Formal languages and automatas

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Outline

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1. History & Motivation
2. Formal Languages Definitions, Examples
3. Prove not all Languages are rational (regular)
4. Automatas Definitions, Examples
5. Prove: Finite automaton accept rational languages
6. Prove: Every rational language is accepted by a finite automaton
7. Overview Weighted Automatons
8. Conclusion

An Alphabet Σ is a set of characters.
Examples:

1. $\Sigma = \{a, b, c\}$
2. $\Sigma = \{x\}$
3. $\Sigma = \emptyset$

A Language L is a set of words with characters out of an Alphabet Σ.
Examples:

1. $\Sigma = \{a, b, c\} \rightsquigarrow L = \{aaa, bbb, ccc, abc\}$
2. $\Sigma = \{x\} \rightsquigarrow L = \{x, xx, xxx, xxxx\}$
3. $\Sigma = \emptyset \rightsquigarrow L = \{\epsilon\}$

$\rightsquigarrow L \subset \Sigma^*$

Formal series

We can also write a Language with a formal series, with a fixed Σ:

$$L = \sum (L, w) w$$

$$\leftrightsquigarrow L = \bigcup_{w \in \Sigma^*} \underbrace{(L, w)}_{\text{0 or 1}} \{w\}$$

Examples:

1. $\Sigma = \{a, b, c\} \rightsquigarrow L = \{aaa, abc\}$
   $\rightsquigarrow L = 1aaa + 1abc + 0a + 0b + 0c + \dots$
2. $\Sigma = \{x\} \rightsquigarrow L = \{x, xx\}$
   $\rightsquigarrow L = 1x + 1xx + 0xxx + 0xxxx + 0xxxxx + \dots$
3. $\Sigma = \emptyset \rightsquigarrow L = \{\epsilon\}$
   $\rightsquigarrow L = 1\epsilon$

Now we can define Addition on formal series:

$$U + V = \sum (\underbrace{(U, w) + (V, w)}_{\text{Boolean addition}})w$$

$$\leftrightsquigarrow U + V = U \cup V$$

Exmaple:
Let $U = \{x, xx\}$ and $V = \{aaa, abc\}$ languages:

$$\begin{aligned} U + V &= (1+0)x + (1+0)xx + (0+1)aaa + (0+1)abc + (0+0)a + \dots \\ &= 1x + 1xx + 1aaa + 1abc + 0a + \dots \end{aligned}$$

Multiplication

Next we can define multiplication on formal series:

$$U \cdot V = \sum (\underbrace{(U, s) \cdot (V, t)}_{\text{Boolean multiplication}}) w, \text{ such that } st = w$$

$$\leftrightsquigarrow U \cdot V = \{ \ st \mid s \in U \wedge t \in V\}$$

Exmaple:
Let $U = \{x, xx\}$ and $V = \{aaa, abc\}$ languages:

$$\begin{aligned} U \cdot V = (1 \cdot 1)xaaa + (1 \cdot 1)xxaaa + (1 \cdot 1)xabc + (1 \cdot 1)xxabc \\ + (0 \cdot 0)axxx + \dots \\ = 1xaaa + 1xxaaa + 1xabc + 1xxabc + 0axxx + \dots \end{aligned}$$

With these definitions, all languages with a fixed alphabet Σ form an algebra $\mathbb{B}\langle\Sigma\rangle$ over $\mathbb{B}$.

Kleene star:
Let U be a Language.

$$U^* = \epsilon + U + U^2 + U^3 + \dots$$

Exmaple:
Let $U = x$.

$$U^* = \epsilon + x + x^2 + x^3 + x^4 + \dots$$

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All languages generated by a finite number of additions, multiplications, and kleene star is a rational (regular) language.

Examples:

$\Sigma = \{x, y, z\}$

$$L_1 = x + y \tag{1}$$

$$L_2 = (x + y + z)^* \tag{2}$$

$$L_3 = (x + y^*)^* z^* \tag{3}$$

$$L_4 = (xyz)^*(y + x^* zxyx^*)^* \tag{4}$$

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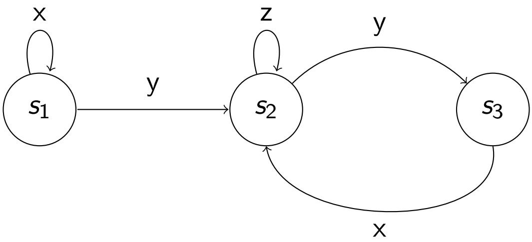
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Thanks for your attention!