732A97 Multivariate Statistics Lab 1

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Question 1: Describing individual variables

Consider the data set in the T1-9.dat file, National track records for women. For 55 different countries we have the national records for 7 variables (100, 200, 400, 800, 1500, 3000m and marathon). Use R to do the following analyses.

input

```
trackrcs <- read.table("T1-9.dat")
colnames(trackrcs) <- c("countries", "x100m", "x200m", "x400m", "x800m", "x1500m", "x3000m", "marathon")

trackrcs2 <- (trackrcs)[,-1]
rownames(trackrcs2) <- trackrcs[,1]
```

a) Describe the 7 variables with mean values, standard deviations e.t.c

mean values

```
colMeans((trackrcs)[,-1])
```

```
##      x100m      x200m      x400m      x800m     x1500m     x3000m
##  11.357778  23.118519  51.989074   2.022407   4.189444   9.080741
##   marathon
## 153.619259
```

median

```
apply((trackrcs)[,-1], 2, median)
```

```
##    x100m    x200m    x400m    x800m   x1500m   x3000m marathon
##   11.325   22.980   51.645    2.005    4.100    8.845  148.430
```

standard deviation

```
apply((trackrcs)[,-1], 2, sd)
```

```
##       x100m       x200m       x400m       x800m      x1500m      x3000m
##  0.39410116  0.92902547  2.59720188  0.08687304  0.27236502  0.81532689
##    marathon
## 16.43989508
```

maximum

```
apply((trackrcs)[,-1], 2, max)
```

```
##     x100m    x200m    x400m    x800m   x1500m   x3000m marathon
##     12.52    25.91    61.65     2.29     5.42    13.12   221.14
```

