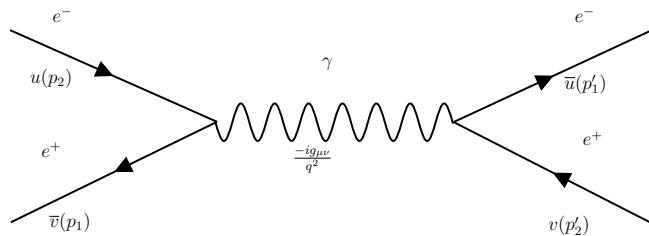


Figure 2.2: Bhabha scattering Feynman diagram

2.7 Adding it all up

Combining the three groups $U(1)_Y \otimes SU(2)_L \otimes SU(3)_C$ as well as introducing the Higgs field from Eq. (2.9) gives us the SM Lagrangian

$$\mathcal{L}_{SM} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\not{D}\psi + \psi_i y_{ij}\psi_j\phi + h.c. + |D_\mu\phi|^2 - V(\phi)$$

All of this is great at explaining what we know so far, which is depicted in Figure 2.3

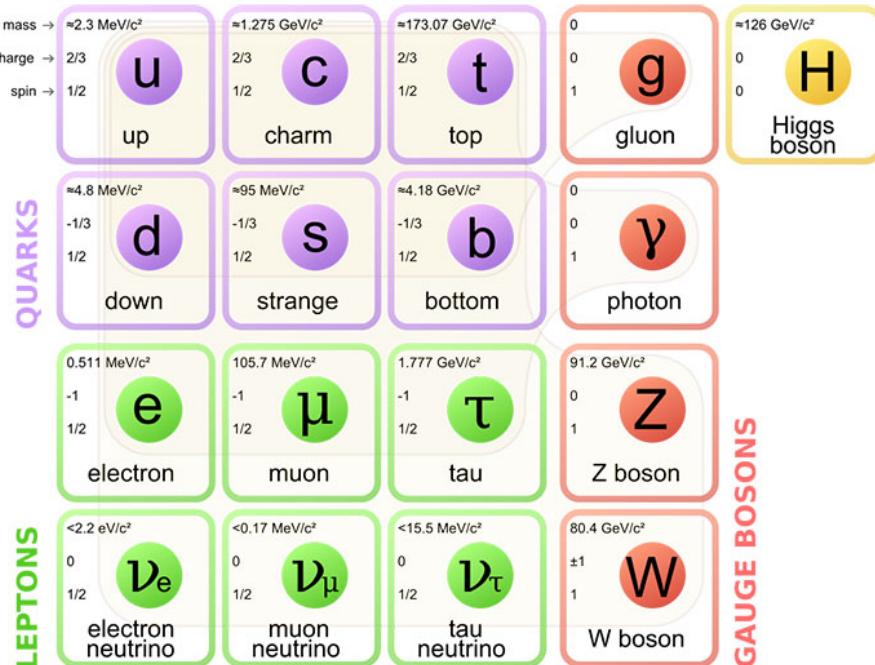


Figure 2.3: The Standard Model of particle physics. Image taken from Wikipedia [\[how to cite\]](#)

From Figure 2.3 we can see all the particles on the standard model. These include the quarks u, d, c, s, t and b , the leptons e, μ and τ as well as their neutrino partner³, the carrier of the strong force g , the carriers of the weak force W^\pm and Z , the carrier of the electromagnetic force γ and the Higgs boson H . From the figure we can also see which particles can interact with each of the bosons.

These are the building blocks of nature, as mesons, subatomic particles consisting of two quarks, and baryons, meaning subatomic particles consisting of three quarks (such as the proton and neutron) can further build atoms with electrons to furthermore create chemistry via quantum mechanical processes.

³Since neutrinos only interact via the weak force it is only present in a field doublet

Chapter 3

Dark Matter

While the standard model of particle physics has been incredibly successful in describing the behavior of subatomic particles, it is not a complete description of the universe. For example, the standard model cannot explain the mysterious gravitational forces that hold galaxies together, and it fails to account for the abundance of matter that we observe in the universe.

These mysteries have led scientists to propose the existence of dark matter, a mysterious substance that makes up a significant fraction of the matter in the universe. Dark matter does not interact with light or other forms of electromagnetic radiation, which makes it extremely difficult to detect directly. However, its presence can be inferred through its gravitational effects on visible matter, such as stars and galaxies.

The search for dark matter is one of the most pressing challenges facing modern physics. If we can understand the nature of dark matter, we will have a much better understanding of the fundamental forces that govern the universe. Moreover, the discovery of dark matter could have profound implications for our understanding of the evolution and fate of the universe as a whole.

3.1 Observations of existence

Proof? Here [3, 4]

3.1.1 Cosmology

3.1.2 WIMP

3.2 Beyond Standard Model candidates

3.2.1 New gauge boson

Z' baby [5, 6, 7]

3.2.2 Supersimmetree

Chapter 4

Production, Detection and Analysis

Now that we have established the necessary theoretical groundwork of particle physics in Chapter 2 and 3, it's time to explore how this knowledge can be applied. This leads us to ask important questions such as, how can we measure what we have learned? How do we put it into practice? Most importantly, how can we use this understanding to uncover new discoveries?

will be
added
later

To answer these questions, we have divided this chapter into three sections, each of which will focus on a different aspect of experimental particle physics. The first section will delve into particle production, followed by an examination of particle detection with the ATLAS detector at the LHC, and finally, we will explore the intricacies of data analysis in particle physics.

By exploring these areas, we hope to provide a comprehensive understanding of the theoretical underpinnings of particle physics, while also highlighting the practical applications of this knowledge.

I am struggling a bit to motivate the different parts of the thesis! So if you have any extra things I can add here feel free to do so!

4.1 Particle production

As we have already seen the shape of the SM we are now ready to dive into the subject of how we can produce the particles that we wish to detect. In this chapter we will start from the basic kinematics of particles and then move to more complex variables that will be of use when analysing data from detectors. The material for the first section is based on Thomson book Modern Particle Physics [2], Jacksons "Kinematics" [8] and Vadlas PhD. thesis [9].

4.1.1 Particle kinematics

Is this comment unnecessary?

As proved by Einstein, everything in spacetime can be described by four-vectors. For the purposes of particle physics, where we are mainly interested in the motion of particles, we will look at the four-momentum. Instead of using general variables as Einstein did, we will describe the particles using the four-momentum in terms of the geometry of the detectors, that means we will use the polar angle, θ , and the azimuthal angle, ϕ , such that we have

$$p^\mu = (E, p_x, p_y, p_z) \xrightarrow{Lab} (E, p_T \cos \phi, p_T \sin \phi, |\mathbf{p}| \cos \theta) \quad (4.1)$$

where p_T is the *transverse momentum* expressed as

$$p_T \equiv \sqrt{p_x^2 + p_y^2} = |\mathbf{p}| \sin \theta \quad (4.2)$$

The energy and momentum can be expressed in relativistic quantities, $E = \gamma\beta$ and $\mathbf{p} = \gamma m\boldsymbol{\beta}$, where $\gamma = 1/\sqrt{1 - \beta^2}$ and $\boldsymbol{\beta} = \mathbf{v}/c$ ¹ where m is the mass of the particle and c is the speed of light in vacuum. By contracting² two four-momentum we get the important Lorentz invariant square of the *invariant mass*

$$m^2 = p_\mu p^\mu = E^2 - |\mathbf{p}|^2$$

which can be generalized for a system containing n particles as

$$m^2 = p_\mu p^\mu = \left(\sum_{i=1}^n E_i \right)^2 - \left(\sum_{i=1}^n \mathbf{p}_i \right)^2 \quad (4.3)$$

¹As this is a particle physics thesis I will convert to Natural Units where we set $c = 1$

²Using the particle physicists convention of the minkowsky metric tensor $\eta_{\mu\nu}, (+, -, -, -)$

As this thesis will focus on a dilepton (and missing transverse energy) final state, which is of the type $2 \rightarrow 2 (+MET)$ then the invariant mass of the two leptons in the final state will be of interest, we will denote this as m_{ll} . From this we can also get another interesting variable, the *transverse energy*. This follows directly from the same equation

$$E_T = \sqrt{m^2 + p_T^2} \quad (4.4)$$

The invariant mass is what we measure in the final state only. But as we are going to use data³ from the LHC, from which the initial state is controlled, it will be of interest to see what the total energy and momentum of the two protons are. The term for this is called the *centre-of-mass* energy, \sqrt{s} , where s is defined by the same formula in Eq. (4.3), with the difference being that we look at the initial particles. For this thesis we will look at data and simulations of Run II from the LHC, which had $\sqrt{s} = 13$ TeV.

As this thesis aims to search for DM, which we know does not interact with matter in the same way as neutrinos, meaning it leaves no signal in detectors. As we know both the centre-of-mass energy, \sqrt{s} , and the invariant mass of all particles in the final state, Eq. (4.3). Then the presence of the non-interacting particles can often be inferred from the presence of *missing transverse energy*⁴ (MET), which is defined by

$$E_T^{miss} = \mathbf{p}_{miss} \equiv - \sum_i \mathbf{p}_{T,i} \quad (4.5)$$

do I need
sources
for the
neutrino
claim?

where the sum extends over the measured momenta of all the observed particles in an event. From this formula, if all particles produced in the collision have been detected, then this sum should be zero. Meaning that significant MET is therefore indicative of the presence of an undetected particle.

Another useful kinematic variable is the *hadronic activity* which is the scalar sum of the transverse momentum of all jets in an event, defined as

$$H_T = \sum_{i \in \{jets\}} ||\mathbf{p}_{T,i}|| \quad (4.6)$$

³And mostly simulations mimicking the ATLAS detector

⁴Also called *missing momentum*

this gives a measurement of the hadronic energy scale of an event. Another handy trick comes from the realization that the centre-of-mass frame is between the hadrons, where the total momentum is given as a function of the energy of the hadron. This means that the final state particles are boosted along the beam axis. With this realization we can now introduce a Lorentz invariant⁵ kinematic property known as the *rapidity*, y used to express the lepton angles

$$y \equiv \frac{1}{2} \ln \left(\frac{E + p_z}{E - p_z} \right) \quad (4.7)$$

We can use that $p_Z = E \cos \theta$ in the high-energy limit as the mass is negligible. In this limit we can use the *pseudorapidity*, η , defined by

$$\eta \equiv -\ln \left(\tan \frac{\theta}{2} \right) \quad (4.8)$$

The pseudorapidity is an interesting variable as it can tell us how close to the beam the final state particles are, where the higher $|\eta|$ means closer to the beam. This variable can also be negative, meaning backwards scattering. From this we can define a new variable which will come handy with particle identification, which is called the *R-cone*, that defines a circle in (η, ϕ) -space surrounding the object of interest. It is defined as

$$\Delta R = \sqrt{(\Delta \eta)^2 + (\Delta \phi)^2} \quad (4.9)$$

Another interesting variable is the *transverse mass*, defined as

$$m_T^2 = m^2 + p_T^2 \quad (4.10)$$

where m^2 is the invariant mass defined in Eq. (4.3). What is interesting with this variable is that it is the equivalent of the invariant mass equation, that takes into account invisible particles! We can take this further by looking at a supersymmetric version of the transverse mass, which calculates a transverse mass for two leptons by distributing the total p_T^{miss} among the two systems, and minimizing the maximum of the two transverse masses by varying the distribution of the p_T^{miss} -vector in terms of the size of p_T . This is

⁵Under boosts along the beam axis

called the *s*transverse mass and is defined by

$$m_{T2}^2(\chi) = \min_{\mathbf{q}_T^{(1)} + \mathbf{q}_T^{(2)} = \mathbf{p}_T} \left[\max \left\{ m_T^2 \left(\mathbf{p}_T^{\ell_1}, \mathbf{q}_T^{(1)}; \chi \right), m_T^2 \left(\mathbf{p}_T^{\ell_2}, \mathbf{q}_T^{(2)}; \chi \right) \right\} \right] \quad (4.11)$$

where \mathbf{q}_T are "dummy 2-vectors", χ is a free parameter used to "guess" the mass of the invisible particle, and $m_T^2(\mathbf{p}_T, \mathbf{q}_T)$ is an application of Eq. (4.10) using two particles:

$$m_T^2(\mathbf{p}_T, \mathbf{q}_T) = 2(p_T q_T - \mathbf{p}_T \cdot \mathbf{q}_T)$$

For a more detailed explanation and interpretation of the stransverse mass I refer the reader to the paper by Barr et.al. [10]. Even though the stransverse mass was made with neutralinos in mind, it can still be used to calculate SM processes. For example, if we want to reduce WW background events, we can first recall that each boson can decay as $W \rightarrow l + \nu_l$ with the W mass as an endpoint. Meaning that we can use m_{T2} to reduce the WW events in a dilepton final state by requiring that $m_{T2} > m_W$.

4.1.2 Proton-proton collisions

With all the kinematics out of the way the question of how the particles are produced still remains. The answer to that is protons. The way we produce elementary particles to study is by colliding two protons together. The reason as to why this works is because protons are also made of elementary particles, two *up* and one *down* to be specific. Because of this it is not hard to realize that the Feynman rules acquired from the SM (Chapter 2) also apply here. In this subsection we will study how different effects of pp -collisions affect the cross section, and therefore the expected number of events to occur, from a kinematical point of view.

Parton Distribution Functions

Although the proton is made up for two up quarks and one down quark, called the *valence quarks*. The proton also consists of gluons and other quarks, called partons. These partons become important in deep inelastic scatterings, where the proton breaks apart due to the high energies in the collisions. As we accelerate the protons before colliding them, we also accelerate the quarks and gluons inside of it, each of the quarks carry a momentum fraction x , called the Bjorken x . We can then calculate the invariant mass of two colliding partons q_1 and q_2 , with the momentum fraction x_1 and x_2 , from the protons momentum p_1 and p_2 respectively as

$$m^2 = x_1 x_2 s$$

where s is the centre-of-mass energy squared of the pp -system. The valence quarks in the proton do not only interact with the other valence quarks in the other proton, but they might also emit gluons which decay into $q\bar{q}$ -pairs, making a “sea” of gluons and quarks around the valence quarks. The momentum of the partons inside of the proton are dependent on the momentum transfer Q^2 and is represented by an experimentally determined momentum distribution, known as the *parton distribution function* (PDF) [11] $f(x, Q^2)$. **Question: Should I say more about PDFs than this? I feel like it is not relevant for my thesis, but is still good to know.** In other words, the PDFs give the probability of a parton to collide with the momentum fraction x . The shape and form the PDFs play an important role of estimating the processes that occur

after the proton collisions, and therefore are crucial when simulating events using Monte Carlo [12]. If we take as an example the process $pp \rightarrow l^+l^- + X$ where X denotes any hadrons formed by the remaining quarks. Figure 4.1 showcases the process.

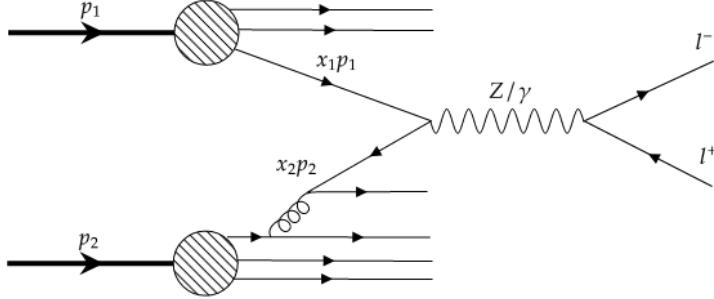


Figure 4.1: Feynmann diagram depicting the $pp \rightarrow l^+l^- + X$ process where X denotes any hadrons formed by the remaining quarks not taking part in the $q\bar{q}$ -collisions carrying x_1 and x_2 momentum from the protons p_1 and p_2 respectively

The cross section for the process is

$$\sigma(p(p_1)p(p_2) \rightarrow l^+l^- + X) = \int_0^1 \int_0^1 \sum_f f_f(x_1) f_{\bar{f}}(x_2) \cdot \sigma(q_f(x_1 p_1) + \bar{q}_f(x_2 p_2) \rightarrow l^+l^-) dx_2 dx_1 \quad (4.12)$$

where $\sigma(q_f(x_1 p_1) + \bar{q}_f(x_2 p_2) \rightarrow l^+l^-)$ is calculated using the rules explained in Chapter 2, and f are the PDFs. As we can see this can drastically change the cross section, and with that the probability, for an event we want to study.

Breit-Wigner resonance

Another important aspect when looking at particle collisions is the *Breit-Wigner resonance*. All unstable particles have a decay rate, Γ , (given as inverse of the lifetime τ) which is present at the wavefunction, $\Psi \propto e^{-i(m-i\Gamma/2)}$. This decay rate also becomes apparent when the unstable particle is the propagator of an event we are studying

$$\sigma \propto \frac{1}{(s - m^2)^2 - m^2\Gamma^2} \quad (4.13)$$

From this we can see that as the square of the centre-of-mass energy s , approaches the unstable particles mass m , there will be a resonance at the invariant mass of the unstable particle, this is called the Breit-Wigner resonance. It is because of resonances like this that we can identify particles such as the Z boson. This resonance will also be present when studying some of the BSM DM models in this thesis.

should I cite the original paper or Z discovery?

Expected events

The most important value we need to know when studying pp -collisions is the number of interactions taking place, this is defined as

$$N = \sigma \int \mathcal{L}(t) dt, \quad \text{where } \mathcal{L} = f \frac{n_1 n_2}{\sigma_x \sigma_y} \quad (4.14)$$

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where σ is the cross section of the interaction as expressed in Eq ... while also taking into account the effects from Eq. (4.12) and Eq. (4.13). The last three symbols come from accelerator kinematics where $\sigma_{x,y}$ denotes the beam size, f is the frequency of bunch crossings, and $n_{1,2}$ is the number of particles in bunches.

4.2 The ATLAS detector

We have so far on this chapter discussed how particles are produced. But we so far not yet explained how we actually detect them, arguably the most important matter in the field of high energy particle physics! This section of the chapter is just about that, and we will explain how the detection happens in A Toroidal LHC ApparatuS, or more commonly known as the ATLAS detector. Figure 4.2 showcases the detector and its size. The information of this section is largely based on the original ATLAS Collaboration introduction paper presented to CERN [13]. (and Knuts thesis [9] that described it in a more understandable way).

from my
under-
standing,
correct me
if I'm wrong!

The ATLAS detector is a general multi-purpose⁶ detector located at the LHC and covers nearly the entire solid angle around the collision point, as described with the (η, ϕ) -coordinates in Section 4.1.1. The ATLAS detector consists of four main subdetectors; (i) an inner tracking detector (ID), an (ii) electromagnetic calorimeter (ECAL), and a (iii) hadronic calorimeter (HCAL), and lastly (iv) a muon spectrometer (MS). Figure 4.3 visualises the four (i) – (iv) main sub-detectors, along with how the different particle types interact with each layer. A brief description of each layer is explained in this section.

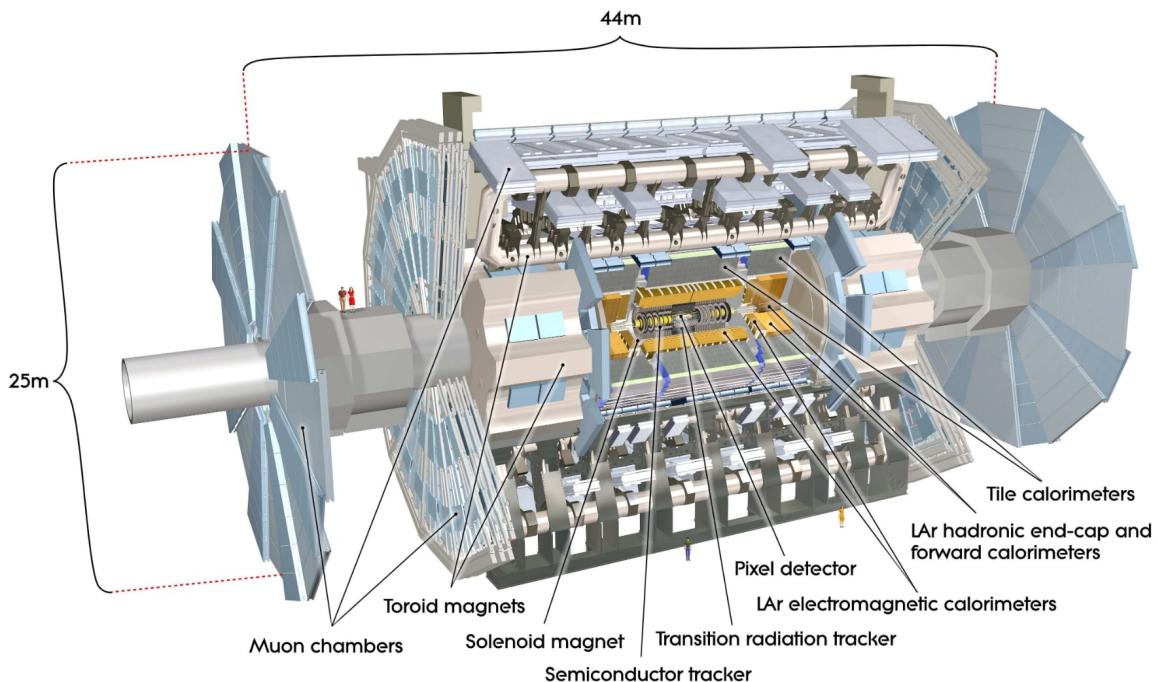


Figure 4.2: Cut-away view of the ATLAS detector, image taken from Ref. [13]

⁶Probing $pp-$ and $AA-$ (heavy ions) collisions.

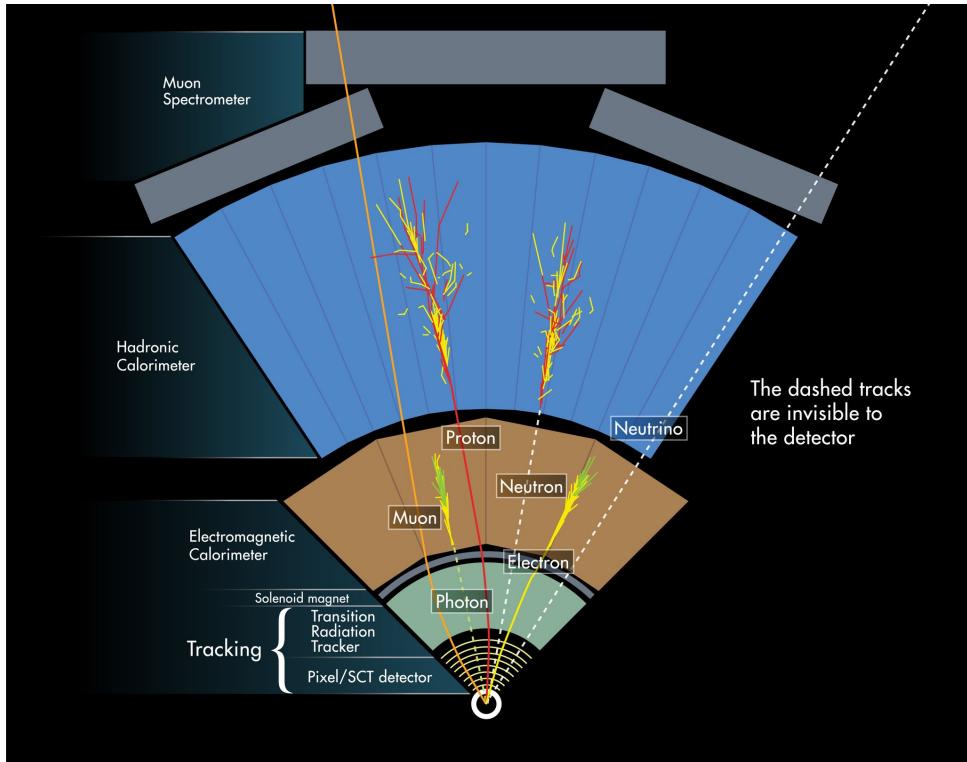


Figure 4.3: Illustration of the ATLAS detector layers, image taken from Ref. [14]

4.2.1 Inner detector

The inner-detector (ID) system is immersed in a 2T axial magnetic field and provides charged-particle tracking in the range $|\eta| < 2.5$. The ID provides the first measurements of the momentum and identification of electrically charged particles, as these can be determined by the curvature of their reconstructed tracks. The ID is made of three independent systems; the pixel detector, the semiconductor tracker (SCT) and the transition radiation tracker (TRT).

The pixel detector is made up of 80 million silicon pixel sensors, each of size $50 \times 400 \mu\text{m}^2$, and spread over multiple layers. Outside the pixel layers are the SCTs, which consist of silicon microstrips trackers, also placed on multiple layers. The SCT covers the pseudo-rapidity region $|\eta| < 2.5$. Furthest away from the interaction point lies the TRTs, they consist of 4mm in diameter straw tubes, which enables track-following up to $|\eta| = 2.0$.

4.2.2 The calorimeters

The ATLAS detector has two types of calorimeters, the *electromagnetic calorimeter* (ECAL) and the *hadronic calorimeter* (HCAL), both designed to fully stop certain types of particles. Both calorimeters covers a pseudorapidity range $|\eta| < 4.9$. The ECAL is immediately surrounding the inner detector and is divided into a barrel part ($|\eta| < 1.475$) and two end-cap components ($1.375 < |\eta| < 3.2$). The ECAL consists of absorbing lead plates, with liquid Argon (LAr) in between. The thickness of the calorimeter is made to fully measure the shower of photons and electron/positrons. The muons will only lose a small fraction of their energy as they have longer interaction lengths with lead.

The HCAL is immediately surrounding the ECAL on all sides and consists of two types of detectors. In the barrel ($|\eta| < 1.0$) and the extended barrel regions ($0.8 < |\eta| < 1.7$), the HCAL is made of steel plates with plastic scintillator tiles as active material. While on the end-cap regions ($1.5 < |\eta| < 3.2$) there are hadronic LAr detectors, with absorbing copper plates as active material; in the forward region ($3.1 < |\eta| < 4.9$) a combination of copper and tungsten plates are used as active material. The active materials are chosen to maximize the interaction cross-section with hadrons, such as neutrons, protons and pions. The depth of the HCALs is also designed to fully stop the particles and their showers in order to measure their total energies. Hadrons are efficiently stopped at the HCALs, meaning that only muons and invisible particles, such as neutrinos and potentially dark matter, leave the HCAL.

4.2.3 Muon spectrometer

The outermost layer of the ATLAS detector is the *muon spectrometer* (MS), dedicated to the measurement of the muons momenta. The MS, similar to the ID, consists of multiple layers of detector material, and is immersed in a strong magnetic field to bend the trajectories of the charged muons. The MS is made of four different types of detector component: (i) Monitored Drift Tubes (MDTs) on the barrel, (ii) Cathode Strip Chambers (CSCs) dealing with the events closer to the beam line in the end cap, (iii) Resistive Plate Chambers (RPCs) in the barrel and (iv) Thin Gap Chambers (TGCs) in the end caps. The MDTs and CSCs are used for tracking while the RPCs and TGCs are used

for tiggers. The tracking is provided for pseudorapidities up to $|\eta| < 2.7$, and the trigger system only extends to $|\eta| < 2.4$.

4.2.4 Summary of ATLAS

The main four parts of the ATLAS detector; *inner-detector* (ID), *electromagnetic calorimeter* (ECAL), *hadronic calorimeter* (HCAL), and the *muon spectrometer* (MS), have briefly been explained. For an easier understanding on the different parts I refer the reader to Figure 4.3 which shows visually how different particles interact with the detector. We can start by noting that there are only three types of particles that interact with the ID, muons, protons and electrons (there might also be other electrically charged hadrons that interact withg the ID). From the ID pixel and SCT detectors we can get information about ϕ, η, p_T and from the TRT we can get to know the *the charge* of the particles (depending on how the magnet bends their trajectories)⁷. Going to the next layer we see that in the ECAL the electron is completely stopped, such that we can know its energy, E . We can also note that the photons will interact with matter here and also be completely stopped. It is worth noting that in just the two first detector parts we have all the information we need to calculate interesting kinematic variables for the electron and photons. Going further into the detector, we see that the HCAL completely stops the proton and that now the detector interacts with the neutrons, giving us information about their energies (and location for neutrons). Lastly we see that the muon flies through all of this and finally hits the MS where we can get information about its energy. ***Note/Question to Farid/Eirik: Why does it not stop?***

The noteworthy particle in Figure 4.3, which was not mentioned in the summary above either, is the *neutrino*. The reason for this is because the neutrino, just like DM, is an *invisible particle*, meaning that it does not interact with matter. We can still reconstruct information about the neutrino and DM howeverver, we do this as explained in Section 4.1.1 with variables such as the MET, Eq. (4.5), as we know the centre-of-mass energy that went into the collision.

⁷So as we know that the proton is positively charged, then we know that the muon and electron in Figure 4.3 have negative charge

Another thing to add is that the explanation of the detector is brief in this thesis. The reality is much more complicated than this as there are, i.e. *triggers* (which might be excluding DM events) choosing which events are "important enough to record" as the *hardware and data storage* is a big problem considering the magnitude of events that happen each second. There are as well as algorithmic procedures to calculate the variables aforementioned from the detector signal which both consume energy and time. As this is not the focus of my thesis I will refer the reader to further literature explaining the real hardships when collecting data at the ATLAS detector and other LHC projects. [15]

Question: Is it okay for me to do this and move on? Or should I write about this?

4.3 Data analysis

The time has come to explore how we can search for new physics phenomena, now that it has been established how particles are produced and how we detect them. In this section we will take into account the classic way of searching for new physics which is called the *cut and count method*, but there are other methods to search for new physics, such as Machine Learning (ML) which is the method pursued in this thesis. There might be other methods, such as Quantum ML, but as of today we are still in an early stage of the technology [16]. To give a short description of the cut and count method, it makes kinematical *cuts* such that we isolate signal from background. The way the signal and background are made is by Monte Carlo (MC) simulations, such that we can more easily identify SM background from DM and then compare our results with real data. As it wouldn't be a real experimental physics discovery without making a statistical analysis of the results we will also explain how we utilize this tool. To guide us through this process I will use $ZZ^{(*)}$ channel in the discovery of the Higgs Boson in 2012 [17] as an example of the success of this method.

4.3.1 Cut and count

The cut and count method is what is currently the standard method of doing data analysis with CERN related research. As the name implies, the cut and count method works by making cuts on kinematical variables and afterwards counting how many events are left. The goal of using this method is to make cuts such that we remove as many background events as possible while also keeping as many signal events as possible. For example, if we were to study a new physics model with a new light vector boson behaving similarly to the Z boson, but with a higher mass, then a good kinematical cut to remove many background processes would be to require that $m_{ll} > 100$ GeV, as this would remove the majority of Z -resonance from the final state, making it easier to "find" the new physics model.

To more thoroughly explore the cut and count method we can look at the Higgs discovery, in particular the $H \rightarrow ZZ^{(*)} \rightarrow 4l^8$ channel. In section 4.1 of the original article [17]

⁸where l is for lepton, but only means e^\pm or μ^\pm . question: should i explain why not τ ?

can I include this?

there is something called *event selection*. The event selection are the kinematical cuts used. The kinematical cuts used in the article, aside from the *standard selection critera* (see Chapter 6.3), were:

- Single-lepton or dilepton triggers
- Four leptons on final state with $p_T > 20, 15, 10, 7$ GeV, in the order of most energetic to least
- Higgs-boson candidates are formed by selecting two same flavour opposite charge lepton pairs

The first "cut" is to make sure that we have leptons on both the data and simulations. The second cut is that there needs to be four leptons, with different p_T cuts to remove background while keeping as much signal as possible. And lastly we want to have two lepton pairs of the same-flavour with opposite-charge, this is to make sure that the leptons that we observe actually decay from Z -bosons. What now remains is to explore the "count" part of this method. When counting the events that pass the event selection one usually counts the background events that pass, the data points that pass, and also the signal events that pass. For the 2012 Higgs discovery, the Higgs channel with a Higgs mass of 125 GeV was used as signal. The results of the 2012 discovery is shown Figure 4.4.

Altough this section made the process look simple, it was through the effort of many scientist working togheter that made this happen. The great computational power needed to *correctly* simulate events and reconstruct objects from detector signal was, and still is a big challenge. Not to mention the state-of-the-art technology to be able to be to both accelerate the protons to an energy high enough to "create" new physics, and to actually be able to detect it. But this alone was not enough to claim the discovery, as we're physicits we need to be completely sure that what we show is true. We do this with statistics which is the next section.

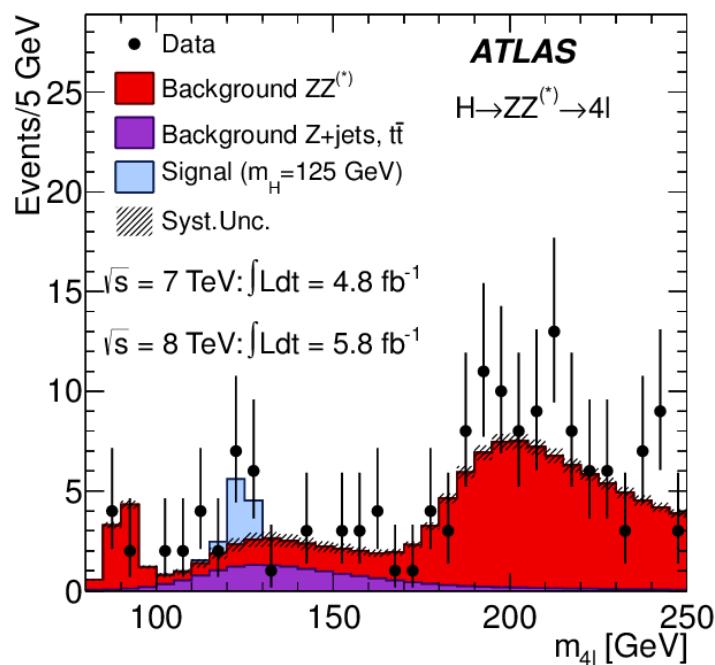


Figure 4.4: The Higgs discovery on the $ZZ^{(*)}$ channel, image taken from Ref. [17]

4.3.2 Statistical analysis

To make any sort of claim in modern physics we should be absolutely certain that what we are claiming is true, as just making the cuts and isolating a signal to background is not enough. To be specific we need to be *at least 5σ sure* to claim any new discoveries. But as not everything in experimental high energy particle physics is a new discovery, but is in fact for the most part *exclusions*. Then we need to explain what an exclusion is, and how to make them.

This thesis is following Bugges lecture notes on statistics [**how do I cite his powerpoint from FYS5555**] and the example set by the ATLAS Collaboration article "Search for new particles in events with one lepton and missing transverse momentum in pp collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector" [18], where a Bayesian analysis is performed to set limits on the studied processes. Using the signal+background hypothesis, the expected number of events in each channel of a process we are studying is

$$N_{\text{exp}} = \varepsilon_{\text{sig}} L_{\text{int}} \sigma B + N_{\text{bkg}}$$

where L_{int} is the integrated luminosity, ε_{sig} is the signal selection efficiency defined as the fraction of signal events that satisfy the event selection criteria, N_{bkg} is the expected number of background events, and σB is the cross-section times branching ratio of the process. Using Poisson statistics, the likelihood to observe N_{obs} events is

$$\mathcal{L}(N_{\text{obs}}|\sigma B) = \frac{(N_{\text{exp}})^{N_{\text{obs}}} e^{-N_{\text{exp}}}}{N_{\text{obs}}!} \quad (4.15)$$

We include uncertainties by introducing nuisance parameters θ_i , each with a probability density function $g_i(\theta_i)$, and integrating the product of the Poisson likelihood with the probability density function. The integrated likelihood is

$$\mathcal{L}_B(N_{\text{obs}}|\sigma B) = \int \mathcal{L}(N_{\text{obs}}|\sigma B) \prod g_i(\theta_i) d\theta_i \quad (4.16)$$

where a log-normal distribution is used for the $g_i(\theta_i)$. The nuisance parameters are taken to be: L_{int} , ε_{sig} and N_{bkg} , with the appropriate correlation accounted for between the first and the third parameters. The measurements of the two decay channels (muon or

electron final state for $H \rightarrow lll'l'$) are combined assuming the same branching fraction for each, thus Eq. 4.16 remains valid with the Poisson likelihood replaced by the product of the Poisson likelihoods for the two channels. The integrated luminosities for the electron and muon channels are fully correlated. We can further use Bayes' theorem which gives the posterior probability that the signal has signal strength σB :

$$P_{\text{post}}(\sigma B | N_{\text{obs}}) = N \mathcal{L}_B(N_{\text{obs}} | \sigma B) P_{\text{prior}}(\sigma B) \quad (4.17)$$

where $P_{\text{prior}}(\sigma B)$ is the assumed prior probability, here chosen to be flat in σB , for $\sigma B > 0$. The constant factor N normalises the total probability to one. The posterior probability is evaluated for each mass and decay channel as well as for their combination, and then used to set a limit on σB .

As we can see, the inputs for the evaluation of \mathcal{L}_B (and P_{post}) are ε_{sig} , L_{int} , N_{bkg} and N_{obs} and the uncertainties of the first three. The uncertainties for these should account for experimental and theoretical systematic effects as well as the statistics of the simulated samples. For this thesis the systematic uncertainties will not be calculated, but will rather be assumed to be $\pm 30\%$ of the background, as this is roughly the standard value of ATLAS papers.

To make exclusions we can use Eq. (4.17) to establish a *confidence limit* (CL). CLs are defined as the probability to observe the number of events observed in an experiment, N_{obs} , or *less* given signal+background. We usually define a signal+background hypothesis to be excluded when $\text{CL}_{s+b} < 5\%$. Meaning a 95% CL. Such that the probability to falsely exclude an existing signal(+background) is 5%. The standard practice in particle physics when using CL is to *set limits* on theoretical models, rather than exclude them.

On the other side, to claim any discovery in particle physics we need to know the *significance* of any statistical fluctuation. Before getting to the significance we can discuss the *p-value*, defined as the probability to observe the number of events observed in the

experiment, n_{obs} , or *more* given only background

$$p = P(N \geq N_{\text{obs}} | \lambda = N_{\text{bkg}}) = \sum_{k=N_{\text{obs}}}^{\infty} \mathcal{L}(k | N_{\text{bkg}}) \quad (4.18)$$

The smaller the p -value, the less compatible an observation is with the background only hypothesis, meaning more likely to be a discovery! From this we can find the significance Z by

$$p = \int_{-\infty}^{-Z} \frac{e^{-x^2/2}}{\sqrt{2\pi}} dx$$

As mentioned in the start of this subsection, a discovery in particle physics is defined to be at least a $Z = 5\sigma$ deviation from the background hypothesis, meaning that we would have a p -value of $p \leq 2.87 \times 10^{-7}$. In other words, with a 5σ deviation, the probability to falsely discover something is at worst **one in roughly 3.5 million**.

As the significance is an interesting quantity we can give it its own definition. In this thesis we will use the low statistics formula for the significance, as this is the most general one. We can either define the significance as the *observed significance* by

$$Z = \sqrt{2 \left[N_{\text{obs}} \ln \frac{N_{\text{obs}}}{N_{\text{bkg}}} - N_{\text{obs}} + N_{\text{bkg}} \right]} \quad (4.19)$$

or as the *expected significance* by changing $N_{\text{obs}} \rightarrow N_{\text{sig}} + N_{\text{bkg}}$, where N_{sig} is the number of signal events

$$Z = \sqrt{2 \left[(N_{\text{sig}} + N_{\text{bkg}}) \ln \left(1 + \frac{N_{\text{sig}}}{N_{\text{bkg}}} \right) - N_{\text{sig}} \right]} \quad (4.20)$$

However, Eq. (4.18) did not include any nuisance parameters, it used Eq. (4.15) instead of Eq. (4.16). We want to express the significance with uncertainties. From "Discovery sensitivity for a counting experiment with background uncertainty" from Glen Cowan [19], we can use Eq. (17) on his paper that reads

$$Z = \left[-2 \left(N_{\text{obs}} \ln \left[\frac{N_{\text{obs}} + m}{(1 + \tau)N_{\text{obs}}} \right] + m \ln \left[\frac{\tau(N_{\text{obs}} + m)}{(1 + \tau)m} \right] \right) \right]^{1/2}$$

where $m = \tau N_{\text{bkg}}$ and Eq. (19) that reads

$$\tau = \frac{N_{\text{bkg}}}{\sigma_{\text{bkg}}^2}$$

where σ_{bkg} is the uncertainty of the background. This gives us

$$Z = \sqrt{-2 \left(N_{\text{obs}} \ln \left[\frac{N_{\text{obs}} + \frac{N_{\text{bkg}}^2}{\sigma_{\text{bkg}}^2}}{(1 + \frac{N_{\text{bkg}}}{\sigma_{\text{bkg}}^2}) N_{\text{obs}}} \right] + \frac{N_{\text{bkg}}^2}{\sigma_{\text{bkg}}^2} \ln \left[\frac{\frac{N_{\text{bkg}}}{\sigma_{\text{bkg}}^2} (N_{\text{obs}} + \frac{N_{\text{bkg}}^2}{\sigma_{\text{bkg}}^2})}{(1 + \frac{N_{\text{bkg}}}{\sigma_{\text{bkg}}^2}) \frac{N_{\text{bkg}}^2}{\sigma_{\text{bkg}}^2}} \right] \right)} \quad (4.21)$$

Which makes for a better estimate of the significance one has in reality. *Should I add plots showing CLs and significance?*

4.4 Summary

In this chapter we have studied how one can calculate the number of expected events from a pp -collision can lead to the discovery of new particles. Using special relativity as a tool, we can express the four-momentum of the particles with detector-coordinates, $p^\mu = (E, p_T \cos \phi, p_T \sin \phi, |p| \cos \theta)$. From this four-momentum vector we can thereafter calculate interesting kinematic variables such as the invariant mass m_{ll} , missing transverse energy E_T^{miss} , stransverse momentum, m_{T2} , among others. Using the advanced ATLAS detector the four-momentum can be recorded from the accelerated protons at the LHC. With the recorded data and MC simulations taking into account the experimental features such as the PDFs and Breit-Wigner resonance, as well as the ATLAS kinematics, we can compare how the simulated events fare with the data recorded. By playing around with the kinematical variables of the particles and making cuts to isolate new physics signal we can see if there is a discrepancy between the data recorded and the SM background. After creating this signal region with the cut and count method we conduct a statistical analysis to see how the new theory/observed data deviates from our current understanding of physics.

This state-of-the-art method is what currently is being used at CERN and has led to a great advancement in the field. However with the rise of new technologies, such as *machine learning*, which excel at classification tasks, a door has been opened to try new methods. In this thesis we will use ML to hopefully create a better and more general signal region than what the current cut and count method does. Before describing how, we will explain what machine learning is, this is the subject of the next chapter.

I don't know if this summary is too dense, or if the opening to ML is too superficial. Feel free to comment as harshly as you want here :-)

Chapter 5

Machine Learning

Machine Learning (ML) has emerged as a powerful tool for analyzing complex datasets and making accurate predictions. Its applications span across various fields, from natural language processing to image recognition, and it has been used successfully to solve a range of problems.

