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Formal Languages and Automatas

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Seminar Paper on Formal Languages and Automatas
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Disclaimer

I confirm that this seminar paper is my own work and I have documented all sources and material used.

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Abstract

Abstract... **Keywords**: Math

Contents

1 Introduction

2 Backround

First of all, we need to define some simple structures and ideas, that will be really important in this paper. We will define formal languages in two ways and then automatas arcordingly. A formal language is just a set of words, where words are a series of characters from an alphabet.

Definition 1 *An Alphabet is an abritrary non empty finite set denoted with* Σ

Example 2.1 *Simple examples for Alphabets would be:*

1. $\Sigma = \{x\}$
2. $\Sigma = \{a, b, c, d, \ldots z\}$
3. $\Sigma = \{1, 2, 3, \ldots n\}$ *where* $n \in \mathbb{N}$

This definition is rather general, because an alphabet Σ can really be anything. We could also define an Alphabet as subset of the natural numbers $\Sigma \subset \mathbb{N}$. This is equalivant to the previous definition, becaus we can always find an isomorphism between a finite amount of objects and the natural numbers. In other words we can always just number a finite amount of objects. The above definition results in more readable words, thus resulting in more readable problems. Furhtermore its unintuitive to have words full of numbers and languages full of number series. That is why im chosing this definition, like most of the literature.
Now with our first building block defined we can define words, which are just a series of characters in an alphabet.

Definition 2 *Let* Σ *be an fixed alphabet, then* w *is a series of characters out of* Σ. *The empty words which contains no characters, is denoted by* ε

Example 2.2 *Simple examples for words would be:*

1. $\Sigma = \{a, b, c\} \rightsquigarrow w_1 = abc$
2. $\Sigma = \{x\} \rightsquigarrow w_2 = xxxx$
3. $\Sigma = \{1, 2, 3\} \rightsquigarrow w_3 = 112233$
4. $\Sigma = \{1, 2, 3\} \rightsquigarrow w_4 = \varepsilon$

The empty word ε is analogous to the empty set in Set theory. It has a lot uf uses, but one obvious is that now one can define a Monoid over words $(M, \cdot, \Sigma, \varepsilon)$. M is a set of words and plus is concetanation of words, then ε is the neutral element of the Monoid. Which brings us to the next definition.

Definition 3 *Let* Σ *be an fixed alphabet and* w, v *words with characters out of* Σ. *Then the contenation is defined by:*

$$w \cdot v := wv$$

sometimes just written as wv. *With powers:*

$$w^n = \underbrace{w \cdot w \cdot \ldots w}_{n \text{ times}} \text{ where } n \in \mathbb{N}$$

In simple terms a concatenation of two words is just a new word with the two words chained togehter.
Now we can again build on words to define langauges.

Definition 4 *Let* Σ *be an fixed alphabet, then a formal language is a set of words, with characters in* Σ.

Example 2.3 *Simple examples for formal languages would be:*

1. $\Sigma = \{a, b, c\} \rightsquigarrow L = \{aaa, bbb, ccc, abc\}$
2. $\Sigma = \{x\} \rightsquigarrow L = \{x, xx, xxx, xxxx\}$
3. $\Sigma = \{1, 2, 3\} \rightsquigarrow L = \{11, 22, 33, 123, \varepsilon\}$

This definition is really clear and simple. We will see a less intuitive way to define language shortly. Obviously the simpler one has some downsides, which we will discuss later in the paper.
The kleene star * is a really powerful operator, to define it we need to first define contenation for languages.

Definition 5 *Let* Σ *be an fixed alphabet and* L, V *languages with words constructed out of* Σ. *Then the contenation of formal languages is defined by:*

$$L \cdot V := \{l \cdot v | l \in L \vee v \in V\}$$

sometimes just written as LV. *With powers defined recursively:*

$$\begin{aligned} L^0 &:= \{\varepsilon\} \\ L^1 &:= L \\ L^{i+1} &:= \{l_i \cdot l | l_i \in L^i \vee l \in L\} \end{aligned}$$

In simple terms we are creating a new language by compbining every word of the first language with every word of the second. Now we are able to define the kleene star operator.

Definition 6 *Let* Σ *be an fixed alphabet and* L *a language with words constructed out of* Σ. *Then the kleene star of a formal language is defined by:*

$$L^* := \bigcup_{i \in \mathbb{N}_0} L^i = L^0 \cup L^1 \cup L^2 \cup \ldots$$

In simple terms it generates all possible permutation of abriatary length of a formal language. This is really powerful, because Σ^* is the biggest language one can construct from the alphabet Σ. Thus every language L generated by Σ is just a subset of Σ^*.
Unintuively, we can also write a formal language as a formal power series.