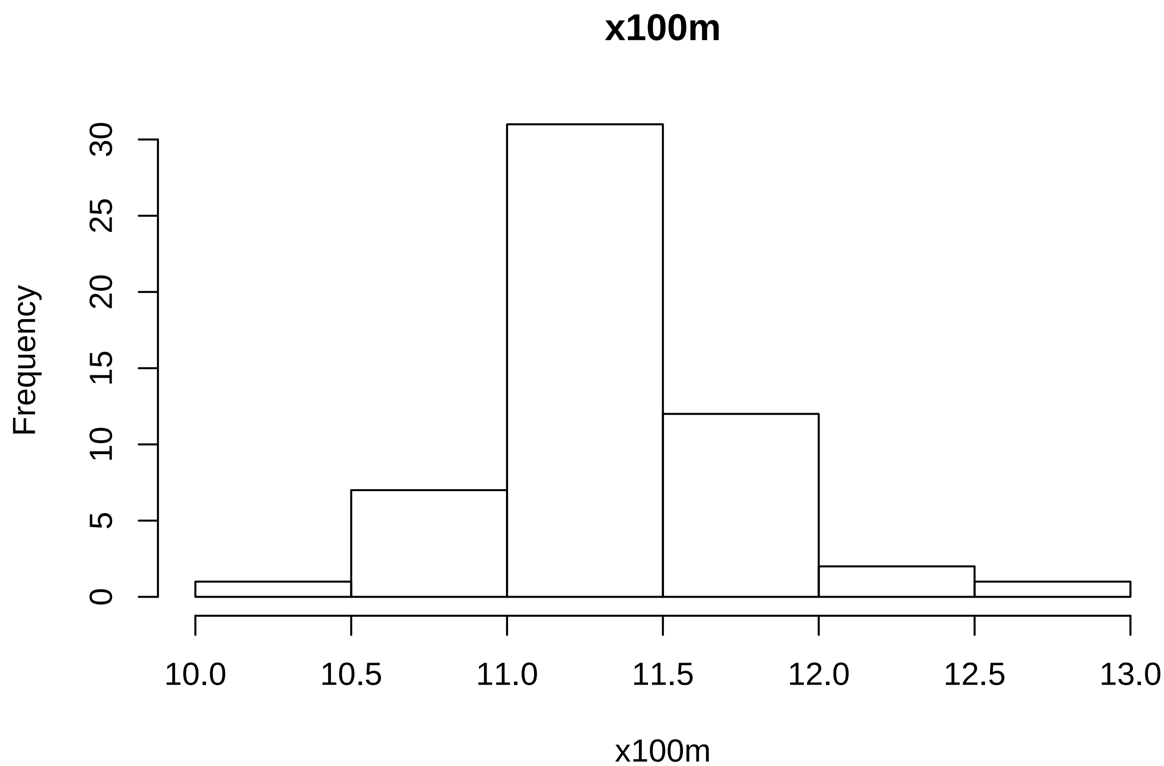
minimum

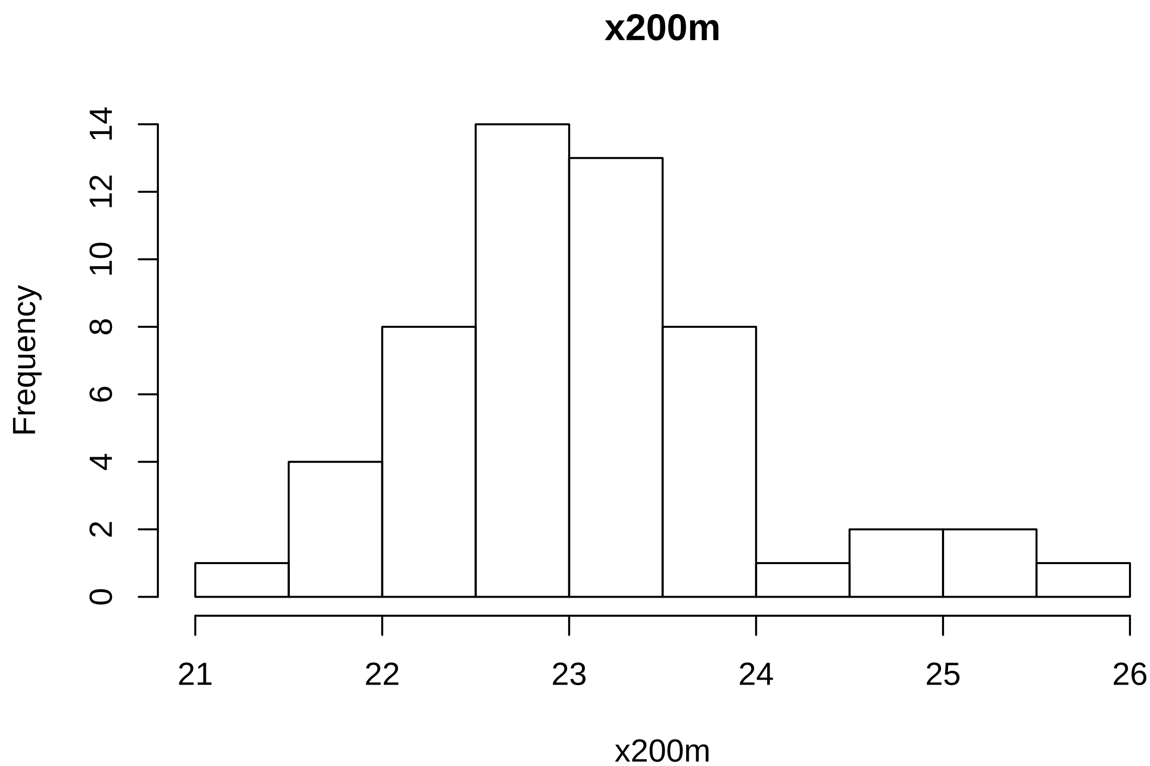
```
apply((trackrcs)[,-1], 2, min)
```

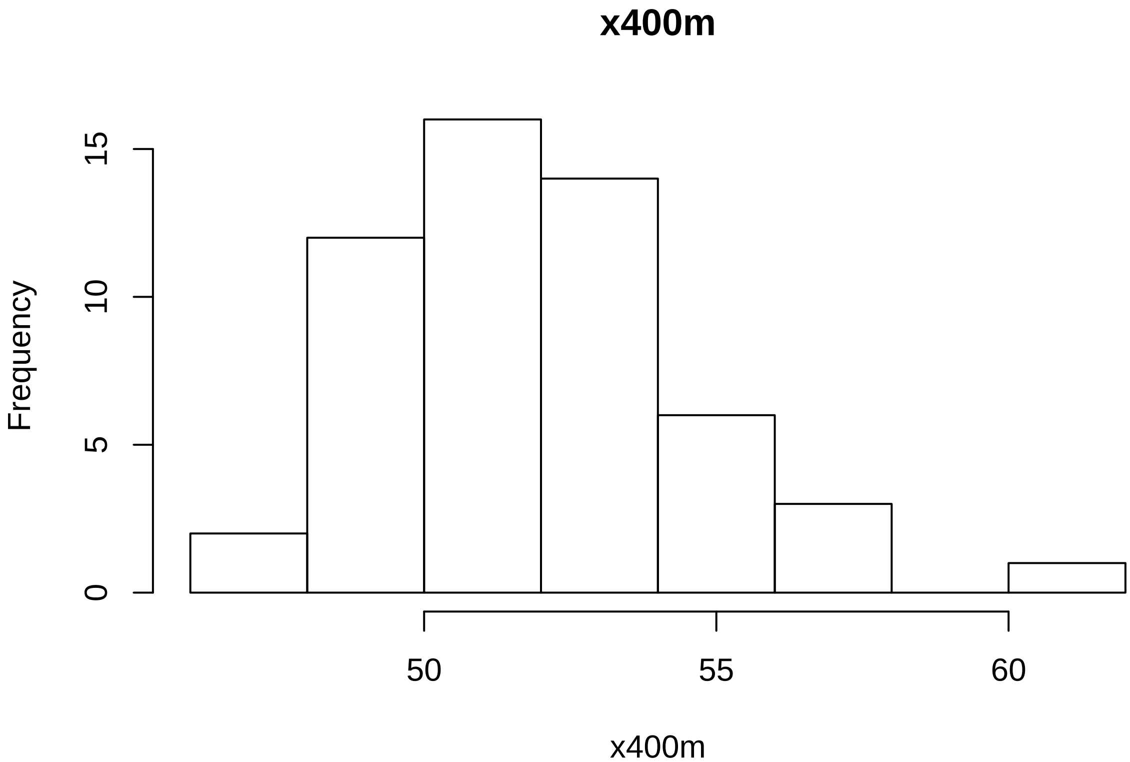
```
##     x100m    x200m    x400m    x800m   x1500m   x3000m marathon
##     10.49    21.34    47.60     1.89     3.84     8.10   135.25
```