The main approach of this thesis will be to use ML as its popular rise has proven to be effective at binary classification tasks [20, 21] in High Energy Particle Physics. For our purposes it will be a powerful tool to attempt to classify events as SM background or as DM signal.

To give a short description of the essence of ML we can start by considering a general parameter $\beta = \{\beta_1, \beta_2, \dots, \beta_n\}$ for a n-dimensional problem, which for our purposes can be seen as what are called *weights* and *biases* ($\beta = \{w, b\}$), the goal is to choose these parameters β such that we minimize a cost (also called loss) function $C(\beta)$ with respect to a set of data points given by a matrix \mathbf{X} . This matrix \mathbf{X} will be our dataset containing n *features* for each event m , and is of the following form

$$\mathbf{X} = \begin{pmatrix} x_{11} & x_{21} & x_{31} & \cdots & x_{n1} \\ x_{12} & x_{22} & x_{32} & \cdots & x_{n2} \\ x_{13} & x_{23} & x_{33} & \cdots & x_{n3} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ x_{1m} & x_{2m} & x_{3m} & \cdots & x_{nm} \end{pmatrix} \quad (5.1)$$

This project will only focus on Supervised Learning, meaning that we know what the output is a binary representation of signal and background, such that we can use target values \mathbf{t} . Then we give the network a score depending on how close the predicted output is to real values \mathbf{t} . Then we repeat the process after tweaking the parameters β and see if the score gets better.

To this end, we will first provide a mathematical foundation for ML, starting with a brief overview of the different types of ML algorithms we will study, such as Neural Networks (NN) and Boosted Decision Trees (BDT). Other aspects such as the importance of feature selection and feature engineering in preparing the data for ML algorithms we will, as well as the concept of model evaluation and optimization will be discussed in Chapter 7. This project will take from granted that the reader is comfortable with linear algebra and jump straight into the mathematical fundations of ML.

5.1 Neural Networks

The theoretical foundation for this chapter is mainly based of Hjort-Jensen lecture notes [22, 23, 24]. Before begining we can briefly explain the idea behind NNs. As stated by Hjorth-Jensen in [22]:

The idea of NN is to mimic the neural networks in the human brain, which is composed of billions of neurons that communicate with each other by sending electrical signals. Each neuron accumulates its incoming signals, which must exceed an activation threshold to yield an output. If the threshold is not overcome, the neuron remains inactive, i.e. has zero output

That takes us to what a *neuron* is.

5.1.1 Artificial neurons

To describe the behaviour of a neuron mathematically we can use the following model that mimics how one neuron works

$$y = f \left(\sum_{i=1}^n w_i x_i \right) = f(z) \quad (5.2)$$

Where y , the output of the neuron, is the value of its *activation function* (See section 5.1.3), which has the weighted, w_i , sum of signals x_1, \dots, x_n received by n neurons that are in a preceding layer.

The goal of NNs is to mimic the biological nervous system by letting each neuron interact with each other by sending signals, for us is in the form of a mathematical function between each layer, called the activation function. Most NNs consist of an input layer, an output layer and intermediate layers, called hidden layers. All the layers can contain an arbitrary number of neurons, and each connection between two neurons is associated with a weight variable w_i . The goal of using NNs is to teach the network the patterns of the data to then predict some target. In the context of our search for DM, by giving a NN our data set of events as its input layer, we can then train the network to classify events as signal or background.

Explained in greater detail if we were to look at a single event of the data, we start with an input with all the relevant features of the event, \mathbf{X} . Using Eq. (5.2) on every neuron on the next layer we can teach the network if there are any connections between the features, we can repeat this process for n layers. As an output we want a single neuron to see if it has predicted the event to be a signal or background, since this is binary output. After analysing the prediction we can use the labels on the target data \mathbf{t} to tell (the network) whether it predicted correctly or wrong. We can then use a *cost function* and a specific *metric* to evaluate numerically how well the network predicted the output by giving it a score. Seeing how the results fare we can then back-propagate to shift the weights and biases and repeat the process until we are satisfied with our result. Each of these iterations is called an epoch.

To generalize our artificial neuron to a whole network we can look at a Multilayer Perceptron (MLP). An MLP is a network consisting of at least three layers of neurons, the input, one or more hidden layers, and an output. The number of neurons can vary for each layer. The above explanation is a very dense and simplified one. In reality it is complicated to find out which cost function, activation function, metric, etc. are best suited to the given problem. But before we get into the gory details we can explore the mathematical model that illustrates what was tried to be explained above.

5.1.2 Optimizers

The way we "tweak the parameters β to see if the network prediction gets better" is by using something called an *optimizer*. We will mainly focus on the theory behind the *Stochastic Gradient Descent* (SDG) optimizer as it is more easy to digest. Before explaining the SDG we have to look at the Gradient Descent (GD).

Given a cost function $C(\beta)$ we can get closer to the minimum by calculating the gradient $\nabla_\beta C(\beta)$ wrt. the unknown parameters from the NN β . If we were to calculate the gradient at a specific point β_i in the parameter space, the negative gradient would correspond to the direction where a small change $d\beta$ in this parameter space would result in the biggest decrease in the cost function. In the same way we in physics would determine where the local (or global) minima at a complex multidimensional potential numerically.

In GD we can chose (which needs to be optimized) a step size η related to the size of an iteration in the parameter space; this is called the *learning rate*. The mathematical function for an iteration in parameter space to optimize the parameter β such that it minimizes the cost function is given as

$$\beta_{i+1} = \beta_i - \eta \nabla_\beta C(\beta_i) \quad (5.3)$$

To converge towards a minimum we should select a learning rate η small enough to not "step over" the minimum point of the cost-function-space, but also not too small to get stuck on a local minimum rather than the global minimum. Thus using the learning rate as a hyperparameter in a grid search is a good way to optimize a NN for a given task.

In GD one computes the cost function and its gradient globally for all data points. This quickly becomes computationally heavy when dealing with large datasets. Thus a common approach is to compute the gradient over batches of the data. For our purposes it would be optimal to use GD, but our data size is massive, of the order of 10^8 events (13 GB), becoming computationally impossible. Thus instead of making a $n \times 10^8$ matrix (see Eq. (5.1)), we could for example split it into ten smaller matrices of $n \times 10^7$ to then perform a parameter update, making the computation possible. This is where SGD comes in, for each step, or epoch the data is divided randomly into N batches of size n . Then for each batch we use Eq. (5.3) to update the parameters, thus updating β_{i+1} N -times for each epoch. The idea of SGD comes from the observation that the cost function can almost always be written as a sum over n data points. The main advantage of SGD is not to ease the computation however, as using more batches also reduces the risk of getting stuck in a local minimum since it introduces a randomness of which part of the parameter space we move through, but this is at the cost of reducing statistics which might not be ideal for every problem.

There are other optimization algorithms we could use, such as the popular ADAM [25]. The Adaptive Moment Estimation (ADAM) is a more advanced optimization algorithm that uses adaptive learning rates. It computes individual learning rates for each weight based on the average of past gradients and their variances. ADAM also uses momentum to accelerate the convergence of the optimization algorithm. We will test both ADAM

and SGD when optimizing our networks.

5.1.3 Activation functions

As seen in Section 5.1.1, an important aspect of NNs are activation functions and cost functions. As shall become apparent in Section 5.1.4, when evaluating an activation function we get the neuron output, but what are these activation functions? Mathematically speaking, activation functions are: Non-constant, Bounded, Monotonically-increasing and continuous functions. In this project we will utilize two different activation functions at different layers. The first one is a sigmoid activation function

$$f(x) = \frac{1}{1 + e^{-x}} \quad (5.4)$$

which is the most basic activation function, this will be used from the input layer to our first hidden layer as the risks of having little neuron activation is minimal here. On all other layers I will utilise a Rectified Linear Unit (ReLU)

$$f(x) = x^+ = \max(0, x) = \begin{cases} x & \text{if } x > 0, \\ 0. & \text{otherwise.} \end{cases} \quad (5.5)$$

which has better gradient propagation, meaning that there are fewer vanishing gradient problems compared to the sigmoidal function.

5.1.4 Feed Forward network

To describe how the network "guesses" outputs in a mathematical model we can start by looking at Eq. (5.2) where we got an output y from an activation function f that receives x_i as input. We can expand the function as as following

$$y = f \left(\sum_{i=1}^n w_i x_i + b_i \right) = f(z) \quad (5.6)$$

where w_i is still the weight and we now use the previously introduced bias b_i which is normally needed in case of zero activation weights or inputs. The difference comes now in the interpretation; where in the activation $z = (\sum_{i=1}^n w_i x_i + b_i)$ the inputs x_i are now the

outputs of the neurons in the preceding layer. Furthermore an MLP is fully-connected, meaning that each neuron received a weighted sum of the output of **all** neurons in the previous layer. To expand Eq. (5.6) we can first look at the output of every neuron i in a weighted sum z_i^1 for each input x_j on a layer

$$z_i^1 = \sum_{j=1}^M w_{ij}^1 x_j + b_i^1 \quad (5.7)$$

Where M stands for all possible inputs to a given neuron i in the *first* layer. Such that if we evaluate the weighted sum in an activation function f_i for each neuron i , then the output of all neurons in layer 1 is y_i^1

$$y_i^1 = f(z_i^1) = f\left(\sum_{j=1}^M w_{ij}^1 x_j + b_i^1\right)$$

To generalize this for l -layers, which may have different activation functions, we write it as

$$y_i^l = f^l(z_i^l) = f^l\left(\sum_{j=1}^{N_{l-1}} w_{ij}^l y_j^{l-1} + b_i^l\right)$$

Where N_l is the number of neurons in layer l . Thus when the output of all the nodes in the first hidden layer is computed, the values of the subsequent layer can be calculated and so forth until the output is obtained. With this we can show that we only need the inputs x_n to calculate the output with l hidden layers

$$y^{l+1} = f^{l+1}\left[\sum_{j=1}^{N_l} w_{ij}^{l+1} f^l\left(\sum_{k=1}^{N_{l-1}} w_{jk}^l \left(\cdots f^1\left(\sum_{n=1}^{N_0} w_{mn}^1 x_n + b_m^1\right) \cdots\right) + b_j^l\right) + b_i^{l+1}\right] \quad (5.8)$$

This shows that an MLP is nothing more than an analytic function, specifically a mapping of real-valued vectors $\hat{x} \in \mathbb{R}^n \rightarrow \hat{y} \in \mathbb{R}^m$. We can also see that the above equation is essentially a nested sum of scaled activation functions of the form

$$f(x) = c_1 f(c_2 x + c_3) + c_4$$

where the parameters c_i are the weights and biases. By adjusting these parameters we shift the activation function to better match the label we are training the data on, this is the flexibility of a NN. Something else we can note is that Eq. (5.8) can easily be changed

into matrix notation, as hinted with Eq. (5.1). However this realization can help make computing the values a much easier task by for example utilizing TensorFlow or other mathematical packages in Python. An illustration showing the main idea of how a Feed forward network is set up is shown in Figure 5.1.

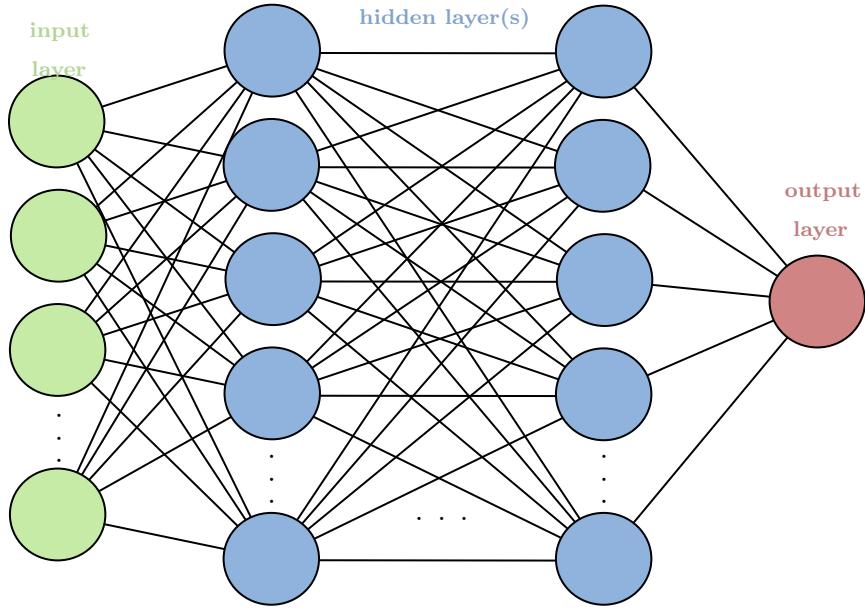


Figure 5.1: Basic illustration of a network with two hidden layers.

5.1.5 Back Propagation algorithm

So far we have only explained Feed Forward networks, which helps us to compute the output of the NN in term of basic linear algebra. We mentioned the possibility to adjust the weight and biases, but never explained how. Now is the time to dive into that subject, as we will explain the back propagation algorithm. What we want to know is how the changes in the biases and weights in the network change the cost function, and how we could use the final output to modify the weights? Before we derive these equations we start by a plain regression problem, using the Mean Squared Error (MSE) as a cost function for pedagogical reasons

$$C(\hat{W}) = \frac{1}{2} \sum_{i=1}^n (y_i - t_i)^2 \quad (5.9)$$

where \hat{W} is the matrix containing all the weights and (more importantly) t_i are the targets, which are the labels of events telling whether we have a signal or background

event. To generalise this we first have to generalise to Eq. (5.7) for a layer l

$$z_i^l = \sum_{j=1}^M w_{ij}^l y_j^{l-1} + b_i^l \Leftrightarrow \hat{z}^l = (\hat{W}^l)^T \hat{y}^{l-1} + \hat{b}^l$$

where the right side is written in matrix notation. From the definition of z_j^l with an activation function, Eq. (5.6), we have

$$\frac{\partial z_j^l}{\partial w_{ij}^l} = y_i^{l-1} \quad (5.10)$$

and

$$\frac{\partial z_j^l}{\partial y_i^{l-1}} = w_{ij}^l$$

which again, with the definition of the sigmoid activation function, Eq. (5.4), gives us

$$\frac{\partial y_j^l}{\partial z_j^l} = y_j^l(1 - y_j^l) = f(z_j^l)(1 - f(z_j^l)) \quad (5.11)$$

Furthermore, we need to take the derivative of Eq. (5.9) with respect to the weights, doing so for a respective layer $l = L$ we have

$$\frac{\partial C(\hat{W}^L)}{\partial w_{jk}^L} = (y_j^L - t_j) \frac{\partial y_j^L}{\partial w_{jk}^L}$$

where the last partial derivative is easily computed using the chain rule with Eq. (5.10) and Eq. (5.11)

$$\frac{\partial y_j^L}{\partial w_{jk}^L} = \frac{\partial y_j^L}{\partial z_j^L} \frac{\partial z_j^L}{\partial w_{jk}^L} = y_j^L(1 - y_j^L)y_k^{L-1}$$

Such that

$$\frac{\partial C(\hat{W}^L)}{\partial w_{jk}^L} = (y_j^L - t_j) y_j^L(1 - y_j^L)y_k^{L-1} := \delta_j^L y_k^{L-1} \quad (5.12)$$

where we have defined the error

$$\delta_j^L := (y_j^L - t_j) y_j^L(1 - y_j^L) = f'(z_j^L) \frac{\partial C}{\partial y_j^L} \quad (5.13)$$

or in matrix form

$$\delta^L = f'(\hat{z}^L) \circ \frac{\partial C}{\partial \hat{y}^L}$$

where on the right hand side we wrote this as a Hadamard product¹. This error δ^L is an important expression, since as we can see in the index form of this expression in Eq. (5.13), we can measure how fast the cost function is changing as a function of the j -th output activation. This means that if the cost function doesn't depend on a particular neuron j , then δ_j^L would be small.

We also notice that everything in Eq. (5.13) is easily computed. Thus we can also see how the weight changes the cost function using Eq. (5.12) quite easily. Another thing we can compute with Eq. (5.13) is

$$\delta_j^L = \frac{\partial C}{\partial z_j^L} = \frac{\partial C}{\partial y_j^L} \frac{\partial y_j^L}{\partial z_j^L}$$

which can be interpreted in terms of the biases b_j^L as

$$\delta_j^L = \frac{\partial C}{\partial b_j^L} \frac{\partial b_j^L}{\partial z_j^L} = \frac{\partial C}{\partial b_j^L} \quad (5.14)$$

where the error δ_j^L is exactly equal to the rate of change of the cost function as a function of the bias.

Something interesting is that when using Eq. (5.12 - 5.14) we see that if a neuron output y_j^L is small, then the gradient term, Eq. (5.12), will also be small. We say then that the weight learns slowly, meaning that the contribution of said neuron is less important "to fix" than those that have a higher weight. Of course this example is a very simple one to wrap our heads around, but the magic comes when the algorithm is evaluating a random neuron in a layer n , after using many layers the NN becomes a **black box** for us to wrap our heads around!

It is also worth noting that when the activation function is flat at some specific values (which also varies with the chosen function!) the derivative will tend towards zero, making the gradient small, meaning the network is learning slow as well. To finish up our back propagation algorithm we still need one equation. We are now going to propagate backwards in order to determine the weights and biases. We start by representing the

¹Also called *element-wise* product, $(A \circ B)_{ij} = (A \odot B)_{ij} = (A)_{ij}(B)_{ij}$

error in the layer before the final one $L - 1$ in term of the errors of the output layer. If we try to express Eq. (5.13) in terms of the output layer $l + 1$, using the chain rule and summing over all k entries we get

$$\delta_j^l = \sum_k \frac{\partial C}{\partial z_k^{l+1}} \frac{\partial z_k^{l+1}}{\partial z_j^l} = \sum_k \delta_k^{l+1} \frac{\partial z_k^{l+1}}{\partial z_j^l}$$

recalling Eq. (5.7) (replacing 1 with $l + 1$) we get

$$\delta_j^l = \sum_k \delta_k^{l+1} w_{kj}^{l+1} f'(z_j^l) \quad (5.15)$$

Which is the final equation we needed to start back propagating.

5.1.6 Summary

To summarize the whole process of the NN

- First take the input data \mathbf{x} and the activation \mathbf{z}_1 of the input later, and then compute the activation function $f(z)$ to get the next neuron outputs \mathbf{y}^1 . Mathematically this is taking the first step of the feed forward algorithm, i.e. choosing $l = 0$ in Eq. (5.8)
- Secondly we commit all the way in Eq. (5.8) and compute all \mathbf{z}_l , activation function and \mathbf{y}^l .
- After that we compute the output error $\boldsymbol{\delta}^L$ by using Eq. (5.13) for all values j .
- Then we back propagate the error for each $l = L - 1, L - 2, \dots, 2$ with Eq. (5.15).
- The last step is then to update the weights and biases using Eq. (5.3) for each l and updating using

$$w_{jk}^l \leftarrow w_{jk}^l - \eta \delta_j^l y_k^{l-1}$$

and

$$b_j^l \leftarrow b_j^l - \eta \delta_j^l$$

This whole procedure is usually called an epoch, which we can repeat as many times as we want to better reduce the cost function in hope of converging to the global minima.

5.2 Boosted Decision Trees

In the previous chapter, we explored how NNs can aid in the task of signal and background classification, such as the one in this thesis where we are looking DM signal in the SM background. In this chapter, we will delve into Boosted Decision Trees (BDTs), another powerful tool for binary classifications. BDTs, unlike neural networks, are not based on simulating biological neurons. Instead, they rely on decision trees, a simple yet powerful idea that has been around for decades. [26, 27] Decision trees are built by recursively splitting data based on the values of features until a stopping condition is met. BDTs take this idea one step further by training an ensemble of decision trees, where each tree attempts to correct the mistakes of its predecessors.

BDTs have several advantages over other machine learning algorithms, such as neural networks, for binary classification problems. They are highly interpretable, which allows us to understand how they arrived at their decisions. This is particularly important in this field, high energy particle physics, where the ability to interpret results is crucial. BDTs are also highly resilient to overfitting and can handle missing data effectively. Given their strengths, BDTs have been widely used in particle physics for binary classification tasks, such as distinguishing signal from background events. The idea of the BDTs used in high energy physics today was proposed as early as 2001 by Friedmann in the paper *Greedy Function Approximation* [28], and since then have become an indispensable tool in the field. In particular, in the ATLAS collaboration challenge *The Higgs boson machine learning challenge* [29] the creation of a BDT package called **XGBoost** [30] increased the popularity even more as they won the challenge. Today **XGBoost** has become a standard ML tool used in the field.

In this chapter, we will explore the theory behind BDTs in the context of *extreme gradient boosting*, for this we are mainly interested in two ways of making DTs, a set of Classification And Regression Trees (CART). The theory is mainly based of Hjort-Jensens lecture notes [31, 32] and section 2 on the original **XGBoost** paper [30].

5.2.1 Decision trees

Starting with Decision Trees (DT) in the context of this DM search. The idea behind DTs is to find the most important features which contain the most information of signal (DM), and then split the dataset along the values of these features with the goal of creating a dataset containing pure signal. As we are going to use kinematical variables as features we will use this to both augment and automate the classical cut and count method (Chapter 4.3), which is based of making kinematical cuts on the variables. To find which kinematical features are most important in splitting the dataset we have to achieve a stopping criteria where we end up on a so-called *leaf node*.

A DT is typically divided into a *root nodes*, *interior nodes* and a final *leaf nodes*, also called leaves, which are all connected by *branches*, hence the name decision *tree*. As mentioned in the start of this chapter, we will look at two ways of making DTs, the first one we will look at is the *regression tree*.

5.2.2 Regression trees

As previously mentioned the leaves contain the prediction of a trained network, and are used to make new predictions on new datasets based on the information the DT learning in the branching process. As how to construct trees, there are mainly two steps we need to follow:

1. Split the predictor space (set of possible values x_1, x_2, \dots, x_p) into J distinct and non-overlapping regions, R_1, R_2, \dots, R_J
2. For every observation that falls into the region R_j , we make the same prediction, which is the mean of the response values for the training observations in R_j

is this too similar to 9.3 in [31]?

But how do we construct the regions R_1, R_2, \dots, R_J ? In theory these regions could have any shape, but for simplicity and pedagogical reasons we will choose to divide the predictor shpe into high-dimensional rectangles, or boxes. This means that the goal is to find boxes R_1, R_2, \dots, R_J that minimize a *cost function*, which again, for pedagogical reasons will

be the a MSE (Eq. (5.9)), defined as

$$\sum_{j=1}^J \sum_{i \in R_j} (y_i - \bar{y}_{R_j})^2$$

where \bar{y}_{R_j} is the mean response for the training observations withing box j . Ideally we would consider every possible partition of the feature space into J boxes, but his is computationally infeasable. Thus the common apporach is to begin at the top of the tree, where all observations belong to a single region, and then split the predictor space; where every split is indicated via two new branches down in the tree. This is greedy (as in *Greedy Function Approximation* [28]) since the best split is mad in the first step, rather than looking ahead and picking a split that might lead to a better tree in a future step.

To make any split we start by selecting the predictor x_j and a cutpoint s that splits the predictor space into two regions R_1 and R_2

$$\{X|x_j < s\} \quad \text{and} \quad \{X|x_j \geq s\}$$

so that we obtain the lowest MSE,

$$\sum_{i:x_i \in R_1} (y_i - \bar{y}_{R_1})^2 + \sum_{i:x_i \in R_2} (y_i - \bar{y}_{R_2})^2$$

where we consider all predictors x_1, x_2, \dots, x_p and each possible value s for each of them, where these values can be randomly assigned numbers. For any j and s we define the pair of half-planes where \bar{y}_{R_1} and \bar{y}_{R_2} are the mean responses for the training observations in $R_1(j, s)$ and $R_2(j, s)$ respectively. The goal is to find the value j and s such that we minimize the cost function, which can be done quickly depending on the number of predictors.

Then we repeat the process looking for the best predictor and best cutpoint, but instead of stopping with two regions, we split one of the two into another two regions creating a total of three regions. This is called the depth of a tree, and we can in principle continue to split indefinetely, however one usually sets a stopping criterion on how many events should be at a leaf.

The method explored above is straight forward, but often leads to overfitting and unnecessarily large and complicated trees. To mediate this one usually uses a so-called *Cost complexity* pruning algorithm. For regression procedures, the algorithm, rather than considering every possible subtree, comnsiders a sequence of trees indexed by a nonnegative tuning parameter α . For each value of this α there corresponds a subtree $T \in T_0$ such that

$$\sum_{m=1}^{\bar{T}} \sum_{i:x_i \in R_m} (y_i - \bar{y}_{R_m})^2 + \alpha T \quad (5.16)$$

is as small possible. In the equation above T is the number of terminal nodes of the tree T , R_m is the rectangle corresponding to the m -th terminal node. This tuning parameter α controls a trade-off between the subtree's complexity and its fit to the training data. As α increases, the above equation will give us a higher value, meaning that our trees would normally prioritize less complex models. This procedure is repeated untill we minimize Eq. (5.16) and choose a suitable value for α . This is the essense of regression trees.

5.2.3 Classification trees

The second DT type we will study is the *classification tree*. Classification trees are very similar to regresion trees, except that they are used to predict a qualitative response rather than a quantitative one. This means that classification trees are used for predictive modeling in which the output variable is discrete, such as classifying an email as spam or not spam, or predicting whether an event in our dataset is a DM signal or SM background event.

Like regression trees, classification trees recursively split the data based on the values of input variables until a stopping condition is met. However, the splitting criteria for classification trees are different. Classification trees cannot use the MSE function used in regression trees as a criterion for making binary splits, instead they use the *classification error rate*. The classification error rate, classification trees have discrete outputs, is simply the fraction of the training observations in a region that do not belong to the most common class.

Classification trees use measures such as Gini index or the entropy to determine quality

of a split at each node. The idea behind these quality measures is to select the split that maximizes the separation between the different categories of the output variable.

If a classification tasks takes for example $k = 1, 2, \dots, K$ values as outputs, we can define a probability distribution function p_{mk} that represents the number of observations of k in a region R_m with N_m observations. We can represent this likelihood function in terms of the proportion $I(y_i = k)$ of obervations of this output in region R_m as

$$p_{mk} = \frac{1}{N_m} \sum_{x_i \in R_m} I(y_i = k)$$

where p_{mk} represents the majority class of observations in region m . The tree most common ways of splitting a node are given by: The misclassification error

$$p_{mk} = \frac{1}{N_m} \sum_{x_i \in R_m} I(y_i \neq k) = 1 - p_{mk}$$

The Gini index G

$$G = \sum_{k=1}^K p_{mk}(1 - p_{mk}) \quad (5.17)$$

and the entropy s

$$s = - \sum_{k=1}^K p_{mk} \log(p_{mk})$$

5.2.4 The CART algorithm

As we are going to look at extreme gradient boosting, the main algorithm for this type of DT is the Classification And Regression Tree (CART). For classification, the CART algorithm splits the dataset in two subsets using a single feature k and threshold t_k . This could for example be a threshold set by a value of the invariant mass of an event we are trying to classify as background or signal. The way we optimize the pair (k, t_k) such that we get the purest subset is by for example using the gini factor G . With this the cost function tries to minimize

$$C(k, t_k) = \frac{m_{left}}{m} G_{left} + \frac{m_{right}}{m} G_{right}$$

where $G_{left/right}$ calculates the impurity of the left/right subset using Eq. (5.17) and $m_{left/right}$ is the number of events in the left/right subset. Once the algorithm successfully splits the training set into two, it splits the subsets using the same algorithm, and so on, recursively. For our purposes we will make the DT stop searching for the pair (k, t_k) once we have reaches the maximum depth we chose, or whenever the sum of the weights on a leaf is one. Both of these are hyperparameters that need to be optimized, for that see Chapter 7.3.2.

The CART algorithm for regression works similarly to the one for classification, but instead of trying to split the training set in a way that minimizes the gini or entropy impurity, it tries to split the training set in a way that minimizes the (MSE). Meaning that we have

$$C(k, t_k) = \frac{m_{left}}{m} MSE_{left} + \frac{m_{right}}{m} MSE_{right}$$

with

$$MSE_{node} = \frac{1}{m_{node}} \sum_{i \in node} (\bar{y}_{node} - y_i)^2$$

and

$$\bar{y}_{node} = \frac{1}{m_{node}} \sum_{i \in node} y_i$$

5.2.5 Ensemble modeling and (extreme) gradient boosting

Now that we have explained how DTs work, we still need more information to get the full picture of *Boosted* DTs. For this section of the chapter we. As we are going to look at a massive dataset, a single tree is not strong enough to be used in practice. What is actually used is the *ensemble model*, which sums up the prediction of multiple trees together. What is important of this models is that the different trees we are summing together try to complement each other. For example if the first split on one tree is on the invariant mass of a dilepton pair, while on another tree it is the MET, then we know physically that these are important features that DM + dilepton models have as a signature, meaning that it should be a good combination. Mathematically, we can write

the prediction of an ensemble model in the form

$$\hat{y}_i = \sum_{k=1}^K f_k(x_i), \quad f_k \in \mathcal{F} \quad (5.18)$$

where K is the number of trees we want to sum over, f_k is a function of the functional space \mathcal{F} , where \mathcal{F} is the set of all possible CARTs. With this we want to minimize a the objective function of the form

$$\mathcal{L} = \sum_i^n C(y_i, \hat{y}_i) + \sum_{k=1}^K \Omega(f_k)$$

where we sum over n leaves, C is training cost function (for example it can be the MSE), and $\Omega(f_k)$ is the complexity of the tree f_k , which penalizes the more complex models. We can see from both functions above that we need to learn all trees at once to make predictions, this is however both computationally demanding and intractable. Instead, we can use an additive strategy, where we fix what we have learned, and add a new tree at a time. Using Eq. 5.18 with 0 trees and going up we see that we get

$$\begin{aligned}\hat{y}_i^{(0)} &= 0 \\ \hat{y}_i^{(1)} &= f_1(x_i) = \hat{y}_i^{(0)} + f_1(x_i) \\ \hat{y}_i^{(2)} &= f_1(x_i) + f_2(x_i) = \hat{y}_i^{(1)} + f_2(x_i) \\ \hat{y}_i^{(3)} &= f_1(x_i) + f_2(x_i) + f_3(x_i) = \hat{y}_i^{(2)} + f_3(x_i)\end{aligned}$$

and so on. Mathematically we see that a prediction at step t as in $\hat{y}_i^{(t)}$ takes the form

$$\hat{y}_i^{(t)} = \sum_{k=1}^t f_k(x_i) = \hat{y}_i^{(t-1)} + f_t(x_i) \quad (5.19)$$

Such that we can now write the objective to be

$$\mathcal{L}^{(t)} = \sum_i^n C(y_i, \hat{y}_i^{(t-1)} + f_t(x_i)) + \Omega(f_t) \quad (5.20)$$

from which we can set up our choice for cost function. In general case we take the Taylor expansion of the loss function up to the second order such that we get

$$\mathcal{L}^{(t)} = \sum_i^n \left[C(y_i, \hat{y}_i^{(t-1)}) + g_i f_t(x_i) + \frac{1}{2} h_i f_t^2(x_i) \right] + \Omega(f_t) + const$$

where

$$g_i \equiv \partial_{\hat{y}_i^{(t-1)}} C(y_i, \hat{y}_i^{(t-1)}) \quad \text{and} \quad h_i \equiv \partial_{\hat{y}_i^{(t-1)}}^2 C(y_i, \hat{y}_i^{(t-1)})$$

Such that the specific objective at step t becomes

$$\sum_{i=1}^n \left[g_i f_t(x_i) + \frac{1}{2} h_i f_t^2(x_i) \right] + \Omega(f_t) \quad (5.21)$$

where we removed all constants. Such that Eq (5.21) becomes the optimization goal for the new tree. As we can see the above equation can take into account any cost function as it is written in a general form with a Taylor expansion. Furthermore, we can rewrite this by expanding

$$\Omega(f_t) \rightarrow \alpha T + \frac{1}{2} \lambda \sum_{j=1}^T w_j^2$$

where the first term is the same as in Eq. (5.16), and the second is called the *regularization term* (see Chapter 5.3.1). Inserting this into Eq. (5.21) we get

$$\mathcal{L}^{(t)} = \sum_{j=1}^T \left[\left(\sum_{i=1}^n g_i \right) w_j + \frac{1}{2} \left(\sum_{i=1}^n h_i + \lambda \right) w_j^2 \right] + \alpha T$$

such that with a fixed tree structure, q , we can define the optimal weight w_j^* on a leaf j as

$$w_j^* = -\frac{\sum_{i=1}^n g_i}{\sum_{i=1}^n h_i + \lambda} \quad (5.22)$$

and correspondingly the optimal objective reduction

$$\mathcal{L}^{(t)}(q) = -\frac{1}{2} \sum_{j=1}^T \frac{(\sum_{i=1}^n g_i)^2}{\sum_{i=1}^n h_i + \lambda} + \alpha T \quad (5.23)$$

With both of these equations we have all we need to use BDTs, the extreme part of the gradient boosting is something the **XGBoost** algorithm has implemented in terms of system optimization, such as parallelising the tree boosting task, and can be read in more

detail on section 4 of the original paper [30].

5.2.6 Summary

So in short BDTs split datasets based on different values of the features on the dataset, figure 5.2 shows an illustration of how DTs work.

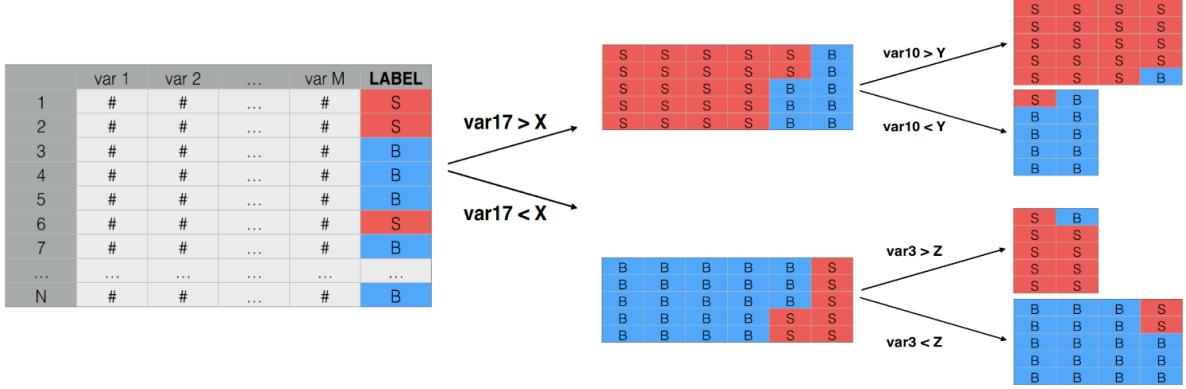


Figure 5.2: Image showcasing how decision trees work by splitting a dataset of N events with M features whenever a threshold X , Y or Z for a feature is passed. The label S and B state whether the event is signal or background respectively.

The above illustration just shows how one tree might work, in reality we can choose to have an arbitrary number of trees and combine their results with the ensembling and boosting method. Then after combining the generated tree structures we use Eq. (5.22) and Eq. (5.23) to minimize the objective to get the most accurate predictions as possible.

5.3 Tools and evaluation methods used for both algorithms

Now that we have explained how both NNs and BDTs work, we will explain how tools used by both algorithms work.

more motivation

5.3.1 Cost functions

Cost functions have been mentioned throughout this chapter, but what are they? Cost functions are what we will utilize to evaluate how well the output of the network fares against the target, i.e. if our network "guesses" right whether an event is signal or background, thus making this a very important part of our network! Before getting into this we first have to look at logistic regression. Since we are studying a binary classification task where the output is either $t_i = 0$ or $t_i = 1$, meaning background or signal. We can introduce a polynomial model of order n as

$$\hat{y}_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \cdots + \beta_n x_i^n$$

where the hat notation, \hat{y} , symbolises a condensed matrix form of the output of a layer. We can define the probabilities of getting $t_i = 0$ or 1 given our input x_i and β with the help of a logistic function. Using this we get the probability as

$$p(t_i = 1|x_i, \beta) = \frac{1}{1 + e^{-t_i}}$$

and

$$p(t_i = 0|x_i, \beta) = 1 - p(t_i = 1|x_i, \beta)$$

We want to then define the total likelihood for all possible outcomes from a dataset $\mathcal{D} = \{(t_i, x_i)\}$, with the binary labels $t_i \in \{0, 1\}$, applying the Maximum Likelihood Estimation (MLE) principle. This gives us

$$P(\mathcal{D}|\beta) = \prod_{i=1}^n [p(t_i = 1|x_i, \beta)]^{t_i} [1 - p(t_i = 1|x_i, \beta)]^{1-t_i}$$

from which we obtain the log-likelihood

$$C(\boldsymbol{\beta}) = \sum_{i=1}^n (t_i \log p(t_i = 1|x_i, \boldsymbol{\beta}) + (1 - t_i) \log[1 - p(t_i = 1|x_i, \boldsymbol{\beta})])$$

By taking the parameter $\boldsymbol{\beta}$ to first order in x_i and reordering the logarithm we get

$$C(\boldsymbol{\beta}) = - \sum_{i=1}^n (y_i(\beta_0 + \beta_1 x_i) - \log(1 + \exp(\beta_0 + \beta_1 x_i))) \quad (5.24)$$

This equation we will use throughout the thesis and is known as the *cross entropy*. The two beta parameters used are the weight and biases, $\beta_1 = w$ and $\beta_0 = b$. The goal is to change these parameters such that it minimizes the cost function.

L2-regularization

Furthermore we will add an extra term to the cost function, proportional to the size of the weights. We do this to constrain the size of the weights, so they don't grow out of control, this is to reduce *overfitting*. We will use the so-called *L2-norm* where the cost function becomes

$$C(\boldsymbol{\beta}) \rightarrow C(\boldsymbol{\beta}) + \lambda \sum_{ij} w_{ij}^2 \quad (5.25)$$

meaning that we add a term where we sum up **all** the weights squared. The factor λ is called the regularization parameter. The L2-norm combats overfitting by forcing the weights to be small, but not making them exactly zero. This is so that less significant features still have some influence over the final prediction, although small.

5.3.2 Sample weight

A "sample weights" array is an array of numbers that specify how much weight each sample in a batch should have in computing the total loss. It is commonly used in imbalanced classification problems (the idea being to give more weight to rarely-seen classes). Taken from the original documentation [33]. To mathematically illustrate how this weight works is by multiplying a constant term χ into the cost/loss function that is used further on the error propagation. As an example we can extend the simple MSE,

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for article?

Eq. (5.9) to

$$C(\hat{W}) = \frac{1}{2} \sum_{i=1}^n \chi_i (y_i - t_i)^2 \quad (5.26)$$

This will become an important feature when discussing network optimization in chapter 7, and especially for particle physicists as we want to re-weight Monte Carlo events to expected events to correctly showcase kinematical distributions.

5.3.3 Area under the ROC-curve

To evaluate how well our networks do. We will use the accuracy, which literally tells us how many events it guessed right. But also take into account a more advanced metrics, this is the Area Under the "Receiver Operating Characteristic (ROC)" Curve (AUC). Before explaining what the AUC is we first need to explore the ROC curves. The ROC curves help us illustrate how successful a network is a *binary classification* (only). The ROC curve, as the name states is a curve, where we plot the *True Positive Rate* (TPR) against the *False Positive Rate* (FPR). The TPR is defined by dividing the *True Positive* (TP), which is when the network correctly guesses a signal event to be a signal event, by the *Positive* (P), which is the total number of signal events in the data.

$$\text{TPR} = \frac{\text{TP}}{\text{P}}$$

On the other hand, the FPR is when we divide the *False Positive* (FP), when the network guesses a background event to be signal event, divided by the *Negatives* (N), the total number of background events in the data.

$$\text{FPR} = \frac{\text{FP}}{\text{N}}$$

To get a numerical value of how well a network classifies data we thus calculate the AUC. The TP and the FP are both going to be predictions from the networks, which for this thesis will give an event a score from 0 to 1², meaning that if we had a perfect network which guesses everything correctly we would only get TPs and thus an AUC of 1. If we were to get an area of 0.5 this would mean that the network is randomly

²Where 0 means that the network believes an event has 0% chance of being a signal event, and 1 means that the network believes with 100% certainty that an event is a signal event.

guessing whether an event is a signal or background, practically making a coin toss for every event. The goal is to have a network to give an AUC score as closely to 1 as possible.

While only using the accuracy as a metric is not a bad start, it is not favorable to use as a metric if datasets are unbalanced. As an example, if we had a dataset with 100 events, where 95 were background events and 5 were signal events, if we only used accuracy as a metric we would be inclined to think that an accuracy of 95% is great. However, as the dataset is unbalanced, the network could easily take a shortcut and guess every event to be a background event. Meaning that the network learned nothing. This is where the AUC comes into play, as this metric highlights whenever a network guesses wrongly in a binary classification. Meaning that if we used the worst case scenario with 95% accuracy when guessing everything to be background, we would have a $\text{TPR} = 0$ and a $\text{FPR} = 0$, if we were to plot this we would get a flat line on the FPR and TPR axis, meaning that it would give us an AUC of 0.5. Highlighting that our network is randomly guessing, and that it needs to be optimized further.

Comment for farid/eirik, I used wikipedia as a source on the theory here, should I cite that?

5.3.4 Validation plots

Another tool that we will use to evaluate how well a network does is by making so-called validation plots. The way these plots work is by first dividing the *already predicted* data set into background and signal. Then making histograms from 0-1 with the prediction score of the network for each event. As a score closer to 1 means that the network predicts an event to be a signal event, and 0 means that the network predicts a background event. Then we would ideally have all the signal events shoved to the right and all the background events shoved to the left. Usually this is not the case in high energy particle physics, as some background events might be similar to signal events. We will therefore utilize this kind of plots to see the distribution of the network predictions.

It is standard practice to make fill the histograms with semi-transparent colors, such that one can visualise the whole spectra for both background and signal. In this thesis we will use this, as well as validation plots where we divide the SM events into their

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respective background channels and fill them with opaque colors, while leaving the signal unfilled. We will in addition include real data points in the latter plots, to see how well SM simulations agree with the data. Ideally we want our data to not agree with the SM simulations when the network predicts a score close to one, as this discrepancy could be a hint of new physics.

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5.3.5 Significance plots

Significance after making cuts on the validation plots.

5.3.6 Feature importance

For BDTs, as we can calculate which features had the most impact when splitting the data, we will also include feature importance plots. As the name states these plots tell us how important the features in the dataset were in the training to separate signal from background. As we are using the `XGBoost` package when working with BDTs, we will use the inbuilt functions of the package to plot the feature importance.

When plotting the feature importance there are several metrics we can use to determine how important they are, for this thesis we will mainly look at

- Gain: which measures the relative contribution of a feature to the model's performance
- Weight: the number of times a feature appears on a tree to split the data
- Cover: the number of samples affected by the split with a feature

Where the first one is the standard used by `XGBoost`.

