Type Search and Choice: True and Adopted Type Mismatch and the Generation of Frames

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Abstract

A model of decision making is built where the type of the decision maker matters for how this process takes place. Individual's type is assumed to be determined by Nature and ignored by individuals. Self-Type ignorance starts a process in which individuals search for a type to adopt. In this search process, individuals take into account the information in their current state, together with a net valuation function and a threshold, to determine when the search process must stop. The type-search process produces an adopted type that may or may not coincide with individual's true type. If the adopted type is different to the true type, this adopted type is shown to function as a frame in an extended choice problem. In our choice framework, adopted types as frames can lead to sub-optimal choices with individual welfare implications. Possible applications of the model are suggested.

Keywords: Choice, Bounded rationality, Frames, Consideration sets, Search, Types, Unknown Type.

1 Introduction

Decision processes can be overwhelming and costly in terms of the time invested and information search and acquisition. Given that there is usually a level of information absence involved in decision making, even consciously selected choices in a bounded setting may or may not (in the end) be the optimal for each individual. This is particularly true if knowledge about self-characteristics, areas of strength or capabilities are not well defined or are non-existent.

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The presence of suboptimal choice among individuals is usually linked to either poor judgement, or to a negative bias in opportunities. For example, when individuals choose which level of education to attain, what health behaviours to follow, how much should be saved, or which goods to acquire, it is often observed that their decisions deviate from their optimal as dictated by rational agent models. These "anomalies" raise questions about the nature of these discrepancies, and why they emerge, in particular among the poor or socially disadvantaged.

As stated by Duflo (2006), a branch of literature has researched the permanence of poverty led by the "poor but efficient hypothesis", based on the work of Schultz et al. (1964), which succinctly states that the poor behave as rationally as their environment allows them to. Nevertheless, further developments in the literature opened a new research avenue where the effort focused on market failures and how they hinder the result of individual's rational behaviour (Duflo, 2006, p. 367). On these grounds, it seems reasonable to conjecture individual characteristics and environmental influences affect agents' behaviour. In this regard, Ray (2006) argues there is more to the behaviour of an individual than its own self, and that "individual desires and standards of behaviour are often defined by experience and observation; they do not exist in social isolation" (Ray, 2006, p. 409). The author refers to the influence that an individual's environment has on her behaviour as *aspirations*, and defines an *aspirations window* as the set of reachable individuals someone can aspire to be.

This paper presents a model that can accommodate these and other behavioural patterns that are commonly observed, and depart from the fully rational individual setting. The approach focuses on phenomena related to education and career choice, with the main motivating example being related to educational choices. However, the generality of the model allows the setting to be extended to other realms such as consumer behaviour, gender bias, crime, and other similar topics. This work builds on existing tools in the literature to present a model that provides insights that can help support policies addressing the negative effects of inequality of opportunities, poverty traps, aspirations, and escalation costs in social mobility, and that can be translated into identity adoption and the literature related to it. Additionally, the introduction of status quo provide means to analyse how the distribution of types can produce traps that affect efficiency.

More precisely, the paper presents a model of type determination for boundedly rational agents. An individual's type in this context is parallel to the concept of identity explored by Akerlof and Kranton (2000). In the model's setting, individuals are unaware of their type, and posses limited information on which type is appropriate for them. This limitation leads them to embark on a type search process guided by a heuristic rule defined by a satisficing criterion based on Simon (1955). The satisficing strategy is particularly determined by payoff thresholds that are influenced by the composition of types in social environments, and that inform the agent when to stop searching and adopt a type . These thresholds can be interpreted as the aspirations individuals posses, where aspirations are understood as in Appadurai (2004) and Ray (2006). The type search and adoption process can lead individuals to choose types that do not correspond to their true types. It is shown in the model how an individual's adopted type can constitute a frame that interferes with choice processes in extended choice problems, providing a link between the model and the literature on choice with frames (Bernheim and Rangel, 2007; Salant and Rubinstein, 2008), choice with search (Dalton and Ghosal, 2012; Horan, 2010; Masatlioglu and Ok, 2005), and other related work. Finally, the results in the model are used to explore the effects that inequality of opportunities and subsidy based policies can have on type adoption.

One can find in the literature a number of other approaches that also investigate non-optimal behaviour. Bounded rationality, for example, is one of the earliest attempts to do so, aiming to model choice behaviour considering boundaries to the unlimited capabilities of the rational man on information processing. A common reference for the origin of this line of research is Simon (1955), where the author exposes the problems and weaknesses of theories based on the rational individual, and defines an agent with less demanding assumptions. Lipman (1995), Selten (1999), and Rubinstein (1998) present a variety of models relying on the concept of bounded rationality.

On the other hand, theoretical research on type unawareness is rather scarce. Murayama (2010), for example, develops a two sided search model where agents are not aware of their type, and discover it by process of rejection and acceptance with implications for welfare in equilibrium. Concerning work related to individual behaviour and choice, Young (2008) builds a model introducing self-image in individual's utility function, in the model's setting individuals do not have a clear idea of what their identity is. He finds that, under certain assumptions, agents may find impossible to define an identity for themselves. Also, Gul and Pesendorfer (2007) develop a model where agent's preferences depend on other individuals' characteristics and personalities; and Calvó-Armengol and Jackson (2009) study the influence that social environments have on both parents and children. This interaction results in overlapping environments determining behaviour correlation among the two agents. Regarding the industrial organisation literature, Boone and Shapiro (2006) build a model where the type of consumer changes over time as a function of previous consumption of goods, giving power to the producer on rent extraction. Learning theory approaches situations similar to those of type ignorance or adjustment usually by departing from models that assume rational equilibrium or that are built in a game theoretical setting (Slembeck, 1998). In learning theory models, individuals adjust their behaviour incorporating information captured by social interaction in a way such that they optimize on payoffs via imitation of better strategies, or at least strategies that seem to be the best.

The model of type search and adoption presented here is closely related to a relatively new line of research that has been coined as identity economics. This topic has been recently developed by Akerlof and Kranton (2000), Bénabou and Tirole (2011), and Fryer and Jackson (2008), among others; and can be traced back to Sen (1985), Folbre (1994), and Kevane (1994). The core idea in this literature is that by developing an identity, an individual's sense of self, agents can see their choices limited, in turn affecting outcomes in their environment, which can work in detriment of their well-being. Individuals subject to identity standards aiming at perceiving a positive payoff, which derives from fitting in a social environment and within particular groups. Within this literature, work is scarce on the formalisation of identity formation, this is an avenue where the present work seeks to contribute. Regarding the applied literature, research has focused on identity (type) awareness instead of unawareness. In these studies, identity influences judgements individuals make about themselves and others, and affects choices that have short and long term impacts on individual and collective life. The relation between identity and these outcomes can be found in the works of Humlum et al. (2012) who use factor analysis methodology to extract how identity influences educational or career choices; Benjamin et al. (2010) who implement experiments to capture the effects of race identity on patience in decision making; Shayo (2009) and Klor and Shayo (2010) develop a model of identity and then test it using experiments to determine the effect of identity on redistribution preferences, the former focusing on payoff maximization behaviour, and the later with class and national identities as focal points; Hoff and Pandey (2006) use two experiments in rural India to consider if social identity of individuals can explain cognitive performance and responses to economic incentives. Although this work focuses on identity and not on types, the latter can be related to the former if type adopted is instead defined as identity (For other approaches see Blume and Durlauf (2001), Ozgur and Bisin (2011): social interactions; Bénabou and Tirole (2011): identity driven by moral behaviour; Jamison and Wegener (2010): multiple selves).

Recently, an interest in the study of aspirations as an influential element in individual behaviour has emerged in economics. In an influential work, Appadurai (2004) argues that aspirations are a result of both individual and social factors, as stated in his own words "Aspirations are never simply individual (as the language of wants and choices inclines us to think). They are always formed in interaction and in the thick of social life" (Appadurai, 2004, p. 67). The concept of aspirations has been used to develop theoretical models that aim at explaining the determinants of poverty from an individual perspective. For example, among those leading this efforts one can find the work of Ray (2006), and Genicot and Ray

(2014). The former presents strong arguments in favour of the inclusion of aspirations in the standard economic framework, while the latter presents a model in which aspirations are determined by the distribution of income in individual's environment, affecting their investment decisions, which consequently affect society's wealth. They use their model to explain how economic outcomes are so persistent. More recently, Dalton et al. (2016) have contributed with a model where poverty deters aspirations and generates poverty traps, in their model all agents, independent of their status, posses the same features, but the lack of resources generates inconsistencies in the way aspirations are formed and considered by individuals. The influence that inequality plays on aspirations (modelled as reference points), subsequently affects investment behaviour on individuals. In their model, too high aspirations truncates investment, while moderately high aspirations spur it.

With respect to research that seeks to extend the classical models choice literature, recent work on choice has focused on the introduction of frames, search processes, and consideration sets in the traditional framework. This new literature has emerged with interesting results that illuminate on possible reasons and processes behind sub-optimal choice among individuals. For example, in the literature on frames and choice, Salant and Rubinstein (2008) model choice with frames, where the pair composed by a frame and the set of alternatives defines an extended choice problem, and axiomatically determine and study the implications for choice behaviour. Bernheim and Rangel (2007) suggest a framework with ancillary conditions that affect choices, focusing on welfare implications. Applying the concept of frames to the economics of imperfect competition, Eliaz and Spiegler (2011) develop a model of consumer choice with consideration sets where entities with market power can affect choice via frames.

With respect to choice involving search processes, Masatlioglu and Nakajima (2013) present a model of iterative search and decision making with reference points leading the search process. Horan (2010) offers a model of choice form lists in which a search process takes place. Dalton and Ghosal (2012) build a model where choices are driven by frames that are endogenously determined with a feedback process involved, they describe choice procedures in their framework and explore the effects on welfare under a number of assumptions that restrict the information on the part of the decision maker. Other related literature includes Masatlioglu and Ok (2005) who expand the classical choice theory to include the influence of status quo in choice behaviour departing from the revealed preference theory. Concerning choice models that make use of the satisficing criterion, Papi (2012) presents an axiomatic model of bounded rationality, making use of the satisficing concept within the revealed preference framework. Additionally, Caplin and Dean (2011) build a model of choice with search in which search is costly, and where one of the search criteria explored involve decision makers having a reservation utility that indicates when to stop searching. In their model the reservation utility is actually a satisficing criteria.

By offering a framework that helps understand why individuals may end up taking actions that seem to be suboptimal from the point of view of a purely rational agent, this paper contributes adding to the existent literature on bounded rationality. The model offers a rational on how types are formed in social environments, this contribution helps filling some gaps in the identity economics literature leadered by Akerlof and Kranton (2000), where efforts have focused on the consequences of identities on outcomes, and little has been done in terms of formalising how identities are formed. When formalising the search process that leads individuals to adopt a type, the concept of satisficing as a stop/choice criteria is borrowed form Simon (1955, 1997). This strategy has been used mainly in the context of the firm, and have not been fully exploited in individual choice settings (Some exemptions are Papi (2012) and Caplin and Dean (2011)).

In the type adoption process presented, thresholds constitute a fundamental part of type adoption, determining the extent of sub-optimality of adopted types. In the model, thresholds can be linked to aspirations by adapting the definition of the latter from the studies of Appadurai (2004) and Ray (2006). Thus the paper also contributes to research in economics on the relationship between aspirations and identity, offering a framework that can be used to analyse identity adoption among individuals in society. Also, while Dalton et al. (2016) model aspirations as individual processes, and in Genicot and Ray (2014) aspirations emerge from individual's social environment, the approach followed here allows for individual and environmental factors to influence individuals' aspirations and outcomes, an addition to the contributions mentioned above.