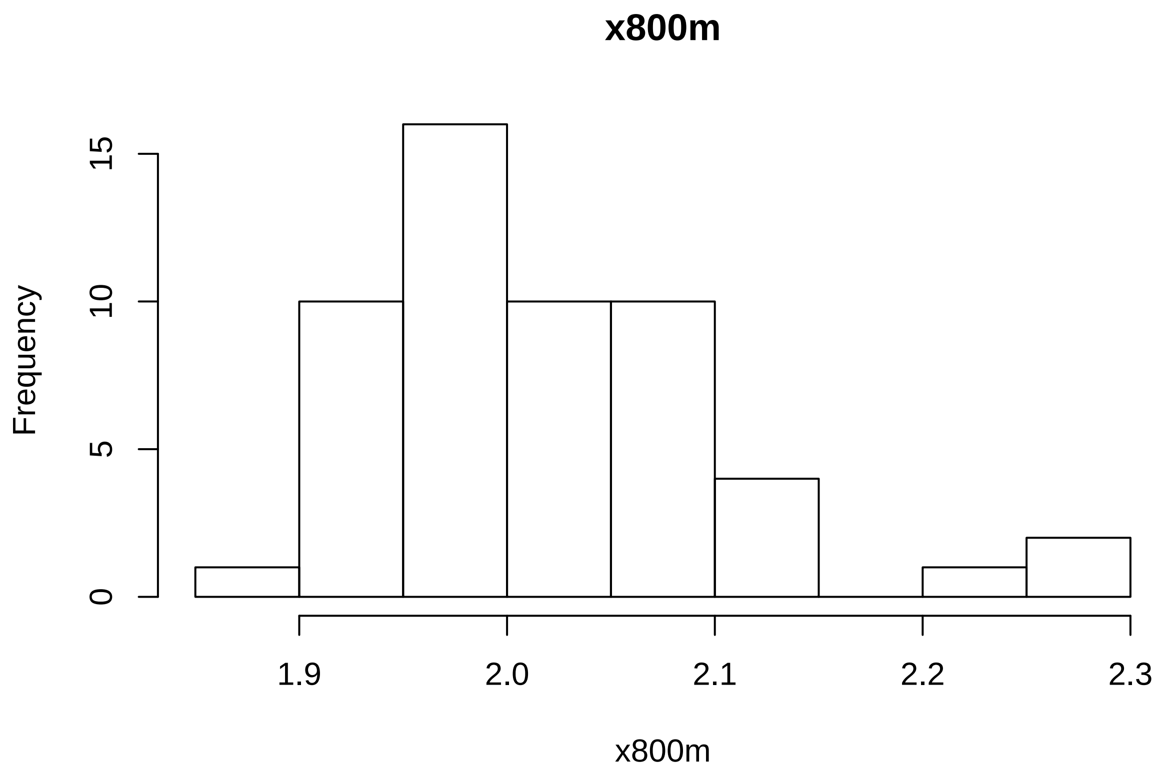
b) Illustrate the variables with different graphs (explore what plotting possibilities R has). Make sure that the graphs look attractive (it is absolutely necessary to look at the labels, font sizes, point types). Are there any apparent extreme values? Do the variables seem normally distributed? Plot the best fitting (match the mean and standard deviation, i.e. method of moments) Gaussian density curve on the datas histogram. For the last part you may be interested in the hist() and density() functions.