Part II

Implementation / Methods

Chapter 6

Data Preparation

Now is the time to focus on the critical task of preparing the data for our DM search using ML. This task is essential because accurately distinguishing between signal and background events is a key challenge in searching for any new signal event, as seen on Section 4.3. In this chapter we will define what *background* and *signal* actually are, as well as selecting appropriate kinematical cuts to define the control region for our search. In addition we will explain the data preparing procedure such that it is in a format that is well-suited for our ML algorithms. The reason this process is of great importance is because we can maximize the chances of our ML algorithm to accurately identify any DM signals. Moreover, by providing a detailed explanation of the data preparation process, we can ensure that our dataset is both reliable and effective, and that our analysis is robust and trustworthy. In short, this chapter represents a critical step towards achieving our goal of using ML to classify DM from SM events using ATLAS data, and it is essential to the success of our overall research goal.

6.1 Standard Model background estimation

As we are going to look at dilepton final states with missing transverse energy to try to teach our ML to learn DM signatures, then we need to take into account all possible SM background events that have these kind of final states. The SM backgrounds processes we will look at are explained in the next subsections. In Appendix A is the full list of dataset IDs for background process used in this thesis.

6.1.1 W and Drell Yan

$W/Z/\gamma^* + \text{jets}$ as all of these can directly decay into two leptons, $W/Z/\gamma \rightarrow ll'$, where the prime marks a different flavour. All the production mechanism for these background processes can be seen in Figure 6.1. As the W boson behaves differently than the Z and γ , since the W boson can decay into neutrinos giving us MET, while the Z and γ processes have the highest cross section. We will divide these background processes into two, W and $Z/\gamma^* + \text{jets}$, also called *Drell Yan* processes. To simulate these background processes Sherpa 2.2.11 [34] was used.

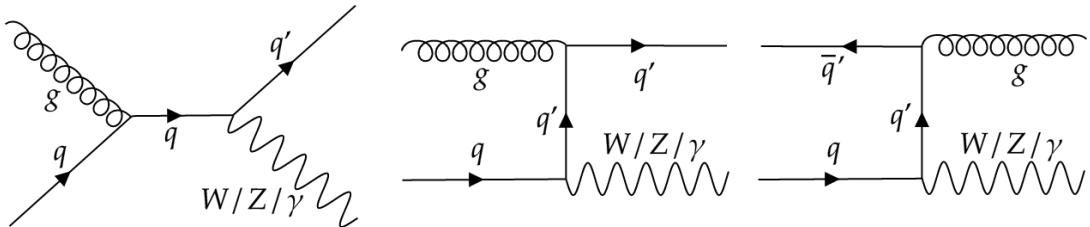
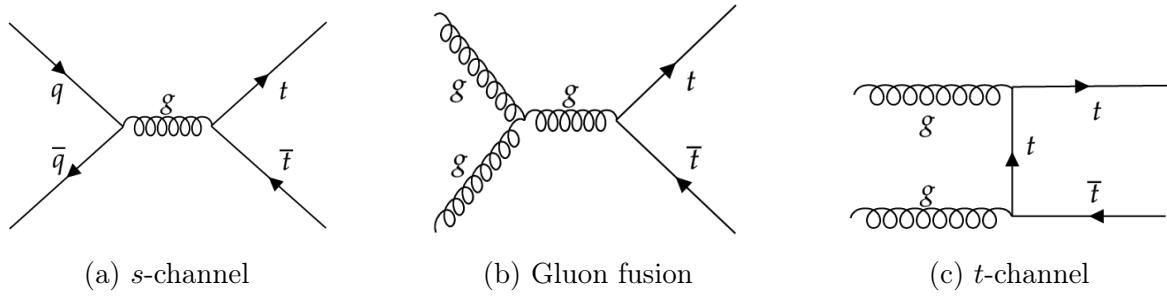


Figure 6.1: Diagrams showcasing SM $W/Z/\gamma + \text{jets}$ production. A quark marked with a prime, indicated that the quark might be a different flavour after interacting with a boson.

6.1.2 TTbar

Another SM background process we will look at are processes that have a $t\bar{t}$ in them. Since the top has a high branching ratio of decaying into a b -quark and a W boson, $t \rightarrow bW^+$, where the W boson can again decay into a neutrino giving us MET. The main production mechanism for these processes can be seen in Figure 6.2. To simulate these background processes Powheg-Box v2 [35] interfaced with Pythia 8 [36] was used.

Figure 6.2: Diagrams showcasing SM $tt\bar{t}$ production

6.1.3 Single top

On the same note we have processes with a single top. Where again the top has a high branching ratio of decaying into a b -quark and a W boson, $t \rightarrow bW^+$, where the W boson can again decay into a neutrino giving us MET. The main production mechanism for these processes can be seen in Figure 6.3. To simulate these background processes Powheg-Box v2 [35] interfaced with Pythia 8 [36] was used.

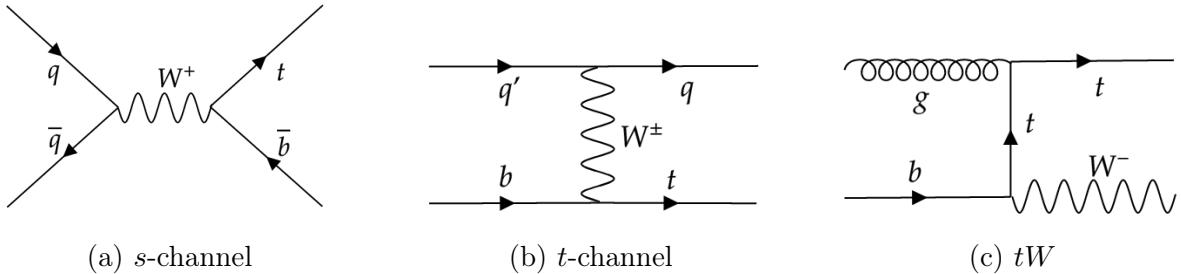


Figure 6.3: Diagrams showcasing SM single top productions. A quark marked with a prime, indicated that the quark might be a different flavour after interacting with a boson.

6.1.4 Diboson

The last SM background process we will look at are processes containing two bosons, called *diboson* backgrounds. The two SM bosons we will consider when looking into these final states are the W and Z , as these can decay as $W/Z \rightarrow ll'/ll$ or $W \rightarrow l\nu_l$, where the prime means a different lepton flavour. The main production mechanism for these processes can be seen in Figure 6.4. To simulate these background processes Sherpa 2.2.11 [34] was used.

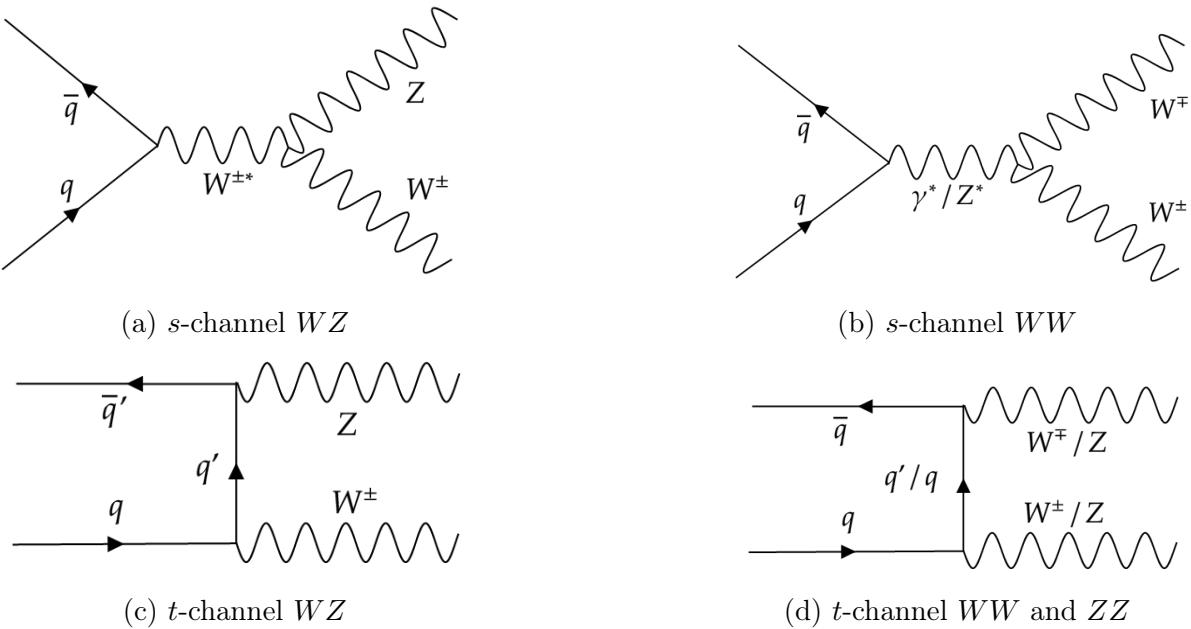


Figure 6.4: Diagrams showcasing SM diboson production. A star superscript on a boson indicates that the boson needs to be virtual and off mass shell. A quark marked with a prime, indicated that the quark might be a different flavour after interacting with a boson.

6.2 Dark Matter samples

6.3 Object selection

Before getting into our DM search and regions we will study we have to define how standard objects are defined. By standard objects we mean electron, muons and jets. As this thesis is made using ATLAS data and simulations we will use the same standard criteria cuts they choose.

The electrons are selected using the criteria on Table 6.1.

Table 6.1: Electron selection criteria.

Feature	Selection criteria
Transverse momentum	$p_T > 25 \text{ GeV}$
Pseudorapidity range	$ \eta < 1.37$ or $1.52 < \eta < 2.47$
d_0 significance cut	$ d_0(\sigma) < 5$
z_0 cut	$ \Delta z_0 \sin \theta < 0.5 \text{ mm}$

The muons are selected using the criteria on Table 6.2.

Table 6.2: Muon selection criteria.

Feature	Selection criteria
Transverse momennum	$p_T > 27(20) \text{ GeV}$ for leading (subleading)
Pseudorapidity cut	$ \eta < 2.5$
d_0 significance cut	$ d_0(\sigma) < 5$
z_0 cut	$ \Delta z_0 \sin \theta < 0.5 \text{ mm}$

The jets are selected using the criteria on Table 6.3.

Table 6.3: Jet selection criteria.

Feature	Selection criteria
Algorithm	Anti- k_t
R -parameter	0.4
Transverse momentum	$p_T > 20 \text{ GeV}$
Pseudorapidity cut	$ \eta < 4.5$
Jet Vertex Tagger	> 0.5 for $p_T < 60 \text{ GeV}$, $ \eta < 2.4$
forward JVT	fJVT < 0.4 and $ \text{timing} < 10 \text{ ns}$ for $p_T < 120 \text{ GeV}$, $2.5 < \eta < 4.5$
b-jet tagging	DL1r score > 0.665 with 85% efficiency

The missing transverse energy is reconstructed from jets selected and calibrated according to the information in Table 6.3. In addition to these a kinematical cut of $m_{ll} > 10 \text{ GeV}$ will be made to exclude hadron productions.

6.4 Control region

Now that we have defined what our background and signal are, the next step is to create a so-called *control region*. The control region is the kinematical region we will use as

a base for our search. As we are conducting a model independent search we want our kinematical cuts to be minimal and as general as possible. As we are looking at a dilepton final state, then we need to first define what we mean by that. If we were only searching for DM with the Z' model, then a sensible definition would be a Same Flavour Opposite Sign (SFOS) leptons ($e^\pm e^\mp, \mu^\pm \mu^\mp$). However to stay as general, and model independent, as possible we will also study other possible combinations as these might be important for theories such as SUSY or Lepton Flavour Violating (LFV) models. These are Different Flavour Same Sign (DFSS), DFOS and SFSS lepton pairs.

Since we are looking for DM, which we expect to behave similarly to a neutrino, then a nice kinematical variable to set a general cut to isolate signal from background is the MET, we can see the distribution of MET on all of Run II in a dilepton final state in Figure 6.5.

As we are conducting a model independent search, we want to use minimal cuts. The MET cut made in this thesis was chosen to be of 50 GeV (violet line in plot), meaning we will *only* look at events where the MET is greater than this. As we can see from Figure 6.5 by making a kinematical cut on 50 GeV we are cutting out a massive part of the background processes while only losing a small part of the signal.

Another kinematical cut that will be included in this thesis is another jet p_T on top of the one from Table 6.3. This is because jet reconstruction is already a hard task, but when pushing the limits of our selection criteria with a MET cut becomes even harder. And the worse the reconstruction of the jets the worse the agreement between MC simulations and real data is, and as we already know, the SM agrees with what we observe with an incredible accuracy. In Figure 6.6 we can see how the data and MC simulations of the SM agree when counting the number of b- and light jets with a p_T cut of ≥ 30 GeV and ≥ 40 GeV respectively. The agreement with different p_T cuts can be seen in Appendix C

Other than the object selection criteria, these are the only cuts that will be used to define the control region. Their summary can be seen in Table 6.4

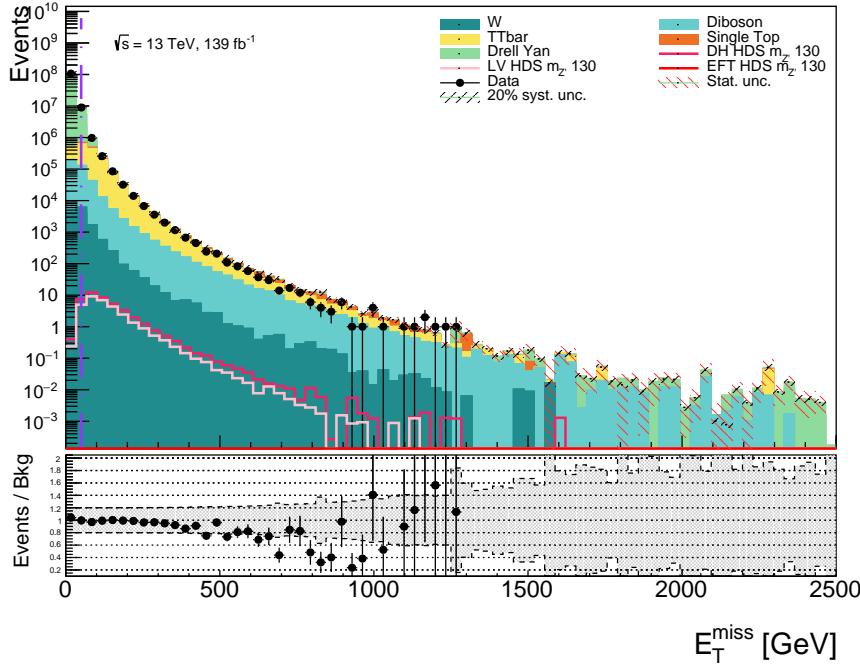


Figure 6.5: Distribution of MET when looking at a dilepton final state in all of Run II. The violet line shows where we will make out MET cut to create a control region. The DM models here have the highest cross section making them the most visible to our purposes.

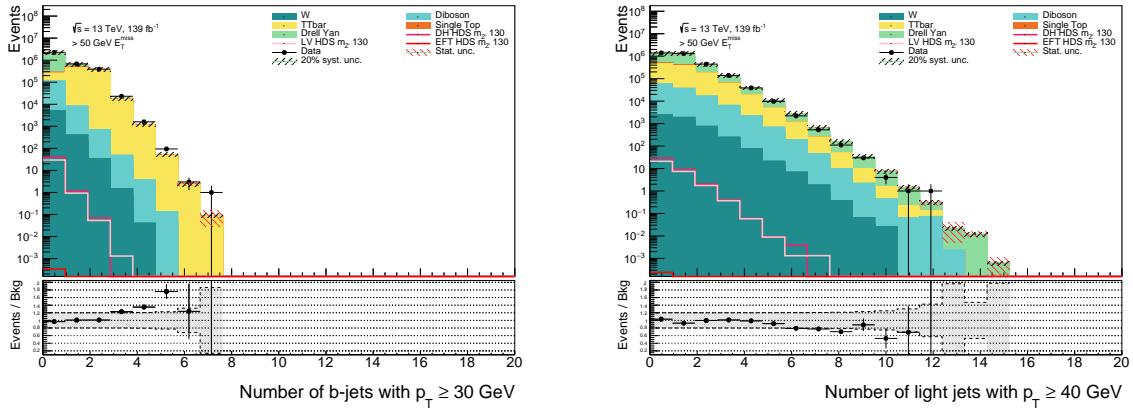


Figure 6.6: b- and light jet distributions with $p_T \geq 30$ and ≥ 40 GeV, respectively.

6.5 Feature selection for ML

For this thesis there are many possible kinematic variables that can be used as features for our ML algorithms, in this section we will make use of the kinematical variables presented in Section 4.1.1 as well as introduce new variables that might help our ML algorithm to correctly learn the patterns of SM background and DM signal events.

Table 6.4: Table showcasing the cuts used to define the control region for our model independent search.

Feature	Selection criteria
Dilepton final state	$\ell^\pm \ell^\mp, \ell^\pm \ell^\pm, \ell^\pm \ell^\mp$ and $\ell^\pm \ell'^\pm$
Missing Transverse Energy b- (light) jets	$E_T^{miss} > 50$ GeV $p_T \geq 30(40)$ GeV

As we are studying a final state with two leptons it is therefore natural to look at the kinematics for both of these as these will give us as general information as possible. The first thing we will look at is the transverse momentum, p_T , of each lepton as defined in Eq. (4.2). We will also look at the azimuthal angle, ϕ and the pseudorapidity, η defined in Eq. (4.8), to know where in the detector the leptons are located. With this we have practically constructed a four-momentum from which we could learn all particle kinematics. However, we want to help our ML algorithm as much as possible in the task of learning SM background and DM signal. A powerful kinematical variable for this is therefore the invariant mass, m_{ll} defined in Eq. (4.3), which can help the ML algorithm sort out resonant models. Another variable of interest is the transverse energy, E_T defined in Eq. (4.4) for the lepton pair. The distribution of the invariant mass can be seen in Figure 6.7.

As we are studying DM, an invisible particle, the most important kinematical variable to distinguish background from signal is the missing transverse energy, E_T^{miss} defined in Eq. (4.5), as this how we expect DM to be recorded. Another version of the MET is the so-called *Object-based E_T^{miss} significance*, or $E_T^{miss,sig}$ for short, this variable is used to deal with artificial or fake E_T^{miss} . The way $E_T^{miss,sig}$ works is by weighing the value of E_T^{miss} by the precision of its reconstruction. It is defined as

$$E_T^{miss,sig} = \frac{E_T^{miss}}{\sigma(E_T^{miss})} \quad (6.1)$$

where $\sigma(E_T^{miss})$ is the uncertainty of the reconstruction of the E_T^{miss} , which consider the individual uncertainties of the objects that enter the E_T^{miss} calculation. The distribution of this variable can be seen in Figure 6.8. In this thesis we will use both E_T^{miss} and $E_T^{miss,sig}$.

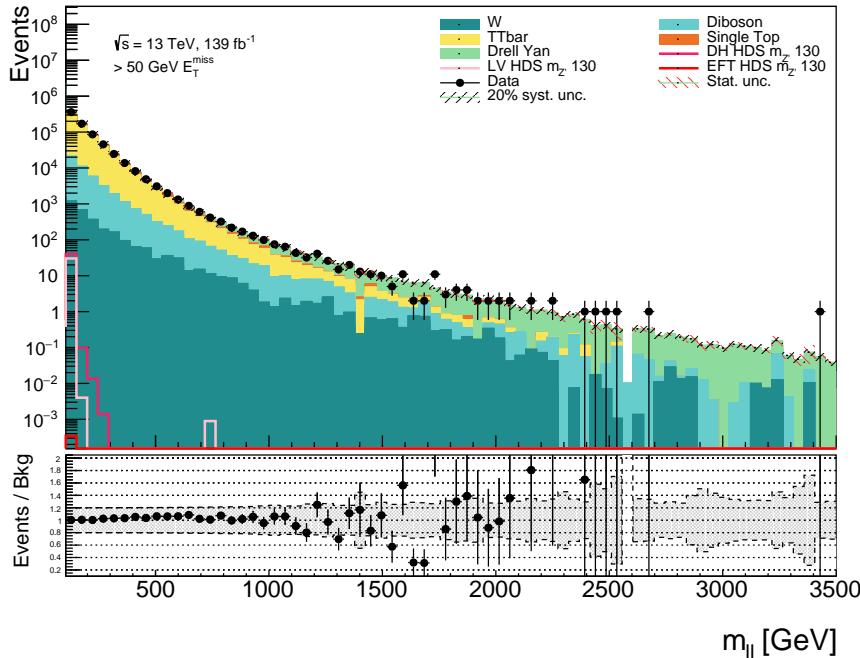


Figure 6.7: Distribution of m_{ll} in control region.

even though they are correlated, as the algorithm might chose in different instances¹ which of these is of more importance.

To keep the trend of variables for invisible particles, we will also study the transverse mass, m_T defined in Eq. (4.10), and stransverse mass, m_{T2} defined in Eq. (4.11) which might help our ML algorithm to sort i.e. W backgrounds from the signal. The distribution of the stransverse mass can be seen in Figure 6.9. Moving on to more jet related variables, we will look at the p_T , η and ϕ of the three most energetic jets, as these have the best MC and data agreement (see Figure 6.6), the invariant mass of the two most energetic jets m_{jj} , as well as count the number of b- and light jets. Altough these jet variables might not sound like the most useful when singleing out DM signal from SM background, there is always the possibility that our ML algorithm sees a pattern which is overlooked by us when conducting these kind of searches. Another varible we will look at is the hadronic activity, H_T defined in Eq. (4.6), from which we will also look at the ratio between the MET and hadronic activity, E_T^{miss}/H_T , as this can help the ML algorithm sort out DM events from background events.

¹Meaning different *signal regions* when conducting a model independent search. See Chapter TO COME

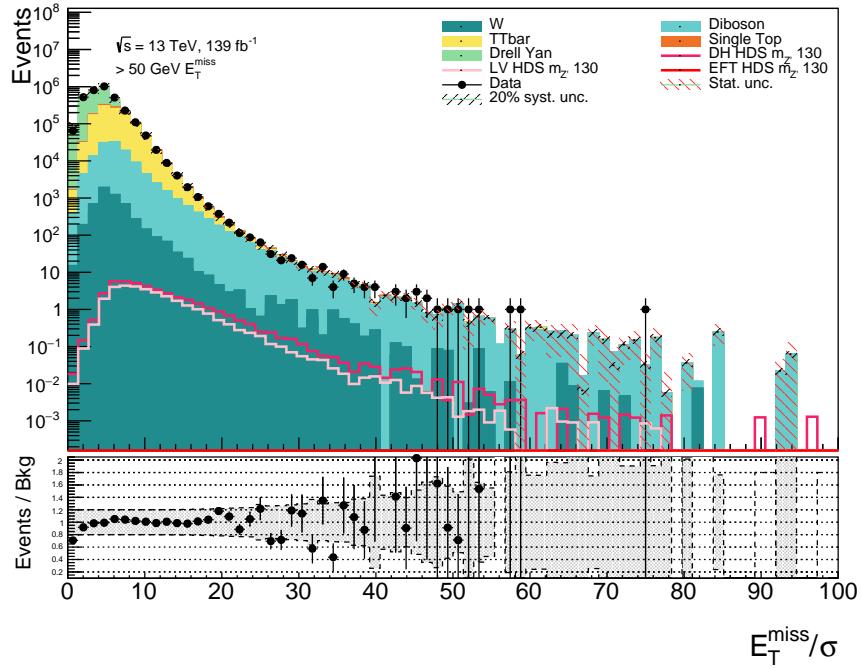


Figure 6.8: Distribution of $E_T^{miss,sig}$ in control region.

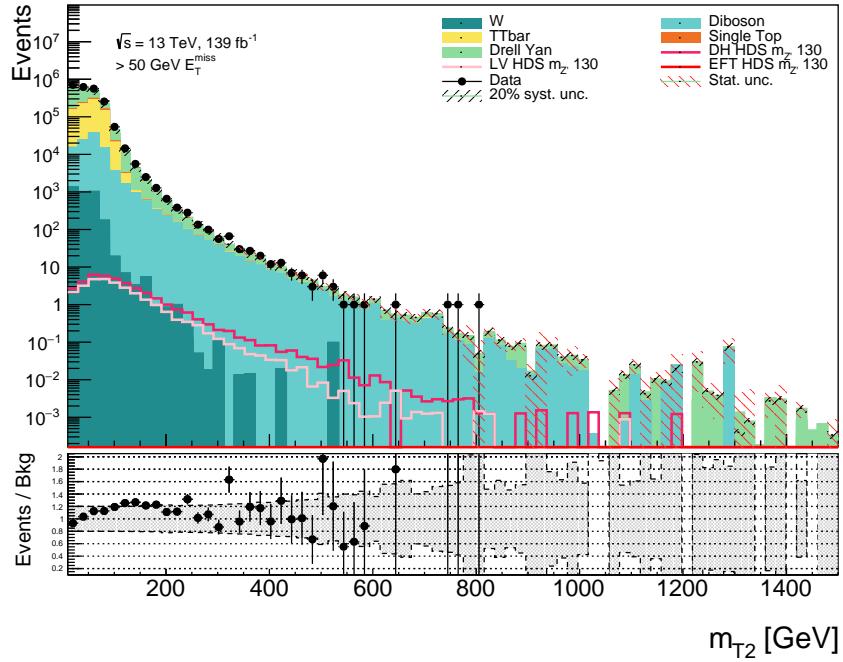


Figure 6.9: Distribution of m_{T2} in control region.

To know the distance between the "lepton-jet" and "MET-jet" we will look at the difference in azimuthal angle between: the lepton pair, $\Delta\phi(l_1, l_2)$, the dilepton jet and MET jet, $\Delta\phi(ll, E_T^{miss})$, the leading lepton and MET jet, $\Delta\phi(l_l, E_T^{miss})$, and the lepton closest

to the MET jet and the MET jet, $\Delta\phi(l_c, E_T^{miss})$. In some of the models we are studying it is expected for DM and the leptons pair to be back to back. The distribution of $\Delta\phi(ll, E_T^{miss})$ can be seen in Figure 6.10.

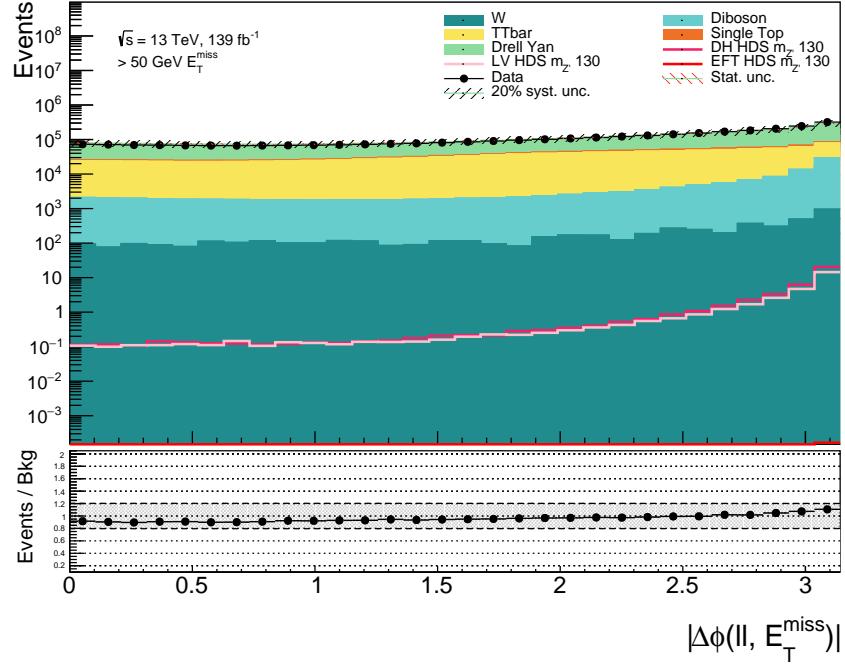


Figure 6.10: Distribution of $\Delta\phi(ll, E_T^{miss})$ in control region.

The kinematic variables used are summarized in Table 6.5 and the distribution of the remaining kinematical variables are shown in Appendix B.

Table 6.5: Table showcasing the kinematic variables that will be used as features on our ML algorithm.

* These have poor MC and data agreement. † These create *jagged arrays*.

Kinematic variable	Feature name
p_T of both leptons	lep1pt & lep2pt
ϕ of both leptons	lep1phi & lep2phi
η of both leptons	lep1eta & lep2eta
Invariant mass of dilepton pair, m_{ll}	mll
Missing transverse energy in event, E_T^{miss}	met
Missing transverse energy significance in event, $E_T^{miss,sig}$	met_sig
Transverse mass in event, m_T	mt
Stransverse mass in event, m_{T2}	mt2
Transverse energy of lepton pair, E_T	et
ϕ between lepton pair, $\Delta\phi(l_1, l_2)^*$	dPhiLeps
ϕ between lepton pair and MET jet, $\Delta\phi(ll, E_T^{miss})$	dPhiLLMet
ϕ between leading lepton and MET jet, $\Delta\phi(l_l, E_T^{miss})$	dPhiLeadMet
ϕ between closest lepton and MET jet, $\Delta\phi(l_c, E_T^{miss})^*$	dPhiCloseMet
Hadronic activity, H_T	ht
Ratio between E_T^{miss} and H_T , E_T^{miss}/H_T	rt
Number of b-jets	nbjets
Number of light jets*	nljets
p_T of three jets with highest p_T †	jet1pt & jet2pt & jet3pt
ϕ of three jets with highest p_T †	jet1phi & jet2phi & jet3phi
η of three jets with highest p_T †	jet1eta & jet2eta & jet3eta
Invariant mass of two jets with highest p_T , m_{jj} †	mjj

6.6 Transfer to ML friendly syntax

We have so far explained how the data will be used and carefully selected, however the question of how the data is made, and the number of samples we will work with still remains unanswered.

As mentioned when defining what our background and signal is, we will mainly look at MC simulations. The MC simulations are made into ROOT [37] NTuples which already have passed the object selection criteria on section 6.3. What remains is to set the kinematical cuts so we have our control region. To do this as well as saving the remaining

events into a new file which will be used for our ML algorithm I utilized the algorithms on `EventSelector`², which also saved the event that passed the event selection criteria as `ROOT` histograms to make plotting the distributions easier. After saving the events that passed the selection criteria I used the algorithms on `DataPrep`³ to plot the actual distributions of kinematical variables, and more importantly converting all of the events that passed the event selection into a ML friendly syntax. For this thesis we are converting from `ROOT NTuples` to `pandas DataFrame` [38] which can furthermore be saved as `h5` files to be read more efficiently. The reason we chose `pandas DataFrame` is because of the easily readable kinematics per event, the easily applicable kinematical cuts to the whole dataset (to more effectively create signal regions), and most importantly because of the compatibility with `XGBoost` [30] and `TensorFlow` [39] which will be the ML packages we will utilize for both BDTs and NNs.

6.7 Problems

There are however still problems when we....

6.7.1 Missing variables

For this thesis, as shown in Table 6.5 with the †, we will record events with up to three jets in the final state. However, there isn't always three jets in the recorded events that we will be studying, this creates *jagged arrays* which we can interpret as arrays with missing values. This is an unwanted feature that we need to avoid when training a ML algorithm, and is a problem that is not common in other ML tasks aside from HEPP. To mediate this problem we chose to set the p_T to zero for the missing jets and m_{jj} to zero if there are less than two jets, this is something that is physically reasonable as it doesn't violate any conservation laws. More problematic however is the η and ϕ when there aren't jets. To mediate this I've set the values to -999, which has no physical meaning and is impossible to achieve, this we did so it becomes easier for us to identify the jagged arrays that need different interventions when preparing both NNs and BDTs. The methods we explored to mediate the jagged array problem is further explained in Chapter 7.

²Available here: <https://github.com/rubenguevara/Master-Thesis/tree/master/EventSelector>

³Available here: <https://github.com/rubenguevara/Master-Thesis/tree/master/DataPrep>

6.7.2 MC and data disagreement

There is also another problem, with the final states that are not SFOS, as the MC generated background on these tend to be lower than the recorded data. The number of events that are not SFOS are minimal though, and we think the reason it doesn't fit the data is

something
more to
add?

because we are not including fake leptons.

Now that we have explained how the data has been prepared and the problems that arise with this we are ready to start playing around with ML! **Question: Anything else to add to this chapter?**

Chapter 7

Machine Learning

We have now presented the dataset and theory behind everything, and are thus ready to start making our ML algorithms. The first part of this chapter will be a presentation of the dataset in more details, included all the events and the difficult variable called the *weights*. We will also explore how we divide our dataset into a training and testing set. Afterwards we will discuss the crucial task of optimizing the architecture and hyperparameters of the ML models for obtaining high-performance classifiers. In this field, the challenge is often to distinguish between signal events from rare physics phenomena, such as our DM interactions, and the much more common SM background events. Since these signals events are extremely rare, the classifiers need to be highly optimized in order to effectively separate them from the background. In addition, the datasets have a large number of features, which can lead to overfitting or poor generalization if not properly optimized. Another hardship is how to mitigate the phenomena of jagged arrays and missing variables described in the previous chapter.

It is therefore essential to carefully choose the model architecture and hyperparameters, as well as the dataset pre-processing techniques, in order to achieve the best possible performance in the search for the rare DM physics phenomena. This can involve optimizing parameters such as the learning rate, the regularization strength, (exclusively for the NNs) the number of layers, the number of neurons per layer, (exclusively for the BDTs) the number of trees, and the tree depth. The second and third part of this chapter will be just about optimization for the NNs and BDTs respectively.

7.1 The datasets

The way we are making datasets in this project, to keep it as close to model independent as possible, will be of the following format

- Make a dataset with all of the SM backgrounds and one DM model.
- Make a dataset with all of the SM backgrounds and all of the DM models, divided into different signal regions.

The idea behind the first one is to start with a model dependent approach, where we train a ML algorithm to learn just one model at a time. The reason for taking this approach and not just having one dataset with all the models included, which would indeed make it a model independent approach, is because of the different phenomenology and experimental signatures of the models we are studying. **Question: Should I give concrete examples on how SUSY vs Z' differs, or is this sentence enough?** To achieve model independence with this dataset, the plan is to combine the results of the different networks into one. **Question: Is it correct to say statistically combine the results? Or what exactly are we thinking of combining using this method?**

For the second one, even if we were to include models with different signatures it would work out in our favour. For example, if we created Signal Regions (SR) in different regions of the m_{ll} phasespace, and trained a ML network in each respective SR, then the network would learn the DM resonance appearing in each area, instead of learning the model resonance. Indeed, achieving model independence. Furthermore, the plan is to combine the results of each respective region to get an overall view of all of Run II.

In this project we will look at real and simulated data from the ATLAS detector from all running periods of Run II. This is from period a, d and e each with an integrated luminosity of 36.4, 44.3 and 58.5 fb^{-1} respectively. In Table 7.1 we can see the overall number of MC samples and expected events for all background channels as well as DM models. In addition to using the features in Table 6.5, we will also include the EventID, dataset ID, period, dilepton final state, label telling us whether an event is signal or background and the *weight* of each event.

Table 7.1: Table showcasing the number of MC samples and expected events for every background channel and DM model that will be used in this thesis.

* The expected number of events for any BSM model has prefixed assumptions about for example cross section, decay width, etc. As we do not empirically know if any of these assumptions are correct, these are not set in stone.

Channel	MC samples	Expected events
W	38,684	5,436.325
Drell Yan	47,201,697	2,198,257.648
TTbar	28,126,697	978,530.817
Single top	729,624	93,898.462
Diboson	10,983,689	116,653.466
Standard model total	87,080,391	3,392,776.718
Z' Dark Higgs Heavy Dark Sector	498,621	*54.917
Z' Dark Higgs Light Dark Sector	892,365	*155.369
Z' Light Vector Heavy Dark Sector	521,759	*39.820
Z' Light Vector Light Dark Sector	470,352	*235.699
Z' Effective Field Theory Heavy Dark Sector	715,409	*0.0004
Z' Effective Field Theory Light Dark Sector	640,963	*0.0007

7.1.1 Weights

The weights are crucial for most of what is to come further on this thesis. The weights only apply to MC simulations and can be interpreted as corrections to simulate real data, which is why the MC samples and expected events on Table 7.1 differ, where the latter has applied weights. The weights on this project are defined as Eq. (7.1) and are in fb^{-1} units.

$$W = w_{mc} \times w_{pu} \times w_{xs} \times w_{lsf} \times w_{nlo,ew} \times w_{ttbar} \times w_{jet} \times w_{bjet} \times \frac{lumi_{period}}{\sum w_{mc,DSID}} \quad (7.1)$$

This equation has a lot of information in it, so every part of this will be discussed. Starting from the raw MC weight, w_{mc}

The pile-up weight w_{pu} , takes into account detector features

The cross section weight w_{xs} , defined as $w_{xs} := xs \times w_{kf} \times g_{eff}$ where xs is the cross section of the process, w_{kf} is the k -factor which has higher order corrections, and g_{eff} is the filter efficiency.

The Lepton and trigger Scale Factor (LSF), w_{lsf} , which...

The Next to Leading Order (NLO) electroweak correction $w_{nlo,ew}$, which ...

The $t\bar{t}$, jet and b-jet weights, w_{ttbar} , w_{jet} and w_{bjet} , which ...

And lastly the luminosity of the period, meaning

$$lumi_{period} = \begin{cases} 36.4, & \text{for } period = a \\ 44.3, & \text{for } period = d \\ 58.5, & \text{for } period = e \end{cases}$$

divided by the Sum Of Weights (SOW) **Question: Sum of every w_{mc} of each event or sum of every W pr event?** of every MC sample for each dataset ID (background process)

7.1.2 Signal regions

Write in
more de-
tails

Will make three regions:

- $m_{ll} > 120$ GeV and $E_T^{miss} \in [50, 100]$ GeV
- $m_{ll} > 120$ GeV and $E_T^{miss} \in [100, 150]$ GeV
- $m_{ll} > 120$ GeV and $E_T^{miss} > 150$ GeV

7.1.3 Train and test split

When training our networks we will use 80% of the dataset, the remaining 20% will be used to test whether the networks are learning any physical patterns or learning the dataset. When splitting the datasets however, we need to be absolutely certain that the distributions follow the same shape and have the same ratio of background channels as well as having good MC and data agreement. In this thesis we will utilize a function from `scikit learn` [40] called `train_test_split` which can easily split our dataset into a training and testing set of our size, while also sorting the label of the event. Another benefit of using this function is that it has included a random seed such that we can always split the datasets in the same shape.

In Figure 7.1 we can see that the distribution remains the same when we split the dataset into 80-20, where we also see the data and MC agree. To see the distribution of every other kinematical variable see **question: Appendix or github repo? The appendix**

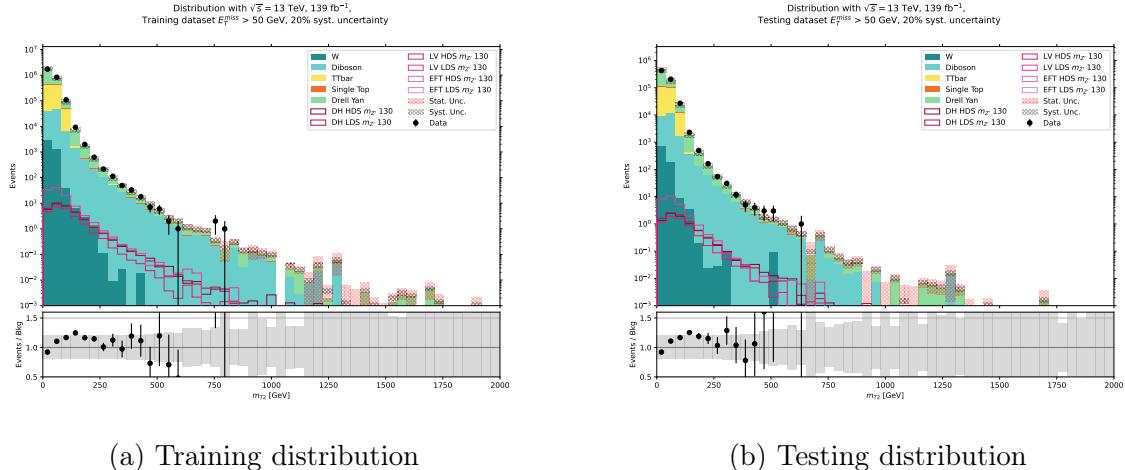


Figure 7.1: Train and test distribution of the stransverse mass when dividing SM background and DM samples into 80% and 20% datasets. The integrated luminosity for each distribution is 80% and 20% of 139 fb^{-1} respectively

is already very long! In addition to splitting the data, when making validation plots (See Chapter 5.3.4), when plotting MC simulations and real data, we nee to make sure that the luminosity of the plot is correct. That means that if we are plotting the network score in the test dataset, as this is 20% of the whole dataset we need to scale everything with a factor of 5 so we get back to 100% and the full luminosity of Run II.

7.2 Neural Network Training

For this thesis we utilize `TensorFlow v. 2.7.1 GPU` for all NNs. After creating the dataset, the first and most important thing is to marinate the `batch_size` whenever we try anything while using the dataset. This is because of both the size of the dataset and because of the imbalance between signal and background, as explained in section 5.1.2 and as seen in Table 7.1.

The highest possible batch size that could be used for this thesis was 2^{24} ¹, leading to roughly 17 million samples per batch. This is the best that a dedicated GPU, NVIDIA A100-PCIE-40GB, could handle. The batch size also decreases the more complex the NN becomes, as this requires greater computational power.

With this out of the way there are still a few problems to solve to optimize our NN. These are described below.

7.2.1 Padding of data

There are two difficulties to overcome when utilizing NNs as compared to BDTs. The hardest to solve, as a reliable solution seems not to exist yet, is the padding of jagged arrays (see Chapter 6.7.1). Padding is the process of filling missing values of a dataset entering a network. This problem doesn't usually appear in other fields than high energy particle physics. To recapitulate what was done in Chapter 6.7.1, the problem arises when the size of some features is not constant. This is the case of number of jets in an event. When creating the dataset the missing values of p_T and m_{jj} were set to zero, while the missing values of η and ϕ were set to -999.

As mentioned before, there is no general consensus of how the padding should be done, and there are many different methods of doing so. The classical data scientist way of solving this problem would be to just take the mean of every feature and use that as a variable for every event with missing values. That means replacing every $p_T, m_{jj} = 0$

¹It has to be a power of two because of the alignment of virtual processors (VP) onto physical processors (PP) of the GPU. As the number of PP is often a power of 2, using a number of VP different from a power of 2 leads to poor performance.

and $\eta, \phi = -999$ with the mean of every p_T , m_{jj} , η and ϕ (excluding the 0's and -999's respectively). However this is not popular among physicists since it breaks conservation laws when we say there are jets present in an event when in reality there are none. Another approach is to use Bayesian statistics or ML to estimate the missing values, these options will not be pursued in this thesis due to time, but might be of interest for future projects. Another approach, is setting all the missing values to zero, as this might mean that there isn't anything there, but this also breaks conservation laws since $\eta, \phi = 0$ have physical meaning, this is also highly looked down upon by data scientists since this could affect the weighting when training the network and potentially create a false pattern for the network to follow which would lead to a bias.

The jet p_T and m_{jj} being 0 is a valid form of padding the dataset, as this doesn't break any fundamental law of physics. However setting ϕ as something outside of $[-\pi, \pi]$ doesn't make much sense as this is the angle around the detector. Setting a high value of $|\eta|$ might be physically possible (in the future) but as of today the ATLAS detector has a $|\eta| < 4.9$ (see Chapter 4.2) as the pseudorapidity states how close to the beamline the objects recorded are. However having a p_T of a jet equal to zero while still recording the η and ϕ breaks the laws of physics, so this is a problem that needs to be fixed.