Additionally, by linking the model's search and adoption type process to the work on frames in extended choice problems developed by Bernheim and Rangel (2007) and Salant and Rubinstein (2008), among others, the paper contributes with the inclusion of self-type unawareness as an explanation of how frames are formed. Furthermore, in the model type unawareness and type adoption emerge as a leading cause of rationally bounded behaviour. Concerning policy implications, two applications of the model offer interesting results. In one the effects of inequality of opportunities on type selection are analysed, formally showing how inequality produces low aspirations, and how them lead to sub-optimal type selection. Another application shows how certain policies that reduce the costs of type search, oriented at improving type selection can backfire leading individuals to opposite behaviour to the one intended.

In the following sections the main elements of the model of type search are presented, followed by the result of such search process. Then, the extended choice model is developed with adopted types as frames. Finally, some possible applications of the model are offered. A section with final comments including future extensions closes the paper.

2 Two motivating examples

Before the model is presented it is convenient to offer some motivating examples of how aspirations and the social environment of an individual can influence choice behaviour. The examples are purely anecdotal, nevertheless they offer insight into the core idea of the model.

2.1 Aspirational changes

This first case exemplifies how aspirations can be influenced by environments. The case presented involves an orphanages, run by catholic nuns, where the following events where observed. The orphanage was exclusively for girls, and offered primary and secondary education to the girls in the orphanage in a school located in the same premises that the orphanage occupied, schooling was also open to external children. The institution took care of the girls until they finished secondary school, then they had to decide how to continue their lives, some of them choosing a career path to follow. It was noticed that most of the girls that opted for a career path where choosing either to become school teachers, clerical staff, or nuns, precisely the type of activities they observed in their social environment composed predominantly by nuns, external teachers, and administrative staff.

The director of the orphanage decided to start a programme in which families would "godparent" one or more girls from the orphanage, inviting the girls to live with them for short periods of time, usually a couple of weekends per month, or for the whole summer vacation. Not all the girls entered into the programme, and the ones that entered where selected in a rather random fashion. After a period of time, it was observed that the girls who where in the programme started to choose paths different from the historical trend observed among other girls from the same institution. These paths included studying dentistry, law, business administration, among others. Although both groups continued to choose the common paths, one could easily observe the programme was changing the aspirations of the girls that participated in it, which was precisely one of the goals of the programme.

2.2 Environmental change

A second example illustrates how changes in environment could lead to behavioural changes. Countries usually differ in their laws, and also on how strictly those laws are enforced. When differences are marked between two countries sharing a border, it is often observed that individuals change behaviour when moving from one country to another. Take for example border sharing countries in Europe and America. In some countries the highway codes differ, and also the degree to which the codes are followed by drivers tend to be dissimilar. It is common to hear comments on how residents of one country drive carelessly, and engage in littering behaviour, and do not respect speed limits for example. But as soon as they cross the border to another country where residents show the opposite behaviour, those same individuals that usually misconduct change their behaviour once they move from one environment to the other. In this case, it is the environment that changes, and possibly the expected costs of engaging in inappropriate behaviour.

The elements mentioned in the previous examples: aspirations, social environment, and predominant behaviour, together with the benefits and costs experienced by individuals will be part of the key elements introduced in the model. The examples presented here do not exactly match the theoretical model, nor are they intended to claim that they are determined by the factors mentioned here, surely there is a complex line of causation in those behaviours including a variety of factors, still they illustrate the main features of the model, as well as the issues to be address in the model presented in what follows.

3 Model

The model develops as follows, agents are born in random environments without an identity, but in possession of an initial signal that constitutes incomplete information on their true type. They use this information to start a type search process that will end with the adoption of a type. In order to adopt a type, agents need to acquire characteristics, which are defined for each type and can be obtained by agents at a cost. To complement the initial signal, agents take into account information about the status quo of types in their environment, together with a sense of distance between types that allows them to distinguish how far types are from each other.

The rationale behind the inclusion of diverse environments is to take into account the fact that individuals are born under dissimilar circumstances, which provide them with social and institutional support that may vary in quantity and quality among different existent environments. By introducing the possibility of having various status quos across environments, the model takes into account that agents can have different reference types at which they can aim, mimicking the influence that certain types have on agent's aspiration formation, given the environment where they are immersed.

To adopt a type agents take into account the payoff they will perceive from adopting a given type, the cost of adopting such a type, and a *satisficing* criterion that consists in at least perceiving certain level of net payoff, this latter defined as the difference between the payoff and the cost of adopting a type. The assumption of agents following this criterion is in the spirit of the concept of *satisficing* introduced by Simon (1955).[2]

Concerning the payoff perceived from adopted types, assume that agents preferences over types are representable by a function $\pi : \Theta \rightarrow \mathbb{R}_+$ of class C^2, for which $\pi_\theta > 0,\ \pi_{\theta^2} \leq 0$, and such that $\pi_i = \pi(\theta_i^{'}) = \pi(\rho(\lambda_i^{'}))$. That is, $\pi(\cdot)$ has a positive and increasing at a decreasing rate valuation for types, and the values attained by the function for every type are specified for each characteristic that produces such types.

Type adoption is not a costless action. There are costs produced by the adoption of a type that are generated by the resources exerted to reach the possibility of adopting the type. On the one hand, acquiring characteristics is costly, e.g. obtaining a formal education diploma requires at least time and effort. On the other, not all environments possess the same provisions to help individuals in the enterprise of pursuing a type to adopt, just as not all towns in a country have a university campus at a walking distance. Also, even people in the same environment experience the world in different ways, they have different perceptions over what is achievable, and what is not, and how much effort is needed to reach a given goal. To introduce these features in the model, it is assumed that agents' costs depend on the characteristics, the status quo of their environment, and the distance defined over types. Let the costs of adopting type θ_k be represented by $\mathcal{C}_i = \mathcal{C}(\rho(\lambda_i^{'}), \theta_{\aleph_i}, \mathcal{M}_i)$, where $\rho(\lambda_i^{'}) = \Theta_i^{'}$. Assume C is a linear function of $\theta_\aleph$, and convex with respect to both θ and $\mathcal{M}$. Furthermore, assume $\mathcal{C}$ has an additive functional form composed by a linear function $C_{\aleph_i} = C_\aleph(\theta_{\aleph_i})$, for which $\dfrac{\partial C_\aleph}{\partial \theta_\aleph} < 0,\ \dfrac{\partial^2 C_\aleph}{\partial \theta_\aleph^2} = 0$, and a class C^2 function $C_i = C(\rho(\lambda_i^{'}), \mathcal{M}_i)$, with $\dfrac{\partial C}{\partial \theta} \geq 0,\ \dfrac{\partial^2 C}{\partial \theta^2} > 0,\ \dfrac{\partial C}{\partial \mathcal{M}} \geq 0,\ \dfrac{\partial^2 C}{\partial \mathcal{M}^2} > 0$. Thus, the general cost function $\mathcal{C}(\rho(\lambda_i^{'}), \theta_{\aleph_i}, \mathcal{M}_i)$ is convex and the signs of the partial derivatives are preserved.[3]

The net valuation agent i has on adopting a type $\theta^{'} \in \rho(\lambda^{'})$ is $\mathcal{V}(\subseteq^{'})$ and is equal to the difference between her valuation and her cost of adopting that particular type, that is $\mathcal{V}(\theta^{'}, \lambda^{'}, \theta_\aleph, \mathcal{M}) = \pi(\theta^{'}) - \mathcal{C}(\rho(\lambda^{'}), \theta_\aleph, \mathcal{M})$. Whenever $\mathcal{V} < 0$ the cost of adopting type $\theta^{'}$ surpasses the payoff of adopting that type, and thus type $\theta^{'}$ is not chosen. Furthermore, as criterion of type choice, we don't only require $\mathcal{V}$ to be positive, but also that it reaches at least a minimum threshold value to capture the idea of agents evaluating the worthiness of adopting a given type not only on the basis of private costs and payoff, but also on the valuation that a type has in each of the environments. Notice that imposing these threshold conditions the

[2]The concept of *satisficing* appears first in (Simon, 1997, p. 118-120). The world is a combination of the words "satisfy" and "suffice", and is meant to represent a heuristic choice procedure that does not necessarily involves an optimising criteria.

[3]That is $\mathcal{C} : \Lambda \times \Theta \times \mathcal{M} \rightarrow \mathbb{R}$. As the sum of convex functions produces a convex function we have $\mathcal{C}_\theta \geq 0, \mathcal{C}_{\theta^2} > 0,\ \mathcal{C}_{\theta_\aleph} > 0,\ \mathcal{C}_{\theta_\aleph^2} > 0,\ \mathcal{C}_\mathcal{M} > 0,\ \mathcal{C}_{\mathcal{M}^2} > 0.$

way individuals guide their type choice behaviour by taking into account their valuation and how their social environments bias the selection of each type, and that, by doing so, the demands on individuals being fully rational optimisers are relaxed. Define this threshold as $\Gamma_i = \Gamma(\lambda_i^t, \theta_{\aleph_i}, \mathcal{M}_i)$ where Γ is increasing in λ, that is, the higher the characteristics in the $\succsim_\Lambda$-ranking the higher the threshold, and also increasing in $\theta_{\aleph_i}$ as an indicator of what is acceptable in each environment, and what the agent should aim to according to what society dictates is the norm (the status quo). Feasibility of types is captured by $\mathcal{M}_i$, the distance from one type to another influences the threshold by informing on how hard it is, from the point of view of the individual, to reach any type from a given point in the types space.

Notice that the threshold just described can be interpreted as the aspirations of the individuals, summarising information on what is available, what is reachable, and what individuals should aim at according to the characteristics they posses, what can be expected from a member of the social environment the individual belongs to, and the beliefs the individual has on how difficult it is for them to reach a given type to adopt.

3.1 Types and characteristics

Let i indicate an individual in a shared environment $\aleph_i$. Let the set $\Theta \ni \theta,\ \theta \in [\underline{\theta}, \overline{\theta}]$ be a compact metric space with typical element θ, henceforth called a *type*, and $\underline{\theta}$ and $\overline{\theta}$ as the respective lower and upper bounds with finitely many types between them. Assume there is a complete partial order $\succsim_\Theta$ on Θ, that represents preferences over the elements of Θ. Define $\Omega := 2^\Theta \setminus \emptyset$ as the set of all non-empty subsets of the set of types Θ, and let $\Theta_h \in \Omega$ be one of those subsets, as Θ is finite Θ_h so is as well. Each Θ_h, as subsets of Θ, are bounded, additionally if the subsets are also closed then they are compact metric spaces as well. Observe that any preference $\succsim_{\Theta_h}$ respects $\succsim_\Theta$.