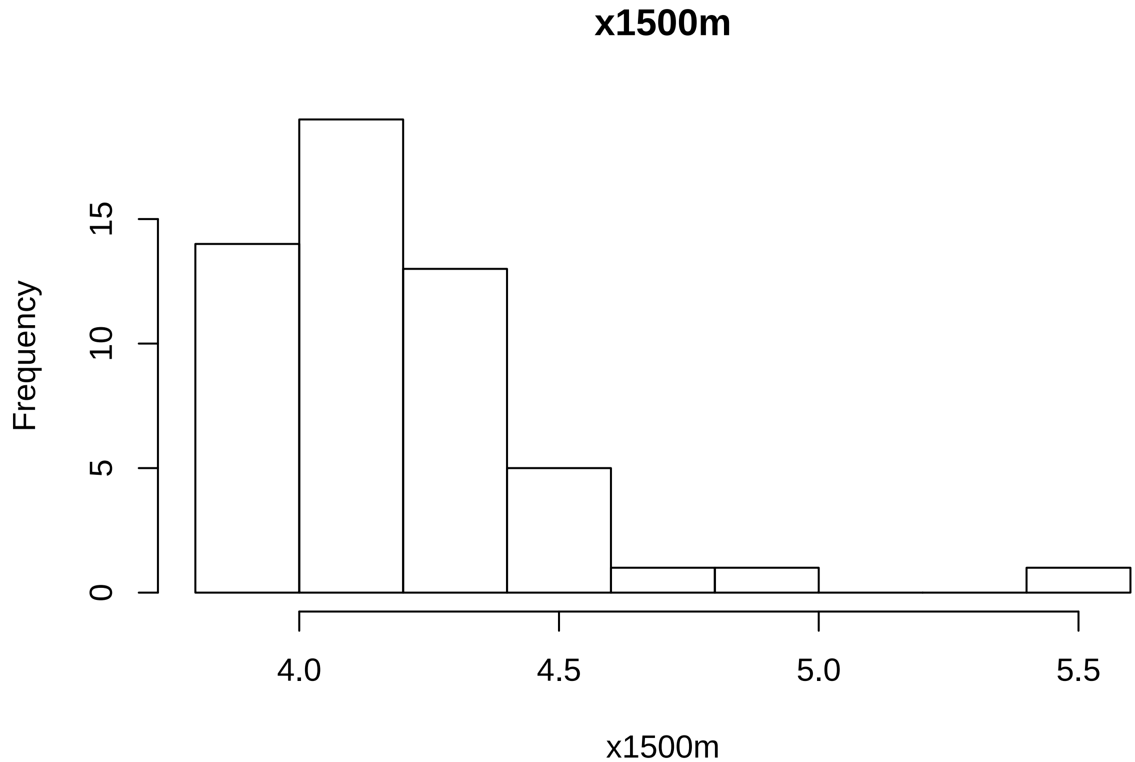
```
j=0; lapply(trackrcs2, FUN = function(i){
  j <<- j+1
  hist(i, main = colnames(trackrcs2)[j], xlab=colnames(trackrcs2)[j])
})
```

