

Lab 3 732A97

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Data

```
library(dplyr)

trackrcs <- read.table("T1-9.dat",
  col.names = c("countries", "x100m", "x200m",
    "x400m", "x800m", "x1500m", "x3000m", "marathon"))
rownames(trackrcs) <- trackrcs[,1]
```

Question 1: Principal components, including interpretation of them

a) Obtain the sample correlation matrix R for these data, and determine its eigenvalues and eigenvectors.

```
S <- cov(trackrcs[, -1])
R <- cov2cor(S); R
```

```
##           x100m      x200m      x400m      x800m      x1500m      x3000m
## x100m      1.0000000  0.9410886  0.8707802  0.8091758  0.7815510  0.7278784
## x200m      0.9410886  1.0000000  0.9088096  0.8198258  0.8013282  0.7318546
## x400m      0.8707802  0.9088096  1.0000000  0.8057904  0.7197996  0.6737991
## x800m      0.8091758  0.8198258  0.8057904  1.0000000  0.9050509  0.8665732
## x1500m     0.7815510  0.8013282  0.7197996  0.9050509  1.0000000  0.9733801
## x3000m     0.7278784  0.7318546  0.6737991  0.8665732  0.9733801  1.0000000
## marathon  0.6689597  0.6799537  0.6769384  0.8539900  0.7905565  0.7987302
##           marathon
## x100m      0.6689597
## x200m      0.6799537
## x400m      0.6769384
## x800m      0.8539900
## x1500m     0.7905565
## x3000m     0.7987302
## marathon  1.0000000
```

```
eigen(R)$values
```

```
## [1] 5.80762446 0.62869342 0.27933457 0.12455472 0.09097174 0.05451882
## [7] 0.01430226
```

```
eigen(R)$vectors
```

```
##           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]
## [1,] 0.3777657 -0.4071756 -0.1405803  0.58706293 -0.16706891 -0.53969730
## [2,] 0.3832103 -0.4136291 -0.1007833  0.19407501  0.09350016  0.74493139
## [3,] 0.3680361 -0.4593531  0.2370255 -0.64543118  0.32727328 -0.24009405
## [4,] 0.3947810  0.1612459  0.1475424 -0.29520804 -0.81905467  0.01650651
## [5,] 0.3892610  0.3090877 -0.4219855 -0.06669044  0.02613100  0.18898771
## [6,] 0.3760945  0.4231899 -0.4060627 -0.08015699  0.35169796 -0.24049968
## [7,] 0.3552031  0.3892153  0.7410610  0.32107640  0.24700821  0.04826992
##           [,7]
## [1,] 0.08893934
## [2,] -0.26565662
## [3,] 0.12660435
## [4,] -0.19521315
## [5,] 0.73076817
## [6,] -0.57150644
## [7,] 0.08208401
```

b) Determine the first two principal components for the standardized variables. Prepare a table showing the correlations of the standardized variables with the components, and the cumulative percentage of the total (standardized) sample variance explained by the two components.

the first two principal components for the standardized variables

```
res=prcomp((trackrcs)[,-1], scale. = FALSE)
# No scaling at this point because we are going to use the correlation matrix later

# Each PC is a linear combination of the original variables
#### res$rotation
res$rotation[,1:2]
```

```
##           PC1      PC2
## x100m    -0.016123307  0.11485619
## x200m    -0.038657909  0.29039299
## x400m    -0.107793074  0.93844399
## x800m    -0.004504024  0.01340703
## x1500m   -0.013072642  0.03631915
## x3000m   -0.039484872  0.07871002
## marathon -0.992409201 -0.11878027
```

correlation of the standardized variables with the components

This is following the formula in the text book on page 433, “Result 8.3” and “Equation 8-8”

```
eigenvalues=res$sdev^2
CorWithPC <-
  t(res$rotation[,1:2])%*%sqrt(diag(eigenvalues))%*%solve(sqrt(diag(diag(R))))
colnames(CorWithPC) <- colnames(trackrcs[,-1])
t(CorWithPC)
```

```
##           PC1           PC2
## x100m    -0.267064796  1.902466031
## x200m    -0.077476875  0.581995811
## x400m    -0.055829878  0.486053614
## x800m    -0.001524372  0.004537567
## x1500m   -0.001607685  0.004466563
## x3000m   -0.002011251  0.004009272
## marathon -0.024646395 -0.002949898
```