Let $\mathcal{L}$ be the total number of existent characteristics, and $\Lambda \subseteq \mathbb{R}_+^{\mathcal{L}}$ be a Riesz space to which the vectors of characteristics $\lambda \in \Lambda$ belong, and whose elements $\{\lambda_1, \lambda_2, \ldots, \lambda_{\mathcal{L}}\}$ indicate the magnitude of each characteristic[4]. Let $\succsim_\Lambda$ be a complete partial order for all elements in Λ. Define the pairs $(\Theta, \succsim_\Theta)$ and $(\Lambda, \succsim_\Lambda)$ as the corresponding complete partially ordered sets. Given that Θ is compact, $(\Theta, \succsim_\Theta)$ forms a complete lattice (Steen (1995, p. 67)), and thus its infimum and supremum exist. Additionally, each type $\theta^{'}$ has a vector of characteristics $\lambda^{'}$ that corresponds to it and accompanies that particular type, indicating the characteristics that each type possesses and in which magnitude, with a type $\theta^{'}$ being

[4]A Riesz space Γ is a real partially ordered vector space, with partial order $\succeq$, that satisfies: i) $\forall\, x, y, z \in A,\ x \geq y \implies x + z \geq y + z$, ii) $\lambda \geq 0, x \geq y \implies \lambda x \geq \lambda y$, iii) For any $x, y \in A$ there exists $sup(x, y) \in A$, $x \wedge y$, for the partial order $\geq$. The value that each characteristic acquires indicates either lack of the characteristic, if the value is equal to zero, or the presence of the characteristic and its magnitude, when different form zero, with higher values being associated to higher magnitudes.

better allotted in terms of $\succsim_\Lambda$-ranked characteristics in comparison to any other type θ'' if and only if $\theta' \succsim_\Theta \theta''$. As higher types are preferred to lower types as ranked by $\succsim_\Theta$, $\underline{\theta}$ contains the lowest $\succsim_\Lambda$-ranked characteristics and $\overline{\theta}$ the highest $\succsim_\Lambda$-ranked characteristics. To formalise these observations the relationship between characteristics and types is specified as a correspondence in the following definition

Definition 1. [Characteristics to types correspondence] Let $\rho : \Lambda \rightrightarrows 2^\Theta \setminus \emptyset$ be an order-preserving mapping from the set of characteristics to the set of types, that is, ρ defines which vector $\lambda' = \{\lambda_1', \lambda_2', \ldots, \lambda_{\mathcal{L}}'\}$ of characteristics corresponds to the subset of types $\Theta' \subseteq \Theta$.

The assumption of ρ being order-preserving is imposed to assure that if $\rho(\lambda') \ni \theta'$ and $\rho(\lambda'') \ni \theta''$, then $\theta' \succsim_\Theta \theta''$ if and only if $\lambda' \succsim_\Lambda \lambda''$; that is ρ will assign a higher $\succsim_\lambda$-ranked vector of characteristics to higher $\succsim_\Theta$-ranked types. The intuition behind this is simple: for higher types, more characteristics and/or characteristics of higher magnitude are needed, as higher types are preferred to lower types, vectors of characteristics that lead to higher types must be preferred to those that lead to lower types. Definition 1 specifies the *bridge* between characteristics and types aiming to represent a mental process on the part of the agents, but such processes correspond to observations that could potentially be confirmed by data sets.

Types in Θ are distributed across agents according to a density function $g(\theta)$ with c.d.f $G(\theta)$, each type with the corresponding vector of characteristics according to $\rho(\lambda_i) \ni \theta_i$. It is assumed that own types are unknown to the agents, however, they possess information about the distribution of types. In the model, own types will be referred to as the *true types*. Although the true type is unknown to each agent i, the agent receives a signal λ_i^o of her endowed characteristics, still, this signal is not complete and is not taken as the final set of characteristics that i possesses.

Assume $\Theta_i \in \Omega$ is the set of all types i could adopt given her characteristics. In order to complete the signal λ_i^o, each i searches for information on the types, and thus the characteristics, of other agents in the agent's current environment $\aleph_i$. Notice that, as types are different among themselves, individuals should be able to capture these differences in a way that is consistent to the ranking of types. Also information coming from each environment may be of different relevance to each agent depending on their own environment and on how close environments are to each other. A definition of agent's perceived distance between types is specified as follows

Definition 2. [Type-to-type distance] Let $\mathcal{M} : \Theta \times \Theta \to \mathbb{R}_+$ be a metric on Θ that completes the metric space $(\Theta, \mathcal{M})$. Define the type-to-type distance as the distance between two given types θ' and θ'' and denote this by $\mathcal{M}(\theta', \theta'')$.

The value of $\mathcal{M}(\cdot)$ gives a measure of the proximity, or the lack of, between types. A value of $\mathcal{M}(\theta^{'}, \theta^{''})$ close to $\mathcal{M}(\underline{\theta}, \overline{\theta})$ indicates that the difference between the types $\theta^{'}$ and $\theta^{''}$ is as big as possible, indicating that one of the two is either close to the top or the bottom, and the other near to the opposite end. Similarly, if the difference is close to zero, then we can infer that the two environments are close to each other according to this criterion. Thus, $\mathcal{M}$ gives a non-negative measure of how apart types are from each other, including representative types of each environment (status quo), these measures will be particularized to each agent i to focus on the perspectives of the agents. Notice as well that these measures are one-to-one comparisons and do not aggregate information, however aggregation can easily be done by summation over the status quo of all environments or particular types.

As will be seen later, it is argued that the distances between types influence the determination of agents' adopted types, together with i's initial signal λ_i^o. As both elements carry relevant information both should bear some weight in agent's type determinacy explanation.

3.2 Search environments

Assume now that the agent is randomly allocated to an environment placed in a continuum of environments $\aleph = [\underline{\aleph}, \overline{\aleph}] \ni \aleph_j$, with environments indexed by $j \in \mathcal{J}$. Each agent is assigned, at a starting period, to a particular environment $\aleph_j$ according to a continuous differentiable cumulative distribution function $F : \aleph \to [0, 1]$ with density f.

Define $\eta_{i,j}$ as the fraction of types θ_i located in group $\aleph_j$. Each environment could have one or various types with higher frequency than the rest of the types present in such environment, for simplicity assume there is only one such type. Such over represented type constitutes the status quo in that given environment, this is specified in the following definition

Definition 3. [Predominant type (Status quo)] A predominant type $\theta_{\aleph_j}$ in an environment $\aleph_j$ is a type defined as

$$\theta_{\aleph_j} = \{\theta_i \in \Theta \ \mid \ \eta_i(\aleph_j) \ > \ \eta_{i' \neq i}(\aleph_j) \, \forall \, \eta_{i' \neq i}(\aleph_j) \in \aleph_j\} \tag{1}$$

The type $\theta_{\aleph_j}$ represents the status quo of types in environment $\aleph_j$; where the status quo is the type of reference of those belonging to environment $\aleph_j$. For simplicity, and without loss of generality, $\eta_i(\aleph_j) \ = \ \eta_{i'' \neq i}(\aleph_j)$ for some $\eta_{i'' \neq i}(\aleph_j) \in \aleph_j$, that is more than one status quo existing in a given environment, is a possibility discarded in the definition.

Predominant types work as *aggregators* of information regarding the composition of environments, indicating not only which type is the most representative in terms of number, but also a way to rank types in terms of representativity within and across environments.

Using Definition 2, we can also define the status quo distance between the status quos in environment $\aleph_i$ and environment $\aleph_j$ as $\mathcal{M}(\theta_{\aleph_i}, \theta_{\aleph_j})$. Similarly, if $\aleph_i < \aleph_j$ then $\mathcal{M}(\overline{\theta}_{\aleph_i}, \underline{\theta}_{\aleph_j})$ gives an indication of the differences across environments i and j if $\overline{\theta}_{\aleph_j}, \underline{\theta}_{\aleph_i}$ are respectively the highest type in environment i and the lowest type in environment j respectively.

Now a description of how agents order the information about the types available in environments is introduced. It is assumed here that agents have full awareness of the type's space, and that they can form a complete ordering of such types and are able to form a type set list, that is an ordered list L of the elements of the types' set, with the order of the elements corresponding to the order relation $\succsim_\Theta$, the following definition specifies ordered lists in the context of this work

Definition 4. [Type set list] Recall $\Theta_K \subseteq \Omega := 2^\Theta \setminus \emptyset$. A list $L_K = L\Big(\Theta_K, \succsim_\Theta\Big) = \{\theta_k, \theta_{k+1}, \ldots, \theta_K\}$ on the set of types Θ^K is a sequential order of every $\theta \in \Theta_K$, using $\succsim_\Theta$ as criterion of order, and meeting the condition that whenever $\theta^{'}$ is placed after $\theta^{''}$ in $L_K(\cdot)$, $\mathcal{M}(\underline{\theta}_K, \theta^{'}) > \mathcal{M}(\underline{\theta}_K, \theta^{''})$. Let L_K^t be a list under consideration at stage t.

To make exposition clearer, the superscript t in lists, which indicates the stage at which the list is being considered, will be omitted unless it is necessary to specify it.

From Definition 4 we can derive a property of list and the sub-lists that can be formed from its elements. This property concerns the transferability of order and rank from sets of types to corresponding lists of types

Lemma 1. ***[List and sub-list elements order]*** *Fix L as the list of all elements in the set Θ ordered in accordance to $\succsim$. For any $\Theta_H, \Theta_K \subseteq \Omega$ and $L_H, L_K \in L$; if $\Theta_H \subset \Theta_K$ then $L_H \subset L_K$. If $\theta^{'}, \theta^{''} \in L_K, \theta^{'} \succsim \theta^{''}$, it is the case that $\theta^{'} \succsim \theta^{''}$ whenever $\theta^{'}, \theta^{''} \in L_H$. Call such a list $L_H \subseteq L_K$ a sub-list of L_K.*

Proof of Lemma 1. Let $\Theta_H \subset \Theta_K$, then there exists a subset $\{\theta\}^{'} \subset \Theta_K$, with at least one type, such that $\{\theta\}^{'} \not\subset \Theta_H,\ \{\theta\}^{'} \cup \Theta_H \subseteq \Theta_K$. From Definition 4 a list L_D contains only the elements of set Θ_D in ascending order of preference. Since for the set of types Θ_K, corresponding to the list L_K, and the set of types Θ_H, corresponding to the list L_H, we have that for all $\theta^{'} \in \Theta_H$ it must be the case that $\theta^{'} \in \Theta_K$, but not the converse, then there is a $\{\theta_i\}^{'} \subset L_K, \{\theta_i\}^{'} \not\subset L_H$, with $\{\theta\}^{'} \cup L_H \subseteq L_K$, and then $L_H \subset L_K$ □

These results define how lists can be divided in sub-lists that contain only a fraction of the elements contained in the *universal* list. Notice that $\Theta_H \subset \Theta_K$ implies $card\{\Theta_H\} < card\{\Theta_K\}$, and as $L_H \subset L_K$, it is also the case that $\#L_H < \#L_K$. Also, as lists contain all sub-lists that are exclusively formed by elements in the list, sub-lists inherit the ordering properties of lists that contain them, thus results found for one list, over elements contained in both lists, also hold for the other list. For ease of exposition, sub-lists will be referred to only when the context requires this, but references will be on lists for most of the definitions and results.

3.3 States and beliefs

In the previous sections the main informational elements about individuals and social environments with respect to types and characteristics have been introduced. Also, it has been established that, within the context of the model, this information is partially available to each agent. In this section the degree of information availability is defined for each individual in the form of perceptions hold by them, and are specified in informational states that summarise how individuals find themselves in terms of the information over types that they hold.

The information that each agent i takes into account at each stage t is defined by the *state* $\langle \lambda_i^t, \theta_{\aleph_i}, \mathcal{M}_i \rangle = \sigma_i^t \in \Sigma$, where Σ is the set of all possible states, λ_i^t is the vector of characteristics possesed by i at stage t, $\theta_{\aleph_j}$ is the status quo in i's environment, and $\mathcal{M}_i$ is a metric defined over Θ according to i's perceptions. Notice that the metric is defined for each i and thus we are assuming it can vary across agents, and also that the set of status quos and the metric remain constant with changes in t. Let the initial state σ_i^o be characterised by the triplet $\langle \lambda_i^o, \theta_{\aleph_i}, \mathcal{M}_i \rangle$.