Another approach that will not be pursued in this thesis, would be to just remove all events that have a missing value completely getting rid of the problem, but this reduces the statistics of the dataset which is not desired when searching for new physics, and more importantly completely removes the kinematical distributions that is present in the SM, meaning the NN would learn wrong physics. One could also just remove the features with missing values to conserve statistics, albeit make it harder for the network to see any pattern that we might miss, but this is also not a desirable mitigation.

Instead we have tried to create new kinematic variables that work around the need of padding. These features are just *counting features*, meaning that we count the number of jets that fulfill some criteria, such as the number of b-jets with $p_T > 20$ GeV, the number of light jets with $p_T > 40$ GeV, the number of jets recorded in the central calorimeter ($|\eta| < 2.5$), and the number of jets with $p_T > 50$ GeV recorded in the forward calorimeter

($|\eta| > 2.5$). The p_T cuts are optimized to allow a good agreement between data and simulations, the full distributions with different cuts in the control region can be seen in

[Appendix D](#). A summary of these variables is shown in Table 7.2.

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Table 7.2: Table showcasing plausible kinematic variables that will not need padding.

Kinematic variable	Feature name
Number of b-jets with $p_T > 20$ GeV	n_bjetPt20
Number of light-jets with $p_T > 40$ GeV	n_ljetPt40
Number of jets recorded in Central calorimeter	n_jetsetaCentral
Number of jets recorded in Forward calorimeter with $p_T > 50$ GeV	n_jetsetaForward50

When training our NN with these new variables instead of dropping features with missing variables we hope that the NN learns more physics by hopefully recognising patterns between all high level features.

7.2.2 Normalization of data

Moving onto the second problem, which is not as problematic as the previous one, namely the normalization of data. Since neural networks send a lot of data into multiple neurons and multiple layers using activation functions and carrying weights and biases that change for every backpropagation iteration, it is important to make sure that a neuron output doesn't die when moving around the network. Meaning that a neuron output becomes insignificant compared to others when navigating the loss-phasespace. A fast way for neurons to die off is to not normalize the data and send it through the network as it is available. The reason it might die is because we send in numbers which vary significantly to each other, i.e. the p_T might be as high as thousands GeV, while E_T^{miss}/σ might be as low as 0.1. What might happen when sending such different numbers is that the network might think "obviously the high number is more important than the low number" thus making the activation function worse for the feature, even though this feature is of high importance when looking at MET final states. A way to fix this problem is to normalize all features. There are many ways to do this, one could do *min max scaling* which normalizes every feature from $[0, 1]$, completely solving the problem

above. Mathematically speaking this is done by

$$X_{norm} = \frac{X - X_{min}}{X_{max} - X_{min}} \quad (7.2)$$

Where X is the array containing all events for a given feature, while X_{min} and X_{max} are the lowest and highest values in the said array. Another way to normalize the data is to make the mean of the data 0 and the standard deviation to one, this is called *Z-score normalization*

$$X_{norm} = \frac{X - \bar{X}}{\sqrt{\sigma_X^2}} \quad (7.3)$$

where \bar{X} is the mean of said array and σ_X^2 is the variance. One could also use pre-built functions in TensorFlow that provide a normalization, such as `Batch_normalization` which normalizes the data entering the network per batch. This is usually used in Convolutional NN's as it improves computational speed. Another one, `Normalize`, provides the same as Eq. (7.3) for the whole training set going in. This is however computationally heavy to use. The `Layer_normalization`, which normalizes the activations of the previous layer in a batch *independently*, rather than *across* a batch like `Batch_Normalization`. Both `Batch_Normalization` and `Layer_Normalization` use an optimized version of the Z-score when normalizing the data, meaning that all the features take the form of 0 ± 1 .

There is a big difference when normalizing data ourselves or using TensorFlow. TensorFlow remembers how the data was normalized when training such that the test data will be normalized the same way, making testing easier. While if we use Eq. (7.2) or Eq. (7.3) ourselves, we have to make sure that we use the same values for X_{max} , X_{min} , \bar{X} or σ_X^2 when normalizing the test data.

7.2.3 Balancing of signal and background

A big problem that needs to be addressed in this thesis is what we should use as sample weights (see Section 5.3.2). If we were to not use any form of sample weights to mitigate the unbalance in our data set it could potentially lead *majority class classification* where the networks could get "lazy" and guess that everything is background.

To combat the majority class classification, we will as mentioned make use of the sample

weights. We will study three cases

1. Unweighted training, meaning that we will be setting the sample weight to one
2. Weighted training, meaning that we will be setting the sample weight to the weights used to re-weight MC events to expected events explained in Chapter 7.1.1
3. Balanced training, where we will make set the weight of all signal events to one, while weighing down the background by the ratio of signal events divided by background events, $\frac{N_{sig}}{N_{bkg}}$

where the latter would in terms of weights mimic a 50-50 distribution of signal and background. To test the different balancing methods we will use a dataset containing only SM processes, and train a network to single our the W channel from the other SM backgrounds as this has the lowest statistics.

7.2.4 Re-weighting MC to expected events

Even if the weighting method previously described might help the NN give us better results, we also want to include the weights used to re-weight MC events to expected events. This is desirable in the sense that we want to show our ML networks the true kinematical distributions of each feature. As the re-weighting weights are generated with simulation corrections as well as luminosity and cross section in mind, it is heavily desirable to also apply these corrections when training our networks, such that it can correctly predict new events regardless of their weight. This is specially crucial when using real data on our predictions, as these have no weights.

Ideally one would take into account both the data imbalance between the signal and background as well as the re-weighting weights when training a network. To do this using `TensorFlow` we could make use of two parameters when training the network: `class_weight` and `sample_weight`. `class_weight` works as a dictionary that weighs events that have the same keys as the dictionary. For our purposes we could make a dictionary where we weigh signal and background events differently, this could be the same type of scaling that was done in the previous section. `sample_weight` takes in individual weights for every single event that goes into the network, meaning that it is crucial

that we know that the desired weight matches the desired event. Ideally we would use both weighting methods, `class_weight` to balance the signal to background ratio and `sample_weight` with the re-weighting weights. However there is a bug in TensorFlow (up to version GPU 2.7.1) that makes it so the program doesn't run when using both parameters. This is not a big problem though, as when looking at the source code [41] one can see that what TensorFlow does with both weights is multiply them together.

For this thesis we tried testing four different methods to re-weight and balance the dataset, which expands further than the idea of point 3. in the previous section. Firstly all background events were re-weighted to expected events, and in addition we expanded the balancing ratio, $\frac{N_{sig}}{N_{bkg}}$, to be

1. $\frac{N_{sig,MC}}{N_{bkg,MC}}$ where we weigh down all background events wrt. the number of MC samples
2. $\frac{N_{bkg,MC}}{N_{sig,MC}}$ where we weigh up all signal events wrt. the number of MC samples
3. $\frac{N_{sig,MC}}{N_{bkg,exp}}$ where we weigh down all background events wrt. the number of expected events
4. $\frac{N_{bkg,exp}}{N_{sig,MC}}$ where we weigh up all signal events wrt. the number of expected events

Where we are not re-weighting the signal events to expected signal events when training because this would in principle remove all events from the NN as there are so few events (see Table 7.1). And also because the expected events of the signal take into account assumptions that are not empirically proven, such as the couplings, masses, decay widths, etc. Taken all of these factors into account what remains is to choose a network architecture and which hyperparameters to use to best fit our task.

7.2.5 Architecture and hyperparameter tuning

The architecture of the NN utilized in this project is of the form shown in Figure 7.2.

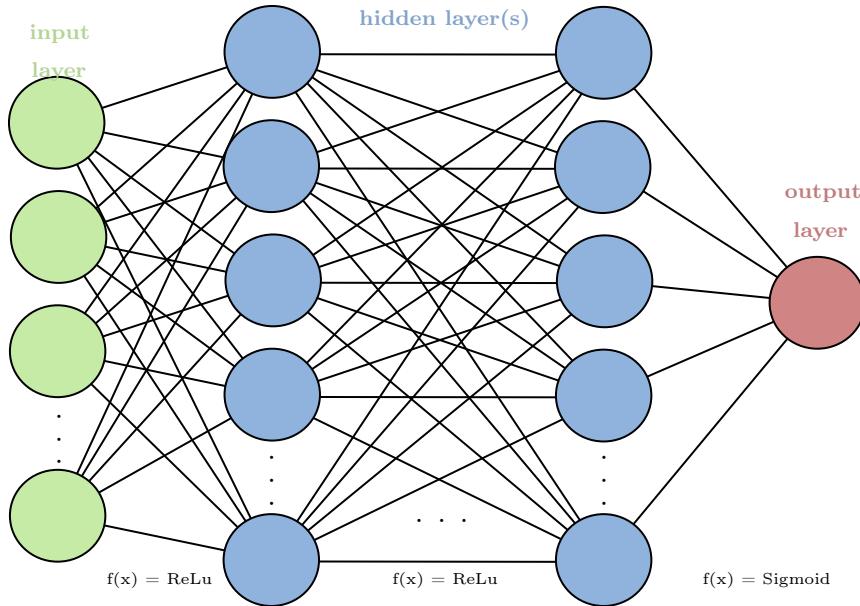


Figure 7.2: Architecture of the NN used on this thesis. The neurons on the hidden layer(s) is a hyperparameter, as well as the number of hidden layers.

Making this type of NN using TensorFlow is easy. An algorithm showing one of the possibilities can be seen in Algorithm 7.1.

Algorithm 7.1: Neural network definition using TensorFlow

```

1  import tensorflow as tf
2  from tensorflow.keras import layers
3
4  def Neural_Network(inputsize, n_layers, n_neuron, eta, lamda):
5
6      model=tf.keras.Sequential()
7
8      for i in range(n_layers):
9          if (i==0):
10              model.add(layers.Dense(n_neuron, activation='relu',
11 kernel_regularizer=
12                  tf.keras.regularizers.l2(lamda), input_dim=inputsized))
13          else:
14              model.add(layers.Dense(n_neuron, activation='relu',
15 kernel_regularizer=
16                  tf.keras.regularizers.l2(lamda)))

```

```

15
16     model.add(layers.Dense(1, activation='sigmoid'))
17
18     sgd=tf.optimizers.SGD(learning_rate=eta)
19
20     model.compile(loss=tf.losses.BinaryCrossentropy(),
21                     optimizer=sgd,
22                     metrics = [tf.keras.metrics.BinaryAccuracy()])
23
24     return model

```

To get the best performance on our NN, we need to find which hyperparameters helps the network reach highest significance. To do this, we need to do a gridsearch. For our neural network we will mainly focus on four hyperparameters explained on section 5.1:

- The learning rate η
- The L2-regressor variable λ
- The number of neurons on each hidden layer `n_neuron`
- Possibly the number of layers `n_layers`, excluding the output. Meaning that `n_layers = 1` means no hidden layer

The metrics that will be used to estimate the best hyperparameters are **AUC**, **binary accuracy** and most importantly **expected significance**. The expected significance for this section has been calculated using the low statistics formula Eq. (4.20) without uncertainties. The expected significance will also be calculated when making a cut on 0.85 on the network prediction score, as explained in Chapter 5.3.5. Meaning we will only look at events which the network rates as signal with 85% confidence and above. For some models where the expected events are too low (EFT models), meaning that the expected significance will be 0 or `NaN`, if this happens we will use the hyperparameters of the network which gives us the highest AUC on the testing set. This will procedure will be done for every single network we will explore.

7.3 Boosted Desicion Tree Training

When working with BDTs we do not run into as many problems as we do with NNs. For example the padding and normalization of data can be completely avoided, making the whole procedure a lot easier when one uses exotic data as we do in high energy particle physics. The weights is still an obstacle that needs to be overcomed when using BDTs. This will be discussed in the next section.

For this project we will as mentioned utilize the eXtreme Gradient Boosting, or **XGBoost** for short, package [30] made for the Higgs ML Challenge [29] whenever we mention BDTs. This project utilized version 1.5.0 without GPU adaptability. **XGBoost** also helps to avoid padding as it is integrated with a `missing_variable` variable where we can simply write the number of the variable that is missing.

7.3.1 Sample weights

For **XGBoost** there is a different problem when it comes to weights. **XGBoost** has a variable called `scale_pos_weight` where we can help the network deal with unbalanced data, such as the one we have. Meaning that the whole problem of combining the re-weighting weights with the balancing weights from Chapter 7.2.4 completely dissapears. Albeit there is a caveat, **XGBoost** does not have to possibility to include negative weights, which this dataset has. The reason **XGBoost** doesn't include weights is because when calculating the number of events in a leaf node, which is made by taking the sum of sample weights, we cannot have a negative value [42, 43]. As for the MC generators, Sherpa [34] takes into account higher order diagrams and needs to add negative weights to "counter" the overcounting of diagrams [44], which are important to correctly scale the simulated events

I have no other sources to this other than spoken words... to real data. In the future this might no longer be a problem as future MC generators might only have positive weights.

A method to mitigate this problem is to use the absolute value of the weights when training. This is however not generally accepted as a solution, and some even say it should be avoided. There are other options however, one of these options is to not include events with negative weights on the training set. This is an okay thing to do, as we

can imagine that if we were to only include events with positive weights on the training, it might be the same as putting the negative weights on the "testing dataset" (Chapter 7.1.3).

Another one that has been used on a published ATLAS (internal) article (chapter 9.3) [45] is to normalize the weights when using the absolute value with respect to the sum of weights over the sum of absolute weights. The reason behind this is because the sum of weights is obviously not the same when we take the absolute value. Mathematically speaking, if we have an array of weights W , we can update this like

$$W \rightarrow |W| \frac{\sum_i W_i}{\sum_i |W_i|} \quad (7.4)$$

such that the weights are at least in the same scale.

7.3.2 Architecture and hyperparameter tuning

Making a BDT for our purposes is fairly easy as well using XGBoost. One way to do it is using the code below in Algorithm 7.2

Algorithm 7.2: Boosted Decision Tree definition using XGBoost

```

1 import xgboost as xgb
2
3 Boosted_Decision_Tree = xgb.XGBClassifier(
4     max_depth,
5     use_label_encoder=False,
6     n_estimators,
7     learning_rate,
8     reg_lambda,
9     predictor = 'cpu_predictor',
10    tree_method = 'hist',
11    scale_pos_weight = sow_bkg/sow_sig,
12    objective = 'binary:logistic',
13    eval_metric = 'auc',
14    min_child_weight = 1,
15    missing = -999,
16    random_state = 42,
17    verbosity = 1)

```

To get the best performance on our BDT we have to do a grid search here as well. The trainable hyperparameters here are different than for NNs though. With XGBoost we have the following hyperparameters

- Tree depth: how many times we split the data
- Number of estimators: how many trees we use to do the gradient boosting
- The learning rate η
- L2-regressor λ , to stop overtraining

The metrics that will be used to estimate the best hyperparameters are **AUC**, **binary accuracy** and most importantly **expected significance**. The expected significance for this section has been calculated using the low statistics formula Eq. (4.20) without uncertainties. The expected significance will also be calculated when making a cut on 0.85 on the network prediction score, as explained in Chapter 5.3.5. Meaning we will only look at events which the network rates as signal with 85% confidence and above. For some models where the expected events are too low (EFT models), meaning that the expected significance will be 0 or NaN, if this happens we will use the hyperparameters of the network which gives us the highest AUC on the testing set. This will procedure will be done for every single network we will explore.

Part III

Results

Chapter 8

Network optimization

Something something Time to prepare our ML algorithms, statistics etc..

8.1 Neural Network Training

For most of the NN optimization methods we trained a NN with the following hyperparameters:

- One hidden layer
- 100 neurons in the hidden layer
- 0.1 learning rate η
- 10^{-5} L2-regularization parameter λ
- The ADAM optimizer

We will mention whenever these parameters were not used. The results for the different network optimization methods explained in Chapter 7.2 follow from here. Startngg with the normalization of data.

8.1.1 Normalization of data

We trained a network using 80% of the whole SM background events as well as 80% of all the Z' DH HDS samples. As sample weights we only balanced the signal and background using MC events. We did this by using method 3. on Chapter 7.2.3. We tested on the remaining 20% of the SM background events, as well as 20% of Z' DH HDS events where $m_{Z'} = 130$ GeV. The different normalization methods explained in Chapter 7.2.2 have been tested and can be seen in the Figure 8.1.

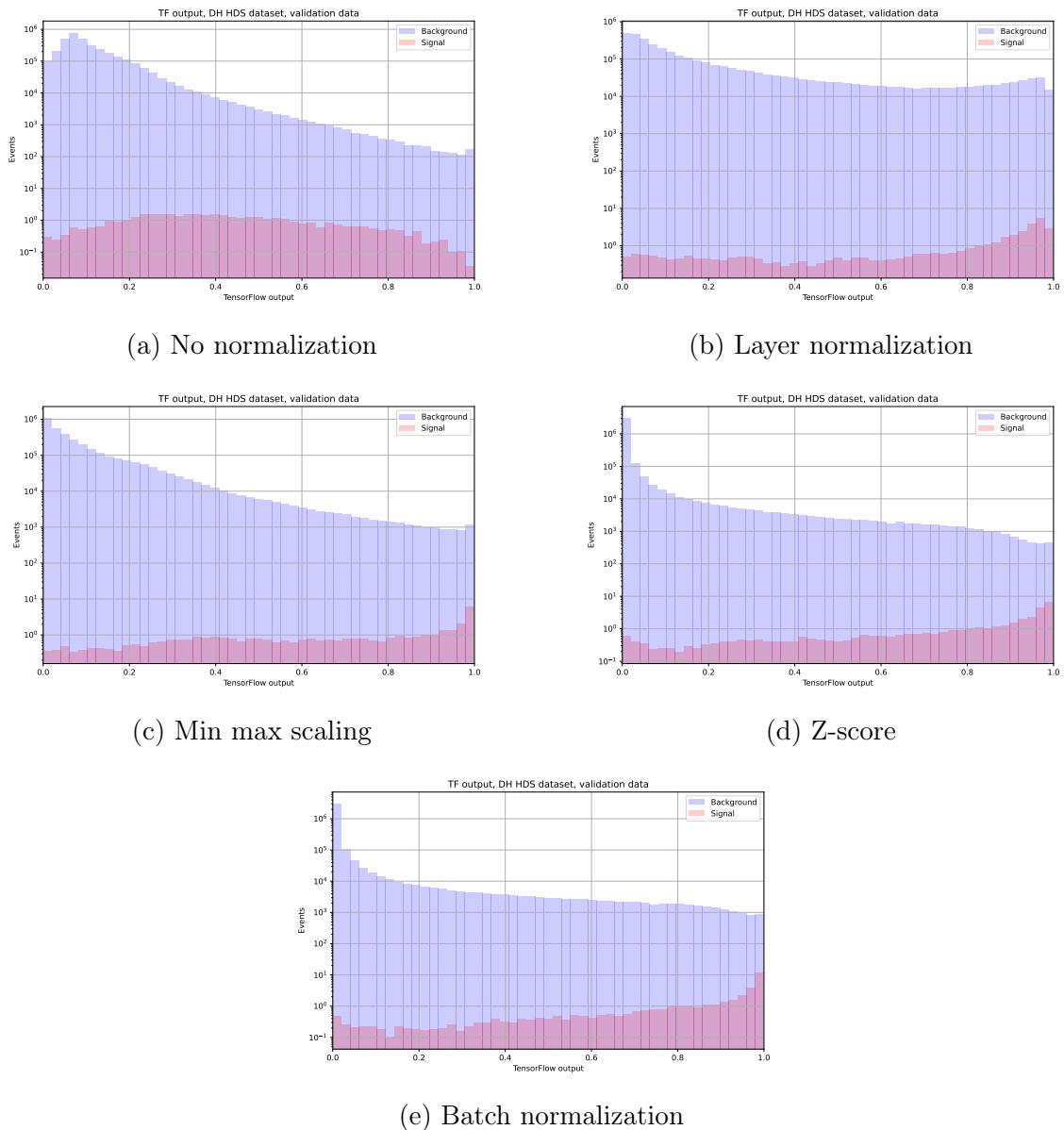


Figure 8.1: NN prediction when using different normalization methods. This is testing a dataset with 20% of the Z' DH HDS $m_{Z'} = 130$ GeV events.

Including datapoints as well as uncertainties on the best performing normalization methods, as well as their calculate expected significance as explained in Chapter 5.3.5, yields the plots shown in Figure 8.2. For more Figures showing NN training results see the GitHub repo¹.

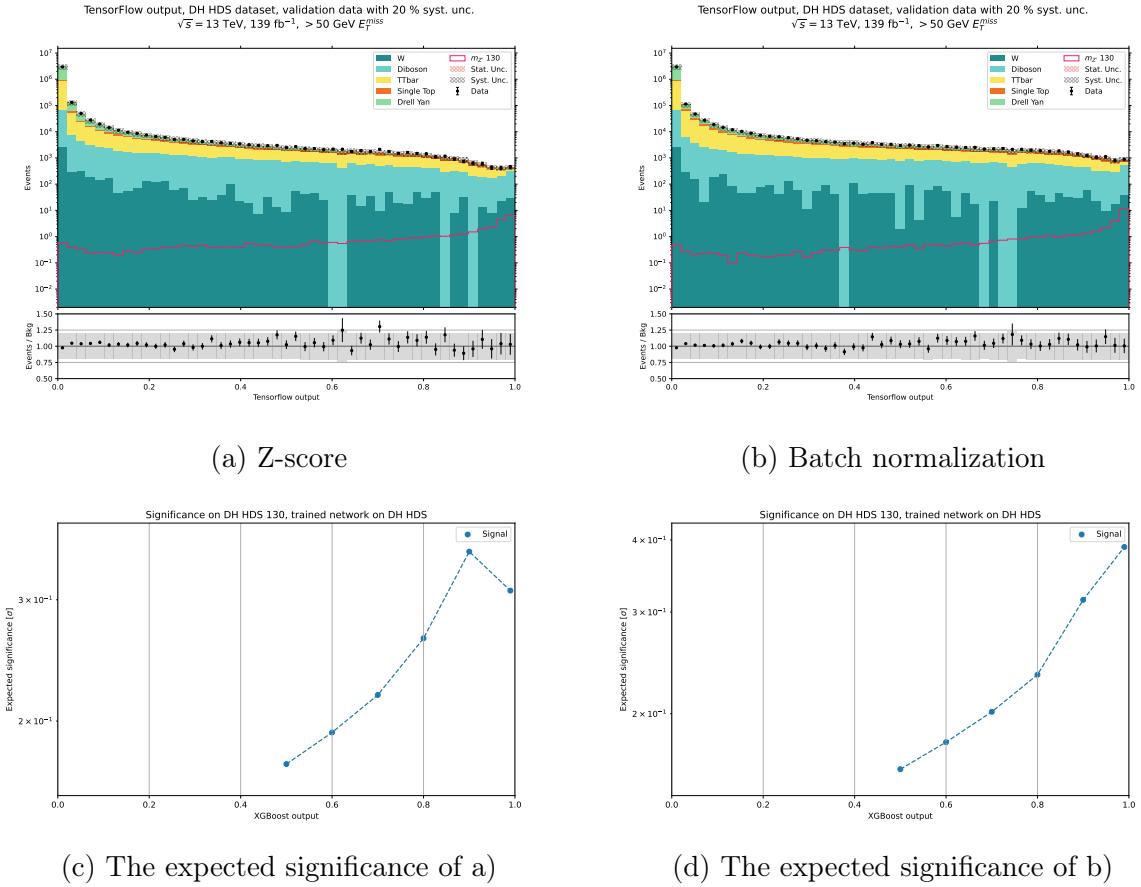


Figure 8.2: Comparison of the best normalization methods. Figure a) and b) show the validation data of both cases, c) and d) show the expected significance of the validation plots when making a cut on the output.

As we can see the **Batch_normalization** method gives us the highest signal and background but is it reasonable to use this method when one is not using a CNN? The reason batch normalization might work best for our case is because when we divide the data by batches it might unevenly represent the SM / signal and their ratio. But by using batch normalization it takes the average of all the batches creating a closer to real distribution. For the following examples in this chapter we will use batch normalization to make the optimal network.

¹ Available here: https://github.com/rubenguevara/Master-Thesis/tree/master/Plots/NeuralNetwork/Normalization_method

8.1.2 Balancing of signal and background

To try the different sample weight methods explained in Chapter 7.2.3 we used a dataset consisting of only SM events where the goal was to treat the W channel as signal and try to isolate it from other SM processes. To train we used `Batch_normalization` and 80% of the SM background events. To test we used the remaining 20% of SM events. We also tested the difference in performance when using the SGD and ADAM optimizers. The difference in distributions when using different optimizers can be seen in Figure 8.3, here the balancing method (3. on Chapter 7.2.3) is used.

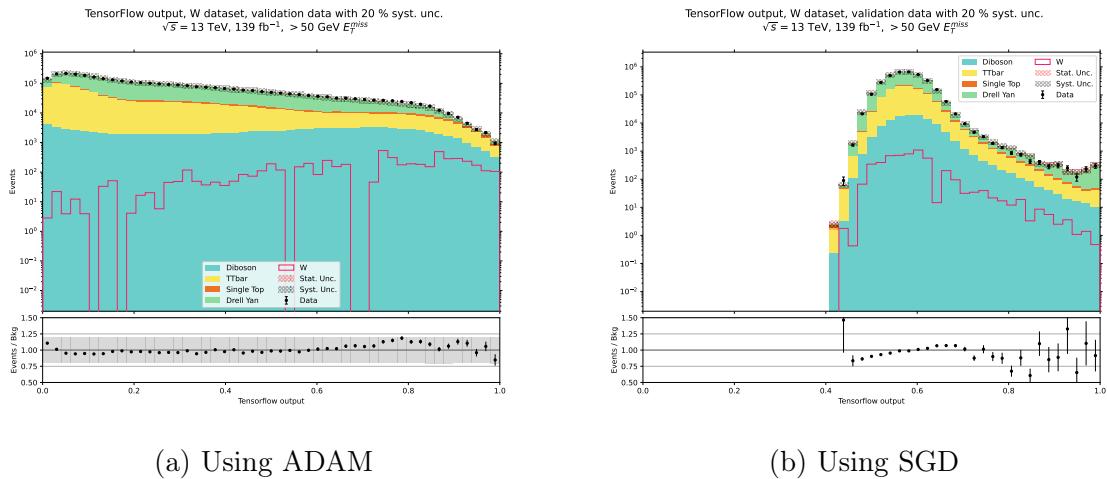


Figure 8.3: Validation plots using SGD and ADAM. This was done using a dataset where the goal was to isolate the W background process from other SM background processes

As ADAM is far better at sorting signal from background we will only use this optimizer further. The results for the different weighting methods can be seen in Figure 8.4, which shows the validation plots and in Figure 8.5 which shows the ROC score. For more Figures showing NN training results see the GitHub repo².

From the figures we see that the only way the network does not predict every event to be a background event³ is when we introduce the balancing method. We also see that the AUC increases more as well. Meaning that we must balance our dataset. Something else to mention, as to why the network does such a poor job at classifying the W background, is that the network here was not optimized for the search. If we were to conduct a thorough

² Available here: <https://github.com/rubenguevara/Master-Thesis/tree/master/Plots/NeuralNetwork/W>

³ Since the output is the score from 0-1 our network gives every event, where 0 means that the network predicts 0% chance for an event to be signal

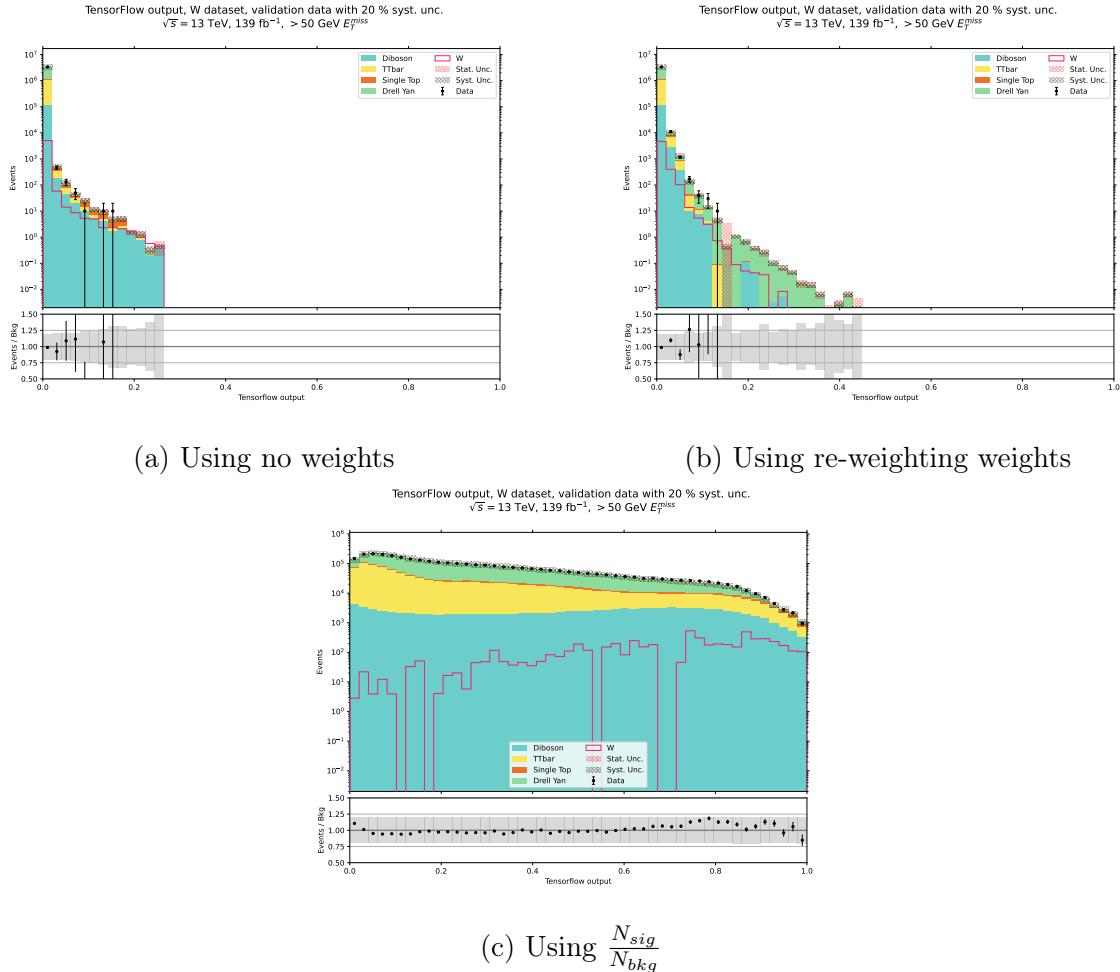


Figure 8.4: Validation plots of different balancing methods. This was done using a dataset where the goal was to isolate the W background process from other SM background processes

grid search of all hyperparameters it would yield greater results, but as this chapter is for testing methods rather analysing data we will not delve further into it for now,

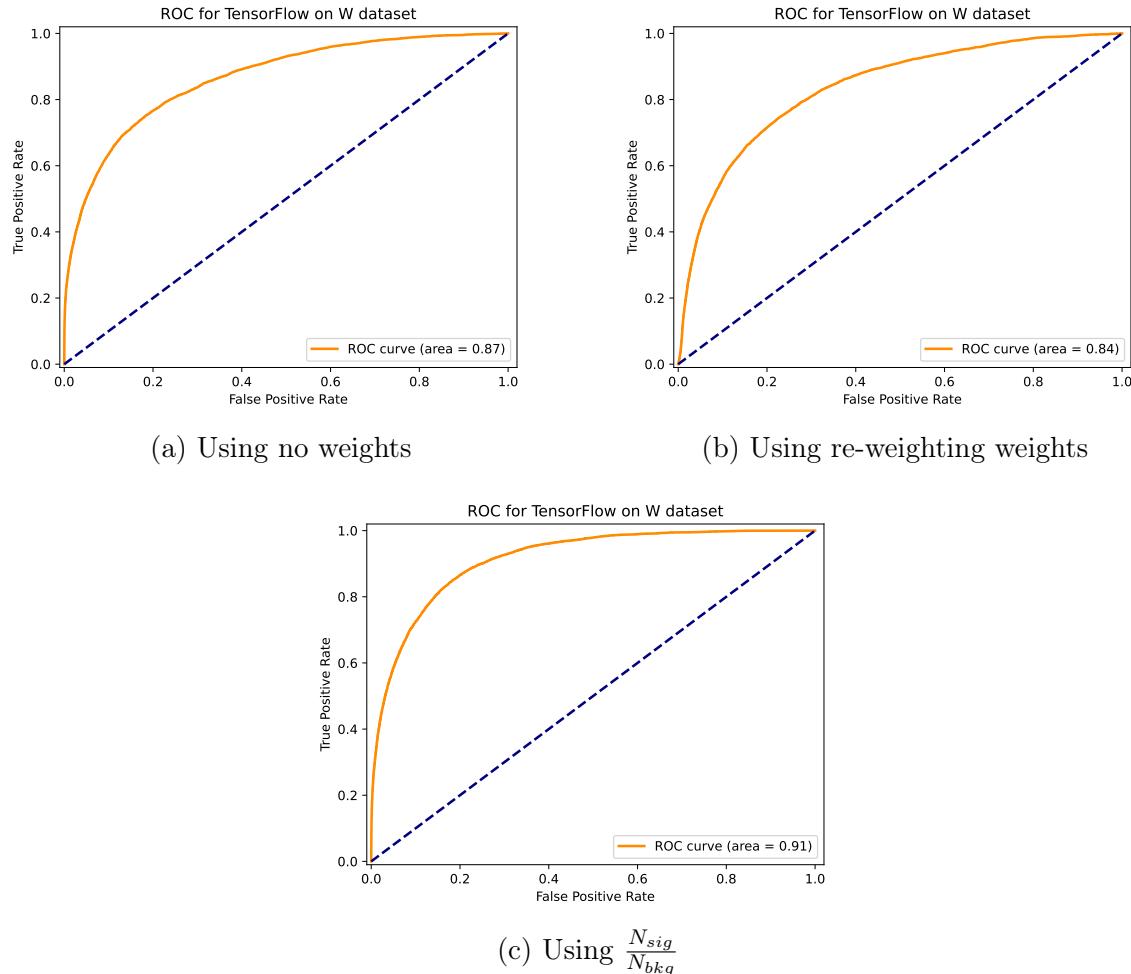


Figure 8.5: ROC plots of different balancing methods. This was done using a dataset where the goal was to isolate the W background process from other SM background processes

8.1.3 Sample weights to get expected events

To try the different sample weight methods explained in Chapter 7.2.4 which include the weights (Chapter 7.1.1), we used a dataset consisting of only SM events where the goal was to treat the W channel as signal and try to isolate it from other SM processes. To train we used `Batch_normalization` and 80% of the SM background events. To test we used the remaining 20% of SM events. The results can be seen in Figure 8.6, which shows the validation plots and in Figure 8.7 which shows the ROC score. For more Figures showing NN training results see the GitHub repo⁴.

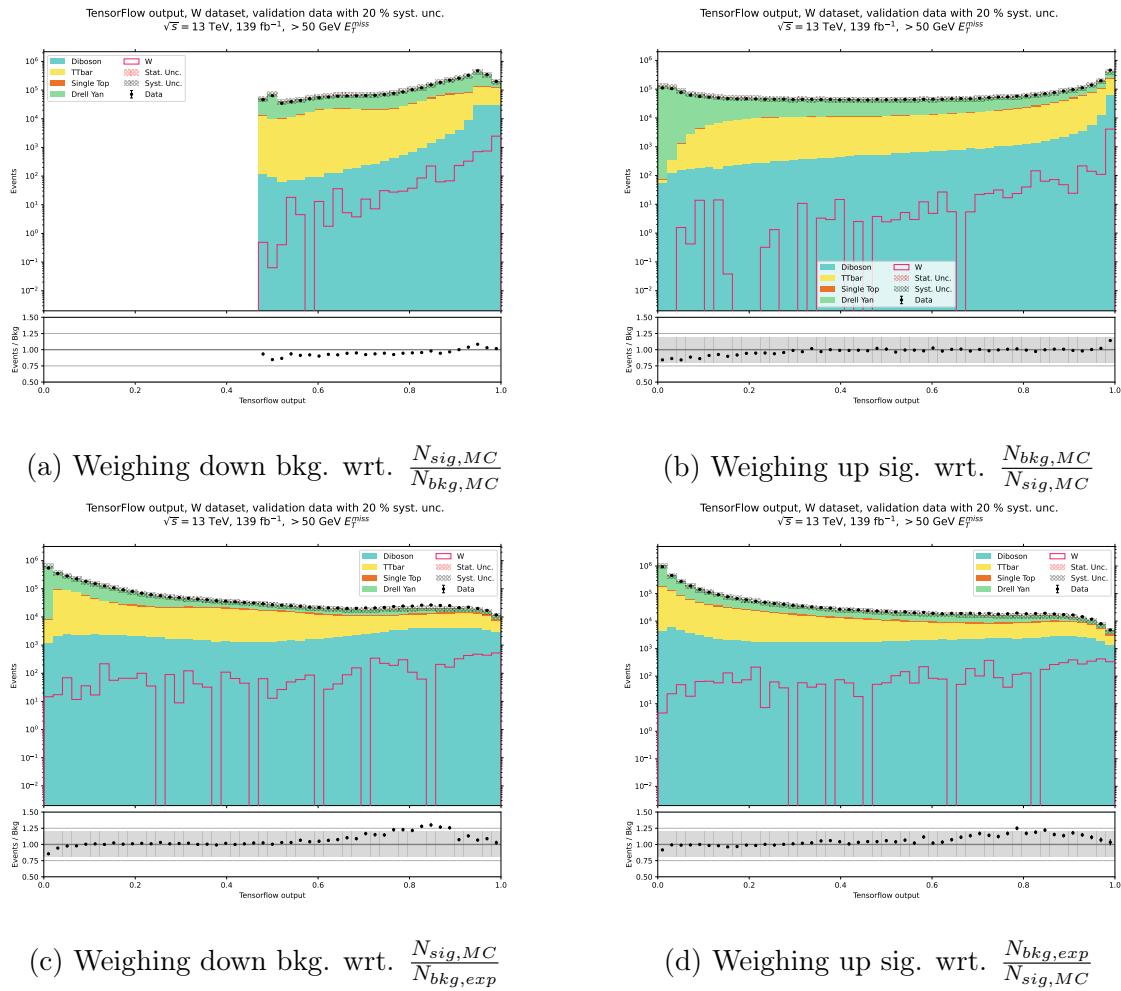


Figure 8.6: Validation plots of different balancing methods when re-weighting background events to expected events. This was done using a dataset where the goal was to isolate the W background process from other SM background processes

As we are only re-weighting the background events, we can see from the figures that we get the best results when balancing with respect to the expected number of background

⁴ Available here: <https://github.com/rubenguevara/Master-Thesis/tree/master/Plots/NeuralNetwork/W>

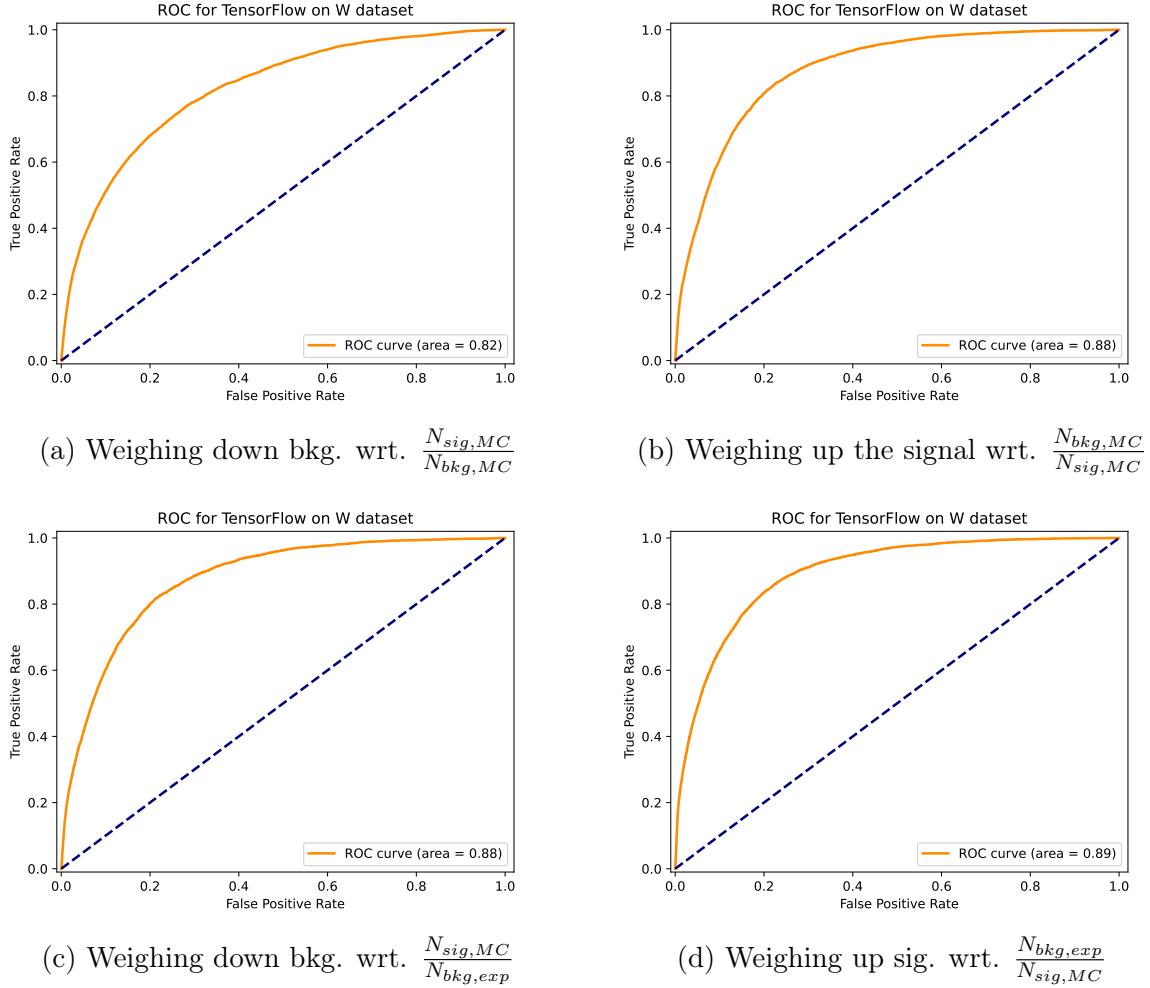


Figure 8.7: ROC plots of different balancing methods when re-weighting background events to expected events. This was done using a dataset where the goal was to isolate the W background process from other SM background processes

events. Which is optimal, as this is what goes into the network. Whether it is better to weigh down the background or weight up the signal is not clear however, from the AUC of the ROC curves it is slightly better to weigh up the signal. To check which gives a higher expected significance however we can look at the expected significance, this is shown in Figure 8.8. Here we see that there is a greater expected significance when weighing up the signal events.