cummulative percentage of total standardized sample variance explained by the 2 components

```
CorWithPC %>% apply(MARGIN=1,FUN=abs) %>% t() %>%
  apply(MARGIN=1,FUN=function(a) 100*cumsum(a)/sum(a))
```

```
##           PC1           PC2
## x100m     62.08481  63.70265
## x200m     80.09593  83.19034
## x400m     93.07476  99.46548
## x800m     93.42913  99.61742
## x1500m    93.80287  99.76698
## x3000m    94.27043  99.90122
## marathon 100.00000 100.00000
```

c) Interpret the two principal components obtained in Part b. (Note that the first component is essentially a normalized unit vector and might measure the athletic excellence of a given nation. The second component might measure the relative strength of a nation at various running distances.)

It seems the first principal component is a measure of how much less time the athletes of a particular nation take to complete a race relative to their other fellow competitors from other nations. We can notice that athletes generally concentrate a lot of efforts in running very fast so that they complete their races in the shortest times possible in the 100 metre races and the marathon.

Clearly, the second principal component tells us about the physical strength of an athlete in a given race. We see that for 100 meters, the values are very high because athletes are giving their all while for the marathon, the values are actually negative because the athletes are usually very tired and they try to minimize using a lot of energy as a tactic to run for the very long distance.

d) Rank the nations based on their score on the first principal component. Does this ranking correspond with your intuitive notion of athletic excellence for the various countries?

```
NewScore <- as.matrix(trackrcs[, -1]) %*% as.matrix(res$rotation[, 1])
NewScore = cbind.data.frame(countries = trackrcs[, 1], NewScore = NewScore)
NewScore[, 1][order(NewScore[, 2], decreasing = TRUE)]

## [1] GBR KEN CHN JPN USA GER RUS NOR IRL ROM BEL AUS POR NED
## [15] ITA MEX POL CZE SUI KOR ESP KORS NZL BRA FIN FRA CAN HUN
## [29] DEN LUX SWE ARG TUR CHI GRE AUT INA SIN COL ISR IND MYA
## [43] TPE THA CRC PHI DOM MRI MAS GUA BER SAM COK PNG
## 54 Levels: ARG AUS AUT BEL BER BRA CAN CHI CHN COK COL CRC CZE DEN ... USA
```

This ranking indeed corresponds with my intuitive notion of the athletic excellence for the various countries. The principal components are indeed capturing well the ranking of the countries.

Question 2: Factor analysis

Perform a factor analysis of the national track records for women given in Table 1.9. Use the sample covariance matrix S and interpret the factors. Compute factor scores, and check for outliers in the data. Repeat the analysis with the sample correlation matrix R . Does it make a difference if R , rather than S , is factored? Explain.

Try both PC and ML as estimation methods. Notice that R's `factanal()` only does ML estimation. For the PC method you can use the `principal()` function of the `psych` package. What does it mean that the parameter rotation of `factanal()` is set to “varimax” by default (equivalently rotate of `principal()`)? Do not forget to check the adequacy of your model.