σ_i^t specifies the information held by agent i at t. Thus, at each stage t the agent updates her informational state given the current status, incorporating updated information provided by the triplet that defines σ_i^t, that is, information updating is deterministic and depends on the vector of characteristics, the status quo, and the metric on type distance at each stage. Notice that, for an agent i, $\rho(\lambda_i^t)$ reports a subset of types Θ_i^t, thus it is implicitly assumed that the probability from the point of view of the agent of having a true type $\Theta_k \subseteq \Theta$ given state σ_i^t is not zero even if Θ_k is a singleton.[5] This is not considered a strong assumption, as if a vector of characteristics produces only one possible type, then it should be clear for the individual that the type produced is a possibility.

3.4 Type search process

Agent's type search starts at each stage t with the information available to the agent at that stage. As already specified, this information defines a current *state* that is described

[5]Thus if the subset of types θ_k includes types $\{\underline{\theta}_k, \ldots, \overline{\theta}_k\}$, then the probability from the point of view of agent i of being of type θ_k is given by $Pr(\theta_i = \theta_k \mid \sigma_i) = \int_{\underline{\theta}_k}^{\overline{\theta}_k} \theta f(\theta) d\theta$. This observation is not necessarily redundant as it permits to discard agents not considering the possibility of adopting a type because of the lack of probability of that type being one they can adopt, or even their true type.

by the characteristics at t, the status quo in the agent's environment, and the type-to-type distances. Agents use the information available at $t = 0$ to determine a *point of departure* from which they start their type-search process, and use updated informational structures to define a *search-departure type* at each stage t. The type search process finishes with a final product $\tilde{\theta}_i$ that is the *adopted-type* that i takes as a *satisfactory measure* of her type.

A search type process should also specify the direction of search that individuals take within the type's space that is being searched, in this case a list of types. Search processes can be assumed to proceed in different formats. For example, agents can search randomly through opportunity sets, testing types with no discernible order, or search unidirectionally, with the search process being determined by a departure point and a direction of search according to an established order. For the present setting it is assumed that agents search sequentially, either progressively or regressively, within a list of types as the one described in Definition 4. This assumption implicitly requires individuals having perfect recall, that is, agents know exactly where in the list they are positioned, where they have been, and retain all information derived from their past search in the list. This is clarified further below starting with the following assumption on sequential searching

Assumption 1. [Sequential type search process] Agent i's type search process on a list $L\Big(\Theta, \succsim_\Theta\Big)$ is sequential departing from a given type θ_i^t, continuing progressively, $\delta \nearrow \overline{\theta}$, by testing types of higher order $\theta_i^{t+1} > \theta_i^t$, or regressively, $\delta \searrow \underline{\theta}$, by testing types of lower order $\theta_i^{t-1} < \theta_i^t$. This search process starting from an initial type search $\theta_i^o \in L^o$, where L^o is an initial list.

Departing from Assumption 1, i's *search-departure type* at any stage t, given state σ_i^t, can be defined as follows

Definition 5. [Search-departure type] Let agent i's search departure type at stage t be the type from which agent i initiates her type search process, at that stage, given beliefs σ_i^t, and define it as

$$\begin{aligned} \theta_i^t &= inf\{\rho(\lambda_i^t \mid \sigma_i^t)\} \text{ if } \delta \nearrow \overline{\theta} \\ \theta_i^t &= sup\{\rho(\lambda_i^t \mid \sigma_i^t)\} \text{ if } \delta \searrow \underline{\theta} \end{aligned} \tag{2}$$

Let θ_i^o be the *initial search-departure type* for $t = 0$ and $\rho(\lambda_i^o \mid \sigma_i^o)$ in the specification above.

From Definition 5, all the information i possesses at t is contained in σ_i^t, this information gives i a (biased) perspective on the distribution of types across environments and is used to determine her search-departure type θ_i^t.

The valuation over types, the cost of adopting a type, and the threshold, described above are assumed to guide a type search process for the individuals. Each i searches the type's space for a type to adopt taking into account the information available to her at each state. Define now a search and stopping rule indicating when the agent is to continue searching for a type or stop and adopt the type reached at that stage of the type-search process. Clearly this stopping rule should require, to be convenient to the agent, the net valuation to be positive, this is imposed also as a requirement for the type adopted by any i. This is a form of bounded rationality, and is less restrictive than full rational behaviour, allowing for near optimising choice, without demanding from the individuals a choosing rule based on a strict optimisation process (Simon, 1955).

The search rule is specified as a heuristic criteria $\Phi\Big(L\Big(\Theta, \succsim_\Theta\Big), \mathcal{V}, \Gamma\Big)$, that takes into account the types available to the individual, in the form of a list L, whose elements $\theta \in \Theta$ are ordered according to the preference ordering $\succsim_\Theta$, the net valuation over types $\mathcal{V}$, and the threshold Γ. $\Phi(\cdot)$ is a heuristic rule that indicates if search is to be stopped or continued, and is based on the idea of *satisficing* criteria as described by Simon (1955), specifying the type that should be adopted under the considered parameters. Notice that at each stage for which no type has been adopted, the payoff perceived by the agent is zero, and the cost incurred in acquiring new characteristics, different from the once the agent already possesses, starts form zero as well, as characteristics already obtained in previous stages do not need to be acquired again.

Taking into account the elements described so far, an intuitive rule for type selection requires the net payoff to be positive when valued at the potential type to be adopted, and it should also be at least equal to the threshold value, this observation is formalised in the following assumption

Assumption 2. [Type search rule] A type-search rule $\Phi\Big(L\Big(\Theta, \succsim_\Theta\Big), \mathcal{V}, \Gamma\Big)$ indicates to the agent, in a given state $\sigma_i^t = \langle \lambda_i^t, \theta_{\aleph_i}, \mathcal{M}_i \rangle$, whether to continue or to stop searching for a type to adopt based on the type alternatives at stake and the preferences over them and the availability criteria represented by the list $L\Big(\Theta, \succsim_\Theta\Big)$, the net payoff $\mathcal{V}(\sigma_i^t)$, and the threshold $\Gamma(\sigma_i^t)$, determining search behaviour as follows

$\Phi(L(\Theta, \succsim_\Theta), \mathcal{V}, \Gamma) =$

$$\begin{cases} \text{i) Continue type search at stage } t \text{ if, for } \lambda^t \in \sigma_i^t,\ \theta^t = \rho(\lambda^t) \text{ is such that } \Gamma(\sigma_i^t) > \mathcal{V}(\sigma_i^t) \\ \\ \text{ii) Stop type search if : } \begin{cases} iia)\ \theta^t \text{ is such that } \mathcal{V}(\sigma_i^t) \geq \Gamma_i(\sigma_i^t); \text{adopt } \tilde{\theta}_i = \theta^t \in L^t(\Theta, \succsim_\Theta) \\ iib)\ \Gamma > \mathcal{V}\ \forall\ \theta \in \Theta; \text{adopt } \tilde{\theta}_i = \theta^* \in L^t(\Theta, \succsim_\Theta), \theta^* \in \operatorname{argmax} \mathcal{V} \end{cases} \end{cases} \quad (3)$$

Rule 3 in Assumption 2 describes individual's search behaviour. It indicates to stop searching for a type if either the threshold has been satisfied by the net valuation, or if it is never satisfied for any existing type. In the former case, the type to adopt, according to the search rule, is the first one for which the threshold is satisfied, for the latter the type to adopt is the one that renders the highest net valuation possible. In this case the intuition is evident, if an individual has a threshold large enough, she will search for a type to adopt through all those types up to the one that maximises $\mathcal{V}$, that is $\theta^* \in argmax\ \mathcal{V}$, and possibly one more to allow the agent to realise that θ^* is the type that optimises $\mathcal{V}$ as she searches though types on a list. This does not implies i is finding θ^* via mathematical optimisation (full rationality), but by trial and error, covering enough types until she discovers the one that maximises $\mathcal{V}$.

The description on how search processes take place, in terms of direction, as stated previously in Assumption 1 does not provide information on how the search process will actually take place. In particular, it is relevant to determine the direction of search to characterise type search and adoption behaviour of individuals. It has been already defined where the type search process starts, under which rules it operates, and over which space it takes place. The following result shows in which cases the agent will search progressively or regressively given the behavioural rules already specified

Proposition 2. ***[Search direction]*** *Given a state σ_i^t, if $\Gamma^t > \mathcal{V}^t$, the type-search direction $\delta(\sigma_i^t)$ is from above and towards $\underline{\theta}$ ($\searrow \underline{\theta}$) if condition $\dfrac{\partial \mathcal{V}(\theta^t)}{\partial \theta} < 0$ holds for $\rho(\lambda_i^t) = \theta^t$, with $\theta^t > \theta^{t+1}$. Conversely type-search direction is from below and towards $\overline{\theta}$ ($\nearrow \overline{\theta}$) if condition $\dfrac{\partial \mathcal{V}(\theta^t)}{\partial \theta} > 0$ is met, with $\theta^t < \theta^{t+1}$.*

Proof of Proposition 2. First notice that, according to Assumption 2, for any state σ_i^t, $\dfrac{\partial \mathcal{V}(\theta^t)}{\partial \theta}$ and θ, if $\Gamma \leq \mathcal{V}$ then the search process stops as the agent has either exactly reached or surpassed the threshold value, thus to have a search direction condition $\Gamma > \mathcal{V}$ is needed. This shows why the first part of Proposition 2 is needed. Now notice that, at θ^* condition $\dfrac{\partial \mathcal{V}(\theta^*)}{\partial \theta} = 0$ holds, with $\dfrac{\partial \mathcal{V}(\theta^t)}{\partial \theta} > 0$ holding before it, and $\dfrac{\partial \mathcal{V}(\theta^t)}{\partial \theta} < 0$ happening after that point, for both $\delta \nearrow \overline{\theta}$ and $\delta \searrow \underline{\theta}$.

When $\Gamma > \mathcal{V}$, the agent has not reached a *satisfactory* type and the agent's search process continues pursuing either higher types, $\theta^t < \theta^{t+1}$, or lower types, $\theta^t > \theta^{t+1}$, these two cases are covered in what follows in the proof.

Case 1. Assume first that $\theta^t < \theta^{t+1}$, then two outcomes are possible, either the net valuation increases or it decreases when testing a type at $t+1$, that is either $\mathcal{V}(\theta^t) > \mathcal{V}(\theta^{t+1}) > 0$ or $0 < \mathcal{V}(\theta^t) < \mathcal{V}(\theta^{t+1})$ is observed by the agent.

Case 1a. Assume the individual observes $\mathcal{V}(\theta^t) > \mathcal{V}(\theta^{t+1}) > 0$, that is $\frac{\partial \mathcal{V}(\theta^t)}{\partial \theta} < 0$, then the gap between π and $\mathcal{C}$ is closing from t to $t+1$, and thus i is searching in a neighbourhood of types located after θ^*. Notice that the difference between π and $\mathcal{C}$ is positive. By assumption $slope(\pi)$ is strictly decreasing and $slope(\mathcal{C})$ is strictly increasing in θ, then the gap that produces $\mathcal{V}$ will continue to close as θ increases, thus increases in θ work in detriment of $\mathcal{V}$. This trend will lead the agent to switch the direction of search either immediately or after some iterations in the same direction, with switching direction implying that search direction is $\searrow \underline{\theta}$.