As a last note for the testing of these methods, the networks, while still getting over 5σ expected significance (without errors) on the W channel, do not have the best distribution on the validation plots. The reason for this might be because we did not optimize the networks we tested, but rather used the same network for test.

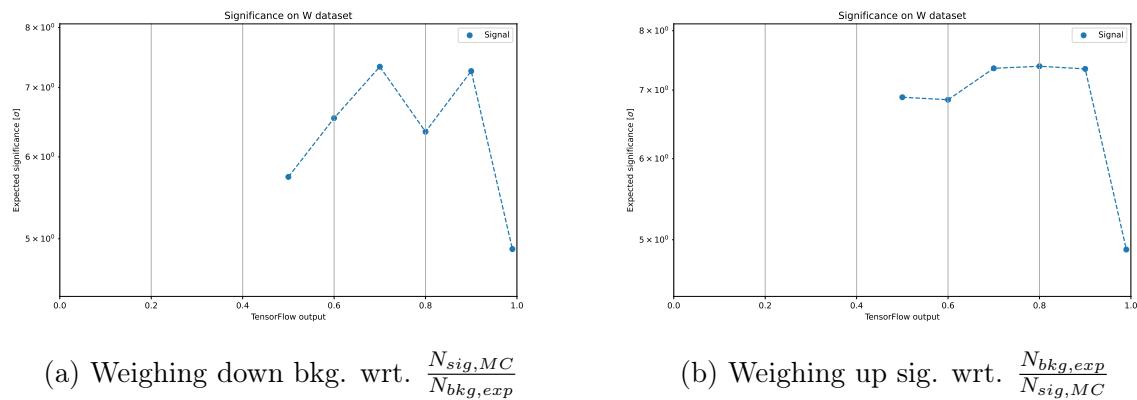


Figure 8.8: Expected significance plots of the best balancing methods when re-weighting background events to expected events. This was done using a dataset where the goal was to isolate the W background process from other SM background processes

8.1.4 Padding of data

For the padding problem. We will as explained in Chapter 7.2.1 try the new variables presented in Table 7.2. The other method we tried was to remove the features with jagged arrays, that means the p_T, η, ϕ of the three most energetic jets, as well as the invariant mass of the two most energetic ones, m_{jj} . We trained a network using 80% of the whole SM background events as well as 80% of all the Z' DH HDS samples. As sample weights we used the best method from the previous section, which was to re-weight every background event and balance the dataset by weighing up all signal events by the ratio of expected number of background events over signal MC events, $\frac{N_{bkg,exp}}{N_{sig,MC}}$. As the best normalization method was `Batch_normalization`, this method was used here. We also utilized the ADAM optimizer instead of SGD.

As changing features changes the whole dataset, then to get the best results as possible we went through a full grid search following the steps in Chapter 7.2.5 for both networks. The result for the hyperparameters that gave the highest significance can be seen in Figure 8.9.

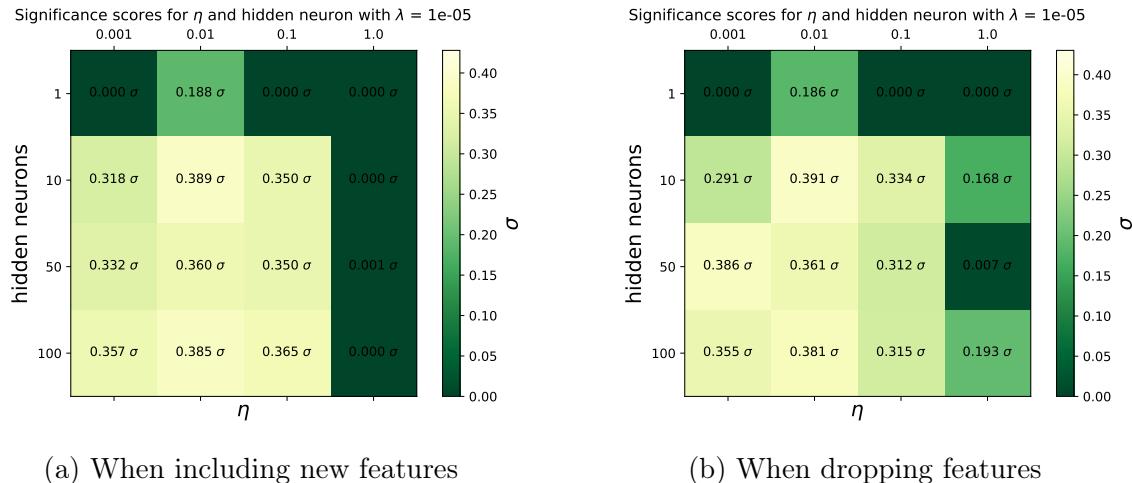


Figure 8.9: Grid search result for pad testing on NN. This is training a dataset with 80% of all Z' DH HDS events.

This means that the best hyperparameters for both networks coincidentally is the same, meaning: `n_neuron = 10`, `eta = 0.01`, `lambda = 1e-5`. The loss, AUC and binary accuracy over epochs for the best networks can be seen in Figure 8.10 and Figure 8.11.

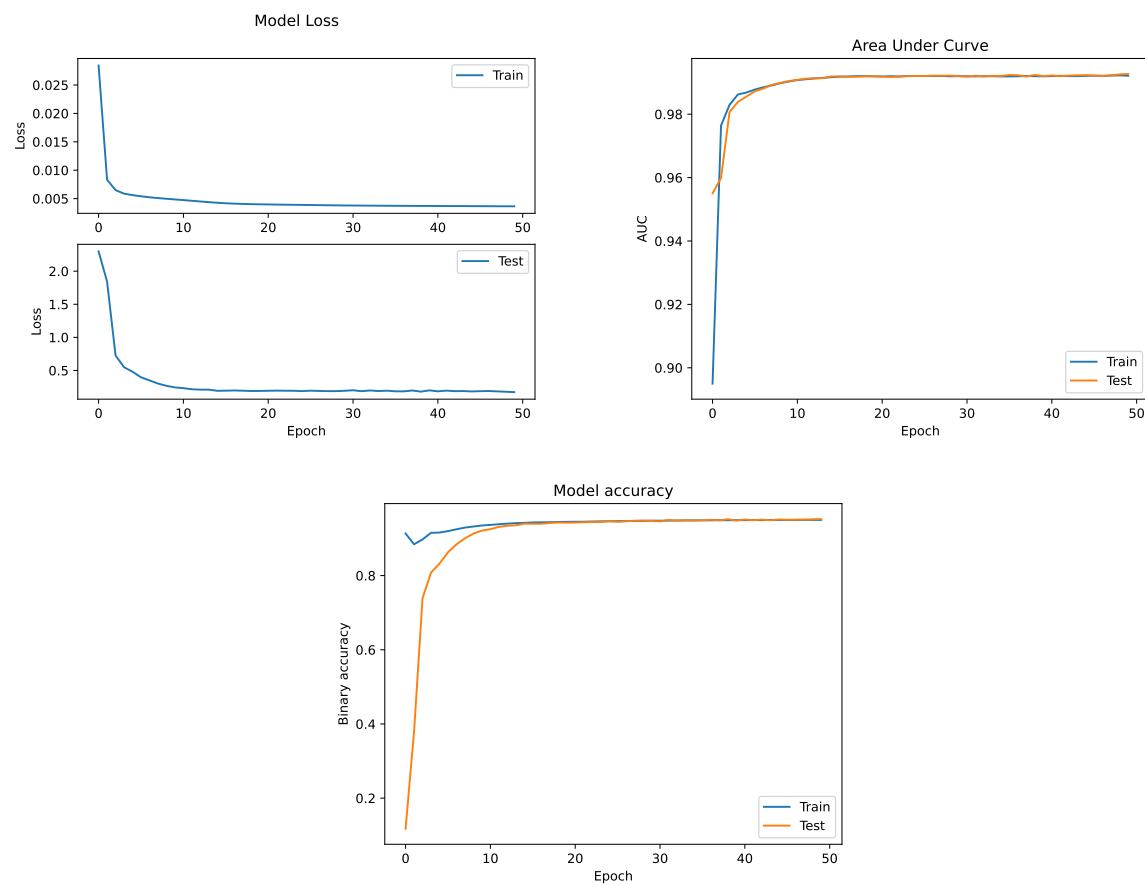


Figure 8.10: NN parameters after 50 epochs with new features. This is training a dataset with 80% of all Z' DH HDS events.

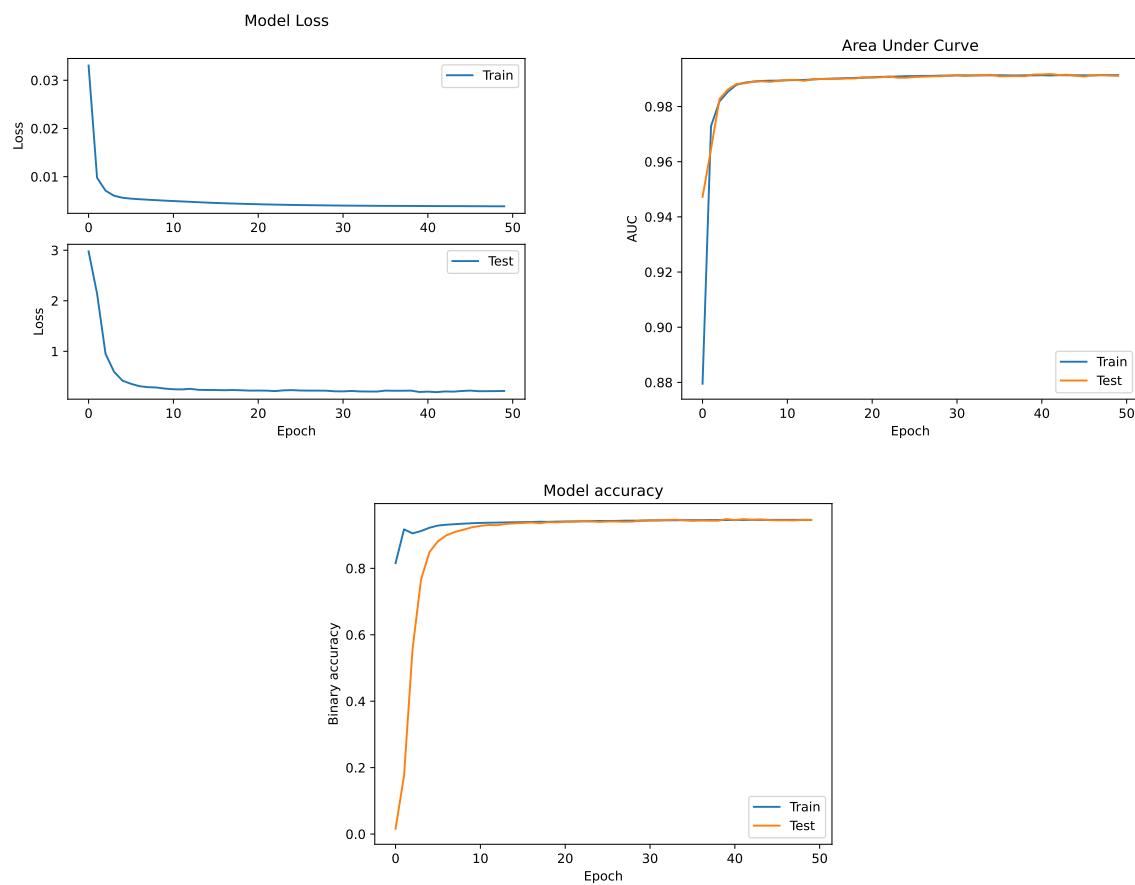


Figure 8.11: NN parameters after 50 epochs when dropping features. This is training a dataset with 80% of all Z' DH HDS events.

We tested on the remaining 20% of the SM background events, as well as 20% of Z' DH HDS events where $m_{Z'} = 130$ GeV. The ROC scores for each network can be seen in Figure 8.12. The validation plots can be seen in Figure 8.13

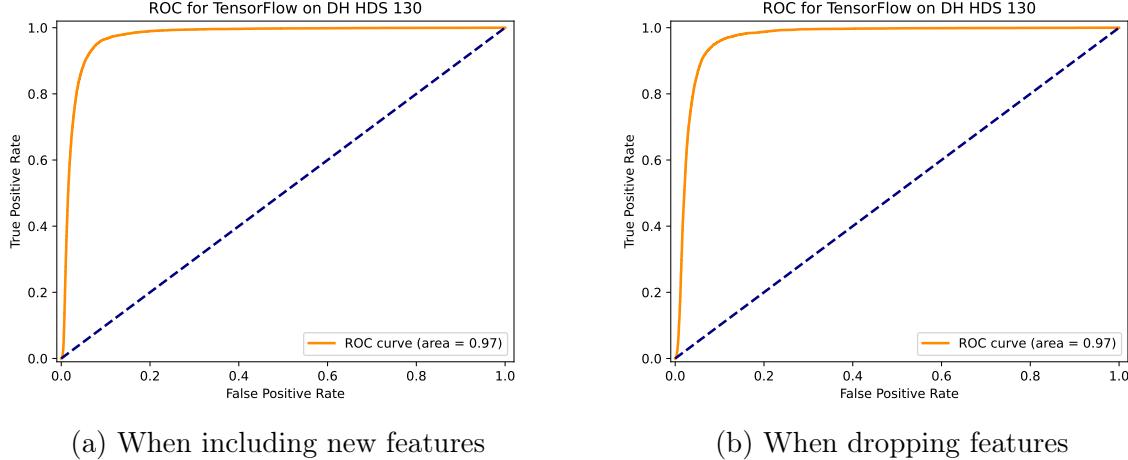


Figure 8.12: ROC plots for both padding methods. This is testing a dataset with 20% of the Z' DH HDS $m_{Z'} = 130$ GeV events.

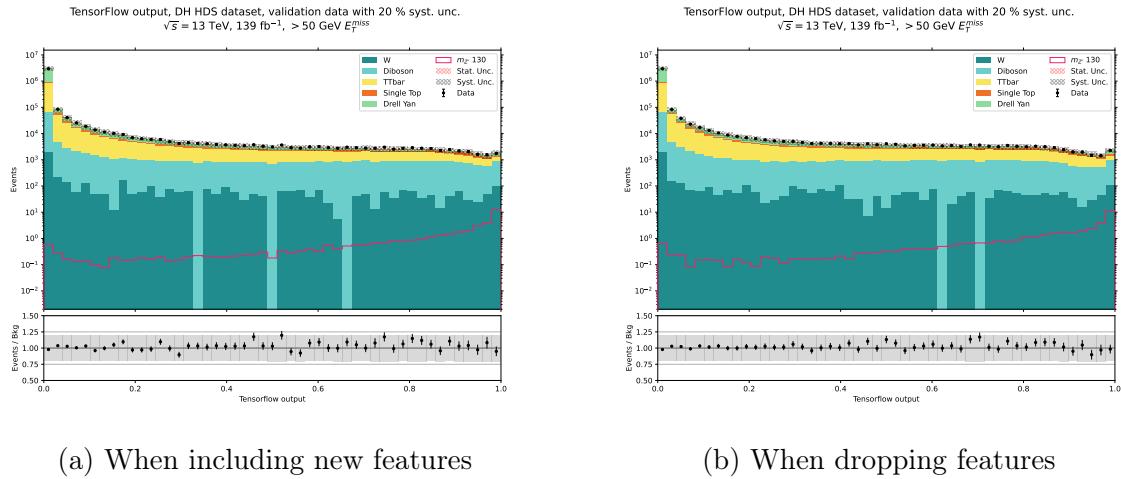


Figure 8.13: Validation plots for both padding methods. This is testing a dataset with 20% of the Z' DH HDS $m_{Z'} = 130$ GeV events.

As we can see the performance of both methods is the same, to check if the sensitivity increases more with the new padding method or not we can check the significance of each model. This can be seen in Figure 8.14, which shows a slight improvement on the sensitivity of the network when using the features.

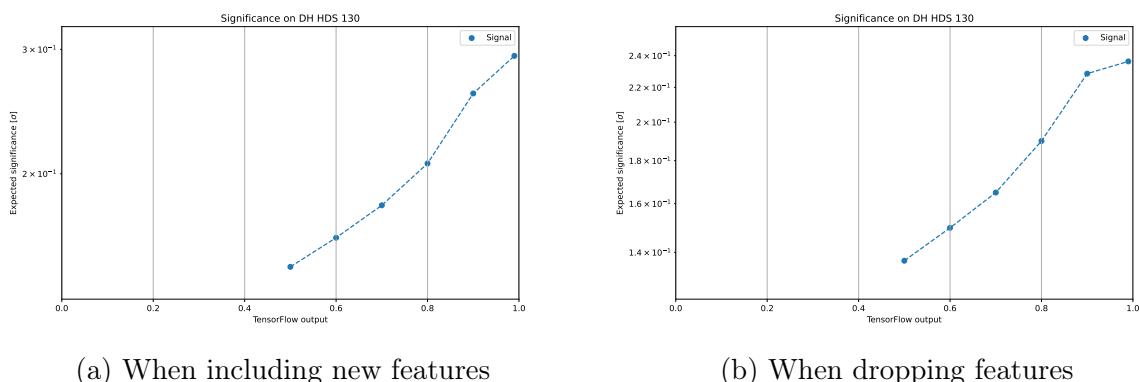


Figure 8.14: Significance plots for both padding methods. This is testing a dataset with 20% of the Z' DH HDS $m_{Z'} = 130$ GeV events.

8.2 Boosted Desicion Tree Training (draft)

8.2.1 Weights

We have tried all of the weighting options and the results can be seen in Figure 8.15.

Remake
the results
and write
better!

As we can see it makes a significant difference whether we use the weights to re-weight MC events to expected events. But there is no mathematical reason as to why we should include this re-weighting weight as sample weights, as the reason to use sample weights is to *only* balance signal and bacground. In theory it makes no sense whatsoevert to include these weights either as the network doesn't really care for cross sections or luminosity, and what we are doing in principle is making it harder for the network to learn anything. But as seen on the results, the BDT learns the background extremely well when using the weights, while it also does a poorer job in learning the signal we are testing.

In a sense this is not a negative thing for our purposes, but stricly speaking we are invoking a semi-unsupervised learning method by punishing the network if it learns the signal too quickly. To get to the point as why this is good for our purposes, we are indirectly making our network more model indepentent! Because of this, the method that will be pursued further in this thesis will be to take the positive weights and balance the data.

Add that
ATLAS
used it, so
because
of that so
can I?

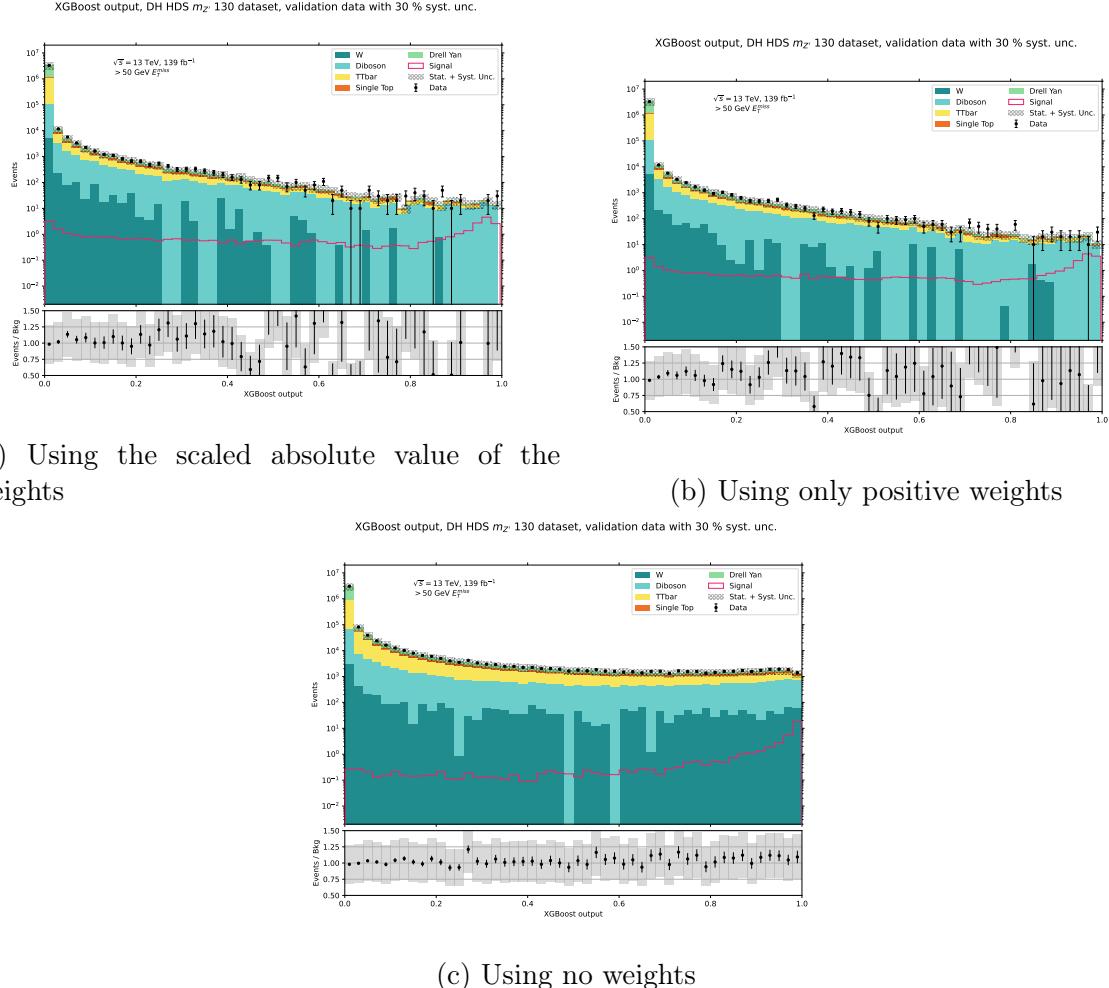


Figure 8.15: Difference when using different weighting methods. All networks were trained using the balancing method explained in Section 7.1.1

8.2.2 Grid Search

As the previous results are to be taken with a heavy grain of salt, I conducted another grid search. On the second grid search I set the values of $\eta = 0.1$ as the trend showed this giving the best results with less overtraining, and $\lambda = 10^{-5}$. This grid search had $n_{\text{estimators}} \in [10, 100, 500, 1000]$ and depth $\in [3, 4, 5, 6]$. The expected significance is shown in Figure 8.16. The testing and training AUC can be seen in Figure 8.17.

should I conduct a new grid search with different λ and loss functions?

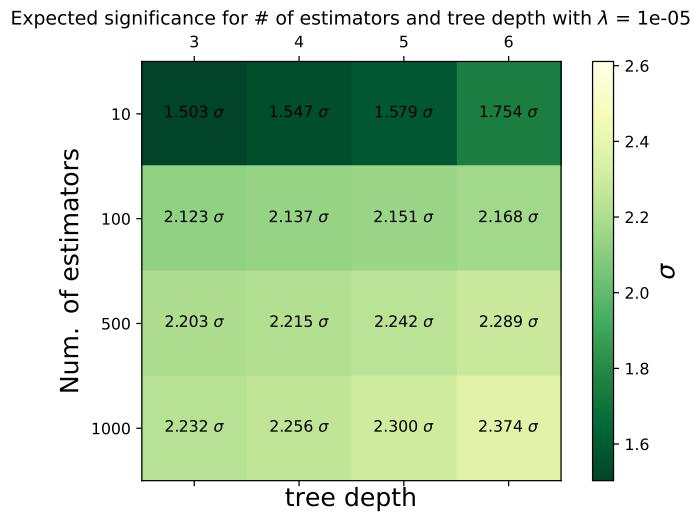


Figure 8.16: Grid search expected significance when setting $\lambda = 10^{-5}$ and $\eta = 0.1$

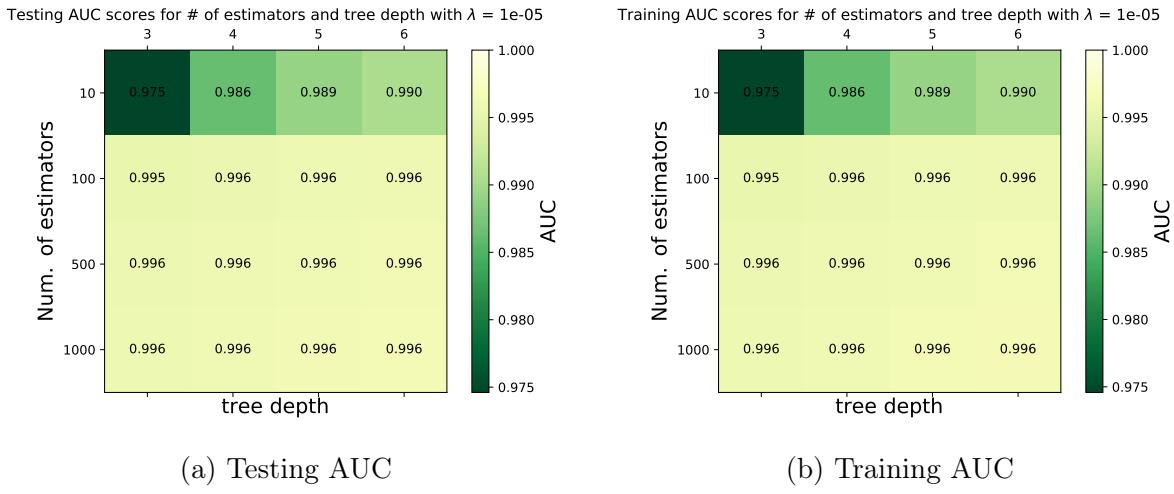


Figure 8.17: Grid search AUC when setting $\lambda = 10^{-5}$ and $\eta = 0.1$

When testing the best network with a depth of 6 and 1000 estimators on the same DH HDS $m_{Z'} = 130$ GeV model we get the feature importance plots shown in Figure 8.18. Here we see that the "weight" metric gives us the expected features as most important.

But the "cover" metric seems to be less of what we expect since the jet kinematic variables score higher, this might just be a curiosity rather than something to be suspect of.

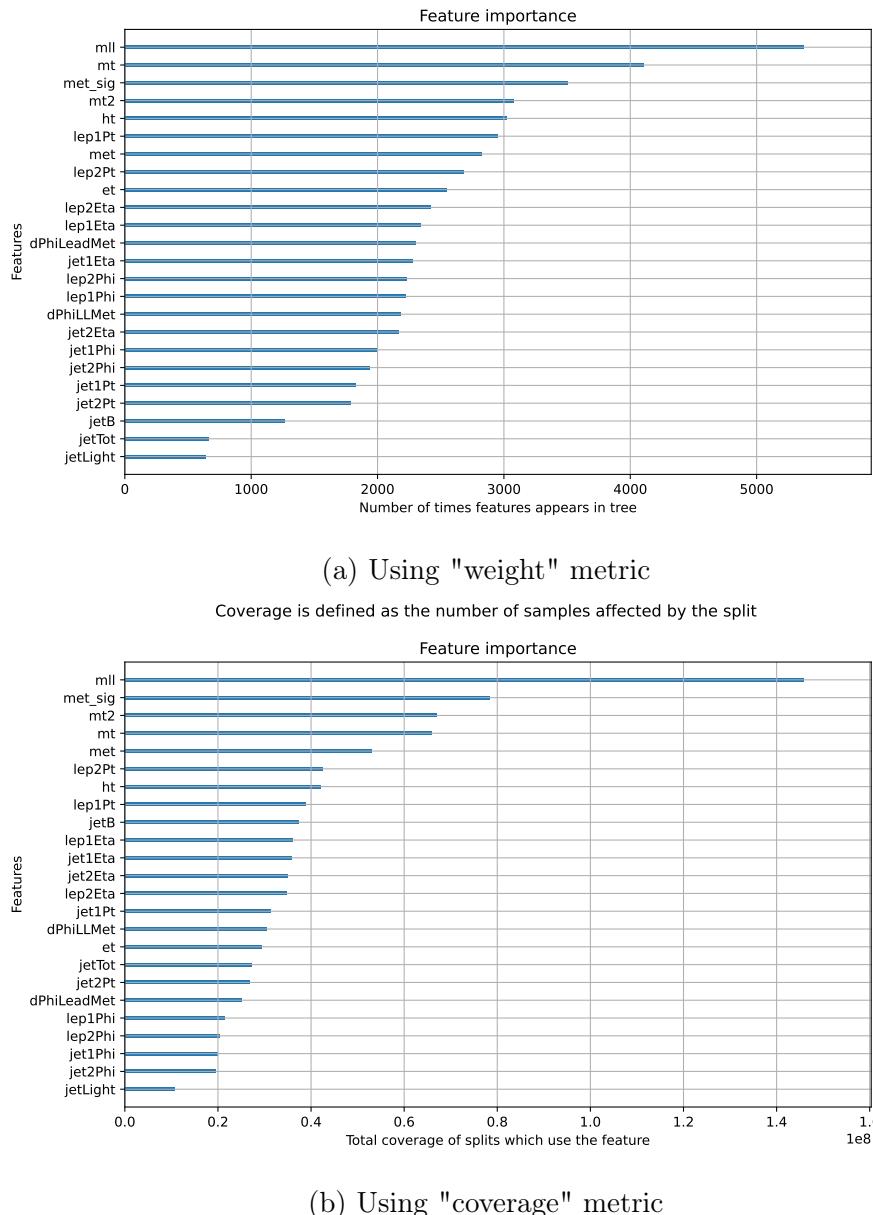


Figure 8.18: Feature importance of depth 6 network trained on FULL Z' DM data set when testing it on DH HDS $m_{Z'} = 130$ GeV model.

8.3 Discussion (draft)

The last thing to be noted is that having a DNN completely removes the possibility of combining the results of multiple networks trained on a single model, as the imbalance becomes too much for the network to see anything. A solution to this however, is

Comment
on lack
of time
and DNN
thoughts.

that instead of combining the results of multiple networks trained on a singular model, one could try the Parametrized NN approach used by Baldi et. al. [20], which could potentially avoid the imbalance problem, but this is proposed as a plausible new research project due to time constrain on this thesis.

XGBoost » TensorFlow

Something something Time to prepare our ML algorithms, statistics etc..

Comment
on fully
converting
to XG-
Boost.

Chapter 9

Model dependent approach

Now that we have established what type of ML network we will utilize as well as optimizing everything such that ...

[write more](#)

9.1 The datasets

Do I need to explain it again if I have it on Table [7.1](#)

9.2 Dark Higgs Heavy Dark Sector

Trained a network using all of the SM background samples and every different Z' mass of this model. Here are the results

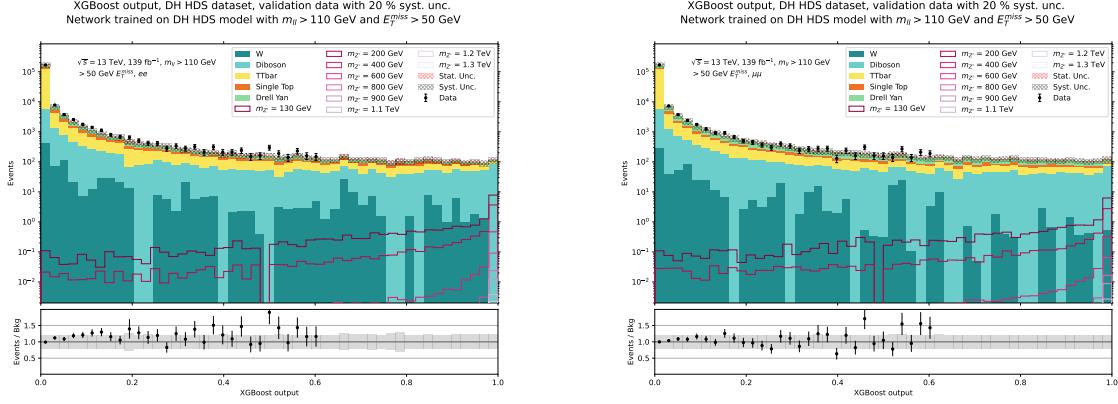


Figure 9.1: Validation plots for network trained on Z' DH HDS

With the ROC for each mass point seen in Figure 9.2.

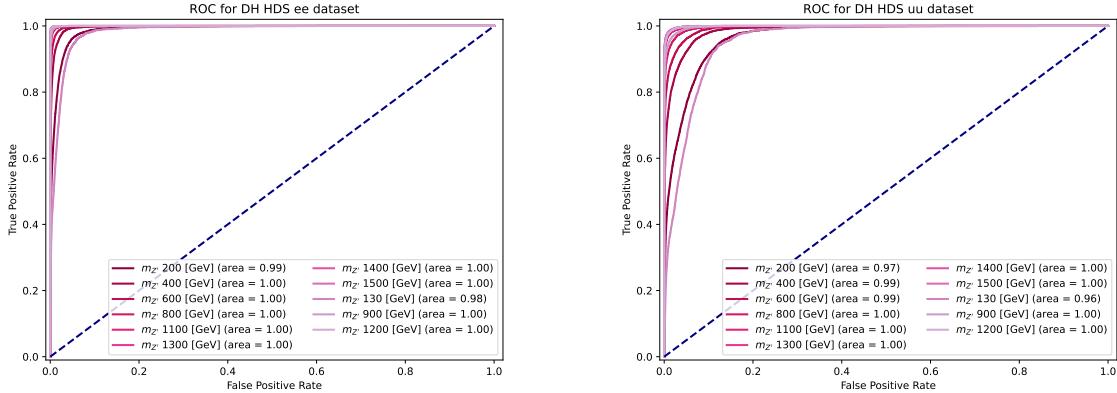


Figure 9.2: ROC plots for every Z' mass point on network trained on Z' DH HDS

Plotting the significance of the models given the binning we get the results from Figure 9.3

Using the last bin as the significance is greatest there, such that we effectively make a cut based on the BDT score. Using the last bin, we can calculate a mass exclusion for both electron and muon channel.

To do so we need to count the number of signal and background events that are on the last bin, as well as their uncertainties. Additionally we will include the number of

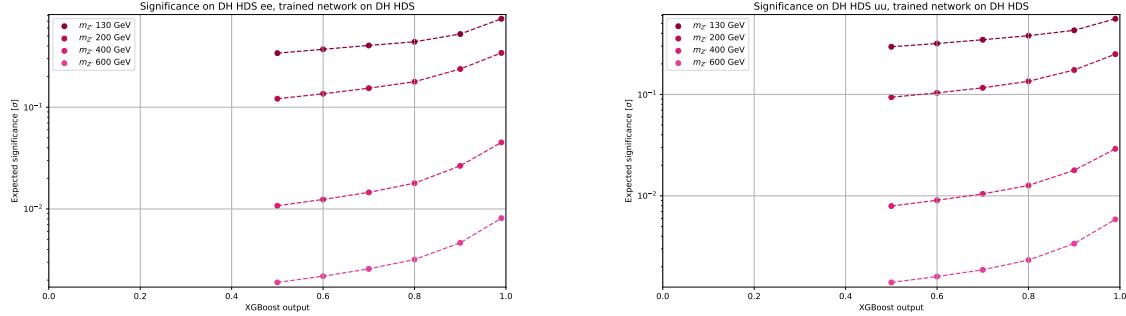


Figure 9.3: Expected significance plots for Z' mass points on network trained on Z' DH HDS

real data events that are there such that we can follow the method explained in Chapter 4.3.2. In Table 9.1 we see the values for each Z' mass point

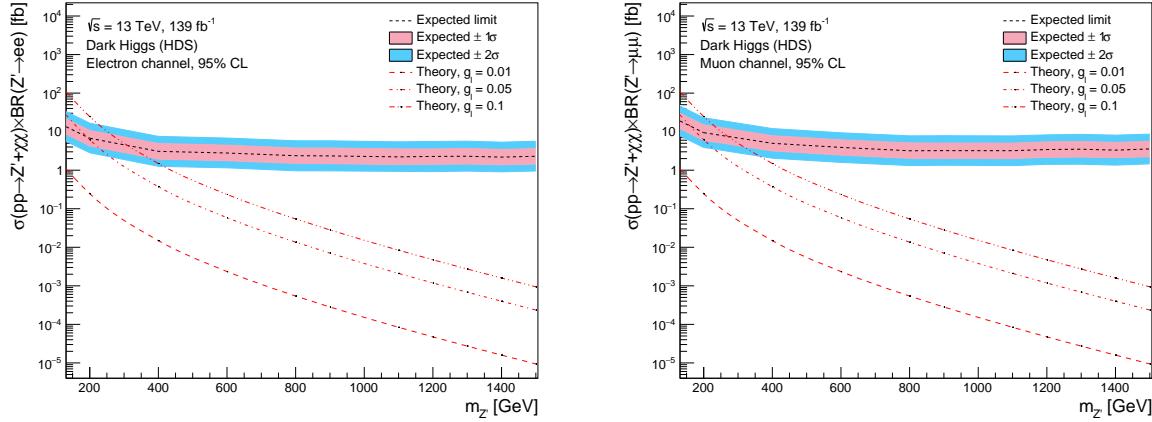


Figure 9.4: Mass exclusion limits of ee and $\mu\mu$ channel for all Z' DH HDS model

As the lepton coupling was chosen to be $g_l = 0.001$ when simulating the data, by the assumption that the number of events that survived the cuts is the same, we can increase this coupling to see how the mass limits changes.

Table 9.1: Inputs for the $Z'h_D \rightarrow l^+l^-\chi\bar{\chi}$ HDS σB calculations. The first three columns are the Z' mass, the theoretical cross section times branching ratio σB , and what Z' decay channel we are looking at. The next two are ε_{sig} , which is the signal selection efficiency, and N_{sig} , which is the theoretical number of signal events after the cuts. The last two columns are the number of background events, N_{bkg} , and the events observed in the data, N_{obs} . The uncertainties of ε_{sig} , N_{sig} and N_{bkg} are statistical with an assumed 20% systematic uncertainty. The MET threshold is $E_{\text{T},\text{min}}^{\text{miss}} = 50\text{GeV}$ and the invariant mass threshold is $m_{ll}^{\text{min}} = 110\text{GeV}$ and is the same for all inputs.

$m_{Z'}$ [GeV]	σB [fb]	Channel	ε_{sig} [$\times 10^{-1}$]	N_{sig}	N_{bkg}
130	1.11	ee	0.25 ± 0.05	7.80 ± 1.58	108.4 ± 23.0
		$\mu\mu$	0.20 ± 0.04	6.28 ± 1.27	124.9 ± 26.1
200	2.46×10^{-1}	ee	0.54 ± 0.11	3.67 ± 0.74	114.1 ± 24.4
		$\mu\mu$	0.41 ± 0.08	2.78 ± 0.56	123.2 ± 25.8
400	1.49×10^{-2}	ee	1.13 ± 0.23	$4.67 \times 10^{-1} \pm 9.37 \times 10^{-2}$	107.0 ± 23.4
		$\mu\mu$	0.79 ± 0.16	$3.29 \times 10^{-1} \pm 6.60 \times 10^{-2}$	127.5 ± 26.6
600	2.35×10^{-3}	ee	1.40 ± 0.28	$9.12 \times 10^{-2} \pm 1.83 \times 10^{-2}$	126.3 ± 26.7
		$\mu\mu$	1.01 ± 0.20	$6.59 \times 10^{-2} \pm 1.32 \times 10^{-2}$	126.3 ± 26.3
800	5.43×10^{-4}	ee	1.59 ± 0.32	$2.40 \times 10^{-2} \pm 4.81 \times 10^{-3}$	118.8 ± 25.6
		$\mu\mu$	1.11 ± 0.22	$1.67 \times 10^{-2} \pm 3.36 \times 10^{-3}$	113.2 ± 23.6
900	2.82×10^{-4}	ee	1.60 ± 0.32	$1.25 \times 10^{-2} \pm 2.51 \times 10^{-3}$	119.3 ± 25.7
		$\mu\mu$	1.12 ± 0.22	$8.78 \times 10^{-3} \pm 1.76 \times 10^{-3}$	114.8 ± 24.0
1100	8.40×10^{-5}	ee	1.63 ± 0.33	$3.81 \times 10^{-3} \pm 7.64 \times 10^{-4}$	114.3 ± 24.4
		$\mu\mu$	1.16 ± 0.23	$2.70 \times 10^{-3} \pm 5.42 \times 10^{-4}$	118.6 ± 24.7
1200	4.75×10^{-5}	ee	1.65 ± 0.33	$2.18 \times 10^{-3} \pm 4.37 \times 10^{-4}$	118.4 ± 25.1
		$\mu\mu$	1.14 ± 0.23	$1.50 \times 10^{-3} \pm 3.01 \times 10^{-4}$	125.6 ± 26.3
1300	2.73×10^{-5}	ee	1.69 ± 0.34	$1.28 \times 10^{-3} \pm 2.57 \times 10^{-4}$	123.9 ± 26.5
		$\mu\mu$	1.15 ± 0.23	$8.72 \times 10^{-4} \pm 1.75 \times 10^{-4}$	130.7 ± 27.2
1400	1.60×10^{-5}	ee	1.67 ± 0.33	$7.43 \times 10^{-4} \pm 1.49 \times 10^{-4}$	115.8 ± 24.5
		$\mu\mu$	1.16 ± 0.23	$5.15 \times 10^{-4} \pm 1.03 \times 10^{-4}$	125.4 ± 26.1
1500	9.42×10^{-6}	ee	1.69 ± 0.34	$4.44 \times 10^{-4} \pm 8.90 \times 10^{-5}$	123.9 ± 26.5
		$\mu\mu$	1.13 ± 0.23	$2.97 \times 10^{-4} \pm 5.96 \times 10^{-5}$	130.7 ± 27.2

9.3 Dark Higgs Light Dark Sector

Trained a network using all of the SM background samples and every different Z' mass of this model. Here are the results

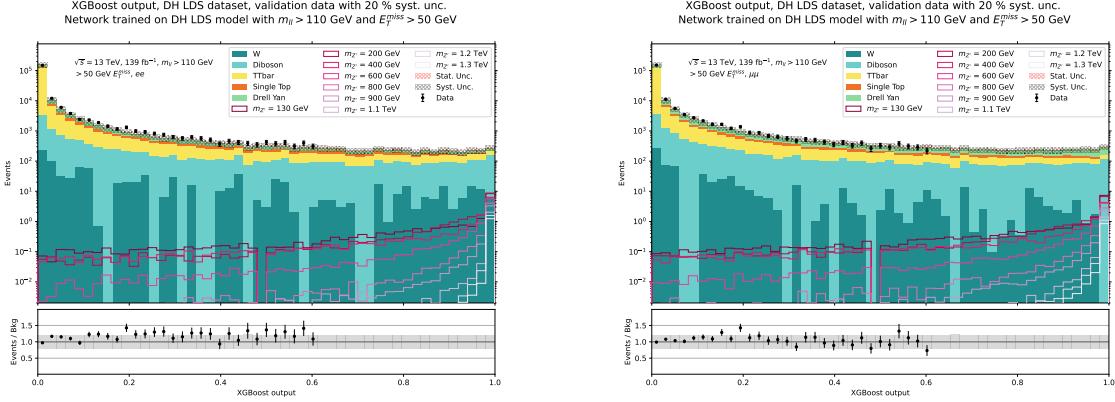


Figure 9.5: Validation plots for network trained on Z' DH LDS

With the ROC for each mass point seen in Figure 9.6.