Definition 7 *Let Σ be an fixed alphabet, then a formal language can be written as:*

$$L = \sum_{w \in \Sigma^*} (L,w)w \texttt{ where } (L,w) \in \mathbb{B} = \{0,1\}$$

The sum is iterating over all possible words $w \in \Sigma^*$ and (L,w) is an indicating function, which is one if w should be in the language and ohterwise zero. Thus all words with an one infront are in the language.
The representation with sets has the andvantage that a formal language will inherit all operations on sets like complement, union, and intersection. Those operations are really useful, thus we will define similar on the power series version.

Definition 8 *Let Σ be an fixed alphabet L,V formal languages, L and V can be written*

$$L = \sum_{w \in \Sigma^*} (L,w)w \quad \textit{and} \quad V = \sum_{w \in \Sigma^*} (V,w)w.$$

Addition and multiplication on formal power series is defined as follows:

$$L + V := \sum_{w \in \Sigma^*} ((L,w) + (V,w))w \quad \textit{and} \quad L \cdot V := \sum_{w \in \Sigma^*} ((L,u) \cdot (V,v))w\texttt{,} \quad \texttt{where } w = uv.$$

This definition is not recursive, because the right addition and muliplication operater are from the Boolean Algebra $\mathbb{B} = \{0,1\}$.

$+$	*0*	*1*
0	*0*	*1*
1	*1*	*1*

$\times$	*0*	*1*
0	*0*	*0*
1	*0*	*1*

With powers defined recursively:

$$\begin{aligned} L^0 &:= \varepsilon \\ L^1 &:= L \\ L^{i+1} &:= \sum_{w \in \Sigma^*} ((L^i,u) \cdot (L,v))w, \quad \textit{where} \quad w = uv. \end{aligned}$$

Kleene Star can now also be defined for formal power series:

$$L^* := L^0 + L^1 + L^2 + L^3 + \cdots = \sum_{i \in \mathbb{N}_0} L^i.$$

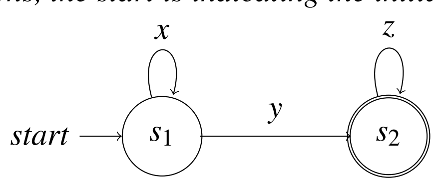
With these operation defined our two versions of formal languages are powerful tools, which will be compared in the following sections. In order to conpare them we will also need finite automatas.

Simillar to formal languages there are multiple ways one can define finite automatas. In this paper we will define two. One is more suitable for combining it with the set version of formal languages and the other one is more siutable for the formal power series representation. First we will define the one more suitable for the set version.

Definition 9 *A finite automata is a quintuple $M = (Q, \Sigma, \delta, q_0, F)$, where*

1. *Q is a finite set of states,*
2. *Σ is an alphabet, all inputs are constructed from,*
3. *$\delta : Q \times \Sigma \to Q$ is the transition function, carrying the automata from one state to another,*
4. *$q_0 \in Q$ is the initial state of the automata,*
5. *$F \subset Q$ are the accepting states of the automata.*

Example 2.4 *Here is a simple automata depicted, where the circles are states , the arrows are transitions, the start is indicating the initial state, and the double lined circle is the accepting state.*

$M = ()$ 

3 Formal Languages: Sets vs. Formal Power Series

4 Maximum Gap Lemma

5 Pumping Lemma

6 Not all Formal languages are regular

7 Pumping Lemma vs. Maximum Gap Lemma

8 Kleene's theorem (Power series, Matrix Automata)

9 Kleene's theorem (Set, Tuple)

10 Kleene's theorem vs. Kleene's theorem

11 Kleene Schützenberg theorem and weigthed automatas

12 Conclusion

12.1 Citation

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`\citep`	(Hale & White Eagle 1980)	(Sprouse et al. 2011)
`\citep*`	(Hale und White Eagle 1980)	(Sprouse and colleagues 2011)
`\citealt`	Hale & White Eagle 1980	Sprouse et al. 2011
`\citealt*`	Hale und White Eagle 1980	Sprouse and colleagues 2011
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`\citealp*`	Hale und White Eagle 1980	Sprouse and colleagues 2011
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12.2 Bibliography

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