Case 1b. If instead $0 < \mathcal{V}(\theta^t) < \mathcal{V}(\theta^{t+1})$ is observed by the individual, or equivalently $\frac{\partial \mathcal{V}(\theta^t)}{\partial \theta} > 0$, then the agent is searching in a neighbourhood of types located before θ^*. Searching for higher types provides i with enough incentives to keep searching as increases in θ produce increases in $\mathcal{V}$. These changes inform the agent the gap between π and $\mathcal{C}$ is becoming wider. Notice that, if the gap is increasing then the individual has not reached θ^* and then this trend will be sustained, but just up to θ^* as by assumption $slope(\pi)$ is strictly decreasing and $slope(\mathcal{C})$ is strictly increasing in θ. This provides incentives to the agent to keep searching in the direction to which θ increases. Thus, the direction of search in this case will be $\nearrow \overline{\theta}$.

The proof for the case in which $\theta^t > \theta^{t+1}$ is symmetric to Case 1, and is provided for completeness.

Case 2. Assume that $\theta^t > \theta^{t+1}$, two outcomes are possible at $t+1$, case 2a: $\mathcal{V}(\theta^t) > \mathcal{V}(\theta^{t+1}) > 0$, or case 2b: $0 < \mathcal{V}(\theta^t) < \mathcal{V}(\theta^{t+1})$.

Case 2a. Assume $\mathcal{V}(\theta^t) > \mathcal{V}(\theta^{t+1}) > 0$, then the gap between π and $\mathcal{C}$ is closing from t to $t+1$. By assumption $slope(\pi)$ is strictly decreasing and $slope(\mathcal{C})$ is strictly increasing in θ, and thus $\mathcal{V}$ will continue to close as θ decreases. This trend will lead the agent to switch the direction of search after some iterations (or none) in the same direction, towards searching for higher types, that is $\nearrow \overline{\theta}$.

Case 2b. Assume $0 < \mathcal{V}(\theta^t) < \mathcal{V}(\theta^{t+1})$, then the agent has incentives to keep searching in the same direction, that is in the direction in which θ decreases, as these changes produce increases in $\mathcal{V}$. Again, this trend will be sustained up to θ^*, as $slope(\pi)$ is strictly decreasing and $slope(\mathcal{C})$ is strictly increasing in θ by assumption. This trend generates incentives for the agent to keep searching in the direction to which θ decreases. Thus, the direction of search in this case will be $\searrow \underline{\theta}$.

$\square$

Proposition 2 implies individuals will define their direction of choice based on how convenient, in terms of perceived net payoffs, it is to search in one direction compared to the opposite one. The result also states the direction will be preserved all along the search process, adding consistency to the search process by preventing individuals from searching types they have already discarded as not appropriate for type adoption.

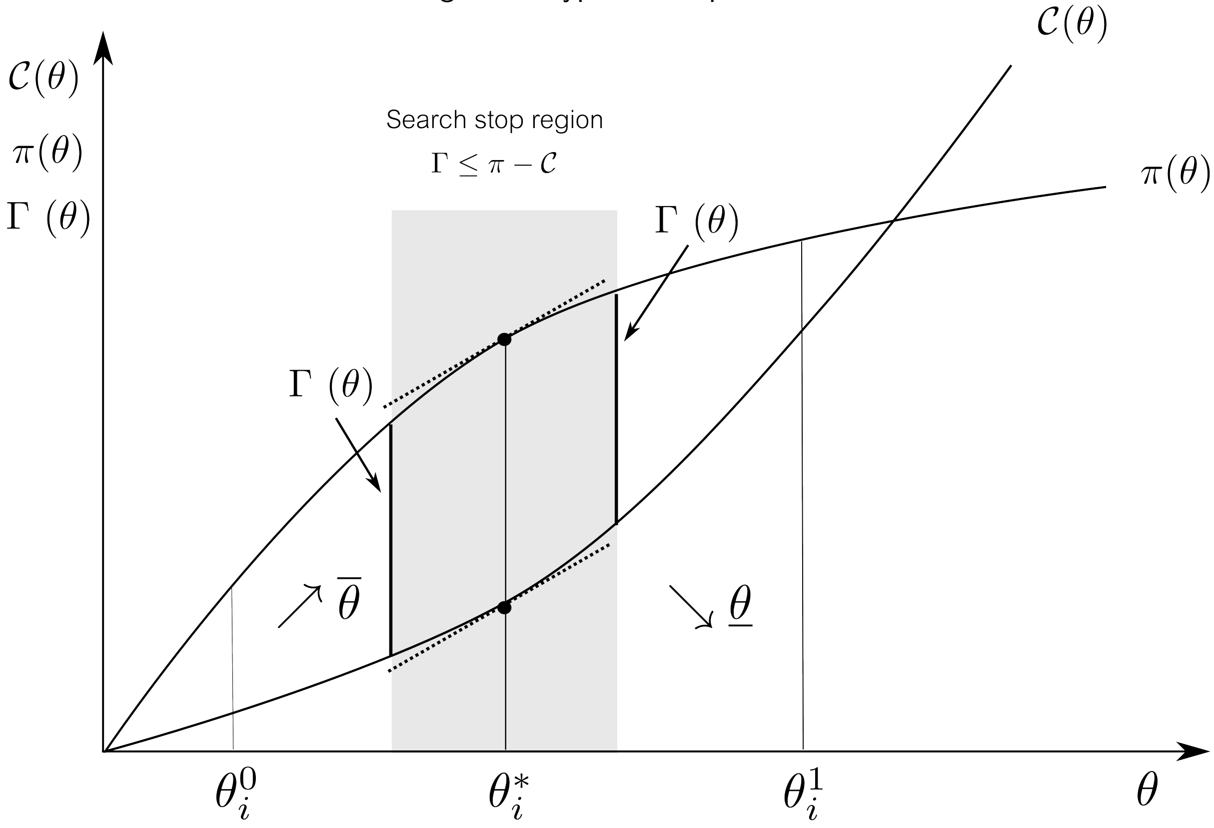
Figure 1 presents in a graphical manner the search process as previously described with all its components, it also shows the results obtained in Proposition 2. In the figure, we can appreciate how the convex cost function $\mathcal{C}(\cdot)$ and the concave payoff function $\pi(\cdot)$ form a locus within which a positive net payoff, $\mathcal{V}$, is perceived by agent i. The shadowed square shows the area in which condition $\Gamma \leq \pi - \mathcal{C}$ holds, that is the area in which searching stops. The optimal type to adopt is θ_i^*, for this type the net payoff is as big as possible. If the agent search departure type is θ_i^0, the agent will search for a type to adopt in direction $\nearrow \overline{\theta}$, while if θ_i^1 is the search departure type the agent will search in direction $\searrow \underline{\theta}$. The idea behind this behaviour is that for those two search departure types, the corresponding directions of search are the ones that produce new types to test for adoption with increasing net payoffs, this is just what Proposition 2 shows.

3.5 Search neighbourhood extension

A situation that can arise during the type-search process is that, for the conditions stated in Assumption 2, the agent may not reach a type to adopt in the list L^t in which the search process is contained at a particular stage t. That is, condition $\Gamma(\lambda^{'}, \theta_{\aleph}, \mathcal{M}) \geq \mathcal{V}(\theta^{'}, \theta_{\aleph}, \mathcal{M})$ for all $\theta^{'} \in L^t \subset L$ may hold for the whole list or search neighbourhood where the search process is taking place at a given stage t. In such a case, i will need to continue the search process in another neighbourhood of types. Here we define a search neighbourhood and a search continuation neighbourhood that describe what happens when these case arises. Also, a result is presented showing the conditions that need to hold for i to continue the search process in a new neighbourhood.

Definition 6. [Search neighbourhood] A *search neighbourhood* for i at stage t is formed by the subset of types Θ_i^t, whose elements correspond to the types reachable by the characteristics possessed by i at that stage. That is, $\Theta_i^t = \{\underline{\theta}_{\Theta_i^t}, \overline{\theta}_{\Theta_i^t}\} = \{\theta \in \Theta : \theta = \rho(\lambda_i^t), \lambda_i^t \in \Lambda\}$. The corresponding type list being $L^t\Big(\Theta_i^t, \succsim_{\Theta}\Big)$.

Figure 1: Type search process



According to Definition 6, the initial search neighbourhood is given by $\Theta_i^o = \{\theta \in \Theta : \theta = \rho(\lambda_i^o)\}$. Indeed this is the subset of types that correspond to the initial characteristics as perceived by the signal λ_i^o. An initial search neighbourhood produces the subset of types $\Theta_i^o \subseteq \Theta$ that corresponds to the signal λ_i^o. Then, given a search direction as specified in Proposition 2, the agent searches $L^o\Big(\Theta_i^o, \succsim_\Theta\Big)$ until she finds $\theta' \in \Theta_i^o$ such that $\mathcal{V}(\theta') \geq \mathit{\Gamma}(\theta')$ as the search rule indicates in Assumption 2. If such a type is found, then i adopts $\tilde{\theta} = \theta'$. Otherwise she continues searching in the next search neighbourhood, which is defined as follows

Definition 7. [Continuation search neighbourhood] Define a *continuation search neighbourhood* for i at stage $t+1$ as $\Theta_i^{t+1} = \{\theta \in \Theta : \theta = \rho(\lambda_i^{t+1})\}$; $\Theta_i^{t+1} \subseteq \Theta \setminus \bigcup\limits_{k \leq t} \Theta_i^k$. With $\lambda_i^{t+1} = \lambda_i^t + \Delta\lambda_i^t$, and $\Delta\lambda_i^t$ equal to the change in λ_i^t.

Definition 7 says that if at any stage t an agent searches within the neighbourhood without reaching a type to adopt conforming to the search rule previously described, then i will search in a new neighbourhood that is a continuation of the types already tested. The initial departure type given beliefs σ_i^o is $\theta_i^o \in \rho(\lambda_i^o \mid \sigma_i^o)$, with $\sigma_i^o = \langle \lambda_i^o, \theta_{\aleph_i}, \mathcal{M}_i \rangle$. A key determinant of this initial stage is the characteristics signal the agent receives, λ_i^o, which partially defines the search departure type θ_i^o, and which is the only variable that can be modified by i in the following stages if the proper incentives exist.[6] Such a change in λ occurring at a stage t is denoted by $\Delta\lambda_i^t$, and the new vector of characteristics is $\lambda_i^{t+1} = \lambda_i^t + \Delta\lambda_i^t$.

These definitions do not state if it is rational for i to acquire the characteristics needed to proceed to the next type search neighbourhood. The following result clarifies on this

Lemma 3. ***[Characteristics acquisition incentives]*** *If a type-search process is in place at t, and no type has been adopted after testing all types in $L^t(\cdot)$. Then, the individual will continue the search process in a new neighbourhood of types if the following conditions hold*

$$\textit{For } \delta(\sigma_i^t) =\nearrow \overline{\theta} : \quad \frac{\partial \pi(\rho(\lambda))}{\partial \lambda} \geq \frac{\partial C(\lambda, \cdot)}{\partial \lambda} \tag{4}$$

$$\textit{For } \delta(\sigma_i^t) =\searrow \overline{\theta} : \quad abs\left(\frac{\partial \pi(\rho(\lambda))}{\partial \lambda}\right) \leq abs\left(\frac{\partial C(\lambda, \cdot)}{\partial \lambda}\right) \tag{5}$$

Proof of Lemma 3. Notice that the conditions stated in Equation 4 and Equation 5 just require the individual to have proper incentives to acquire the characteristics needed to proceed to search in a new neighbourhood of types. If the individual is searching towards higher types, $\delta(\sigma_i^t) =\nearrow \overline{\theta}$, then the increase in payoffs needs to be higher than the increase in cost of acquiring those characteristics. Similarly, if the agent is searching towards lower types, then the condition states that the decrease in payoffs should be smaller than the decrease in costs of acquiring the characteristic needed to have access to those lower types.