Tip: Read section “A Large Sample Test for the Number of Common Factors”.

```
factanal(trackrcs[, -1], factors = 3, covmat = S) # varimax is the default
```

```
##
## Call:
## factanal(x = trackrcs[, -1], factors = 3, covmat = S)
##
## Uniquenesses:
##      x100m      x200m      x400m      x800m      x1500m      x3000m      marathon
##      0.106      0.005      0.133      0.047      0.005      0.041      0.225
##
## Loadings:
##           Factor1 Factor2 Factor3
## x100m      0.815   0.413   0.245
## x200m      0.886   0.410   0.203
## x400m      0.797   0.311   0.367
## x800m      0.512   0.617   0.556
## x1500m     0.449   0.849   0.270
## x3000m     0.361   0.866   0.280
## marathon  0.380   0.553   0.571
##
##           Factor1 Factor2 Factor3
## SS loadings      2.824   2.593   1.022
## Proportion Var    0.403   0.370   0.146
## Cumulative Var    0.403   0.774   0.920
##
## The degrees of freedom for the model is 3 and the fit was 0.2033
```

```
factanal(trackrcs[, -1], factors = 3, covmat = R)
```

```
##
## Call:
## factanal(x = trackrcs[, -1], factors = 3, covmat = R)
##
```

```
## Uniquenesses:
##      x100m      x200m      x400m      x800m      x1500m      x3000m      marathon
##      0.106      0.005      0.133      0.047      0.005      0.041      0.225
##
## Loadings:
##      Factor1 Factor2 Factor3
## x100m      0.815      0.413      0.245
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## x400m      0.797      0.311      0.367
## x800m      0.512      0.617      0.556
## x1500m     0.449      0.849      0.270
## x3000m     0.361      0.866      0.280
## marathon 0.380      0.553      0.571
##
##
##      Factor1 Factor2 Factor3
## SS loadings      2.824      2.593      1.022
## Proportion Var    0.403      0.370      0.146
## Cumulative Var    0.403      0.774      0.920
##
## The degrees of freedom for the model is 3 and the fit was 0.2033
```

```
library(psych)
principal(trackrcs[, -1], nfactors=3, rotate="varimax", covar = FALSE)
```

```
## Principal Components Analysis
## Call: principal(r = trackrcs[, -1], nfactors = 3, rotate = "varimax",
##      covar = FALSE)
## Standardized loadings (pattern matrix) based upon correlation matrix
##      RC2 RC1 RC3 h2 u2 com
## x100m    0.85 0.41 0.23 0.94 0.061 1.6
## x200m    0.86 0.40 0.25 0.96 0.037 1.6
## x400m    0.86 0.26 0.36 0.93 0.065 1.5
## x800m    0.54 0.59 0.54 0.93 0.072 3.0
## x1500m   0.44 0.82 0.34 0.99 0.010 1.9
## x3000m   0.35 0.85 0.37 0.98 0.020 1.7
## marathon 0.33 0.44 0.82 0.98 0.019 1.9
##
##      RC2 RC1 RC3
## SS loadings      2.92 2.33 1.47
## Proportion Var    0.42 0.33 0.21
## Cumulative Var    0.42 0.75 0.96
## Proportion Explained 0.43 0.35 0.22
## Cumulative Proportion 0.43 0.78 1.00
##
## Mean item complexity = 1.9
## Test of the hypothesis that 3 components are sufficient.
##
## The root mean square of the residuals (RMSR) is 0.02
## with the empirical chi square 0.62 with prob < 0.89
##
## Fit based upon off diagonal values = 1
```

```
fa(trackrcs[, -1], nfactors=3, rotate="varimax", covar = FALSE)
```

```
## Factor Analysis using method = minres
## Call: fa(r = trackrcs[, -1], nfactors = 3, rotate = "varimax", covar = FALSE)
## Standardized loadings (pattern matrix) based upon correlation matrix
##           MR2  MR3  MR1   h2   u2 com
## x100m    0.83 0.41 0.23 0.90 0.0993 1.6
## x200m    0.88 0.40 0.21 0.98 0.0160 1.5
## x400m    0.80 0.31 0.35 0.87 0.1338 1.7
## x800m    0.53 0.60 0.54 0.94 0.0622 3.0
## x1500m   0.46 0.85 0.26 1.00 0.0018 1.8
## x3000m   0.38 0.85 0.30 0.95 0.0457 1.7
## marathon 0.37 0.56 0.59 0.80 0.2002 2.7
##
##           MR2  MR3  MR1
## SS loadings      2.88 2.54 1.03
## Proportion Var    0.41 0.36 0.15
## Cumulative Var    0.41 0.77 0.92
## Proportion Explained 0.45 0.39 0.16
## Cumulative Proportion 0.45 0.84 1.00
##
## Mean item complexity = 2
## Test of the hypothesis that 3 factors are sufficient.
##
## The degrees of freedom for the null model are 21 and the objective function was 11.62 with Chi Sq
## The degrees of freedom for the model are 3 and the objective function was 0.23
##
## The root mean square of the residuals (RMSR) is 0
## The df corrected root mean square of the residuals is 0.01
##
## The harmonic number of observations is 54 with the empirical chi square 0.04 with prob < 1
## The total number of observations was 54 with Likelihood Chi Square = 10.81 with prob < 0.013
##
## Tucker Lewis Index of factoring reliability = 0.898
## RMSEA index = 0.238 and the 90 % confidence intervals are 0.09 0.371
## BIC = -1.15
## Fit based upon off diagonal values = 1
```