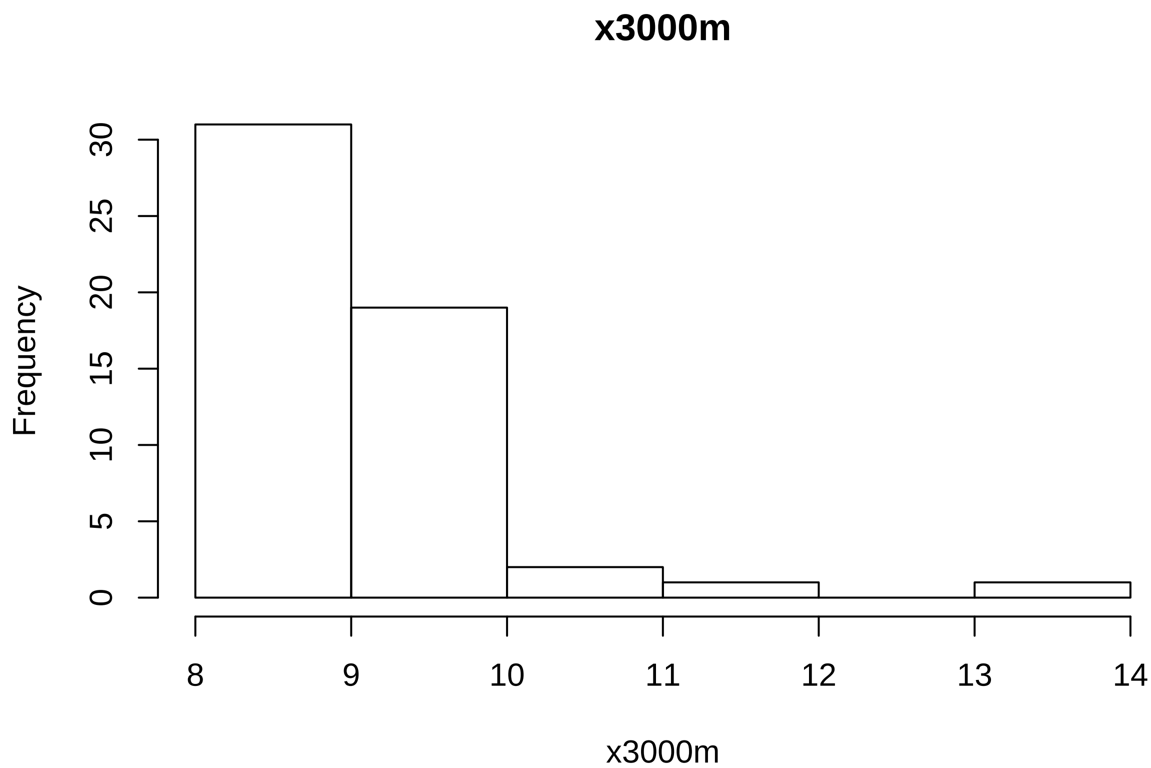

x100m
Frequency
0
5
10
15
20
25
30
10.0
10.5
11.0
11.5
12.0
12.5
13.0
x100m

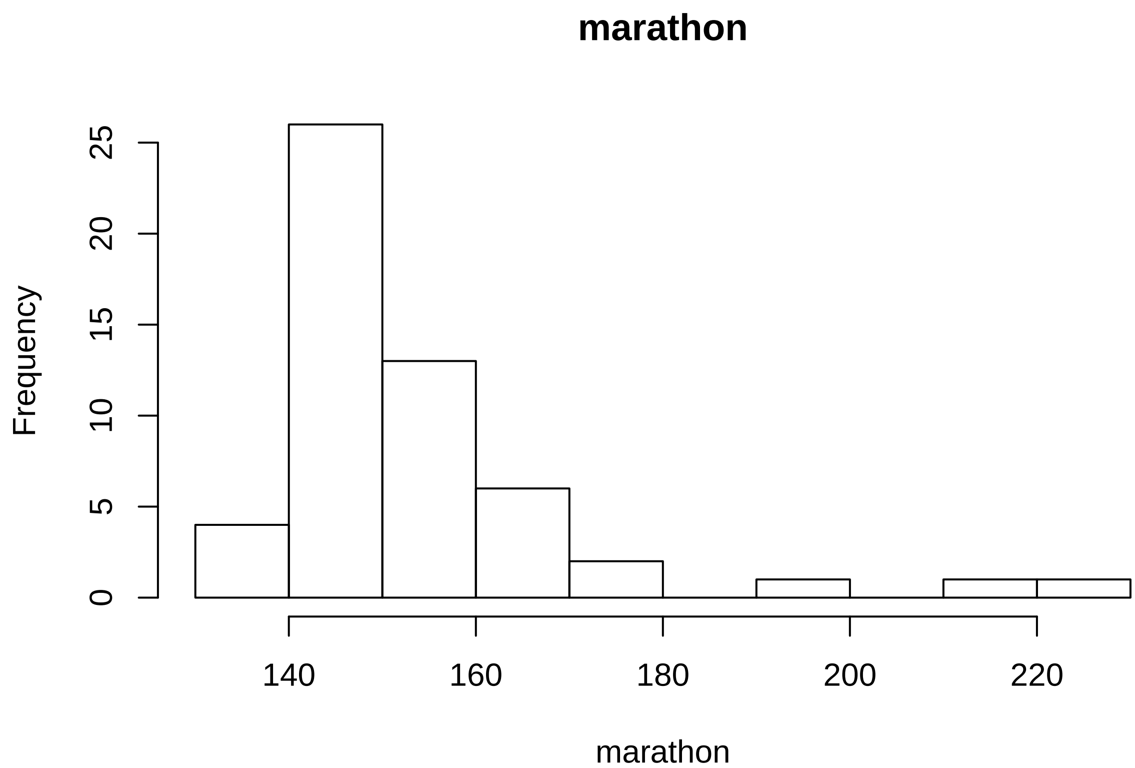

x200m
Frequency
0
2
4
6
8
10
12
14
21
22
23
24
25
26
x200m


x400m
Frequency
0
5
10
15
50
55
60
x400m


x800m
Frequency
0
5
10
15
1.9
2.0
2.1
2.2
2.3
x800m


x1500m
Frequency
0
5
10
15
4.0
4.5
5.0
5.5
x1500m


x3000m
Frequency
0
5
10
15
20
25
30
8
9
10
11
12
13
14
x3000m




```
## $x100m
## $breaks
## [1] 10.0 10.5 11.0 11.5 12.0 12.5 13.0
##
## $counts
## [1]  1  7 31 12  2  1
##
## $density
## [1] 0.03703704 0.25925926 1.14814815 0.44444444 0.07407407 0.03703704
##
## $mids
## [1] 10.25 10.75 11.25 11.75 12.25 12.75
##
## $xname
## [1] "i"
##
## $equidist
## [1] TRUE
##
## attr(,"class")
## [1] "histogram"
##
## $x200m
## $breaks
##  [1] 21.0 21.5 22.0 22.5 23.0 23.5 24.0 24.5 25.0 25.5 26.0
##
```

```
## $counts
##  [1]  1  4  8 14 13  8  1  2  2  1
##
## $density
##  [1] 0.03703704 0.14814815 0.29629630 0.51851852 0.48148148 0.29629630
##  [7] 0.03703704 0.07407407 0.07407407 0.03703704
##
## $mids
##  [1] 21.25 21.75 22.25 22.75 23.25 23.75 24.25 24.75 25.25 25.75
##
## $xname
## [1] "i"
##
## $equidist
## [1] TRUE
##
## attr(,"class")
## [1] "histogram"
##
## $x400m
## $breaks
## [1] 46 48 50 52 54 56 58 60 62
##
## $counts
## [1]  2 12 16 14  6  3  0  1
##
## $density
## [1] 0.018518519 0.111111111 0.148148148 0.129629630 0.055555556 0.027777778
## [7] 0.000000000 0.009259259
##
## $mids
## [1] 47 49 51 53 55 57 59 61
##
## $xname
## [1] "i"
##
## $equidist
## [1] TRUE
##
## attr(,"class")
## [1] "histogram"
##
## $x800m
## $breaks
##  [1] 1.85 1.90 1.95 2.00 2.05 2.10 2.15 2.20 2.25 2.30
##
## $counts
## [1]  1 10 16 10 10  4  0  1  2
##
## $density
## [1] 0.3703704 3.7037037 5.9259259 3.7037037 3.7037037 1.4814815 0.0000000
## [8] 0.3703704 0.7407407
##
## $mids
```

```