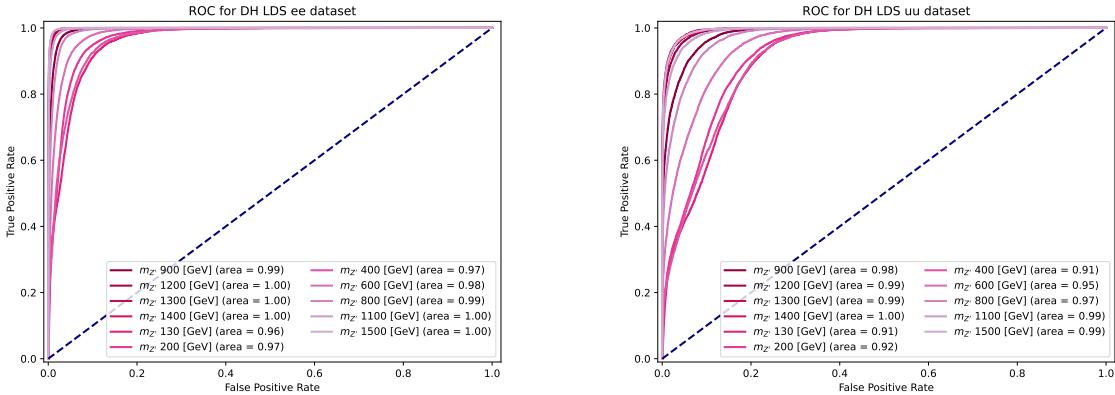


Figure 9.6: ROC plots for every Z' mass point on network trained on Z' DH LDS

Plotting the significance of the models given the binning we get the results from Figure 9.7

Using the last bin as the significance is greatest there, such that we effectively make a cut based on the BDT score. Using the last bin, we can calculate a mass exclusion for both electron and muon channel.

To do so we need to count the number of signal and background events that are on the last bin, as well as their uncertainties. Additionally we will include the number of

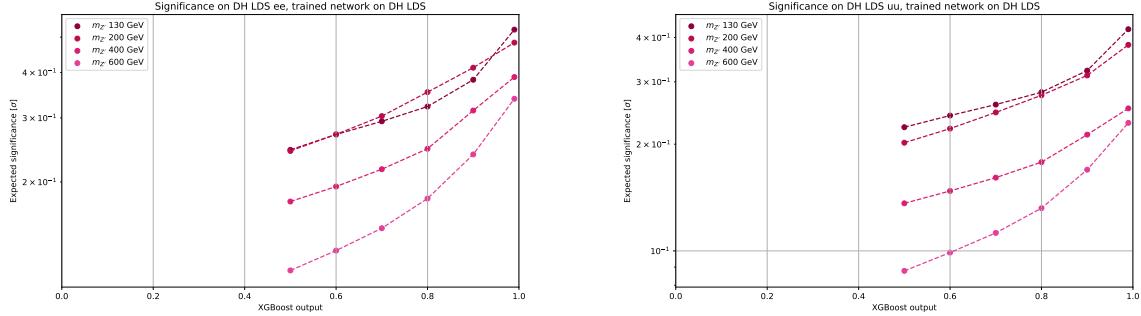


Figure 9.7: Expected significance plots for Z' mass points on network trained on Z' DH LDS

real data events that are there such that we can follow the method explained in Chapter 4.3.2. In Table 9.2 we see the values for each Z' mass point. As the lepton coupling was

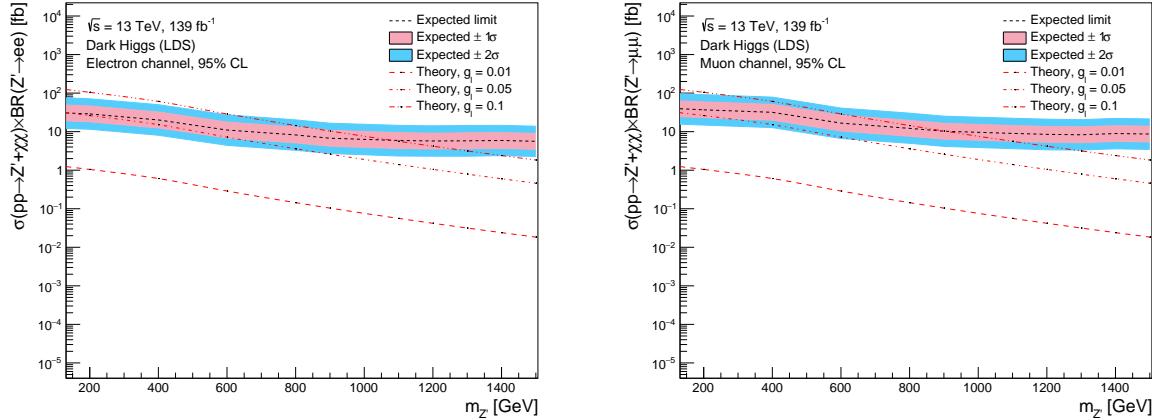


Figure 9.8: Mass exclusion limits of ee and $\mu\mu$ channel for all Z' DH LDS model

chosen to be $g_l = 0.001$ when simulating the data, by the assumption that the number of events that survived the cuts is the same, we can increase this coupling to see how the mass limits changes.

Table 9.2: Inputs for the $Z'h_D \rightarrow l^+l^-\chi\bar{\chi}$ LDS σB calculations. The first three columns are the Z' mass, the theoretical cross section times branching ratio σB , and what Z' decay channel we are looking at. The next two are ε_{sig} , which is the signal selection efficiency, and N_{sig} , which is the theoretical number of signal events after the cuts. The last two columns are the number of background events, N_{bkg} , and the events observed in the data, N_{obs} . The uncertainties of ε_{sig} , N_{sig} and N_{bkg} are statistical with an assumed 20% systematic uncertainty. The MET threshold is $E_{\text{T},\text{min}}^{\text{miss}} = 50\text{GeV}$ and the invariant mass threshold is $m_{ll}^{\text{min}} = 110\text{GeV}$ and is the same for all inputs.

$m_{Z'}$ [GeV]	σB [fb]	Channel	ε_{sig} [$\times 10^{-1}$]	N_{sig}	N_{bkg}
130	1.24	ee	0.25 ± 0.05	8.70 ± 1.75	272.0 ± 56.0
		$\mu\mu$	0.21 ± 0.04	7.33 ± 1.48	299.4 ± 60.7
200	1.05	ee	0.29 ± 0.06	8.34 ± 1.68	294.9 ± 60.3
		$\mu\mu$	0.23 ± 0.05	6.68 ± 1.34	304.4 ± 61.7
400	6.08×10^{-1}	ee	0.38 ± 0.08	6.35 ± 1.28	264.9 ± 55.0
		$\mu\mu$	0.25 ± 0.05	$4.30 \pm 8.66 \times 10^{-1}$	288.9 ± 58.7
600	2.88×10^{-1}	ee	0.70 ± 0.14	5.64 ± 1.13	275.2 ± 56.6
		$\mu\mu$	0.50 ± 0.10	$3.98 \pm 7.98 \times 10^{-1}$	298.3 ± 60.4
800	1.44×10^{-1}	ee	1.01 ± 0.20	$4.05 \pm 8.11 \times 10^{-1}$	295.9 ± 60.5
		$\mu\mu$	0.70 ± 0.14	$2.81 \pm 5.63 \times 10^{-1}$	307.5 ± 62.4
900	1.04×10^{-1}	ee	1.14 ± 0.23	$3.30 \pm 6.61 \times 10^{-1}$	271.3 ± 55.8
		$\mu\mu$	0.80 ± 0.16	$2.30 \pm 4.60 \times 10^{-1}$	293.2 ± 59.4
1100	5.61×10^{-2}	ee	1.30 ± 0.26	$2.03 \pm 4.07 \times 10^{-1}$	270.8 ± 55.7
		$\mu\mu$	0.90 ± 0.18	$1.40 \pm 2.81 \times 10^{-1}$	290.0 ± 58.8
1200	4.19×10^{-2}	ee	1.35 ± 0.27	$1.58 \pm 3.16 \times 10^{-1}$	273.9 ± 56.2
		$\mu\mu$	0.95 ± 0.19	$1.10 \pm 2.21 \times 10^{-1}$	289.4 ± 58.8
1300	3.16×10^{-2}	ee	1.41 ± 0.28	$1.24 \pm 2.48 \times 10^{-1}$	292.8 ± 60.0
		$\mu\mu$	0.97 ± 0.19	$8.49 \times 10^{-1} \pm 1.70 \times 10^{-1}$	288.6 ± 58.5
1400	2.40×10^{-2}	ee	1.43 ± 0.29	$9.51 \times 10^{-1} \pm 1.91 \times 10^{-1}$	298.8 ± 61.2
		$\mu\mu$	0.97 ± 0.19	$6.47 \times 10^{-1} \pm 1.30 \times 10^{-1}$	311.7 ± 63.2
1500	1.84×10^{-2}	ee	1.44 ± 0.29	$7.36 \times 10^{-1} \pm 1.47 \times 10^{-1}$	287.9 ± 59.1
		$\mu\mu$	0.96 ± 0.19	$4.90 \times 10^{-1} \pm 9.82 \times 10^{-2}$	298.7 ± 60.6

9.4 Light Vector Heavy Dark Sector

Trained a network using all of the SM background samples and every different Z' mass of this model. Here are the results

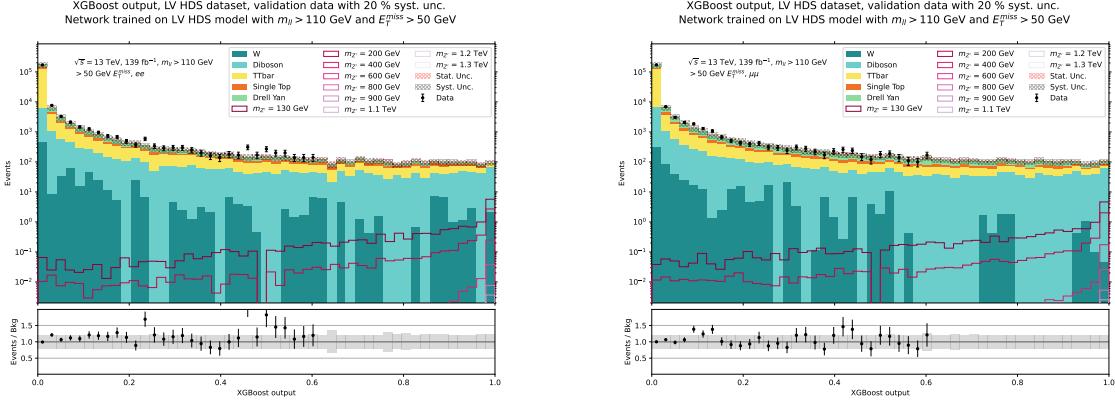


Figure 9.9: Validation plots for network trained on Z' LV HDS

With the ROC for each mass point seen in Figure 9.10.

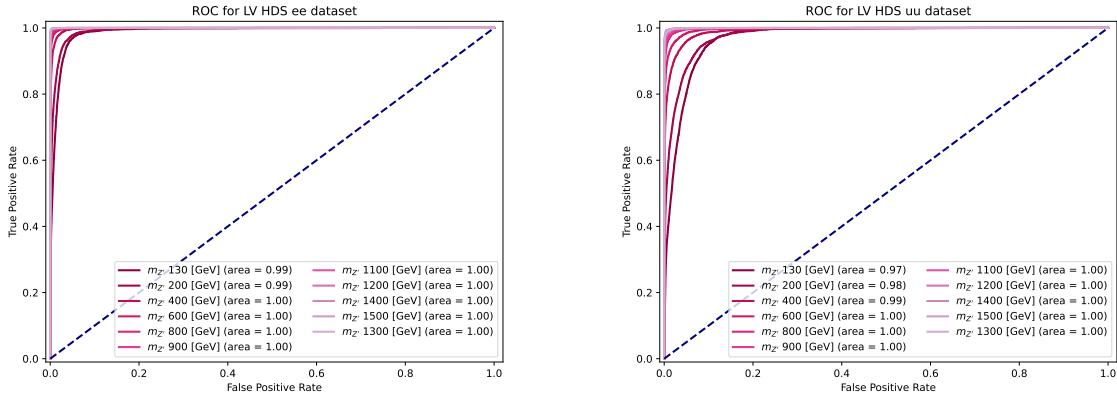


Figure 9.10: ROC plots for every Z' mass point on network trained on Z' LV HDS

Plotting the significance of the models given the binning we get the results from Figure 9.11

Using the last bin as the significance is greatest there, such that we effectively make a cut based on the BDT score. Using the last bin, we can calculate a mass exclusion for both electron and muon channel.

To do so we need to count the number of signal and background events that are on the last bin, as well as their uncertainties. Additionally we will include the number of

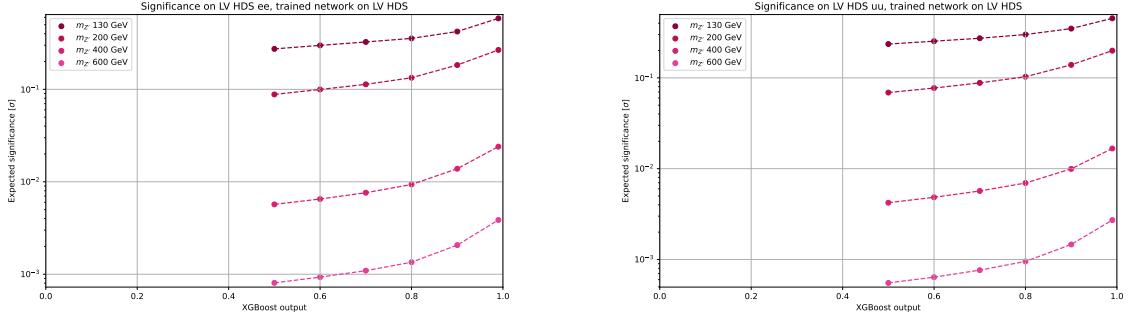


Figure 9.11: Expected significance plots for Z' mass points on network trained on Z' LV HDS

real data events that are there such that we can follow the method explained in Chapter 4.3.2. In Table 9.3 we see the values for each Z' mass point. As the lepton coupling was

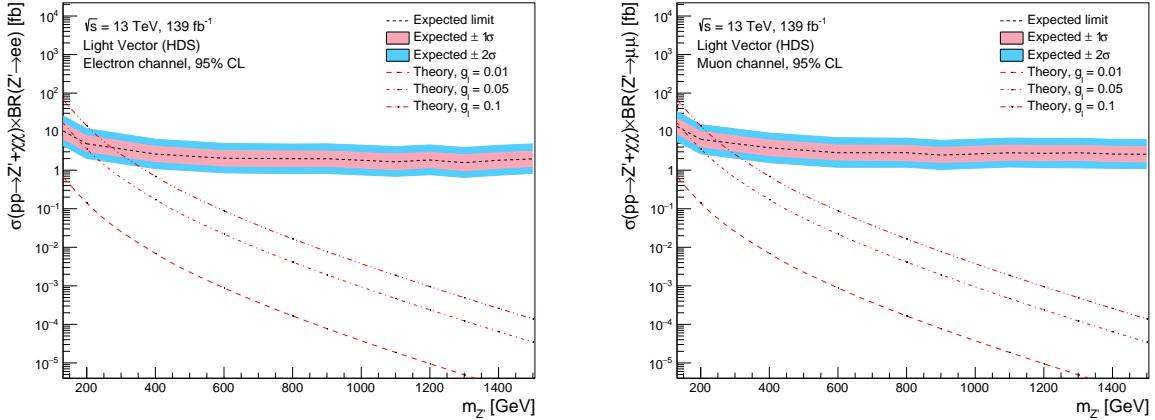


Figure 9.12: Mass exclusion limits of ee and $\mu\mu$ channel for all Z' LV HDS model

chosen to be $g_l = 0.001$ when simulating the data, by the assumption that the number of events that survived the cuts is the same, we can increase this coupling to see how the mass limits changes.

Table 9.3: Inputs for the $Z'V \rightarrow l^+l^-\chi\bar{\chi}$ HDS σB calculations. The first three columns are the Z' mass, the theoretical cross section times branching ratio σB , and what Z' decay channel we are looking at. The next two are ε_{sig} , which is the signal selection efficiency, and N_{sig} , which is the theoretical number of signal events after the cuts. The last two columns are the number of background events, N_{bkg} , and the events observed in the data, N_{obs} . The uncertainties of ε_{sig} , N_{sig} and N_{bkg} are statistical with an assumed 20% systematic uncertainty. The MET threshold is $E_{\text{T},\text{min}}^{\text{miss}} = 50\text{GeV}$ and the invariant mass threshold is $m_{ll}^{\text{min}} = 110\text{GeV}$ and is the same for all inputs.

$m_{Z'}$ [GeV]	σB [fb]	Channel	ε_{sig} [$\times 10^{-1}$]	N_{sig}	N_{bkg}
130	6.93×10^{-1}	ee	0.30 ± 0.06	5.72 ± 1.16	93.6 ± 20.6
		$\mu\mu$	0.24 ± 0.05	$4.66 \pm 9.44 \times 10^{-1}$	104.4 ± 21.9
200	1.41×10^{-1}	ee	0.69 ± 0.14	$2.71 \pm 5.45 \times 10^{-1}$	102.5 ± 22.7
		$\mu\mu$	0.51 ± 0.10	$2.02 \pm 4.08 \times 10^{-1}$	102.4 ± 21.4
400	6.90×10^{-3}	ee	1.30 ± 0.26	$2.48 \times 10^{-1} \pm 4.99 \times 10^{-2}$	107.2 ± 22.7
		$\mu\mu$	0.91 ± 0.18	$1.75 \times 10^{-1} \pm 3.52 \times 10^{-2}$	110.2 ± 23.1
600	8.81×10^{-4}	ee	1.56 ± 0.31	$3.83 \times 10^{-2} \pm 7.67 \times 10^{-3}$	98.2 ± 21.1
		$\mu\mu$	1.06 ± 0.21	$2.60 \times 10^{-2} \pm 5.21 \times 10^{-3}$	91.4 ± 19.7
800	1.66×10^{-4}	ee	1.66 ± 0.33	$7.65 \times 10^{-3} \pm 1.53 \times 10^{-3}$	101.6 ± 21.6
		$\mu\mu$	1.16 ± 0.23	$5.33 \times 10^{-3} \pm 1.07 \times 10^{-3}$	100.5 ± 21.2
900	7.74×10^{-5}	ee	1.71 ± 0.34	$3.67 \times 10^{-3} \pm 7.36 \times 10^{-4}$	106.0 ± 22.5
		$\mu\mu$	1.19 ± 0.24	$2.56 \times 10^{-3} \pm 5.14 \times 10^{-4}$	89.2 ± 19.0
1100	1.87×10^{-5}	ee	1.73 ± 0.35	$9.03 \times 10^{-4} \pm 1.81 \times 10^{-4}$	84.2 ± 18.8
		$\mu\mu$	1.19 ± 0.24	$6.22 \times 10^{-4} \pm 1.25 \times 10^{-4}$	106.1 ± 22.3
1200	9.53×10^{-6}	ee	1.76 ± 0.35	$4.65 \times 10^{-4} \pm 9.33 \times 10^{-5}$	100.2 ± 21.4
		$\mu\mu$	1.22 ± 0.24	$3.22 \times 10^{-4} \pm 6.46 \times 10^{-5}$	105.1 ± 22.0
1300	4.93×10^{-6}	ee	1.73 ± 0.35	$2.38 \times 10^{-4} \pm 4.76 \times 10^{-5}$	77.0 ± 17.7
		$\mu\mu$	1.20 ± 0.24	$1.64 \times 10^{-4} \pm 3.30 \times 10^{-5}$	104.5 ± 22.0
1400	2.59×10^{-6}	ee	1.79 ± 0.36	$1.29 \times 10^{-4} \pm 2.60 \times 10^{-5}$	98.5 ± 21.3
		$\mu\mu$	1.21 ± 0.24	$8.73 \times 10^{-5} \pm 1.75 \times 10^{-5}$	97.8 ± 20.6
1500	1.38×10^{-6}	ee	1.79 ± 0.36	$6.86 \times 10^{-5} \pm 1.38 \times 10^{-5}$	111.1 ± 23.5
		$\mu\mu$	1.20 ± 0.24	$4.61 \times 10^{-5} \pm 9.26 \times 10^{-6}$	96.3 ± 20.2

9.5 Light Vector Light Dark Sector

Trained a network using all of the SM background samples and every different Z' mass of this model. Here are the results

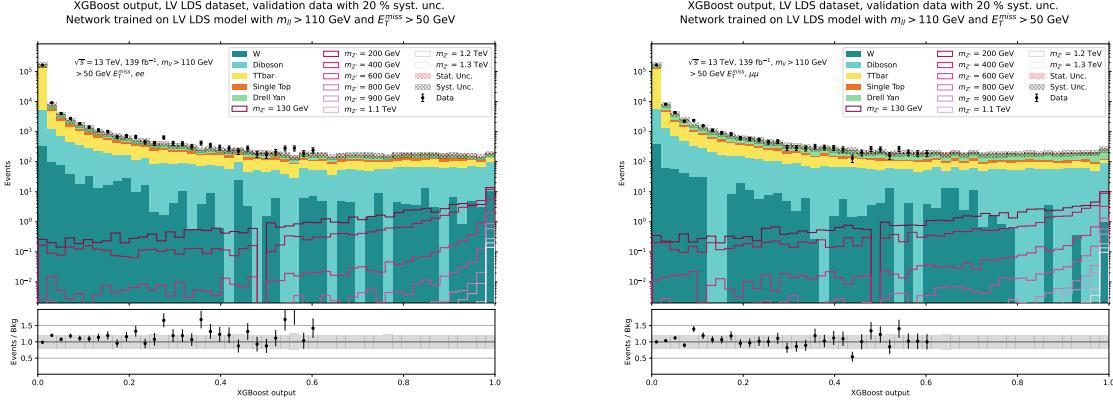


Figure 9.13: Validation plots for network trained on Z' LV LDS

With the ROC for each mass point seen in Figure 9.14.

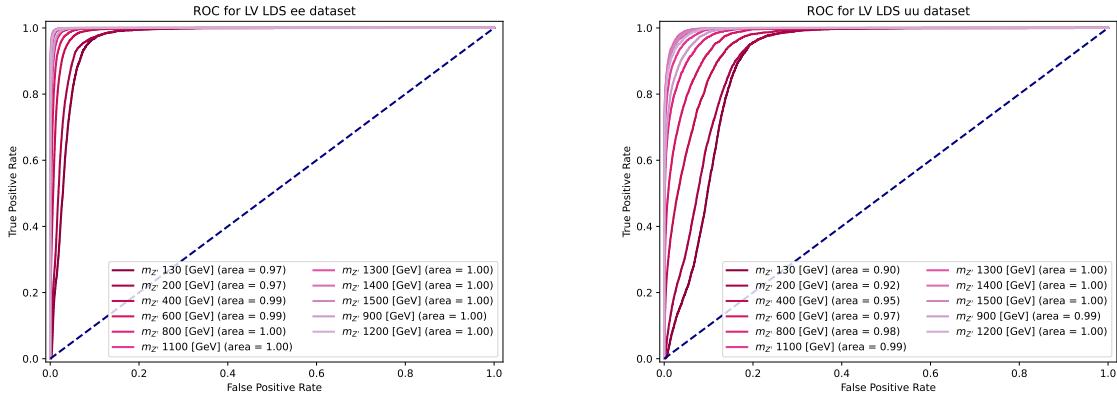


Figure 9.14: ROC plots for every Z' mass point on network trained on Z' LV LDS

Plotting the significance of the models given the binning we get the results from Figure 9.15

Using the last bin as the significance is greatest there, such that we effectively make a cut based on the BDT score. Using the last bin, we can calculate a mass exclusion for both electron and muon channel.

To do so we need to count the number of signal and background events that are on the last bin, as well as their uncertainties. Additionally we will include the number of

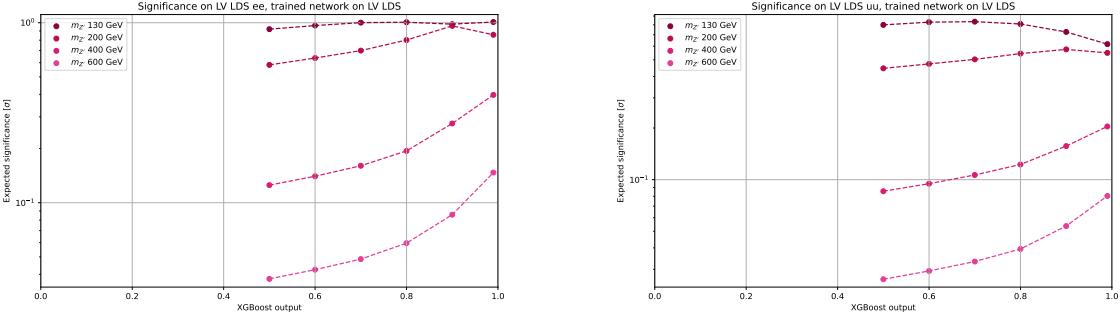


Figure 9.15: Expected significance plots for Z' mass points on network trained on Z' LV LDS

real data events that are there such that we can follow the method explained in Chapter 4.3.2. In Table 9.4 we see the values for each Z' mass point. As the lepton coupling was

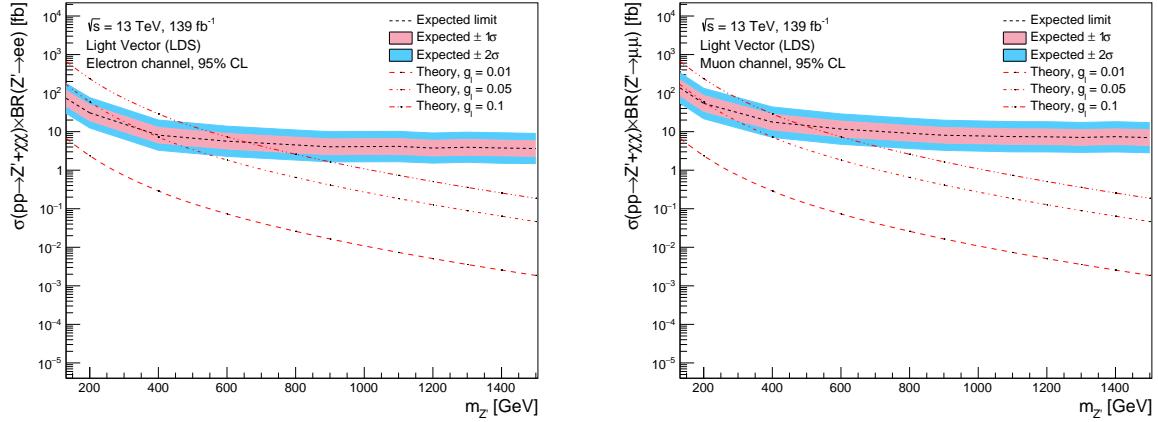


Figure 9.16: Mass exclusion limits of ee and $\mu\mu$ channel for all Z' LV LDS model

chosen to be $g_l = 0.001$ when simulating the data, by the assumption that the number of events that survived the cuts is the same, we can increase this coupling to see how the mass limits changes.

Table 9.4: Inputs for the $Z'V \rightarrow l^+l^-\chi\bar{\chi}$ LDS σB calculations. The first three columns are the Z' mass, the theoretical cross section times branching ratio σB , and what Z' decay channel we are looking at. The next two are ε_{sig} , which is the signal selection efficiency, and N_{sig} , which is the theoretical number of signal events after the cuts. The last two columns are the number of background events, N_{bkg} , and the events observed in the data, N_{obs} . The uncertainties of ε_{sig} , N_{sig} and N_{bkg} are statistical with an assumed 20% systematic uncertainty. The MET threshold is $E_{\text{T},\text{min}}^{\text{miss}} = 50\text{GeV}$ and the invariant mass threshold is $m_{ll}^{\text{min}} = 110\text{GeV}$ and is the same for all inputs.

$m_{Z'}$ [GeV]	σB [fb]	Channel	ε_{sig} [$\times 10^{-1}$]	N_{sig}	N_{bkg}
130	6.80	ee	0.07 ± 0.01	13.8 ± 2.83	182.6 ± 38.0
		$\mu\mu$	0.05 ± 0.01	10.0 ± 2.07	260.9 ± 52.9
200	2.37	ee	0.18 ± 0.04	12.0 ± 2.43	191.4 ± 39.8
		$\mu\mu$	0.13 ± 0.03	8.88 ± 1.81	258.9 ± 52.4
400	2.89×10^{-1}	ee	0.68 ± 0.14	5.45 ± 1.10	186.6 ± 38.7
		$\mu\mu$	0.41 ± 0.08	$3.28 \pm 6.61 \times 10^{-1}$	256.9 ± 52.0
600	7.34×10^{-2}	ee	1.01 ± 0.20	$2.06 \pm 4.13 \times 10^{-1}$	194.8 ± 40.2
		$\mu\mu$	0.65 ± 0.13	$1.32 \pm 2.65 \times 10^{-1}$	268.6 ± 54.4
800	2.59×10^{-2}	ee	1.21 ± 0.24	$8.71 \times 10^{-1} \pm 1.75 \times 10^{-1}$	182.7 ± 38.4
		$\mu\mu$	0.80 ± 0.16	$5.74 \times 10^{-1} \pm 1.15 \times 10^{-1}$	256.9 ± 52.0
900	1.65×10^{-2}	ee	1.29 ± 0.26	$5.90 \times 10^{-1} \pm 1.18 \times 10^{-1}$	176.1 ± 36.5
		$\mu\mu$	0.86 ± 0.17	$3.92 \times 10^{-1} \pm 7.88 \times 10^{-2}$	243.3 ± 49.3
1100	7.34×10^{-3}	ee	1.35 ± 0.27	$2.76 \times 10^{-1} \pm 5.52 \times 10^{-2}$	190.8 ± 39.4
		$\mu\mu$	0.92 ± 0.18	$1.87 \times 10^{-1} \pm 3.75 \times 10^{-2}$	240.9 ± 48.8
1200	5.08×10^{-3}	ee	1.41 ± 0.28	$2.00 \times 10^{-1} \pm 4.00 \times 10^{-2}$	179.8 ± 37.2
		$\mu\mu$	0.96 ± 0.19	$1.36 \times 10^{-1} \pm 2.73 \times 10^{-2}$	250.8 ± 50.8
1300	3.58×10^{-3}	ee	1.43 ± 0.29	$1.42 \times 10^{-1} \pm 2.84 \times 10^{-2}$	190.7 ± 39.5
		$\mu\mu$	0.98 ± 0.20	$9.72 \times 10^{-2} \pm 1.95 \times 10^{-2}$	245.3 ± 49.6
1400	2.56×10^{-3}	ee	1.46 ± 0.29	$1.04 \times 10^{-1} \pm 2.08 \times 10^{-2}$	185.9 ± 38.9
		$\mu\mu$	0.99 ± 0.20	$7.09 \times 10^{-2} \pm 1.42 \times 10^{-2}$	260.4 ± 52.9
1500	1.86×10^{-3}	ee	1.48 ± 0.30	$7.68 \times 10^{-2} \pm 1.54 \times 10^{-2}$	185.1 ± 38.2
		$\mu\mu$	0.98 ± 0.20	$5.10 \times 10^{-2} \pm 1.02 \times 10^{-2}$	245.4 ± 49.7

9.6 Effective Field Theory Heavy Dark Sector

Trained a network using all of the SM background samples and every different Z' mass of this model. Here are the results

With the ROC for each mass point seen in Figure 9.17.

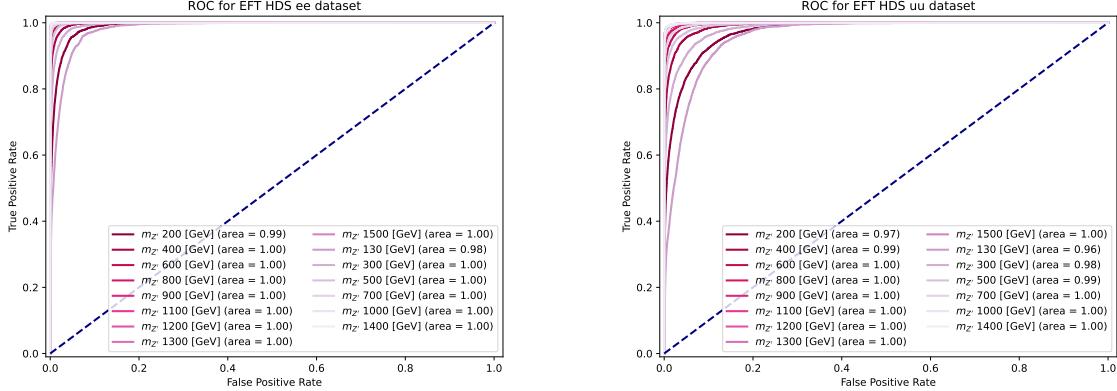


Figure 9.17: ROC plots for every Z' mass point on network trained on Z' EFT HDS

Plotting the significance of the models given the binning we get the results from Figure 9.18

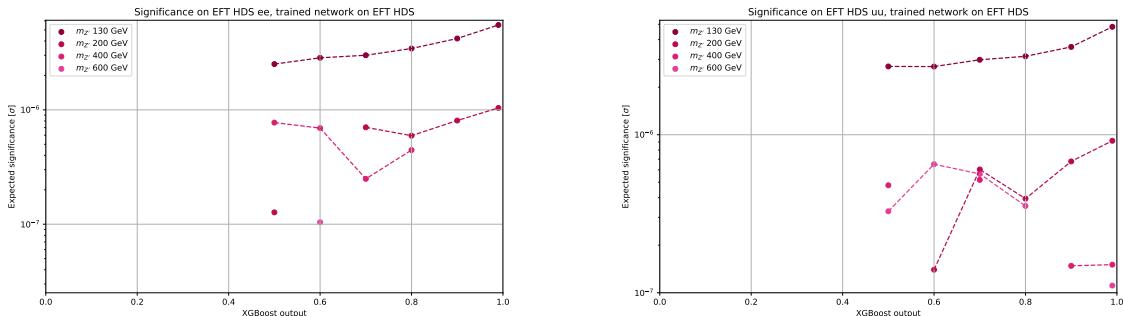


Figure 9.18: Expected significance plots for Z' mass points on network trained on Z' EFT HDS

Using the last bin as the significance is greatest there, such that we effectively make a cut based on the BDT score. Using the last bin, we can calculate a mass exclusion for both electron and muon channel.

To do so we need to count the number of signal and background events that are on the last bin, as well as their uncertainties. Additionally we will include the number of real data events that are there such that we can follow the method explained in Chapter

4.3.2. In Table 9.5 we see the values for each Z' mass point. As the lepton coupling was

Table 9.5: Inputs for the EFT $\rightarrow Z'\chi\bar{\chi}$ HDS σB calculations. The first three columns are the Z' mass, the theoretical cross section times branching ratio σB , and what Z' decay channel we are looking at. The next two are ε_{sig} , which is the signal selection efficiency, and N_{sig} , which is the theoretical number of signal events after the cuts. The last two columns are the number of background events, N_{bkg} , and the events observed in the data, N_{obs} . The uncertainties of ε_{sig} , N_{sig} and N_{bkg} are statistical with an assumed 20% systematic uncertainty. The MET threshold is $E_{\text{T},\text{min}}^{\text{miss}} = 50\text{GeV}$ and the invariant mass threshold is $m_{ll}^{\text{min}} = 110\text{GeV}$ and is the same for all inputs.

$m_{Z'}$ [GeV]	σB [fb]	Channel	ε_{sig} [$\times 10^{-1}$]	N_{sig}	N_{bkg}
130	7.81×10^{-6}	ee	0.28 ± 0.06	$6.08 \times 10^{-5} \pm 1.23 \times 10^{-5}$	121.2 ± 26.6
		$\mu\mu$	0.23 ± 0.05	$4.96 \times 10^{-5} \pm 1.01 \times 10^{-5}$	105.8 ± 22.4
200	6.81×10^{-7}	ee	0.65 ± 0.13	$1.22 \times 10^{-5} \pm 2.46 \times 10^{-6}$	139.3 ± 29.1
		$\mu\mu$	0.49 ± 0.10	$9.19 \times 10^{-6} \pm 1.85 \times 10^{-6}$	103.0 ± 21.6
300	6.01×10^{-8}	ee	1.01 ± 0.20	$1.69 \times 10^{-6} \pm 3.39 \times 10^{-7}$	113.8 ± 25.0
		$\mu\mu$	0.76 ± 0.15	$1.28 \times 10^{-6} \pm 2.57 \times 10^{-7}$	113.3 ± 23.8
400	8.42×10^{-9}	ee	1.30 ± 0.26	$3.06 \times 10^{-7} \pm 6.13 \times 10^{-8}$	118.0 ± 25.7
		$\mu\mu$	0.92 ± 0.18	$2.16 \times 10^{-7} \pm 4.33 \times 10^{-8}$	107.9 ± 22.6
500	1.80×10^{-9}	ee	1.46 ± 0.29	$7.32 \times 10^{-8} \pm 1.47 \times 10^{-8}$	108.6 ± 24.9
		$\mu\mu$	1.03 ± 0.21	$5.16 \times 10^{-8} \pm 1.04 \times 10^{-8}$	97.9 ± 20.7
600	4.83×10^{-10}	ee	1.55 ± 0.31	$2.09 \times 10^{-8} \pm 4.18 \times 10^{-9}$	117.8 ± 25.0
		$\mu\mu$	1.09 ± 0.22	$1.46 \times 10^{-8} \pm 2.92 \times 10^{-9}$	115.5 ± 24.2
700	1.49×10^{-10}	ee	1.64 ± 0.33	$6.82 \times 10^{-9} \pm 1.37 \times 10^{-9}$	122.2 ± 26.1
		$\mu\mu$	1.15 ± 0.23	$4.76 \times 10^{-9} \pm 9.56 \times 10^{-10}$	113.3 ± 23.9
800	5.12×10^{-11}	ee	1.68 ± 0.34	$2.39 \times 10^{-9} \pm 4.80 \times 10^{-10}$	118.5 ± 25.0
		$\mu\mu$	1.17 ± 0.23	$1.66 \times 10^{-9} \pm 3.34 \times 10^{-10}$	105.3 ± 22.1
900	1.90×10^{-11}	ee	1.71 ± 0.34	$9.05 \times 10^{-10} \pm 1.81 \times 10^{-10}$	122.2 ± 25.8
		$\mu\mu$	1.15 ± 0.23	$6.09 \times 10^{-10} \pm 1.22 \times 10^{-10}$	116.5 ± 24.4
1000	7.47×10^{-12}	ee	1.70 ± 0.34	$3.53 \times 10^{-10} \pm 7.07 \times 10^{-11}$	123.0 ± 26.0
		$\mu\mu$	1.17 ± 0.23	$2.44 \times 10^{-10} \pm 4.89 \times 10^{-11}$	117.0 ± 24.6
1100	3.07×10^{-12}	ee	1.74 ± 0.35	$1.48 \times 10^{-10} \pm 2.97 \times 10^{-11}$	108.7 ± 23.3
		$\mu\mu$	1.17 ± 0.23	$1.00 \times 10^{-10} \pm 2.01 \times 10^{-11}$	100.9 ± 21.3
1200	1.31×10^{-12}	ee	1.75 ± 0.35	$6.41 \times 10^{-11} \pm 1.28 \times 10^{-11}$	122.8 ± 26.0
		$\mu\mu$	1.23 ± 0.25	$4.49 \times 10^{-11} \pm 9.01 \times 10^{-12}$	109.7 ± 23.1
1300	5.80×10^{-13}	ee	1.75 ± 0.35	$2.82 \times 10^{-11} \pm 5.65 \times 10^{-12}$	120.9 ± 26.6
		$\mu\mu$	1.23 ± 0.25	$1.99 \times 10^{-11} \pm 3.99 \times 10^{-12}$	115.4 ± 24.4
1400	2.63×10^{-13}	ee	1.74 ± 0.35	$1.27 \times 10^{-11} \pm 2.55 \times 10^{-12}$	123.9 ± 26.1
		$\mu\mu$	1.18 ± 0.24	$8.64 \times 10^{-12} \pm 1.73 \times 10^{-12}$	107.3 ± 22.6
1500	1.22×10^{-13}	ee	1.78 ± 0.36	$6.02 \times 10^{-12} \pm 1.21 \times 10^{-12}$	115.3 ± 24.6
		$\mu\mu$	1.22 ± 0.24	$4.15 \times 10^{-12} \pm 8.32 \times 10^{-13}$	108.3 ± 22.8

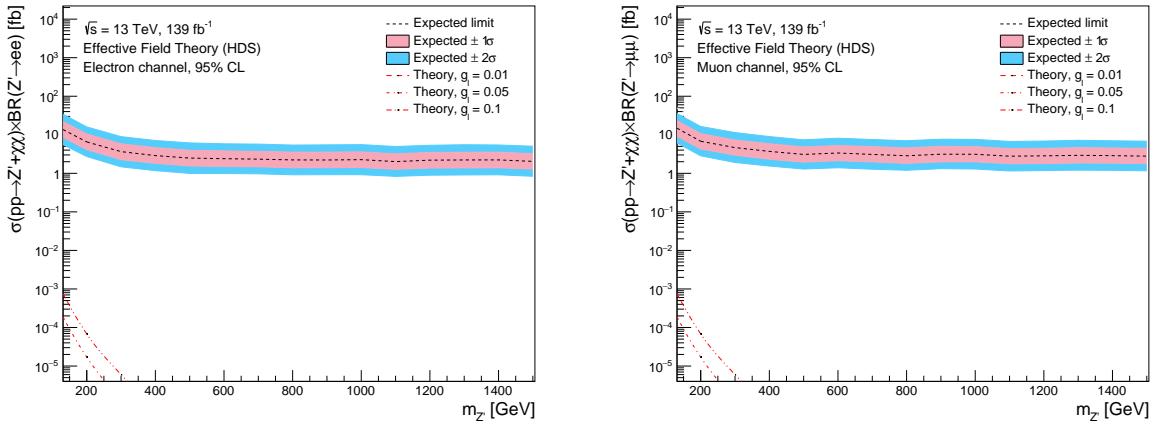


Figure 9.19: Mass exclusion limits of ee and $\mu\mu$ channel for all Z' EFT HDS model

chosen to be $g_l = 0.001$ when simulating the data, by the assumption that the number of events that survived the cuts is the same, we can increase this coupling to see how the mass limits changes.

9.7 Effective Field Theory Light Dark Sector

Trained a network using all of the SM background samples and every different Z' mass of this model. Here are the results

With the ROC for each mass point seen in Figure 9.20.

