# Formal languauges and automatas

Let $\Sigma$ be an Alphabet and $U, V \subset \Sigma^*$ languages.

Lanaguage as a formal series:

$$L = \sum (L, w) w$$

$$\leftrightsquigarrow L = \bigcup_{w \in \Sigma^*} \underbrace{(L, w)}_{0 \text{ or } 1} \{w\}$$

Language defined in FSK:

$$L \subset \Sigma^*$$

Defining plus and multiplication:

$$U + V = \sum ((U, w) + (V, w)) w$$

$$U \cdot V = \sum ((U, s)(V, t)) w, w = st$$

Operations we now out of FSK:

$$U \cup V = \{w \mid w \in U \vee w \in V\}$$

$$U \cdot V = \{\, st \mid s \in U \wedge t \in V\}$$

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$\rightsquigarrow$ With these defintions of $+$ and $\cdot$ all languages with the alphabet $\Sigma$ form an Algebra.
$\rightsquigarrow$ We can then define automatas with matrices $M \in (\Sigma^*)^{n \times n}$
$\rightsquigarrow$ With that we can prove Kleene's theorem with matrix multiplication:

$$L \text{ regular language} \iff L \text{ is accepted by a finite automata}$$

$\rightsquigarrow$ Finally, an Overview over weighted automatas and formal power series