## [1] 1.875 1.925 1.975 2.025 2.075 2.125 2.175 2.225 2.275
##
## $xname
## [1] "i"
##
## $equidist
## [1] TRUE
##
## attr(,"class")
## [1] "histogram"
##
## $x1500m
## $breaks
##  [1] 3.8 4.0 4.2 4.4 4.6 4.8 5.0 5.2 5.4 5.6
##
## $counts
## [1] 14 19 13  5  1  1  0  0  1
##
## $density
## [1] 1.29629630 1.75925926 1.20370370 0.46296296 0.09259259 0.09259259
## [7] 0.00000000 0.00000000 0.09259259
##
## $mids
## [1] 3.9 4.1 4.3 4.5 4.7 4.9 5.1 5.3 5.5
##
## $xname
## [1] "i"
##
## $equidist
## [1] TRUE
##
## attr(,"class")
## [1] "histogram"
##
## $x3000m
## $breaks
## [1]  8  9 10 11 12 13 14
##
## $counts
## [1] 31 19  2  1  0  1
##
## $density
## [1] 0.57407407 0.35185185 0.03703704 0.01851852 0.00000000 0.01851852
##
## $mids
## [1]  8.5  9.5 10.5 11.5 12.5 13.5
##
## $xname
## [1] "i"
##
## $equidist
## [1] TRUE
##
## attr(,"class")
```

```
## [1] "histogram"
##
## $marathon
## $breaks
##  [1] 130 140 150 160 170 180 190 200 210 220 230
##
## $counts
##  [1]  4 26 13  6  2  0  1  0  1  1
##
## $density
##  [1] 0.007407407 0.048148148 0.024074074 0.011111111 0.003703704
##  [6] 0.000000000 0.001851852 0.000000000 0.001851852 0.001851852
##
## $mids
##  [1] 135 145 155 165 175 185 195 205 215 225
##
## $xname
## [1] "i"
##
## $equidist
## [1] TRUE
##
## attr(,"class")
## [1] "histogram"
```

Question 2: Relationships between the variables

a) Compute the covariance and correlation matrices for the 7 variables. Is there any apparent structure in them? Save these matrices for future use.

```
C <- cov((trackrcs)[,-1])
corr_m <- cor((trackrcs)[,-1])

C
```

```
##                x100m      x200m      x400m       x800m      x1500m
## x100m    0.15531572  0.3445608  0.8912960 0.027703564 0.08389119
## x200m    0.34456080  0.8630883  2.1928363 0.066165898 0.20276331
## x400m    0.89129602  2.1928363  6.7454576 0.181807932 0.50917683
## x800m    0.02770356  0.0661659  0.1818079 0.007546925 0.02141457
## x1500m   0.08389119  0.2027633  0.5091768 0.021414570 0.07418270
## x3000m   0.23388281  0.5543502  1.4268158 0.061379315 0.21615514
## marathon 4.33417757 10.3849876 28.9037314 1.219654647 3.53983732
##               x3000m   marathon
## x100m     0.23388281   4.334178
## x200m     0.55435017  10.384988
## x400m     1.42681579  28.903731
## x800m     0.06137932   1.219655
## x1500m    0.21615514   3.539837
## x3000m    0.66475793  10.706091
## marathon 10.70609113 270.270150
```