To show that these are the cases actually observed in the search process, recall form Proposition 2 that at θ^* condition $\dfrac{\partial \mathcal{V}(\theta^*)}{\partial \theta} = 0$ holds. Before point θ^*, from the left and right, we observe that $\mathcal{V}$ is increasing in θ, which means either that π is increasing at a higher rate than C, or that C is decreasing at a higher rate than π. Given convexity and concavity assumptions on C and π we know that to the left of θ^* the payoffs are increasing in θ at a higher rate than the costs, this is the condition stated in Equation 4. Similarly, for the same reasons, to the right of θ^* the costs decrease at a higher rate than that of the payoffs, this is what Equation 5 expresses.

□

What Lemma 3 states is that a necessary condition for $\Delta\lambda_i^t$ to take place is either to observe that when the search direction is towards higher types, the increase in payoffs of acquiring the characteristics needed to have access to the next set of types should be higher

[6] That is the payoff of increasing λ should be at least equal to the costs.

that the increase in costs of acquiring them. The opposite happens if the search direction if towards lower types, in this case the absolute value of the change in cost of acquiring the characteristics should be larger than the absolute change in payoffs that they generate. Observe that, according to the Lemma, these conditions hold when the individual is still searching for a type, otherwise there is no incentive to obtain more characteristics.

3.6 Type adoption

The result of the search process is an adopted type, in this section this outcome is described in a pair of results. These results specify the situations under which there is a match between the adopted type and the true type, and those under which a mismatch emerges. The process depends on the characteristics possessed by the individual (including the initial signal), the set of status quos, and the metric over types. Notice there is an implicit evolution of the agent's characteristics that emerges through stages. All these information is summarised in a history of states $\sigma_i = \{\sigma_i^o, \sigma_i^1, \ldots, \}$ that constitute the perceptions that drive agent i's type choice. Now results for the existence of an adopted type are provided.

Proposition 4. ***[Existence of adopted type]*** *Assume $\rho(\lambda^o) \neq \theta^*$. For all i, at some stage t, given a state σ_i^t, and direction $\delta(\sigma_i^o)$, under a search rule as stated in Assumption 2, there exists a $\tilde{\theta}_i \in \Theta_i^t$ such that $\tilde{\theta}_i$ is i's adopted type. Furthermore, if $\mathit{\Gamma} > \mathcal{V}\ \forall\ \theta \in \Theta$, then there is some $\theta_{\Gamma \geq \mathcal{V}}$ that will be reached at some stage t and will be adopted, with such type being $\theta_{\Gamma \geq \mathcal{V}} = \tilde{\theta}_i = \theta^* \in \operatorname{argmax} \mathcal{V}$.*

Proof of Proposition 4. Notice that conditions $\mathit{\Gamma} \leq \mathcal{V}$ for some $\theta \in \Theta$ or $\mathit{\Gamma} > \mathcal{V}\ \forall\ \theta \in \Theta$ must be reached in the closure of Θ. Also, observe that from the concavity of π and the convexity of $\mathcal{C}$, $\exists\ \theta^*$ for which $\dfrac{\partial \pi(\theta^*)}{\partial \theta} = \dfrac{\partial \mathcal{C}(\theta^*)}{\partial \theta},\ \theta^* \in \operatorname{argmax} \mathcal{V}$.

The proof consists on showing that either there exists $\theta \in \Theta$ such that $\tilde{\theta} = \theta \in \Theta \backslash \theta^*,\ \theta^* \in \operatorname{argmax} \mathcal{V}$, for $\mathit{\Gamma} \leq \mathcal{V}$ or that $\tilde{\theta} = \theta^*$ if $\mathit{\Gamma} > \mathcal{V}\ \forall\ \theta \in \Theta$. In the former case, for both $\delta = \nearrow \overline{\theta},\ \delta = \searrow \underline{\theta}\ , \exists\ \Theta^t \subseteq \Theta$ for some t such that $\mathit{\Gamma} \leq \mathcal{V}$. The proof is made here for $\delta = \nearrow \overline{\theta}$, a parallel proof can be made for $\delta = \searrow \underline{\theta}$ by following a similar argument.

Notice that, if $\inf\{\rho(\lambda_i^o)\} = \theta_i^*$ then $\Gamma(\theta^*) = \subseteq^*$, and the type search process finishes with $\tilde{\theta}_i = \theta_i^*$. This case is excluded from the proposition. Thus a type search process starts with either $\theta_i^o < \theta_i^*$ or $\theta_i^o > \theta_i^*$. Then either $\mathit{\Gamma} \leq \mathcal{V}$ for some $\theta \in \Theta$ should be reached within the interval $(\theta_i^o, \theta_i^*]$ if $\theta_i^o \in [\underline{\theta}, \theta_i^*]$, or the interval $[\theta_i^*, \theta_i^o)$ if $\theta_i^o \in [\theta_i^*, \overline{\theta}]$. Otherwise condition $\mathit{\Gamma} > \mathcal{V}\ \forall\ \theta \in \Theta$ holds. For each case the search rule specified in Assumption 2 indicates which search behaviour i will adopt. Let $\Gamma > \mathcal{V}\ \forall\ \theta \in \Theta$ be Case 1, and $\mathit{\Gamma} \leq \mathcal{V}$ for some $\theta \in \Theta$ be Case 2.

Case 1: Assume $\mathit{\Gamma} > \mathcal{V}\ \forall\ \theta \in \Theta$ holds. Then the maximum for $\mathcal{V}$ that i can reach is precisely $\mathcal{V}(\theta^*)$. In this case, for any $\theta^{'} \in \Theta \setminus \theta^*$ is to the right or to the left of θ^*, the type search procedure will produce a net payoff $\mathcal{V}(\theta) < \mathcal{V}(\theta^*)$, this from the conditions of convexity and concavity of the cost and payoff function. Thus for any of $\delta =\nearrow \overline{\theta}$ or $\delta =\searrow \overline{\theta}$ the type adopted will be $\tilde{\theta} = \theta_{\Gamma > \mathcal{V}} = \theta^*$. In this case, i exhausts the set of all types in the range of types between θ_i^o and θ_i^* searching for a type to adopt, with the search ending with the adoption of θ_i^* according to Assumption 2.

Case 2: Assume $\Gamma(\theta_i^{'}, \cdot) \leq \mathcal{V}(\theta_i^{'}, \cdot)$ for some $\theta_i^{'} \in \Theta$. Either Case 2a: $\theta_i^o \in [\underline{\theta}, \theta_i^*]$ holds, or Case 2b: $\theta_i^o \in [\theta_i^*, \overline{\theta}]$ does.

Case 2a. Assume $\theta_i^o \in [\underline{\theta}, \theta_i^*]$. Then, i will search for a type according to Assumption 2, until finding $\tilde{\theta}_i \in \rho(\lambda_i^t)$ for λ_i^t at some t, including $t = 0$. If $\tilde{\theta}_i \in \rho(\lambda_i^o) = \Theta_i^o$, then i searches within $L(\Theta_i^o, \succsim_\Theta)$ with direction $\delta_i =\nearrow \overline{\theta}$ until condition $\Gamma \leq \mathcal{V}$ holds and the search process finishes with $\tilde{\theta}_i \in \Theta_i^o \subseteq (\theta_i^o, \theta_i^*]$. If $\Gamma(\theta_i^{'}, \cdot) > \mathcal{V}(\theta_i^{'}, \cdot)$ for all $\theta \in \Theta_i^o$, then $\tilde{\theta}_i \notin \Theta_i^o$ and $\Theta_i^o \subsetneq (\theta_i^o, \theta_i^*]$. Under this case i continues the search process at subsequent stages h, with $\tilde{\theta}_i \notin \bigcup\limits_{h<t} \Theta^h$. For conditions stated in Lemma 3, i acquires characteristics λ_i^h at each stage h, continuing her type search in new neighbourhoods $\Theta_i^t \subseteq \Theta \setminus \bigcup\limits_{h<t} \Theta^h$ as specified in Definition 7, until condition $\Gamma(\theta_i^{''}, \cdot) \leq \mathcal{V}(\theta_i^{''}, \cdot)$ holds for some $\theta_i^{''} \in \Theta_i^t \subseteq (\overline{\theta}_{\Theta_i^{t-1}}, \theta_i^*]$, $\overline{\theta}_{\Theta_i^{t-1}}$ the supremum of Θ_i^{t-1}, and $\tilde{\theta}_i = \theta_i^{''}$. Condition $\theta_i^{''} \not> \theta_i^*$ holds, for if this were the case i would realise there is a θ^* and would adopt it, but this is Case 1, with $\mathcal{V}(\theta_i^{''}) < \mathcal{V}(\theta_i^*)$ from the convexity and concavity of the cost and payoff functions.

Case 2b. This part is the symmetric version of Case 2a. Assume $\theta_i^o \in [\theta_i^*, \theta_i^o)$. Then, i will search for a type according to Assumption 2, until finding $\tilde{\theta}_i \in \rho(\lambda_i^t)$ for λ_i^t at some t, including $t = 0$. If $\tilde{\theta}_i \in \rho(\lambda_i^o) = \Theta_i^o$, then i searches within $L(\Theta_i^o, \succsim_\Theta)$ with direction $\delta_i =\searrow \underline{\theta}$ until condition $\Gamma \leq \mathcal{V}$ holds and the search process finishes with $\tilde{\theta}_i \in \Theta_i^o \subseteq [\theta_i^*, \theta_i^o)$. If $\Gamma(\theta_i^{'}, \cdot) > \mathcal{V}(\theta_i^{'}, \cdot)$ for all $\theta \in \Theta_i^o$, then $\tilde{\theta}_i \notin \Theta_i^o$ and $\Theta_i^o \subsetneq (\theta_i^o, \theta_i^*]$. Under this case i continues the search process at subsequent stages h, with $\tilde{\theta}_i \notin \bigcup\limits_{h<t} \Theta^h$. For conditions stated in Lemma 3, i acquires characteristics λ_i^t at each stage t, continuing her type search in new neighbourhoods $\Theta_i^t \subseteq \Theta \setminus \bigcup\limits_{h<t} \Theta^h$ as specified in Definition 7, until condition $\Gamma(\theta_i^{''}, \cdot) \leq \mathcal{V}(\theta_i^{''}, \cdot)$ holds for some $\theta_i^{''} \in \Theta_i^t \subseteq [\theta_i^*, \underline{\theta}_{\Theta_i^{t-1}})$, $\underline{\theta}_{\Theta_i^{t-1}}$ the infimum of Θ_i^{t-1}, and $\tilde{\theta}_i = \theta_i^{''}$. Condition $\theta_i^{''} \not< \theta_i^*$ holds, as if this were the case $\mathcal{V}(\theta_i^{''}) < \mathcal{V}(\theta_i^*)$ from the convexity and concavity of the cost and payoff functions, i will adopt θ_i^* which is Case 1, a possibility already discarded.

□

Corollary. ***[Adopted type mismatch]*** *Any adopted type will be a mismatch, $\tilde{\theta}_i \neq \theta_i^*$, unless $\Gamma(\lambda_i^*, \theta_{\aleph_i}, \mathcal{M}_i) = \mathcal{V}(\theta^*)$, for $\theta_i^* = \rho(\lambda_i^*)$; or $\Gamma(\lambda_i^t, \theta_{\aleph_i}, \mathcal{M}_i) > \mathcal{V}(\theta, \cdot)\ \forall\ t,\ \theta \in \Theta$.*

Proof of Corollary of Proposition 4. These results arise directly form Proposition 4 and Assumption 2. If λ_i^* is such that $\rho(\lambda_i^*)$ produces the true type θ_i^* when condition $\Gamma(\lambda_i^*, \theta_{\aleph_i}, \mathcal{M}_i) = \mathcal{V}(\theta^*)$ is met, then it is clear the adopted type will be her true type indeed. The second part of the claim states the individual will adopt her true type if it exhausts all types up to the optimal type (and possibly one more) searching in one direction according to Assumption 2. As the searching process does not produces a type to adopt then the agent will observe the best option is her true type and will indeed select it as her adopted type. This is Case 1 of Proposition 4.