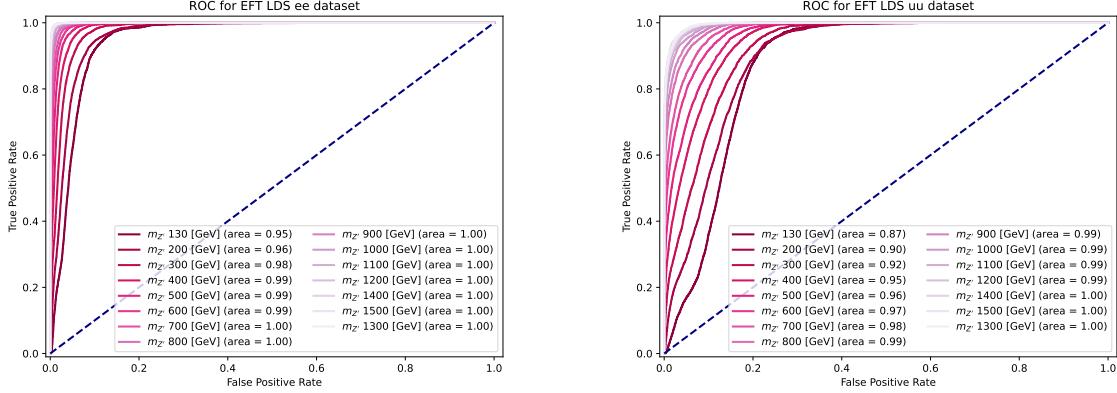


Figure 9.20: ROC plots for every Z' mass point on network trained on Z' EFT LDS

Plotting the significance of the models given the binning we get the results from Figure 9.21

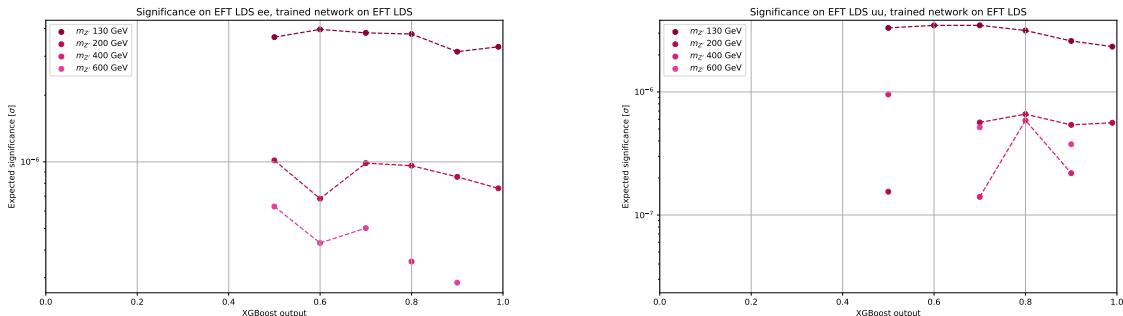


Figure 9.21: Expected significance plots for Z' mass points on network trained on Z' EFT LDS

Using the last bin as the significance is greatest there, such that we effectively make a cut based on the BDT score. Using the last bin, we can calculate a mass exclusion for both electron and muon channel.

To do so we need to count the number of signal and background events that are on the last bin, as well as their uncertainties. Additionally we will include the number of real data events that are there such that we can follow the method explained in Chapter

4.3.2. In Table 9.6 we see the values for each Z' mass point. As the lepton coupling was

Table 9.6: Inputs for the EFT $\rightarrow Z'\chi\bar{\chi}$ LDS σB calculations. The first three columns are the Z' mass, the theoretical cross section times branching ratio σB , and what Z' decay channel we are looking at. The next two are ε_{sig} , which is the signal selection efficiency, and N_{sig} , which is the theoretical number of signal events after the cuts. The last two columns are the number of background events, N_{bkg} , and the events observed in the data, N_{obs} . The uncertainties of ε_{sig} , N_{sig} and N_{bkg} are statistical with an assumed 20% systematic uncertainty. The MET threshold is $E_{\text{T},\text{min}}^{\text{miss}} = 50\text{GeV}$ and the invariant mass threshold is $m_{ll}^{\text{min}} = 110\text{GeV}$ and is the same for all inputs.

$m_{Z'}$ [GeV]	σB [fb]	Channel	ε_{sig} [$\times 10^{-1}$]	N_{sig}	N_{bkg}
130	3.35×10^{-5}	ee	0.06 ± 0.01	$5.20 \times 10^{-5} \pm 1.07 \times 10^{-5}$	248.4 ± 51.7
		$\mu\mu$	0.04 ± 0.01	$3.98 \times 10^{-5} \pm 8.28 \times 10^{-6}$	292.5 ± 59.2
200	2.67×10^{-6}	ee	0.17 ± 0.03	$1.23 \times 10^{-5} \pm 2.50 \times 10^{-6}$	245.0 ± 50.3
		$\mu\mu$	0.12 ± 0.02	$9.11 \times 10^{-6} \pm 1.86 \times 10^{-6}$	291.6 ± 59.0
300	2.06×10^{-7}	ee	0.36 ± 0.07	$2.06 \times 10^{-6} \pm 4.15 \times 10^{-7}$	248.1 ± 51.0
		$\mu\mu$	0.24 ± 0.05	$1.36 \times 10^{-6} \pm 2.75 \times 10^{-7}$	284.7 ± 57.7
400	2.65×10^{-8}	ee	0.61 ± 0.12	$4.52 \times 10^{-7} \pm 9.09 \times 10^{-8}$	250.6 ± 51.7
		$\mu\mu$	0.40 ± 0.08	$2.93 \times 10^{-7} \pm 5.91 \times 10^{-8}$	281.9 ± 57.0
500	5.46×10^{-9}	ee	0.82 ± 0.16	$1.24 \times 10^{-7} \pm 2.48 \times 10^{-8}$	243.1 ± 50.3
		$\mu\mu$	0.51 ± 0.10	$7.69 \times 10^{-8} \pm 1.55 \times 10^{-8}$	293.3 ± 59.4
600	1.47×10^{-9}	ee	1.03 ± 0.21	$4.21 \times 10^{-8} \pm 8.44 \times 10^{-9}$	244.9 ± 50.8
		$\mu\mu$	0.62 ± 0.12	$2.53 \times 10^{-8} \pm 5.08 \times 10^{-9}$	294.2 ± 59.5
700	4.72×10^{-10}	ee	1.13 ± 0.23	$1.48 \times 10^{-8} \pm 2.97 \times 10^{-9}$	256.1 ± 52.4
		$\mu\mu$	0.73 ± 0.15	$9.56 \times 10^{-9} \pm 1.92 \times 10^{-9}$	298.1 ± 60.3
800	1.73×10^{-10}	ee	1.22 ± 0.24	$5.89 \times 10^{-9} \pm 1.18 \times 10^{-9}$	245.5 ± 51.2
		$\mu\mu$	0.82 ± 0.16	$3.93 \times 10^{-9} \pm 7.89 \times 10^{-10}$	301.5 ± 61.1
900	7.02×10^{-11}	ee	1.25 ± 0.25	$2.45 \times 10^{-9} \pm 4.91 \times 10^{-10}$	257.9 ± 52.7
		$\mu\mu$	0.88 ± 0.18	$1.71 \times 10^{-9} \pm 3.43 \times 10^{-10}$	281.0 ± 57.0
1000	3.09×10^{-11}	ee	1.35 ± 0.27	$1.16 \times 10^{-9} \pm 2.32 \times 10^{-10}$	251.1 ± 51.5
		$\mu\mu$	0.89 ± 0.18	$7.66 \times 10^{-10} \pm 1.54 \times 10^{-10}$	290.3 ± 58.8
1100	1.45×10^{-11}	ee	1.40 ± 0.28	$5.63 \times 10^{-10} \pm 1.13 \times 10^{-10}$	238.6 ± 49.0
		$\mu\mu$	0.95 ± 0.19	$3.85 \times 10^{-10} \pm 7.72 \times 10^{-11}$	291.4 ± 59.0
1200	7.19×10^{-12}	ee	1.39 ± 0.28	$2.78 \times 10^{-10} \pm 5.57 \times 10^{-11}$	249.4 ± 51.0
		$\mu\mu$	0.95 ± 0.19	$1.89 \times 10^{-10} \pm 3.80 \times 10^{-11}$	300.7 ± 60.8
1300	3.74×10^{-12}	ee	1.45 ± 0.29	$1.51 \times 10^{-10} \pm 3.02 \times 10^{-11}$	242.4 ± 50.0
		$\mu\mu$	0.97 ± 0.19	$1.01 \times 10^{-10} \pm 2.03 \times 10^{-11}$	307.5 ± 62.2
1400	2.02×10^{-12}	ee	1.48 ± 0.30	$8.31 \times 10^{-11} \pm 1.67 \times 10^{-11}$	244.0 ± 50.7
		$\mu\mu$	1.00 ± 0.20	$5.65 \times 10^{-11} \pm 1.13 \times 10^{-11}$	315.4 ± 63.9
1500	1.13×10^{-12}	ee	1.49 ± 0.30	$4.69 \times 10^{-11} \pm 9.40 \times 10^{-12}$	230.8 ± 47.4
		$\mu\mu$	0.98 ± 0.20	$3.07 \times 10^{-11} \pm 6.16 \times 10^{-12}$	301.5 ± 61.1

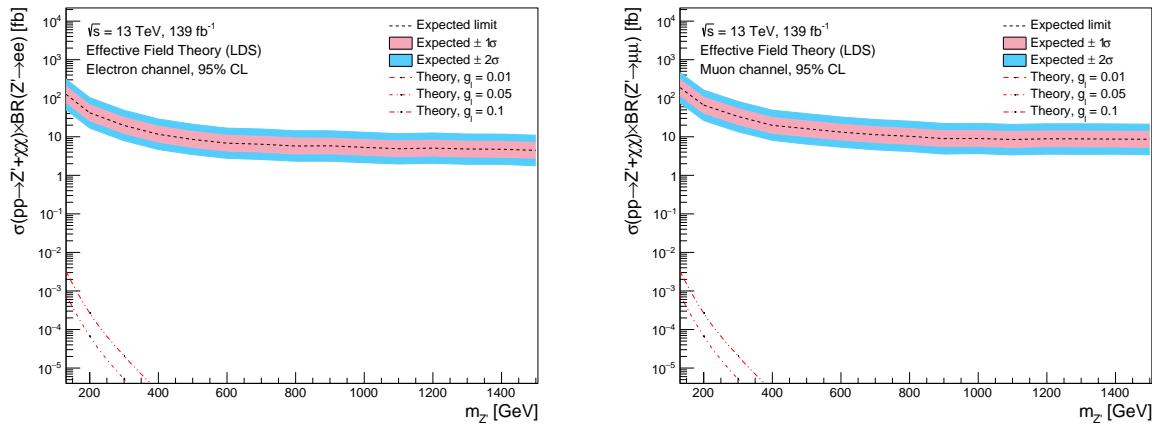
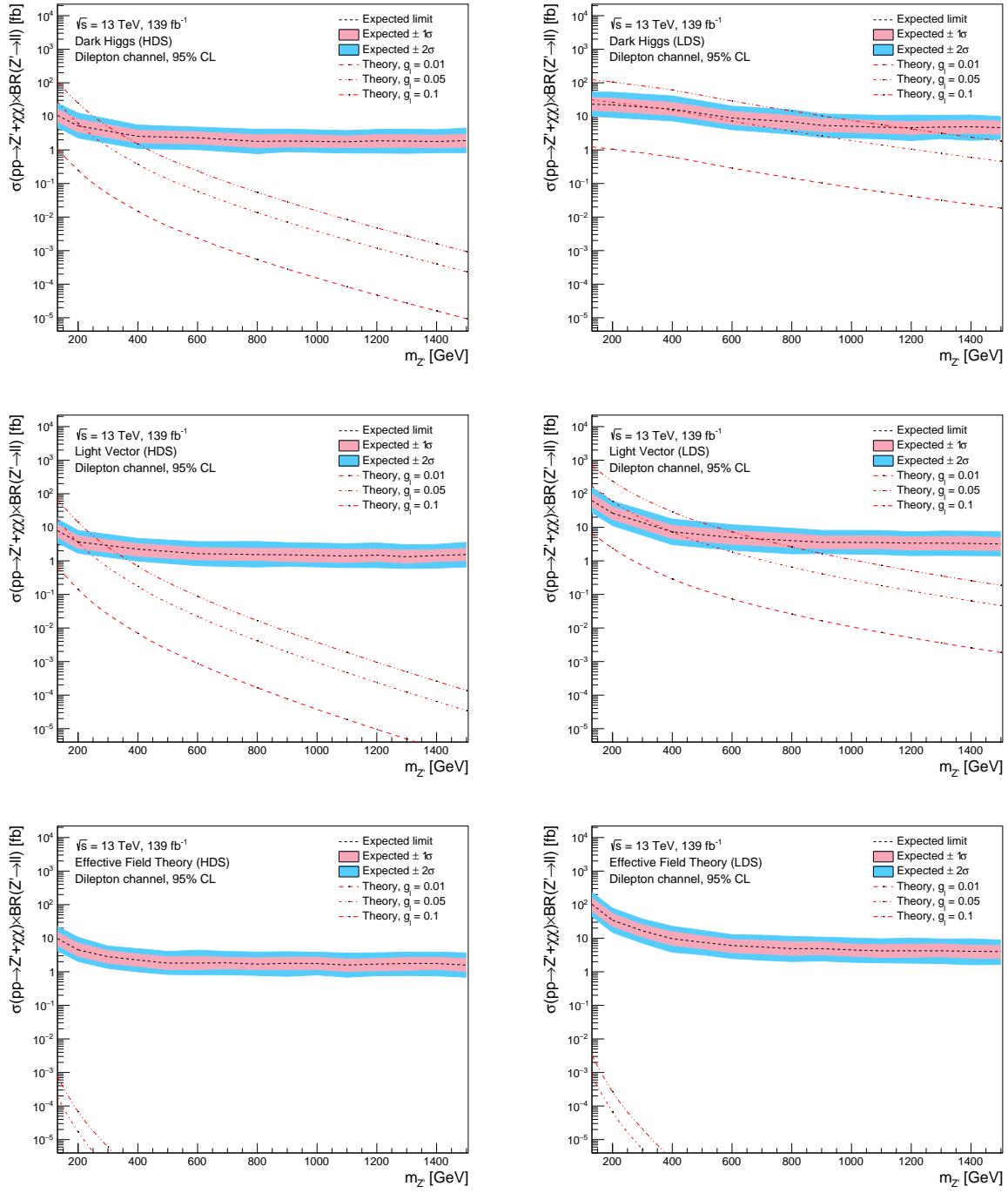


Figure 9.22: Mass exclusion limits of ee and $\mu\mu$ channel for all Z' EFT LDS model

chosen to be $g_l = 0.001$ when simulating the data, by the assumption that the number of events that survived the cuts is the same, we can increase this coupling to see how the mass limits changes.

9.8 Mass exclusion limits for the combined channels

Figure 9.23: Mass exclusion limits of combined ee and $\mu\mu$ channel for all models

Chapter 10

Comparison to cut and count

Testing three models using the classical data analysis way we apply cuts to kinematic variables and try to isolate the signal from the background to then calculate the expected significance. The three models I chose to test are all High Dark Sector models with $m_{Z'} = 130\text{GeV}$. They are a Light Vector (LV), Dark Higgs (DH) and Effective Field Theory (EFT) models. The cuts I made on these are shown in Table 10.1.

Table 10.1: Table showcasing the cuts used when doing the cut and count method.

	Cut
E_T^{miss}/σ	> 10
m_T	$> 160 \text{ GeV}$
m_{ll}	$> 120 \text{ GeV}$
Number of B-jets	0
m_{T2}	$> 110 \text{ GeV}$

Since the cross section to find Dark Matter is really small we have to use the low-statistics expected significance formula to find the closest to correct significance. The formula is

$$Z = \sqrt{2 \left[(s + b) \ln \left(1 + \frac{s}{b} \right) - s \right]} \quad (10.1)$$

Where s is the number of signal events and b is the number of background events. Using this we get the results shown in Table 10.2 for the electron channel and Table 10.3 for the muon channel. Also included on the tables are the number of events. One thing worth mentioning is that when adding another cut on the maximum invariant mass increases the significance. The significance for LV on the electron channel was at 1.2σ when adding

a cut stating that $m_{ll} < 150$ GeV. This makes sense since the models in question all have a $m_{Z'} = 130$ GeV. This cut was not added since we do not want to put a cap on the mass of the propagator, as we don't know what the real mass is.

Table 10.2: Table showcasing the result of the cut and count method for the electron channel.

	LV	DH	EFT	Background
Events before cuts	15	20	0	1,256,624
Events after cuts	4	6	0	117
Expected significance [σ]	0.4	0.6	0	

Table 10.3: Table showcasing the result of the cut and count method for the muon channel.

	LV	DH	EFT	Background
Events before cuts	14	19	0	1,626,098
Events after cuts	3	5	0	108
Expected significance [σ]	0.36	0.51	0	

If we were to compare these results with what our NN and BDT that trained on the full dataset we see that we can calculate the expected significance in different locations for the validation plots. Testing on the networks that trained using the data scientist method on the full DM dataset we get the results shown in Figure 10.1 for XGBoost and Figure 10.2 for the Neural Network.

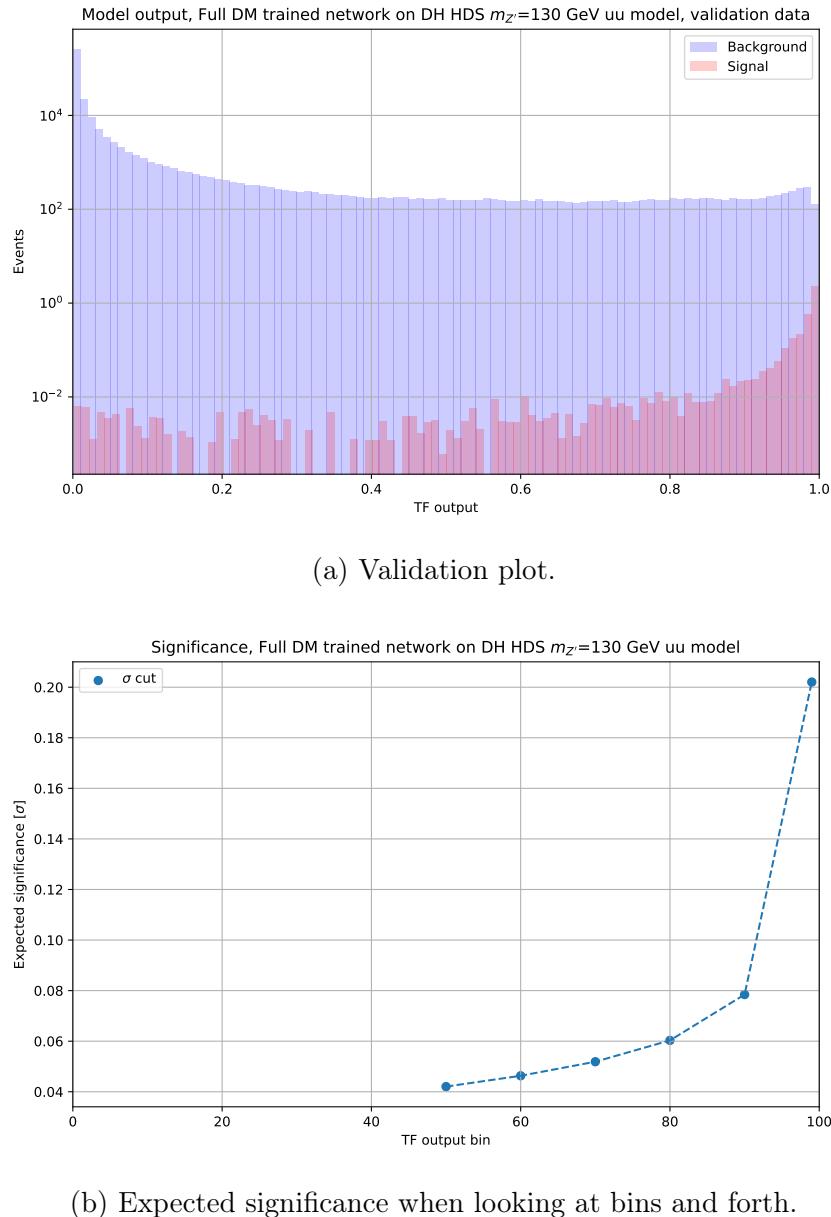
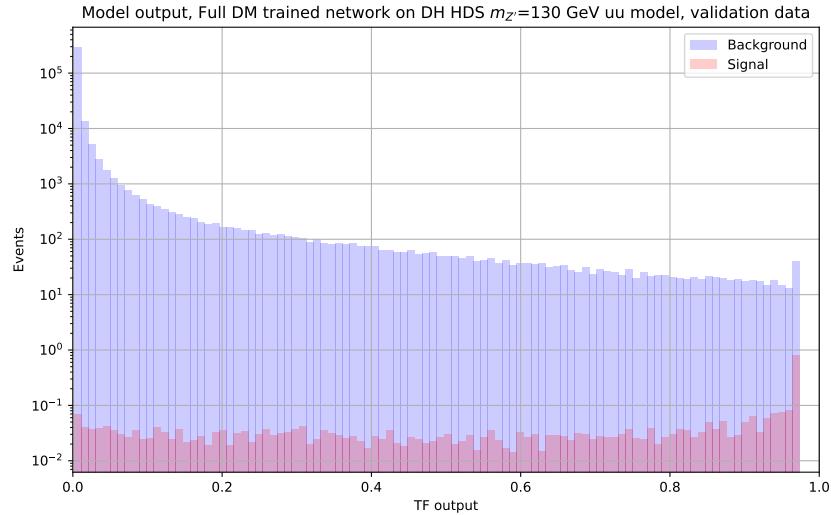
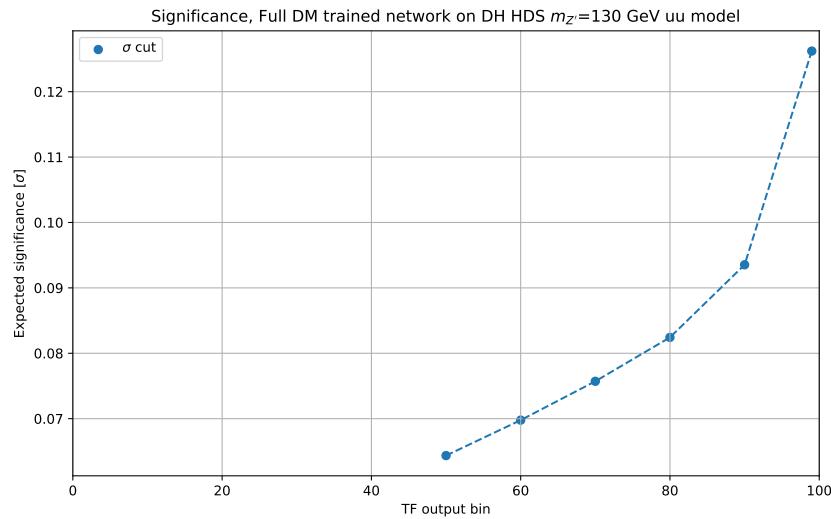


Figure 10.1: Expected significance of XGBoost when trained on the Full DM dataset for the DH HDS $m_{Z'} = 130$ GeV muon model.



(a) Validation plot.



(b) Expected significance when looking at bins and forth.

Figure 10.2: Expected significance of the Neural Network when trained on the Full DM dataset for the DH HDS $m_{Z'} = 130$ GeV muon model.

As we can see the expected significance is lower using ML than a rough cut and count. My theory for why this is the case is because we are testing just *one* sample out of 154 different ones that are included for the three different theories I have acquired so far. And the ML networks shown above have both trained on a dataset including all 154 DM samples. The models that I tested might also not have been one of the "important" models the network learned from. Thus if I were to train the network individually based on the theory it might give better results.

Table 10.4: Table showcasing the result of the cut and count method for the electron channel.

	Signal	Background
MC events	2,990,986	69,664,902
Sum of "Weight"	388.75	2,714,091.3
Sum of generator weights	236.3	55,446,228,776,354.8
Sum of (generator weights*lumi / SOW)	9,167.1	61.1
Sum of (generator weights/SOW)	199.21	1.52

Table 10.5: Table Showcasing how uneven the training dataset is between signal and background. This is on the dataset which incorporates all the different DM MC samples

	Number of events	Sum of weights	Events × SOW [10 ¹³]
Signal	2,991,543	36,327,943.99	1.08
Background	69,664,345	36,327,944.03	25.3

Chapter 11

Conclusion

In conclusion...

which might get us closer to understanding more of spacetime.

Appendix A

Dataset IDs for MC samples

Table A.1: Drell Yan background MC samples

Dataset ID	Process
700320	Zee_maxHTpTV2_BFilter
700321	Zee_maxHTpTV2_CFilterBVeto
700322	Zee_maxHTpTV2_CVetoBVeto
700323	Zmumu_maxHTpTV2_BFilter
700324	Zmumu_maxHTpTV2_CFilterBVeto
700325	Zmumu_maxHTpTV2_CVetoBVeto
700326	Ztautau_LL_maxHTpTV2_BFilter
700327	Ztautau_LL_maxHTpTV2_CFilterBVeto
700328	Ztautau_LL_maxHTpTV2_CVetoBVeto
700452	Zee_mZ_120_ECMS_BFilter
700453	Zee_mZ_120_ECMS_CFilterBVeto
700454	Zee_mZ_120_ECMS_CVetoBVeto
700455	Zmumu_mZ_120_ECMS_BFilter
700456	Zmumu_mZ_120_ECMS_CFilterBVeto
700457	Zmumu_mZ_120_ECMS_CVetoBVeto
700458	Ztautau_mZ_120_ECMS_BFilter
700459	Ztautau_mZ_120_ECMS_CFilterBVeto
700460	Ztautau_mZ_120_ECMS_CVetoBVeto

Table A.2: Single top and TTbar background MC samples

Dataset ID	Process
410472	$t\bar{t}$ dilepton filtered
410644	Single top s -channel (top)
410645	Single top s -channel (anti-top)
410648	$W + t$ associated production dilepton filtered (top)
410649	$W + t$ associated production dilepton filtered (anti-top)
410658	Single top t -channel (top)
410659	Single top t -channel (anti-top)

Table A.3: Diboson background MC samples

Dataset ID	Process
363356	$Z \rightarrow qq + Z \rightarrow ll$
363358	$W \rightarrow qq + Z \rightarrow ll$
363359	$W^+ \rightarrow qq + W^- \rightarrow l\nu$
363360	$W^+ \rightarrow l\nu + W^- \rightarrow qq$
363489	$W \rightarrow l\nu + Z \rightarrow qq$
364250	$llll$
364253	$lll\nu$
364254	$ll\nu\nu$
364255	$l\nu\nu\nu$

Table A.4: W background MC samples

Dataset ID	Process
364156	$W \rightarrow \mu\nu$ maxHTpTV0_70_CVetoBVeto
364157	$W \rightarrow \mu\nu$ maxHTpTV0_70_CFilterBVeto
364158	$W \rightarrow \mu\nu$ maxHTpTV0_70_BFilter
364159	$W \rightarrow \mu\nu$ maxHTpTV70_140_CVetoBVeto
364160	$W \rightarrow \mu\nu$ maxHTpTV70_140_CFilterBVeto
364161	$W \rightarrow \mu\nu$ maxHTpTV70_140_BFilter
364162	$W \rightarrow \mu\nu$ maxHTpTV140_280_CVetoBVeto
364163	$W \rightarrow \mu\nu$ maxHTpTV140_280_CFilterBVeto
364164	$W \rightarrow \mu\nu$ maxHTpTV140_280_BFilter
364165	$W \rightarrow \mu\nu$ maxHTpTV280_500_CVetoBVeto
364166	$W \rightarrow \mu\nu$ maxHTpTV280_500_CFilterBVeto
364167	$W \rightarrow \mu\nu$ maxHTpTV280_500_BFilter
364168	$W \rightarrow \mu\nu$ maxHTpTV500_1000
364169	$W \rightarrow \mu\nu$ maxHTpTV1000_E_CMS
364170	$W \rightarrow e\nu$ maxHTpTV0_70_CVetoBVeto
364171	$W \rightarrow e\nu$ maxHTpTV0_70_CFilterBVeto
364172	$W \rightarrow e\nu$ maxHTpTV0_70_BFilter
364173	$W \rightarrow e\nu$ maxHTpTV70_140_CVetoBVeto
364174	$W \rightarrow e\nu$ maxHTpTV70_140_CFilterBVeto
364175	$W \rightarrow e\nu$ maxHTpTV70_140_BFilter
364176	$W \rightarrow e\nu$ maxHTpTV140_280_CVetoBVeto
364177	$W \rightarrow e\nu$ maxHTpTV140_280_CFilterBVeto
364178	$W \rightarrow e\nu$ maxHTpTV140_280_BFilter
364179	$W \rightarrow e\nu$ maxHTpTV280_500_CVetoBVeto
364180	$W \rightarrow e\nu$ maxHTpTV280_500_CFilterBVeto
364181	$W \rightarrow e\nu$ maxHTpTV280_500_BFilter
364182	$W \rightarrow e\nu$ maxHTpTV500_1000
364183	$W \rightarrow e\nu$ maxHTpTV1000_E_CMS
364184	$W \rightarrow \tau\nu$ maxHTpTV0_70_CVetoBVeto
364185	$W \rightarrow \tau\nu$ maxHTpTV0_70_CFilterBVeto
364186	$W \rightarrow \tau\nu$ maxHTpTV0_70_BFilter
364187	$W \rightarrow \tau\nu$ maxHTpTV70_140_CVetoBVeto
364188	$W \rightarrow \tau\nu$ maxHTpTV70_140_CFilterBVeto
364189	$W \rightarrow \tau\nu$ maxHTpTV70_140_BFilter
364190	$W \rightarrow \tau\nu$ maxHTpTV140_280_CVetoBVeto
364191	$W \rightarrow \tau\nu$ maxHTpTV140_280_CFilterBVeto
364192	$W \rightarrow \tau\nu$ maxHTpTV140_280_BFilter
364193	$W \rightarrow \tau\nu$ maxHTpTV280_500_CVetoBVeto
364194	$W \rightarrow \tau\nu$ maxHTpTV280_500_CFilterBVeto
364195	$W \rightarrow \tau\nu$ maxHTpTV280_500_BFilter
364196	$W \rightarrow \tau\nu$ maxHTpTV500_1000
364197	$W \rightarrow \tau\nu$ maxHTpTV1000_E_CMS

Appendix B

Kinematical variables distribution in control region

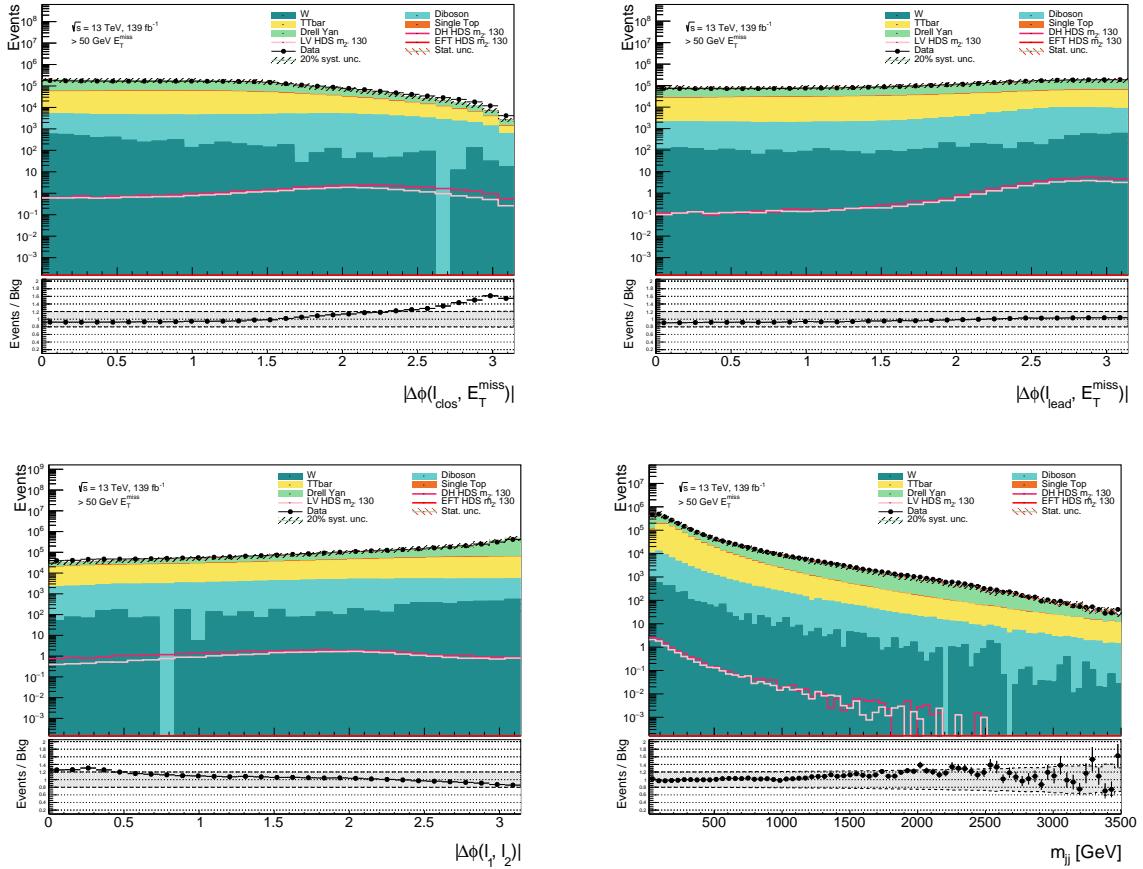
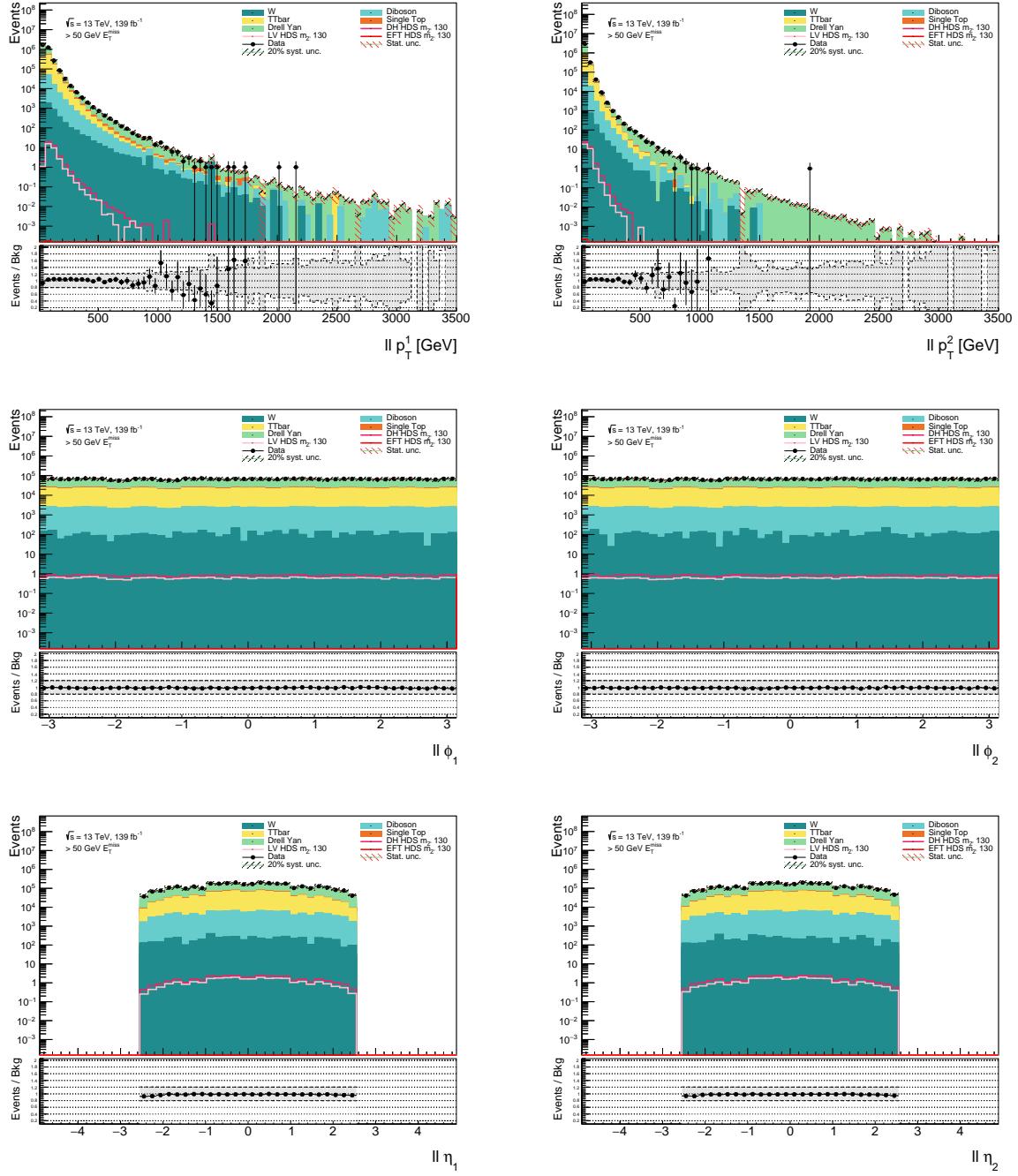


Figure B.1: $\Delta\phi(l_c, E_T^{\text{miss}})$, $\Delta\phi(l_{\text{lead}}, E_T^{\text{miss}})$, $\Delta\phi(l_1, l_2)$ and m_{jj} distribution in the control region.

Figure B.2: Leptons p_T , ϕ and η distribution in the control region.

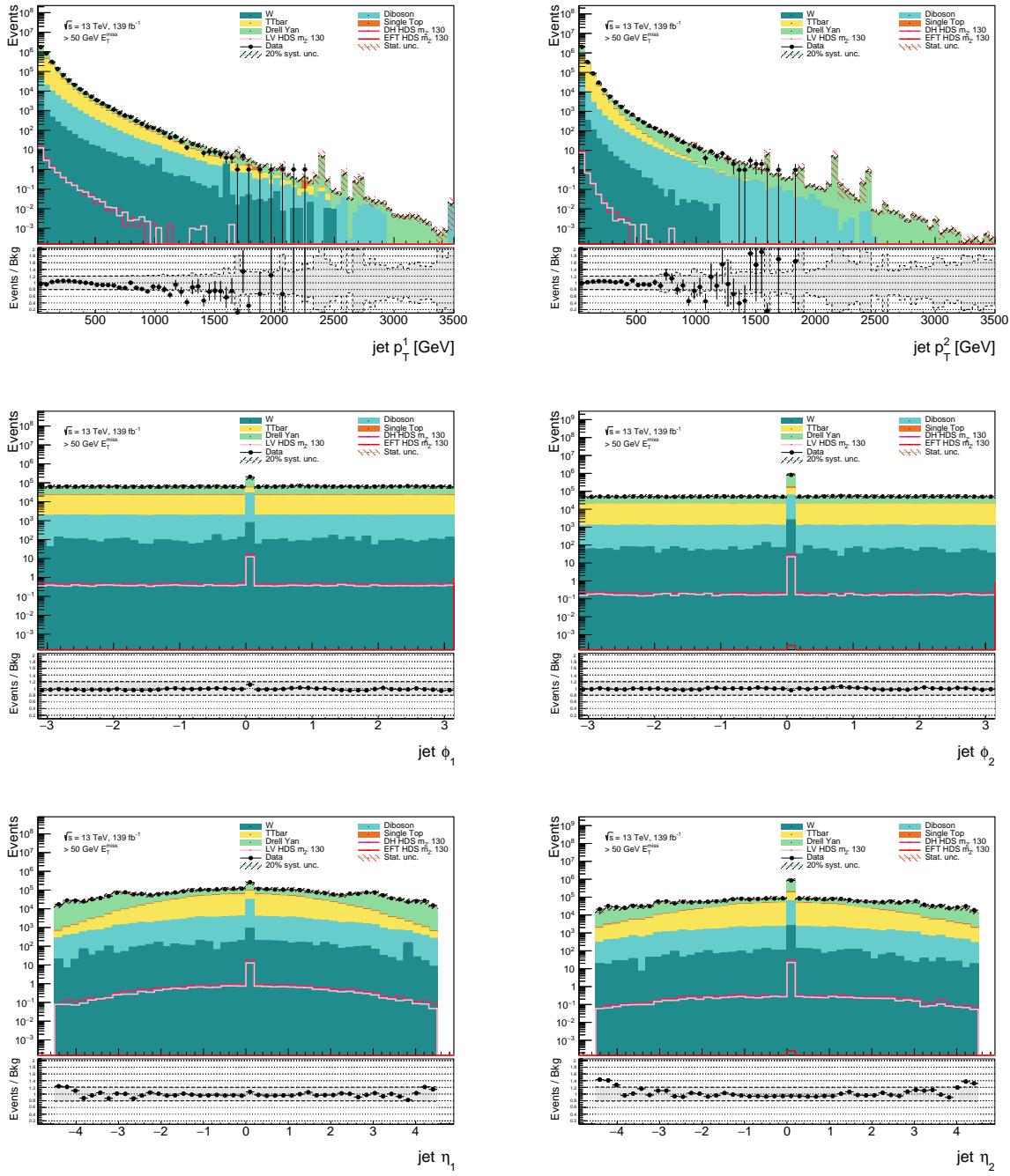


Figure B.3: Leading jets p_T , ϕ and η distribution in the control region.

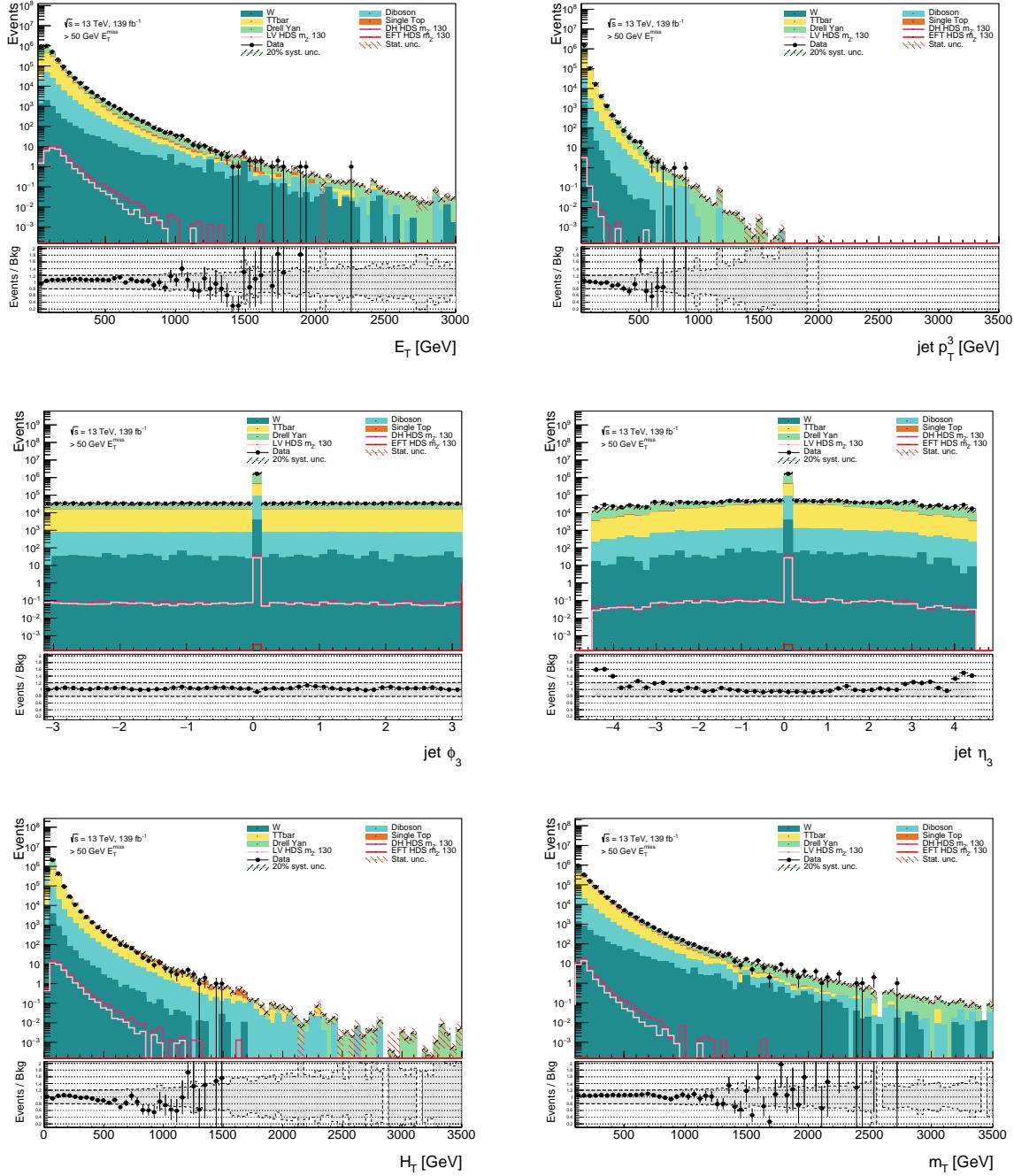


Figure B.4: p_T, ϕ and η of third leading jet and the dilepton pairs E_T , H_T and m_T distribution in the control region.