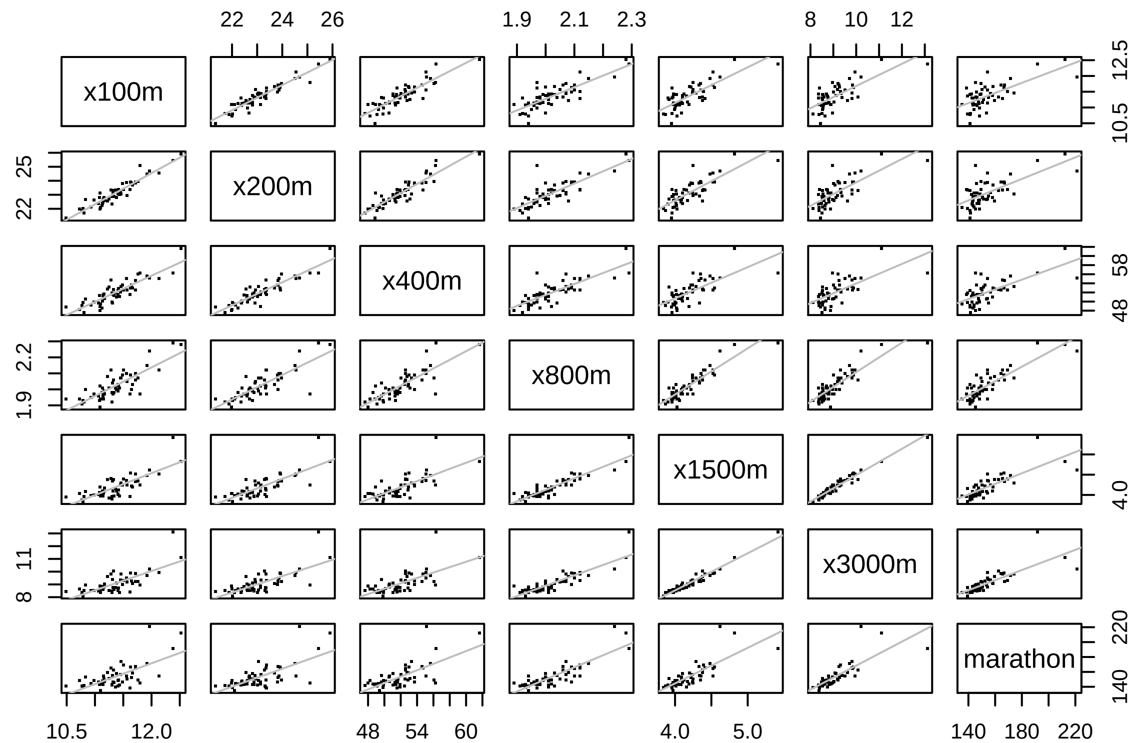
```
corr_m
```

```
##              x100m     x200m     x400m     x800m    x1500m    x3000m
## x100m    1.0000000 0.9410886 0.8707802 0.8091758 0.7815510 0.7278784
## x200m    0.9410886 1.0000000 0.9088096 0.8198258 0.8013282 0.7318546
## x400m    0.8707802 0.9088096 1.0000000 0.8057904 0.7197996 0.6737991
## x800m    0.8091758 0.8198258 0.8057904 1.0000000 0.9050509 0.8665732
## x1500m   0.7815510 0.8013282 0.7197996 0.9050509 1.0000000 0.9733801
## x3000m   0.7278784 0.7318546 0.6737991 0.8665732 0.9733801 1.0000000
## marathon 0.6689597 0.6799537 0.6769384 0.8539900 0.7905565 0.7987302
##           marathon
## x100m    0.6689597
## x200m    0.6799537
## x400m    0.6769384
## x800m    0.8539900
## x1500m   0.7905565
## x3000m   0.7987302
## marathon 1.0000000
```

Both matrices are symmetric. The correlation matrix has ones on the main diagonal.

b) Generate and study the scatterplots between each pair of variables. Any extreme values?

```
pairs(trackrcs[,-1], pch = ".", cex = 1.5, panel = function(x, y, ...){
  points(x, y, ...)
  abline(lm(y ~ x), col = "grey") })
```



The scatterplot matrix tells us that "marathon" is quite an outlier.

c) Explore what other plotting possibilities R offers for multivariate data. Present other (at least two) graphs that you find interesting with respect to this data set.

chernoff face

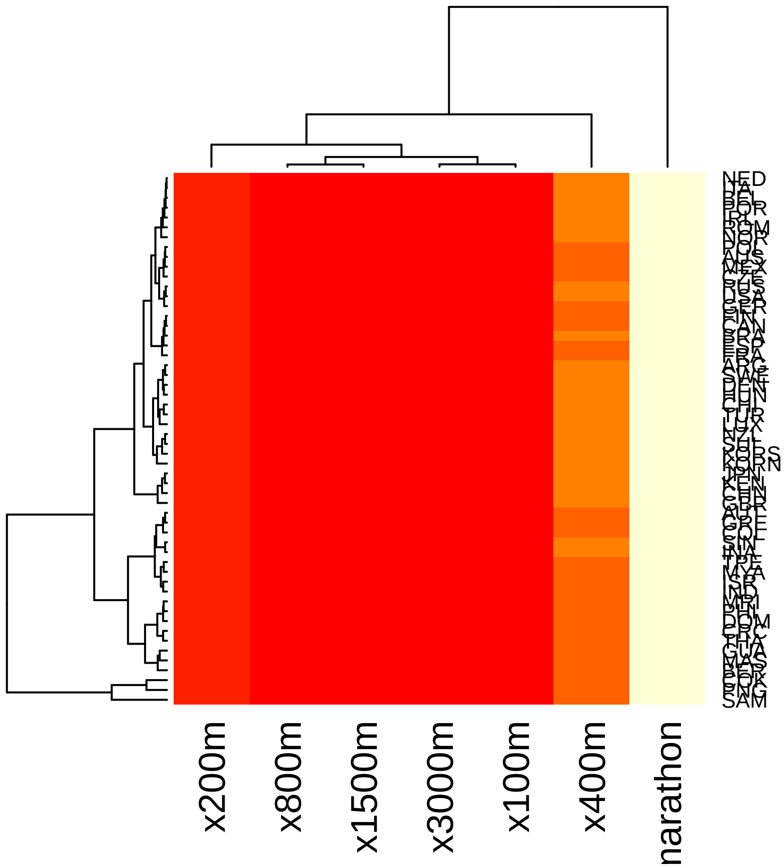
```
library(aplpack)
faces(trackrcs2)
```


```
## effect of variables:
##  modified item       Var
##  "height of face   " "x100m"
##  "width of face    " "x200m"
##  "structure of face" "x400m"
##  "height of mouth  " "x800m"
##  "width of mouth   " "x1500m"
##  "smiling          " "x3000m"
##  "height of eyes   " "marathon"
##  "width of eyes    " "x100m"
##  "height of hair   " "x200m"
##  "width of hair   "  "x400m"
##  "style of hair   "  "x800m"
##  "height of nose  "  "x1500m"
##  "width of nose   "  "x3000m"
##  "width of ear    "  "marathon"
##  "height of ear   "  "x100m"
```