□

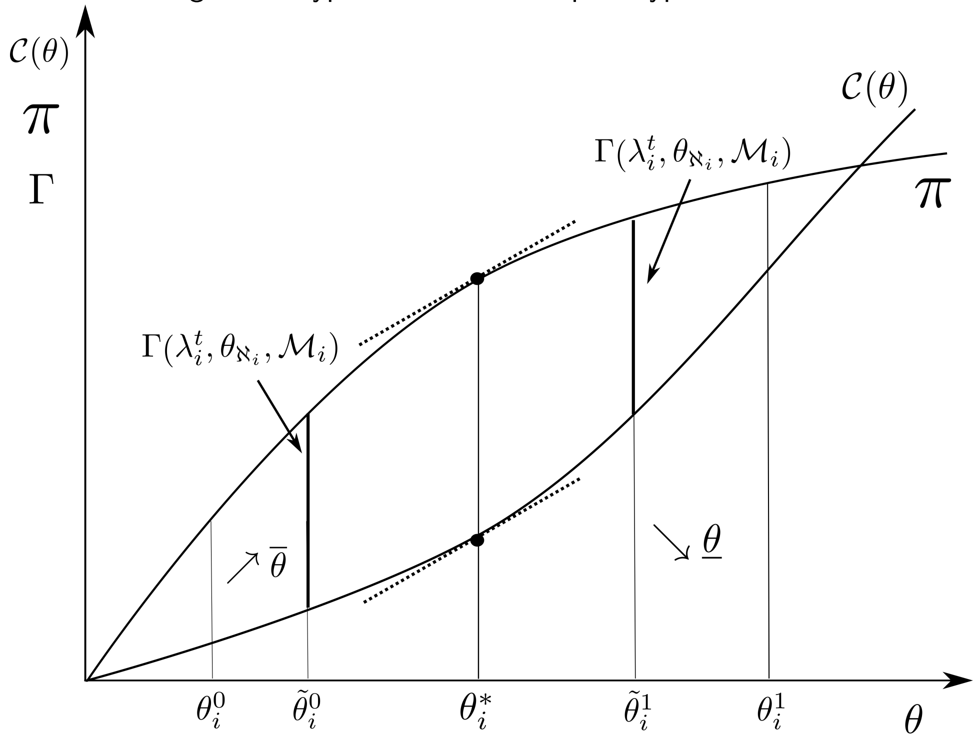
Proposition 4 shows a type will always be adopted under the the stated conditions, and its Corollary specify the conditions that lead to a match between the adopted type and the true type. It was shown that, given a history of states σ_i and a search direction $\delta(\sigma_i^o)$, an agent will adopt a type $\tilde{\theta}_i \in \Theta_i^t$ at some stage t, with the following possible outcomes for a given true type θ^*: either $\tilde{\theta}_i = \theta^*$ or $\tilde{\theta}_i \neq \theta^*$. In the former case no implications arise as i adopts a type that matches her true type and thus the full rationality results apply. On the contrary, the latter case presents a situation in which the type adopted by i differs from i's true type, in this case additional considerations need to be taken in choice analysis as each i in this situation would chose as if her choices did not agree with fully rational behaviour. Indeed, the choice of type corresponds to a boundedly rational behaviour, in which rationality is limited by the search rule that guides i's choice of type. This search rule has as distinctive element a threshold, which resembles the idea of aspirations.

These results are illustrated in Figure 2. In the figure it can be seen that for an initial search type θ_i^0 the direction of search is towards higher types, and with a threshold equal to $\Gamma(\theta)$, the adopted type ends up being $\tilde{\theta}_i^0$, which falls short of the optimal type θ_i^*. Similarly, if the initial search type is θ_i^1 the direction of search is towards lower types, and for a threshold $\Gamma(\theta)$, the adopted type is $\tilde{\theta}_i^1$, which is higher than the optimal type θ_i^*. Both cases present mismatches of adopted type with the true type. Observe that, unless $\Gamma(\cdot) > \mathcal{V}(\cdot)\ \forall\ \theta \in \Theta$ or $\Gamma(\lambda_i^*) = \mathcal{V}(\theta_i^*)$ for $\rho(\lambda_i^*) = \theta_i^* \in \operatorname{argmax} \mathcal{V}(\cdot)$ a mismatch will emerge.

3.7 Choice with types as frames

So far the focus has been placed on type search and type adoption processes. The results obtained indicate that a mismatch between the agent's adopted type and her true type can emerge when the threshold (aspirations) of the individual bound their type selection behaviour. In this section the focus turns to subsequent choices when type adoption acts as a pre-choice process.

Figure 2: Type search with adopted type mismatch



Assume the agent goes trough two phases: e_1 and e_2. In e_1 the individual selects a type $\tilde{\theta}_i$, and in e_2 she faces a choice problem guided by the type output in e_1. e_1 can be a process in which i selects the degree of information she will gather before making a choice in e_2, for example the agent can select a level of formal education to obtain before entering the job market, may search for information regarding a good she wants to acquire, or she may consult a number of specialists before selecting which medical procedure is the one that she will have. In terms of the previous analysis the educational level, the amount of information, or the number of specialists she consults are the adopted type, while the types of jobs selected in the job market, the goods to be chosen, or the selected medical treatment are the choices to be made in e_2.

To proceed with this analysis an extended choice problem is defined. An extended choice problem is formed by an opportunity set, and a set of frames that can alter the choice process without necessarily having any rational fundamental. Both the opportunity set and the set of frames conform the choice problem faced by the individual. This framework has been developed in both Bernheim and Rangel (2007) and Salant and Rubinstein (2008), and is used here to show how such frames can arise in the type adoption model presented.

Following Rubinstein (2012), let X be the finite set of all available alternatives, $\mathcal{X} := 2^X \setminus \{\emptyset\}$, a class in X containing all non-empty subsets of X; and $\mathcal{A} \subseteq \mathcal{X}$ a *consideration set*, that is, a set that contains only the options to be considered by the individual. An *extended choice set* $\{X, f\}$ includes a choice set X and a frame $f \in F$, with the set of all frames denoted by F. An extended choice set, "expands" the standard choice set with the inclusion of an additional criteria of relevance to the agent, when selecting an option from a variety of alternatives. It is thus a useful tool for the analysis of decision making when the individual restricts choice to a consideration set. The extended choice set requires a choice function that contemplates this "extension". An extended choice correspondence $c(\{X, f\})$ selects a unique option $\{x\} \subseteq \mathcal{X}$ from the choice problem $\{X, f\}$, notice that the choice $\{x\}$ can be a singleton or a subset of X. Define a consideration set as follows

Definition 8. [Consideration set] A *consideration set* $\mathcal{A} := \{X, f\} \subset \mathcal{X}$ is a set that contains only the choices from X that will be considered by the individual when facing choice problem $\{X, f\}$ given a frame $f \in F$.

At this point, it is worth clarifying what is considered a frame in this context. Here, as in Salant and Rubinstein (2008), a frame is not additional information that can be of relevance for a rational decision to take place. In the type search framework presented, i's true type can (should) be of relevance when choosing from the set of viable alternatives, as the true type can reveal rational behaviour on the individual, and thus, i's true type is not considered as a frame. A different situation emerges if, an adopted type distinct from the true type is used to define the set of choices to be considered. In this case i's adopted type can lead her to select choices she would not have considered from the set X, had she adopted her true type.

Thus, when frames are absent, under full information, the choice problem is determined by the set of alternatives that are precisely available to i and that are reachable to her. When this is not the case, frames constitute distractors that can lead choice behaviour in a bounded manner. This is introduced here as an assumption explicitly requiring each possible consideration set to be attached to a particular type

Definition 9. [Consideration set by type] Given a type $\theta' \in \Theta$, there exists a unique set $\mathcal{A}_{\theta'} \in \mathcal{X}$ to be referred to as θ' *consideration set*, this set contains only the alternatives that those i's of type θ' will take into account, given that they are of type θ'. For a true type θ_i^*, $\mathcal{A}_{\theta^*}$ denotes i's consideration set when $\tilde{\theta}_i = \theta_i^*$.

According to Definition 9, a consideration set corresponds to every i's true and adopted types. The following results shows when an adopted type can be considered a frame, and if it is the case that such type-frames lead to suboptimal choices under all possible scenarios

Proposition 5. ***[Adopted type as frame]*** *An adopted type $\tilde{\theta}_i$ can be considered a frame f, if and only if it is not equal to i's true type, $\tilde{\theta}_i \neq \theta_i^*$, and $x \in \operatorname{argmax} \succsim_i \notin \mathcal{A}_{\theta_i^*} \cap \mathcal{A}_{\tilde{\theta}_i}$.*

Proof of Proposition 5. Assume $\tilde{\theta}_i \neq \theta_i^*$, then by Definition 9 $\mathcal{A}_{\tilde{\theta}_i} \neq \mathcal{A}_{\theta_i}$. For the two distinct sets, two possibilities arise, either $\mathcal{A}_{\tilde{\theta}_i} \cap \mathcal{A}_{\theta_i^*} = \emptyset$ or $\mathcal{A}_{\tilde{\theta}_i} \cap \mathcal{A}_{\theta_i^*} \neq \emptyset$. If $\mathcal{A}_{\tilde{\theta}_i} \cap \mathcal{A}_{\theta_i^*} = \emptyset$ then $\forall\, x^{'} \in \mathrm{argmax} \succsim_i (\mathcal{A}_{\tilde{\theta}_i}),\ x^{'} \notin \mathrm{argmax} \succsim_i (\mathcal{A}_{\theta_i^*})$, thus it is not possible for i to choose $x^* \in argmax \succsim^i (\mathcal{A}_{\theta_i^*})$ having adopted a type different from her true type.

If $\mathcal{A}_{\tilde{\theta}_i} \cap \mathcal{A}_{\theta_i^*} \neq \emptyset$, then for $x^{'} \in \mathrm{argmax}(\mathcal{A}_{\tilde{\theta}_i})$ either $x^{'} \in \left(\mathcal{A}_{\tilde{\theta}_i} \cap \mathcal{A}_{\theta_i^*}\right)$ or $x^{'} \notin \left(\mathcal{A}_{\tilde{\theta}_i} \cap \mathcal{A}_{\theta_i^*}\right)$. If $x^{'} \in \left(\mathcal{A}_{\tilde{\theta}_i} \cap \mathcal{A}_{\theta_i^*}\right)$, $x^{''} \in \mathrm{argmax} \succsim_i (\mathcal{A}_{\theta_i^*})$ if $x^{''} \in \left(\mathcal{A}_{\tilde{\theta}_i} \cap \mathcal{A}_{\theta_i^*}\right)$ then it must be the case that $x^{'} = x^{''}$, that is $\tilde{\theta}_i$ is not a frame. If on the contrary $x^{''} \notin \left(\mathcal{A}_{\tilde{\theta}_i} \cap \mathcal{A}_{\theta_i^*}\right)$ then i chooses $x^{'} \notin \mathrm{argmax} \succsim_i (\mathcal{A}_{\theta_i^*})$ when adopting a type $\tilde{\theta}_i \neq \theta_i^*$. But then i is not maximising $\succsim_i$ while being able to do so, thus it must be the case that $x^{'} = x^{''}$.