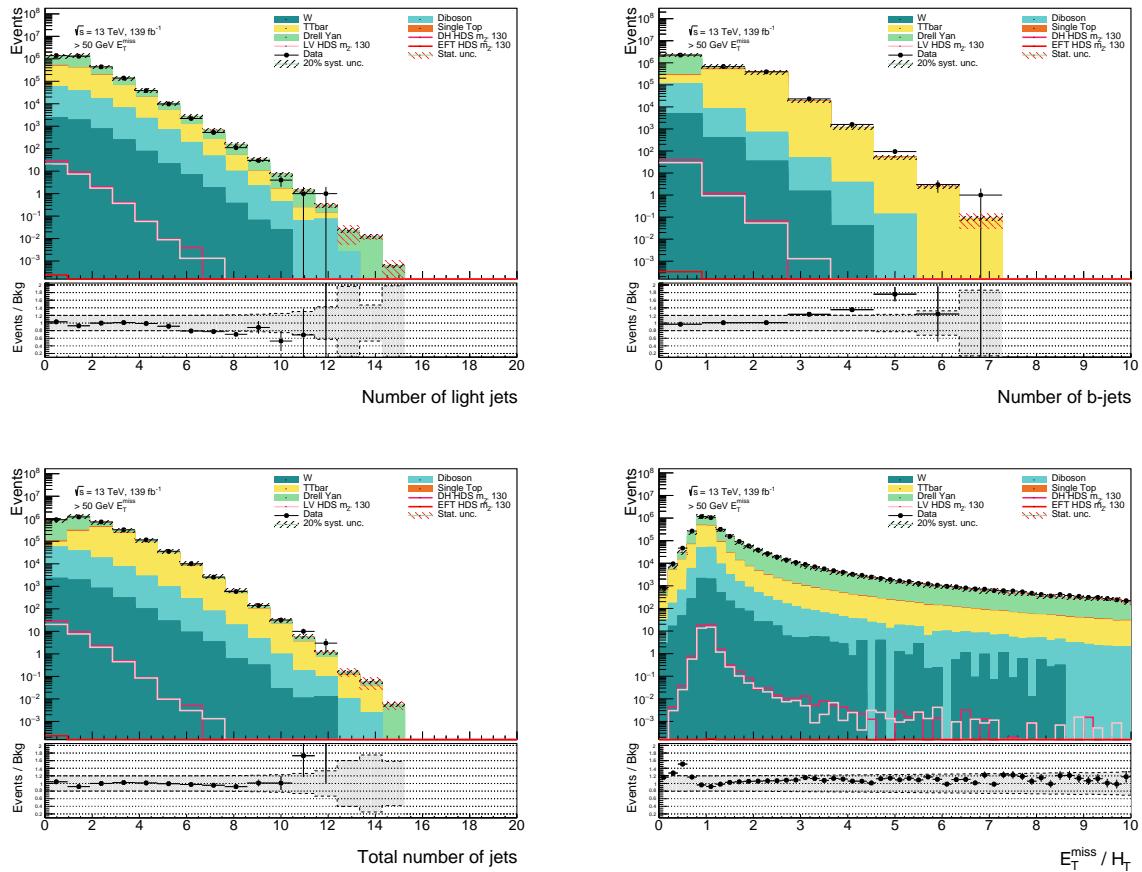


Figure B.5: Number of light, b- and total jets and E_T^{miss} / H_T distribution in the control region.

Appendix C

Data and MC agreement jets CR

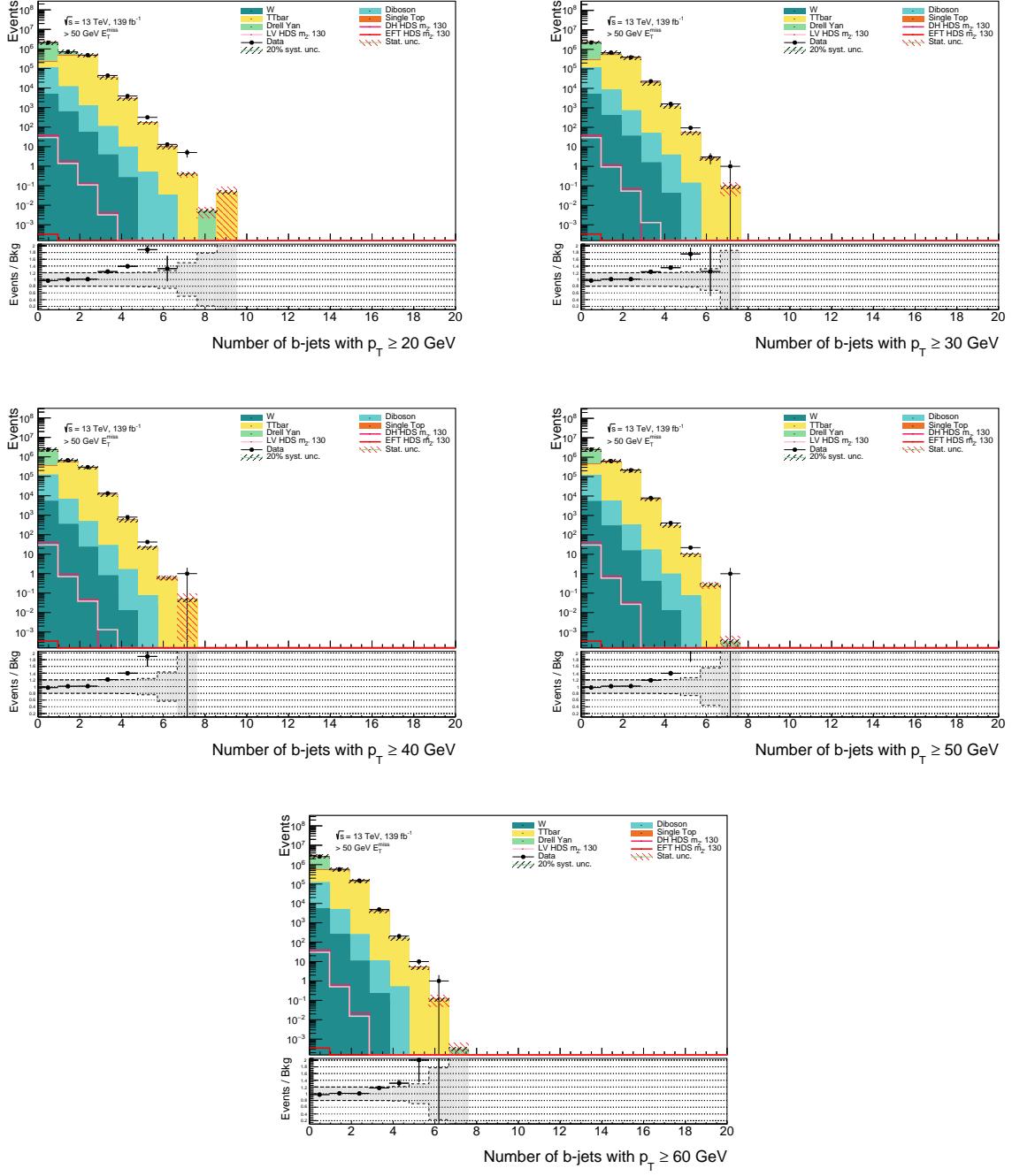


Figure C.1: Data and MC agreement on number of b- jets with different p_T cuts in CR.

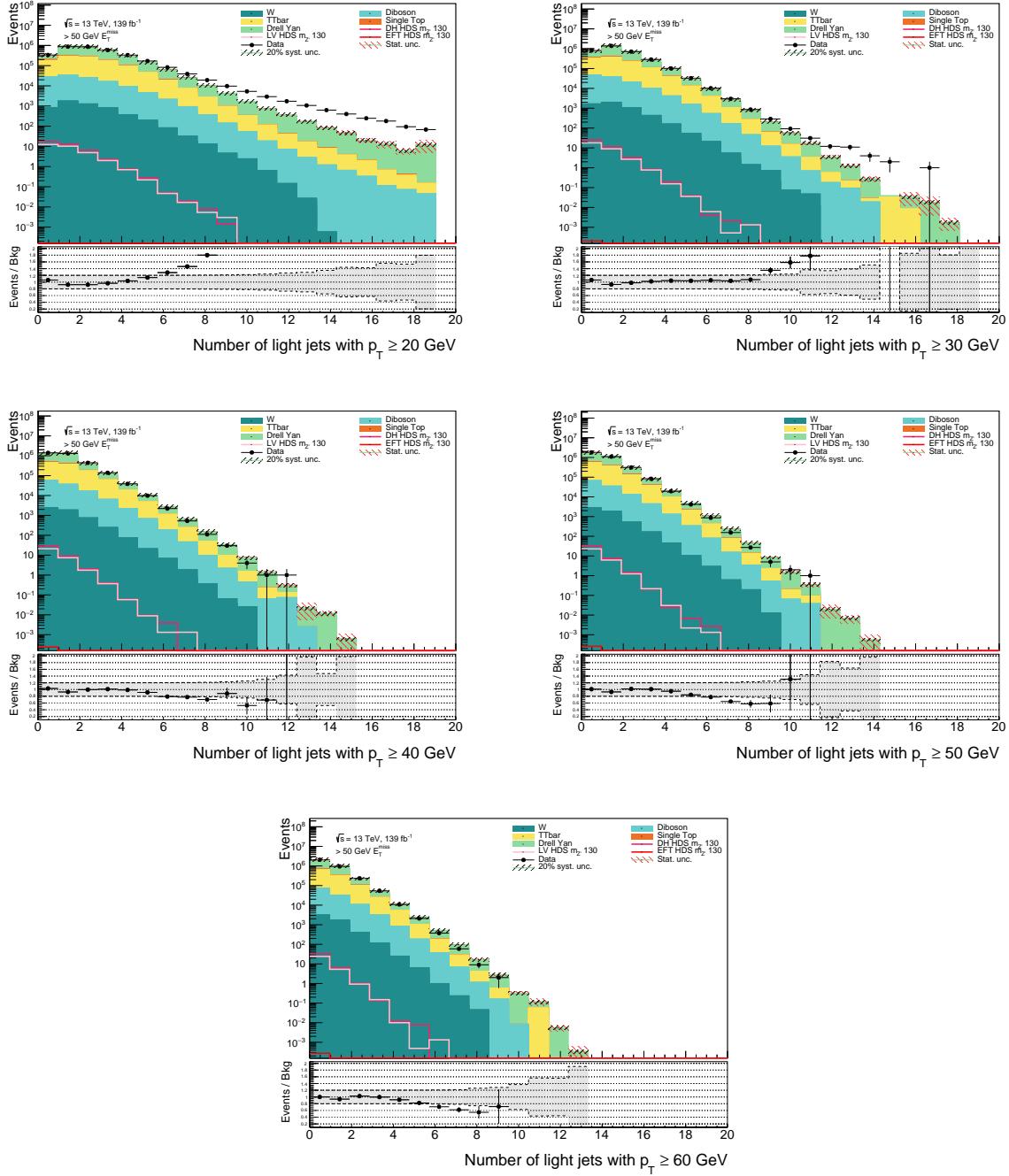


Figure C.2: Data and MC agreement on number of light jets with different p_T cuts in CR.

Appendix D

Data and MC agreement jets CR

to come...

Appendix E

BDT of depth 30

The results can be shown in Figure E.1. This is however highly radical as the convention is to normally not have a depth greater than 7, the reason being that the network is highly likely to overtrain and give wrong predictions. However this was not the case for me as seen for example in Figure E.2.

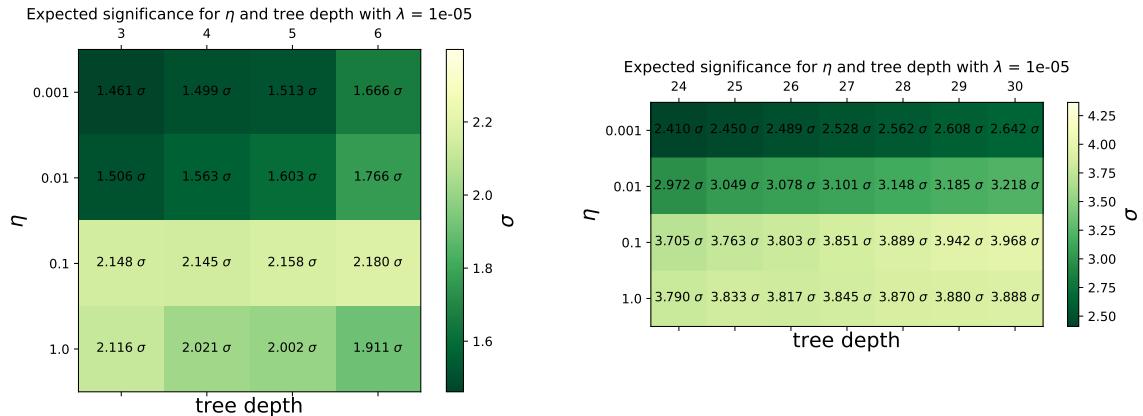
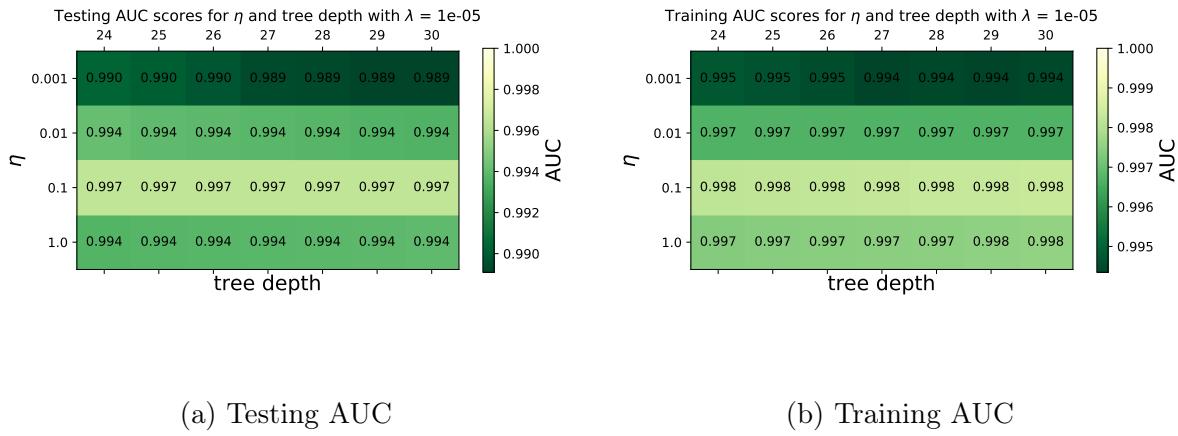


Figure E.1: Grid search expected significance going to a depth of up to 30

I understand however that this is controversial since we are splitting a data set, that is at best of size 2^{27} , 30 times. That means that after a depth of 27 there is exactly one event per branch. So how does a depth of 30 make sense? To help with this we could use a feature in XGBoost to see which features are most important when evaluating a signal. When testing the network trained on the FULL Z' DM data set on a DH HDS $m_{Z'} = 130$ GeV model we get the features shown in Figure E.3 as most important.



(a) Testing AUC

(b) Training AUC

Figure E.2: Grid search AUC going to a depth of up to 30

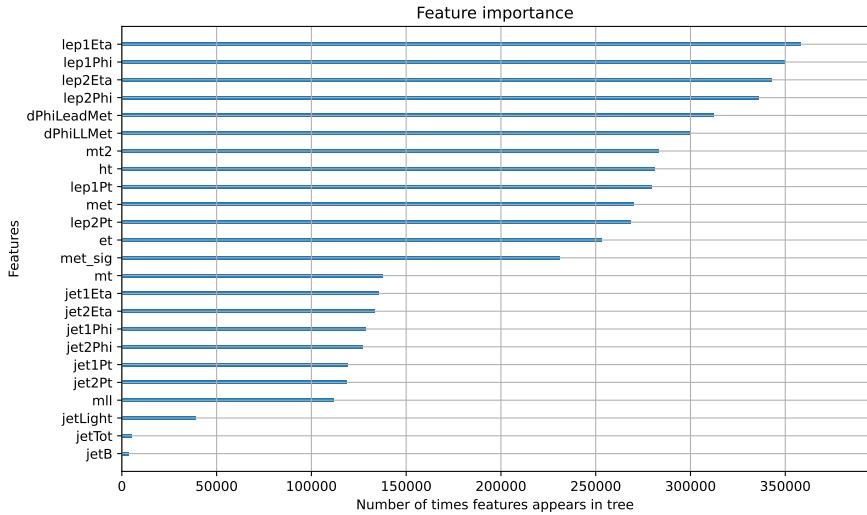
As we can see these features vary a lot depending on which metric we use to evaluate the importance. When using the "coverage" metric, which as stated is defined as the number of samples affected by the split, we get the features we physically expect to be important when trying to single out a DM model. And this metric is arguably the one we need to use to define what features are important. Since the more samples a feature split, the more powerful it is to separate signal from background.

We can see however that when we use "weight" as a metric, which is the XGBoost standard metric, we get completely unexpected features that we physically don't expect to be important when trying to single out a DM model. But as described by the metric, the "weight" is the number of times a feature appears in a tree. Which might explain that the reason the pseudorapidity and ϕ range so high on this list, is simply because the tree is struggling to find a pattern here and is trying extra hard to single out DM from SM.

To showcase the difference in signal recognition between the monstrous 30 depth BDT to the more sensible 6 depth BDT, I again tested the networks on the good old DH HDS $m_{Z'} = 130$ GeV model. The results as well as their expected significance can be seen below.

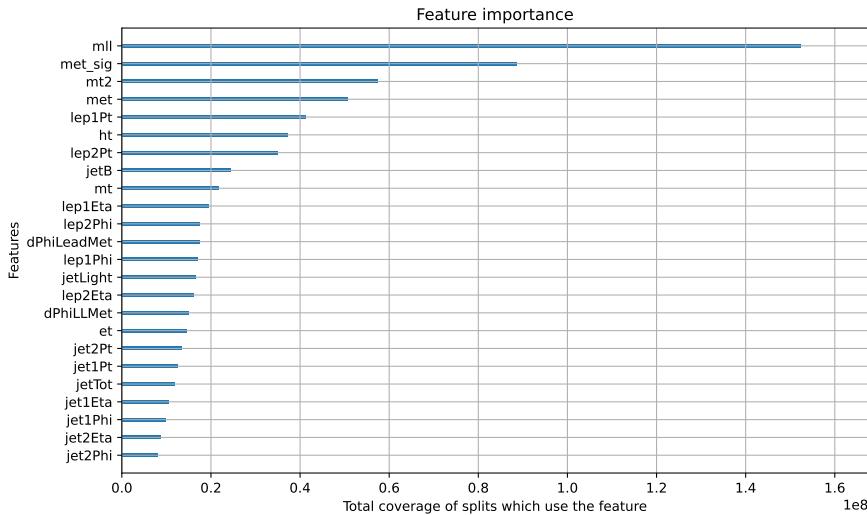
The difference is extreme, when looking at the monster of depth 30 we can get an expected significance of 1.2σ (without uncertainties) on our model of max 15 events, only having made a cut of 50 GeV on the missing transverse energy. We can however see that the data and background do not agree to the same degree of the network with depth 6.

when
should I
use them?



(a) Using "weight" metric

Coverage is defined as the number of samples affected by the split



(b) Using "coverage" metric

Figure E.3: Feature importance of depth 30 network trained on FULL Z' DM data set when testing it on DH HDS $m_{Z'} = 130$ GeV model.

Using purely statistical uncertainty and assuming a systematic uncertainty of 30%, we see that a few data points do not agree with the MC background. These data points are points the network classified as signal, so if we completely trusted the network this would be a hint of new physics! However this is the last thing we should assume, and rather take this as a hint that the network is doing something fishy.

If I had access to XGBoost with built GPU support, I would increase the number of estimators even more to check if this increases the significance while still having a depth

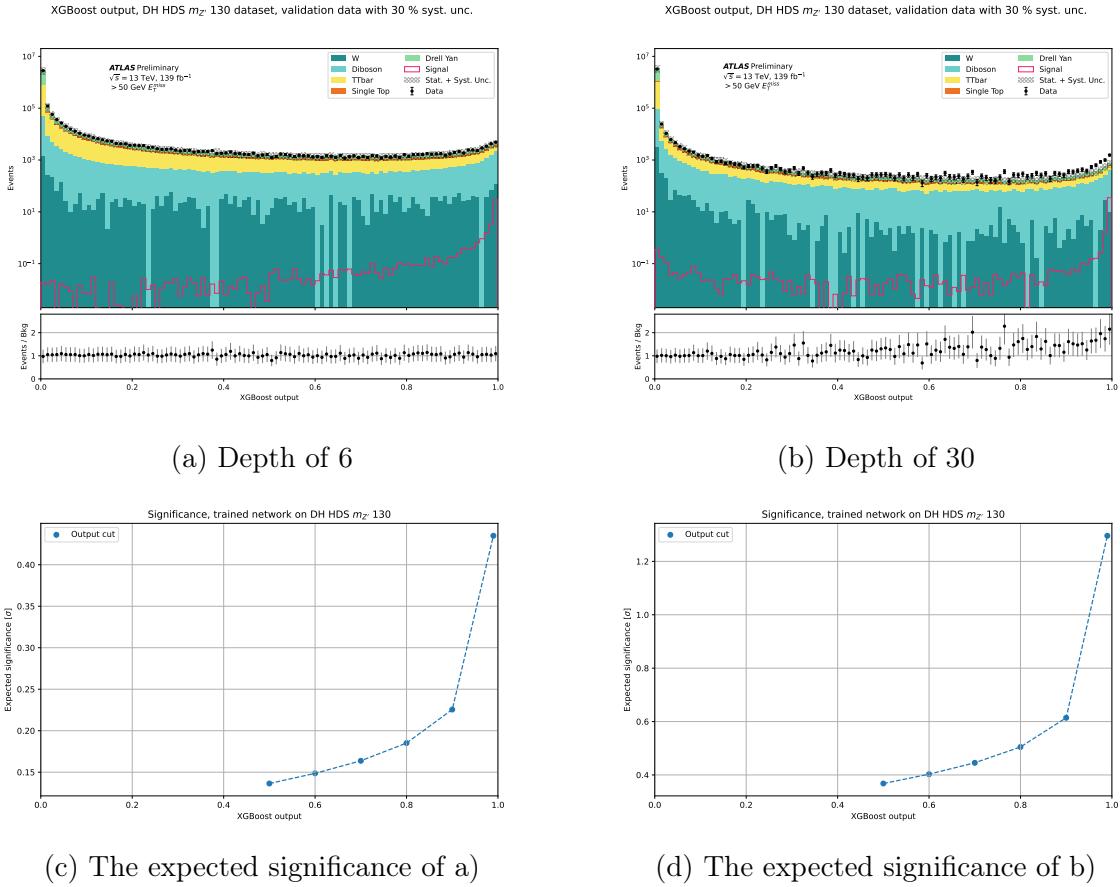


Figure E.4: Comparison of the network performance when having a depth of 6 and 30. Figure a) and b) show the validation data of both cases, c) and d) show the expected significance of the validation plots when making a cut on the output.

of maximum 6. However as of now this is not possible. As the weighting method explained in the previous section was not included here, we will drop going to a tree depth of 30, and have a maximum of 6.

Logbook

We are dropping $\Delta\phi(l_1, l_2)$ and $\Delta\phi(l_c, E_T^{miss})$ due the poor agreement between MC and data. The first one is most likely due us not including fake leptons (and also for all non SFOS final states). The latter is a problem that PhD. Even is being haunted by. There is also the problem of missing variables. For this theis, as shown in Table ??, we will record events with up to three jets in the final state. As mentioned earlier, there isn't always three jets in the final state, to mediate this problem I chose to set the p_T to zero for the missing jets and m_{jj} to zero if there are less than two jets, this is something that is physically reasonable as it doesn't violate any conservation laws. More problematic however is the η and ϕ when there aren't jets. To mediate this I've set the values to -999, which has no physical meaning and is impossible to achieve, this I did so it becomes easier for us to identify the jagged arrays further in the network preparation. The missing variables is not a problem when making BDTs with XGBoost. There is also another problem, albeit less problematic than the previous ones, with the final states that are not SFOS, as the MC generated background on these tend to be lower than the recorded data. The number of events that are not SFOS are minimal though, and we think the reason it doesn't fit the data is because we are not including fake leptons.

I will remove this sentence, but its just so we know

Zp Dark Matter dataset

something more to add?

To train the networks I will utilize two methods. The first one being this where the dataset being sent into the ML network will contain every single DM MC sample available. So far there are 154 different MC samples, these are based on three theories. A Light Vector (LV), Dark Higgs (DH) and Effective Field Theory (EFT) which produces the WIMP DM particles, and a new theoretical particle, Z' , that decays into a lepton pair. The three theories are divided further into MC samples with a Light Dark Sector (LDS) and High Dark Sector (HDS) which tells us the range of the Dark Matter candidate mass.

see next comment for the two methods

And lastly it is divided further into more MC samples with different masses for Z' . This dataset includes all of these samples such that the network learns Dark Matter in a model independent way.

"Ensemble" dataset / Model independence

This subsection and is outdated with our current plan, and this will change when we discuss further how to implement the model-independent aspect.

Another approach is to make multiple datasets and combine the results of every network into a "big network". This is the second approach which I call ensemble modelling. The thought behind this is that when training a network using the full dataset it might only focus on the more resonant models with fixed masses. Also, every different DM sample has different phenomenology, specially in the future when I will be receiving SUSY samples, meaning that it also might not train the network physics. Thus if we were to train a network one sample at a time it would be the perfect scenario. However as will become apparent in Section 7.1.1, the datasets (even the full DM dataset) are extremely unbalanced. To put some numbers, on each DM MC sample there are roughly 40,000 MC events, and for the SM background (with a massive $\text{MET} > 50\text{GeV}$ cut!) there are roughly 87,000,000 MC events. Factoring the weights to re-weight the MC events to expected events gives us an extremely low statistics dataset, which punishes the network for guessing correctly.

THIS IS OUTDATED AND WILL MOST LIKELY BE CHANGED TO THE SIGNAL REGION APPROACH, WHERE WE STATISTICALLY COMBINE WHAT THE NETWORKS LEARN IN A MODEL DEPENDENT WAY Thus making the approach to teach the networks one MC sample at a time is impossible. So far I have tried dividing the the MC samples into 18 different categories. First into their respective theory. Then into LDS or HDS. Then into three $m_{Z'}$ regions, where I've defined the *low mass region* to be $\leq 600 \text{ GeV}$, the *middle mass region* to be $> 600 \cap \leq 1100 \text{ GeV}$ and the *high mass region* to be $\geq 1100 \text{ GeV}$. Using a NN with three hidden layers I get poor results, but changing this into one works! Will repeat with real weights, it didn't work.

Interesting articles

- From "Search for a charged Higgs boson decaying into a top and a bottom quark at $\sqrt{s} = 13$ TeV" [46] they say (chap 4.2): "The signal samples are trained against all backgrounds, which are weighted according to their cross-sections."
-

Bibliography

1. Peskin ME and Schroeder DV. An Introduction to Quantum Field Theory. Reading, USA: Addison-Wesley (1995) 842 p. Westview Press, 1995
2. Thomson M. Modern Particle Physics. Cambridge University Press, 2013
3. Bertone G, Hooper D, and Silk J. Particle dark matter: Evidence, candidates and constraints. *Phys. Rept.* 2005; 405:279–390. DOI: [10.1016/j.physrep.2004.08.031](https://doi.org/10.1016/j.physrep.2004.08.031). arXiv: [hep-ph/0404175](https://arxiv.org/abs/hep-ph/0404175)
4. Einasto J. Dark Matter. 2010. arXiv: [0901.0632 \[astro-ph.CO\]](https://arxiv.org/abs/0901.0632)
5. Elgammal S, Louka M, Ellithi AY, and Hussein MT. Search for the production of dark matter candidates in association with heavy dimuon resonance using the CMS open data for pp collisions at $\sqrt{s} = 8$ TeV. 2021 Sep. arXiv: [2109.11274 \[hep-ex\]](https://arxiv.org/abs/2109.11274)
6. Autran M, Bauer K, Lin T, and Whiteson D. Searches for dark matter in events with a resonance and missing transverse energy. *Phys. Rev. D* 2015 Aug; 92(3):035007. DOI: [10.1103/PhysRevD.92.035007](https://doi.org/10.1103/PhysRevD.92.035007). Available from: <https://link.aps.org/doi/10.1103/PhysRevD.92.035007>
7. Aaboud M et al. Search for dark matter in events with a hadronically decaying vector boson and missing transverse momentum in pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector. *JHEP* 2018; 10:180. DOI: [10.1007/JHEP10\(2018\)180](https://doi.org/10.1007/JHEP10(2018)180). arXiv: [1807.11471 \[hep-ex\]](https://arxiv.org/abs/1807.11471)
8. Jackson JD. "Kinematics". Particle Data Group 2008. Available from: <https://pdg.lbl.gov/2008/reviews/kinemarpp.pdf>
9. Vadla KOH. Search for production of charginos and neutralinos in dilepton final states with the ATLAS detector at the LHC. PhD thesis. University of Oslo, 2022. Available from: <https://www.duo.uio.no/handle/10852/97304>

10. Barr A, Lester C, and Stephens P. A variable for measuring masses at hadron colliders when missing energy is expected; m_{T2} : the truth behind the glamour. *Journal of Physics G: Nuclear and Particle Physics* 2003 Sep; 29:2343–63. DOI: [10.1088/0954-3899/29/10/304](https://doi.org/10.1088/0954-3899/29/10/304). Available from: <https://doi.org/10.1088%2F0954-3899%2F29%2F10%2F304>
11. Martin AD, Stirling WJ, Thorne RS, and Watt G. Parton distributions for the LHC. *The European Physical Journal C* 2009 Jul; 63:189–285. DOI: [10.1140/epjc/s10052-009-1072-5](https://doi.org/10.1140/epjc/s10052-009-1072-5). Available from: <https://doi.org/10.1140%2Fepjc%2Fs10052-009-1072-5>
12. Jadach S, Płaczek W, Sapeta S, Siodmok A, and Skrzypek M. Parton distribution functions in Monte Carlo factorisation scheme. *Eur. Phys. J. C* 2016; 76. Comments: 25 pages, 6 figures:649. DOI: [10.1140/epjc/s10052-016-4508-8](https://doi.org/10.1140/epjc/s10052-016-4508-8). arXiv: [1606.00355](https://arxiv.org/abs/1606.00355). Available from: <https://cds.cern.ch/record/2157419>
13. The ATLAS Collaboration. The ATLAS Experiment at the CERN Large Hadron Collider. *JINST* 2008; 3. Also published by CERN Geneva in 2010:S08003. DOI: [10.1088/1748-0221/3/08/S08003](https://doi.org/10.1088/1748-0221/3/08/S08003). Available from: <https://cds.cern.ch/record/1129811>
14. Pequenao J and Schaffner P. How ATLAS detects particles: diagram of particle paths in the detector. 2013. Available from: <https://cds.cern.ch/record/1505342>
15. ATLAS Collaboration. The ATLAS inner detector trigger performance in pp collisions at 13 TeV during LHC Run 2. *Eur. Phys. J. C* 2022; 82:206. DOI: [10.1140/epjc/s10052-021-09920-0](https://doi.org/10.1140/epjc/s10052-021-09920-0). arXiv: [2107.02485 \[hep-ex\]](https://arxiv.org/abs/2107.02485)
16. Guevara R. The HiggsML and a simple Quantum-enhanced Machine Learning algorithm. 2022. Available from: <https://github.com/rubenguevara/QuantumMachineLearning>
17. ATLAS-Collaboration. Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC. *Physics Letters B* 2012 Sep; 716:1–29. DOI: [10.1016/j.physletb.2012.08.020](https://doi.org/10.1016/j.physletb.2012.08.020). Available from: <https://doi.org/10.1016%2Fj.physletb.2012.08.020>
18. ATLAS Collaboration. Search for new particles in events with one lepton and missing transverse momentum in pp collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector.

- Journal of High Energy Physics 2014 Sep; 2014. DOI: [10.1007/jhep09\(2014\)037](https://doi.org/10.1007/jhep09(2014)037). Available from: <https://doi.org/10.1007%5C%2Fjhep09%5C%282014%5C%29037>
19. Cowan G. Discovery sensitivity for a counting experiment with background uncertainty. Tech. rep. <http://www.pp.rhul.ac.uk/~cowan/stat/medsig/medsigNote.pdf>. London: Royal Holloway, 2012. Available from: <http://www.pp.rhul.ac.uk/~cowan/stat/medsig/medsigNote.pdf>
 20. Baldi P, Cranmer K, Faucett T, Sadowski P, and Whiteson D. Parameterized neural networks for high-energy physics. *The European Physical Journal C* 2016 Apr; 76. DOI: [10.1140/epjc/s10052-016-4099-4](https://doi.org/10.1140/epjc/s10052-016-4099-4). Available from: <https://doi.org/10.1140%2Fepjc%2Fs10052-016-4099-4>
 21. Baldi P, Sadowski P, and Whiteson D. Searching for exotic particles in high-energy physics with deep learning. *Nature Communications* 2014 Jul; 5. DOI: [10.1038/ncomms5308](https://doi.org/10.1038/ncomms5308). Available from: <https://doi.org/10.1038%2Fncomms5308>
 22. Hjorth-Jensen M. Week 40: From Stochastic Gradient Descent to Neural networks. 2021 Nov. Available from: <https://compphysics.github.io/MachineLearning/doc/pub/week40/html/week40.html>
 23. Hjorth-Jensen M. Week 41 Constructing a Neural Network code, Tensor flow and start Convolutional Neural Networks. 2022 Aug. Available from: <https://compphysics.github.io/MachineLearning/doc/pub/week41/html/week41.html>
 24. Hjorth-Jensen M. Applied Data Analysis and Machine Learning, 6. Logistic Regression. 2021 Aug. Available from: https://compphysics.github.io/MachineLearning/doc/LectureNotes/_build/html/chapter4.html
 25. Kingma DP and Ba J. Adam: A Method for Stochastic Optimization. 2017. arXiv: [1412.6980 \[cs.LG\]](https://arxiv.org/abs/1412.6980)
 26. Morgan JN and Sonquist JA. Problems in the Analysis of Survey Data, and a Proposal. *Journal of the American Statistical Association* 1963; 58:415–34
 27. Quinlan JR. Induction of Decision Trees. *Mach Learning* 1986; 1:81–106. DOI: <https://doi.org/10.1007/BF00116251>

28. Friedman JH. Greedy function approximation: A gradient boosting machine. *The Annals of Statistics* 2001; 29:1189–232. DOI: [10.1214/aos/1013203451](https://doi.org/10.1214/aos/1013203451). Available from: <https://doi.org/10.1214/aos/1013203451>
29. Adam-Boudarios C, Cowan G, Germain C, Guyon I, Kégl B, and Rousseau D. The Higgs boson machine learning challenge. *Proceedings of the NIPS 2014 Workshop on High-energy Physics and Machine Learning*. Ed. by Cowan G, Germain C, Guyon I, Kégl B, and Rousseau D. Vol. 42. Proceedings of Machine Learning Research. Montreal, Canada: PMLR, 2015 Dec :19–55. Available from: <https://proceedings.mlr.press/v42/cowa14.html>
30. Chen T and Guestrin C. XGBoost: A Scalable Tree Boosting System. *Proceedings of the 22nd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*. KDD '16. San Francisco, California, USA: ACM, 2016 :785–94. DOI: [10.1145/2939672.2939785](https://doi.org/10.1145/2939672.2939785). Available from: <http://doi.acm.org/10.1145/2939672.2939785>
31. Hjorth-Jensen M. Applied Data Analysis and Machine Learning, 9. Decision trees, overarching aims. 2021 Aug. Available from: https://compphysics.github.io/MachineLearning/doc/LectureNotes/_build/html/chapter6.html#
32. Hjorth-Jensen M. Applied Data Analysis and Machine Learning, 10. Ensemble Methods: From a Single Tree to Many Trees and Extreme Boosting, Meet the Jungle of Methods. 2021 Aug. Available from: https://compphysics.github.io/MachineLearning/doc/LectureNotes/_build/html/chapter7.html
33. TensorFlow. Train and evaluate. Sample weight definition. Available from: https://www.tensorflow.org/guide/keras/train_and_evaluate#sample_weights [Accessed on: 2023 Apr 27]
34. Gleisberg T, Höche S, Krauss F, Schönherr M, Schumann S, Siegert F, and Winter J. Event generation with SHERPA 1.1. *Journal of High Energy Physics* 2009 Feb; 2009:7–7. DOI: [10.1088/1126-6708/2009/02/007](https://doi.org/10.1088/1126-6708/2009/02/007). Available from: <https://doi.org/10.1088%2F1126-6708%2F2009%2F02%2F007>
35. Alioli S, Nason P, Oleari C, and Re E. A general framework for implementing NLO calculations in shower Monte Carlo programs: the POWHEG BOX. *Journal of High*

- Energy Physics 2010 Jun; 2010. doi: [10.1007/jhep06\(2010\)043](https://doi.org/10.1007/jhep06(2010)043). Available from: <https://doi.org/10.1007%2Fjhep06%282010%29043>
36. Sjöstrand T, Ask S, Christiansen JR, Corke R, Desai N, Ilten P, Mrenna S, Prestel S, Rasmussen CO, and Skands PZ. An introduction to PYTHIA 8.2. Computer Physics Communications 2015 Jun; 191:159–77. doi: [10.1016/j.cpc.2015.01.024](https://doi.org/10.1016/j.cpc.2015.01.024). Available from: <https://doi.org/10.1016%2Fj.cpc.2015.01.024>
37. Brun R and Rademakers F. ROOT: An object oriented data analysis framework. Nucl. Instrum. Meth. A 1997; 389. Ed. by Werlen M and Perret-Gallix D:81–6. doi: [10.1016/S0168-9002\(97\)00048-X](https://doi.org/10.1016/S0168-9002(97)00048-X)
38. McKinney W. Data Structures for Statistical Computing in Python. *Proceedings of the 9th Python in Science Conference*. Ed. by Walt S van der and Millman J. 2010 :56–61. doi: [10.25080/Majora-92bf1922-00a](https://doi.org/10.25080/Majora-92bf1922-00a)
39. Martín Abadi, Ashish Agarwal, Paul Barham, Eugene Brevdo, Zhifeng Chen, Craig Citro, Greg S. Corrado, Andy Davis, Jeffrey Dean, Matthieu Devin, Sanjay Ghemawat, Ian Goodfellow, Andrew Harp, Geoffrey Irving, Michael Isard, Jia Y, Rafal Jozefowicz, Lukasz Kaiser, Manjunath Kudlur, Josh Levenberg, Dandelion Mané, Rajat Monga, Sherry Moore, Derek Murray, Chris Olah, Mike Schuster, Jonathon Shlens, Benoit Steiner, Ilya Sutskever, Kunal Talwar, Paul Tucker, Vincent Vanhoucke, Vijay Vasudevan, Fernanda Viégas, Oriol Vinyals, Pete Warden, Martin Wattenberg, Martin Wicke, Yuan Yu, and Xiaoqiang Zheng. TensorFlow: Large-Scale Machine Learning on Heterogeneous Systems. Software available from tensorflow.org. 2015. Available from: <https://www.tensorflow.org/>
40. Pedregosa F, Varoquaux G, Gramfort A, Michel V, Thirion B, Grisel O, Blondel M, Prettenhofer P, Weiss R, Dubourg V, Vanderplas J, Passos A, Cournapeau D, Brucher M, Perrot M, and Duchesnay E. Scikit-learn: Machine Learning in Python. Journal of Machine Learning Research 2011; 12:2825–30
41. Mullenbach J. Keras. Source code. 2023. Available from: <https://github.com/keras-team/keras/blob/master/keras/engine/training.py> [Accessed on: 2023 Apr 27]

42. tedsandler. Does xgboost support *negative* instance weights? Discussion forum. Available from: <https://discuss.xgboost.ai/t/does-xgboost-support-negative-instance-weights/1925> [Accessed on: 2023 Apr 27]
43. hcho3. Forbid negative Hessian values. Issue reported on github. Available from: <https://github.com/dmlc/xgboost/issues/4372> [Accessed on: 2023 Apr 27]
44. Danziger K, Höche S, and Siegert F. Reducing negative weights in Monte Carlo event generation with Sherpa. 2021. DOI: [10.48550/ARXIV.2110.15211](https://doi.org/10.48550/ARXIV.2110.15211). Available from: <https://arxiv.org/abs/2110.15211>
45. Abbott B, Alhroob M, Bhopatkar VS, Hopkins WH, Lambert JE, Metcalfe J, Serkin L, Xu W, and Zhu J. Search for Triboson $W^\pm W^\mp W^\mp$ Production Using 13 TeV pp Collision Data at ATLAS. Tech. rep. Geneva: CERN, 2020. Available from: <https://cds.cern.ch/record/2714377>
46. Benchekroun D, Bosman M, Chadi Z, Mir LM, Riu I, Salvador Salas A, Sato K, Yamauchi H, and Glatzer JMV. Search for a charged Higgs boson decaying into a top and a bottom quarks at $\sqrt{s}=13$ TeV. Tech. rep. Geneva: CERN, 2020. Available from: <https://cds.cern.ch/record/2710015>