A stars plot is not very useful. We can tell from literature that it is also outdated.

heatmaps

```r
heatmap(x=as.matrix(trackrcs2))
```


x200m
x800m
x1500m
x3000m
x100m
x400m
narathon

Question 3: Examining for extreme values

a) Look at the plots (esp. scatterplots) generated in the previous question. Which 3-4 countries appear most extreme? Why do you consider them extreme?

The 3 countries that seem most extreme are Brazil, Russia and the United States.

b) Compute the squared Euclidean distance (i.e. r = 2) of the observation from the sample mean for all 55 countries using R's matrix operations. First center the raw data by the means to get (x arrow - x bar) for each country. Then do a calculation with matrices that will result in a matrix that has on its diagonal the requested squared distance for each country. Copy this diagonal to a vector and report on the five most extreme countries. In this questions you MAY NOT use any loops.

```
x_bar = apply(trackrcs2,1,mean)
d0 = as.matrix(trackrcs2-x_bar); dim(d0)
```

```
## [1] 54  7
```

```
deviation = sqrt( d0%*%t(d0) ); dim(deviation)
```

```
## [1] 54 54
```

```
diagonal_vector <- diag(deviation)
deviation_countries <-
  cbind.data.frame(countries = as.vector(trackrcs[,1]),diagonal_vector)
deviation_countries_ordered <-
   deviation_countries[order(-deviation_countries$diagonal_vector), ]

deviation_countries_ordered[1:5,]
```

```
##     countries diagonal_vector
## PNG       PNG        193.0557
## COK       COK        185.0669
## SAM       SAM        165.7015
## BER       BER        151.3534
## GUA       GUA        148.7706
```

The five most extreme countries are PNG, COK, SAM, BER, GUA.

c)

```r
V <- diag(apply(trackrcs2,2,var))
d_sq_v <- d0%*%solve(V)%*%t(d0)
diagonal_vector2 <- diag(d_sq_v)
deviation_countries2 <-
  cbind.data.frame(countries = as.vector(trackrcs[,1]),diagonal_vector2)
deviation_countries_ordered2 <-
   deviation_countries[order(-deviation_countries2$diagonal_vector), ]

deviation_countries_ordered2[1:5,]
```

```
##     countries diagonal_vector
## COK       COK        185.0669
## PNG       PNG        193.0557
## SAM       SAM        165.7015
## GUA       GUA        148.7706
## BER       BER        151.3534
```

Except Great Britain, still the top five most extreme countries are the same and they are COK, PNG, SAM, GUA, BER

d) Compute the Mahalanobis distance, which countries are most extreme now?

```r
d_sq_m <- d0%*%solve(C)%*%t(d0)
diagonal_vector3 <- diag(d_sq_m)
deviation_countries3 <-
  cbind.data.frame(countries = as.vector(trackrcs[,1]),diagonal_vector2)
deviation_countries_ordered3 <-
   deviation_countries[order(-deviation_countries2$diagonal_vector), ]

deviation_countries_ordered3[1:5,]
```

```
##     countries diagonal_vector
## COK       COK        185.0669
## PNG       PNG        193.0557
## SAM       SAM        165.7015
## GUA       GUA        148.7706
## BER       BER        151.3534
```

Still the top five most extreme countries are the same and they are COK, PNG, SAM, GUA, BER

e) Compare the results in b){d). Some of the countries are in the upper end with all the measures and perhaps they can be classified as extreme. Discuss this. But also notice the different measures give rather different results (how does Sweden behave?). Summarize this graphically. Produce Czekanowski's diagram using e.g. the RMaCzek package. In case of problems please describe them.

Sweden

Czekanowski's diagram

```
library(RMaCzek)
x<-czek_matrix(trackrcs2, n_classes = 7)
plot(x)
plot.czek_matrix(x)
```