□

Proposition 5 shows that although adopted type and true type mismatch is a necessary condition for inefficient choices to arise, it is not a sufficient condition on it's own. In our framework, even if the adopted type does not coincide with the true type, it loses its biasing power if the optimal choice under the absence of frame is still reachable. This latter case emerges when both sets $\mathcal{A}_{\tilde{\theta}_i}$ and $\mathcal{A}_{\theta_i^*}$ have a non-empty intersection, and optimal choices on both lead to the same element. A direct implication for individual well-being from Proposition 5 is presented next

Proposition 6. ***[Weak preference for true-type consideration sets]*** *Define a type extended choice set as $\{X, \theta\}$. If $\tilde{\theta}_i \in F$, then $\tilde{\theta}_i \neq \theta_i^*$ with $\{X, \tilde{\theta}\}$ as consideration set. If $\{X, \theta\} = \{X\}$ then $\tilde{\theta}_i = \theta_i^*$, $\tilde{\theta}_i \notin F$. Given an adopted type $\tilde{\theta}_i \in F$, $\{X\} \succsim_i \{X, \tilde{\theta}\}$, or equivalently $\mathcal{A}_{\theta_i^*} \succsim_i \mathcal{A}_{\tilde{\theta}_i}$.*

Proof of Proposition 6. First notice that if $\mathcal{A}_{\tilde{\theta}_i} = \mathcal{A}_{\theta_i^*}$ then i must be indifferent between the two sets, as they contain exactly the same elements and $x \in argmax\ \mathcal{X} \in A_i^*$. This case is equivalent to the absence of frame. Now for the cases where $\mathcal{A}_{\tilde{\theta}_i} \neq \mathcal{A}_i^*$, assume $\mathcal{A}_{\tilde{\theta}_i} \cap \mathcal{A}_i^* \neq \emptyset$. Then i will be indifferent between any of the two sets if $x^{'} \in \mathcal{A}_{\tilde{\theta}_i} \cap \mathcal{A}_i^*$ and $x^{'} \in\ \mathrm{argmax}(\mathcal{A}_i^*)$, as then we also have $x^{'} \in \mathrm{argmax}\ (\mathcal{A}_{\tilde{\theta}_i})$, with $x^{'} = x^*$. If on the contrary, $\mathcal{A}_{\tilde{\theta}_i} \cap \mathcal{A}_i^* \neq \emptyset$ holds, with $x^{'} \in \mathrm{argmax}(\mathcal{A}_{\tilde{\theta}_i})$ and $x^{'} \notin \mathrm{argmax}(\mathcal{A}_{\theta_i^*})$, then from Definition 9 we have $\mathcal{A}_{\theta_i^*} \succ_i \mathcal{A}_{\tilde{\theta}_i}$. Putting these two outcomes together leads to conclude that $\mathcal{A}_{\theta_i^*} \succsim_i \mathcal{A}_{\tilde{\theta}_i}$. Thus, either $A_{\tilde{\theta}_i} \sim A_{\theta_i^*}$ or $A_{\tilde{\theta}_i} \succ A_{\theta^*}$, and then $\mathcal{A}_{\theta_i^*} \succsim_i \mathcal{A}_{\tilde{\theta}_i}$.

□

The result from Proposition 6 reveals that agents have a weak preference for true-type's consideration sets over those consideration sets that do not correspond to the true-type. The interpretation lies in the fact that when only relevant options are available, there is no possibility for options outside the corresponding true-type consideration set to be considered and chosen. Thus an optimal element should be selected. In opposition, when relevant elements are absent, the case where adopted types are effectively frames, non-optimal options are chosen affecting individual well-being.

The results presented in Proposition 5 and Proposition 6 show how the model of type adoption can be related to choice with frames, and in which cases an adopted type can be considered to work as a frame in an extended choice problem. For a frame to be effective, or for it to actually be a proper frame, it needs to discard from the consideration set those options deem optimal with respect to the preferences of the individuals. This results are interesting as they suggest a step towards a framework to study how frames are formed, and how individual's attention emerges, topics that are becoming more relevant in the literature and practice (Caplin, 2016).

4 Applications

This section presents two applications of the model. The first one analyses the effect that inequality of opportunities has on type adoption, and the other studies how anti poverty policies, exclusively directed to reduce the cost of acquiring a type, can backfire when not accompanied by changes in individual's aspirations.

4.1 Inequality of opportunities and adopted type bias

This section studies the bias effect that higher inequality of opportunities has on the type search process. To do so, the distance between types as perceived by a given agent, the metric $\mathcal{M}_i$, is deemed as an approximation of the inequality of opportunities as perceived by individual i. To do so the analysis proceeds by comparing two states each with different distances between types, that is each with different inequality of opportunities.

The convexity of the cost function $\mathcal{C}(\theta_i'(\lambda_i'), \theta_{\aleph_i}, \mathcal{M}_i)$ with respect to θ and the concavity of the payoff function produce an area for which positive net payoffs can be obtained if the corresponding types are adopted, call this area the type-locus. Formally, [7] the type-locus can be expressed as $\int_{\underline{\theta}}^{\overline{\theta}'} \pi(\theta) - \mathcal{C}(\theta, \mathcal{M}) \; d\theta$, where $\overline{\theta}'$ is the highest value of θ for which the type-locus is positive. Notice that the threshold $\Gamma(\lambda_i^t, \theta_{\aleph_i}, \mathcal{M}_i)$ also depends on $\mathcal{M}$.

Assume that all remains the same but for the distance between types, and to simplify the analysis lets compare only two distinct cases defined by the two metrics $\mathcal{M}_i^0$ and $\mathcal{M}_i^1$. Then an increase in $\mathcal{M}$ will have the effects described in what follows: fix the environment $\aleph$ and

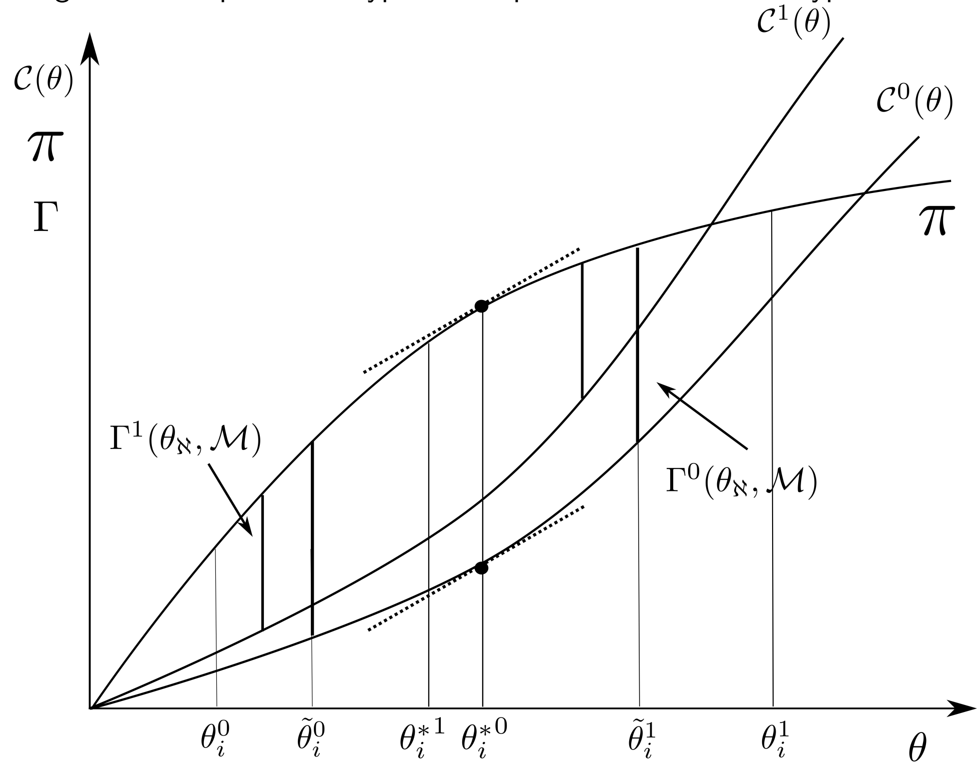
[7] A little abuse of notation is used here avoiding the use of all super-indexes and sub-indexes when no confusion can be created.

the state σ^t, increases in the distance between types, increases in the metric $\mathcal{M}$, shrinks the type-locus and displaces the threshold restricted area in the type-locus towards the origin.

For a given environment $\aleph_j$, assume that $\mathcal{M}_i^0 < \mathcal{M}_i^1$, then $\mathcal{C}(\theta, \mathcal{M}^o) < \mathcal{C}(\theta, \mathcal{M}^1)$ as the bigger the perceived distance between types is, the bigger the costs of being able to select a higher type to adopt. Given that the payoffs do not depend on $\mathcal{M}$, the type-locus decreases[8]

$$\int_{\underline{\theta}}^{\overline{\theta}'} \pi(\theta) - \mathcal{C}(\theta, \mathcal{M}^o)\, d\theta > \int_{\underline{\theta}}^{\overline{\theta}''} \pi(\theta) - \mathcal{C}(\theta, \mathcal{M}^1)\, d\theta$$

Figure 3: Comparison of type-search process with different type distance



On the other hand, $\Gamma(\theta_\aleph, \mathcal{M})$ decreases when $\mathcal{M}$ increases, the intuition behind this effect is that the threshold of the agent will decrease as higher types are less feasible as candidates for type adoption. Figure 3 shows the effects of an increase of $\mathcal{M}$ in both the type-locus and the set of types that lie in the area constrained by the left and right thresholds. The costs increase from $C(\theta)^0$ to $C(\theta)^1$, while the threshold changes from $\Gamma(0)^o$ to $\Gamma(1)^1$. For the case depicted in the figure, the threshold decreases enough to fit inside the new type-locus. If this were not the case, then $\Gamma(\cdot) > \mathcal{V}(\cdot)\ \forall\ \theta \in \Theta$. Assumption 2 indicates that in this case i will adopt her true type θ^*. However, for the case illustrated in Figure 3 the threshold decreases

[8]Even if π depends on $\mathcal{M}$, if $\dfrac{\partial \pi}{\partial \mathcal{M}} < \dfrac{\partial C}{\partial \mathcal{M}}$ the analysis would be the same.

enough to fit inside the type-locus, driving all those individuals whose types lie to the left of $\Gamma(\theta)^1$ to adopt lower types than those whose threshold is $\Gamma(\theta)^0$.

Notice that those that start their search to the right of θ_i^*, when facing $\mathcal{M}_i^1$, adopt types that are closer to their true type, as the increase in costs and the decrease in threshold guides them to satisfy the type search criteria described in Assumption 2 closer to the origin, and to the left of those that face $\mathcal{M}_i^0$ instead. The opposite happens for those whose type search starts to the left of their true type. When facing type distance $\mathcal{M}_i^1$, these individuals see their search criteria satisfied closer to the origin, and further from their true type in comparison with those that face type distance $\mathcal{M}_i^0$. Thus, the effect of higher perceived distance between types, has distinctive effects depending on the relative place where they start their type search process. Notice that this works in detriment of those who start searching to the left of their true type, potentially those in more of a disadvantage, and in favour of those that start searching to the right of their true type, presumably those more advantaged amongst those with higher $\mathcal{M}$.

4.2 Cost improving subsidies

Many anti poverty policies pursue reducing the cost of reaching outcomes via subsidies. For example, a policy may aim at decreasing the costs of accessing secondary education with the objective of having more citizens graduating with such an educational level. The motivation behind these interventions resides on the idea that by reducing the costs of reaching individuals will, *de facto*, reach the outcome.

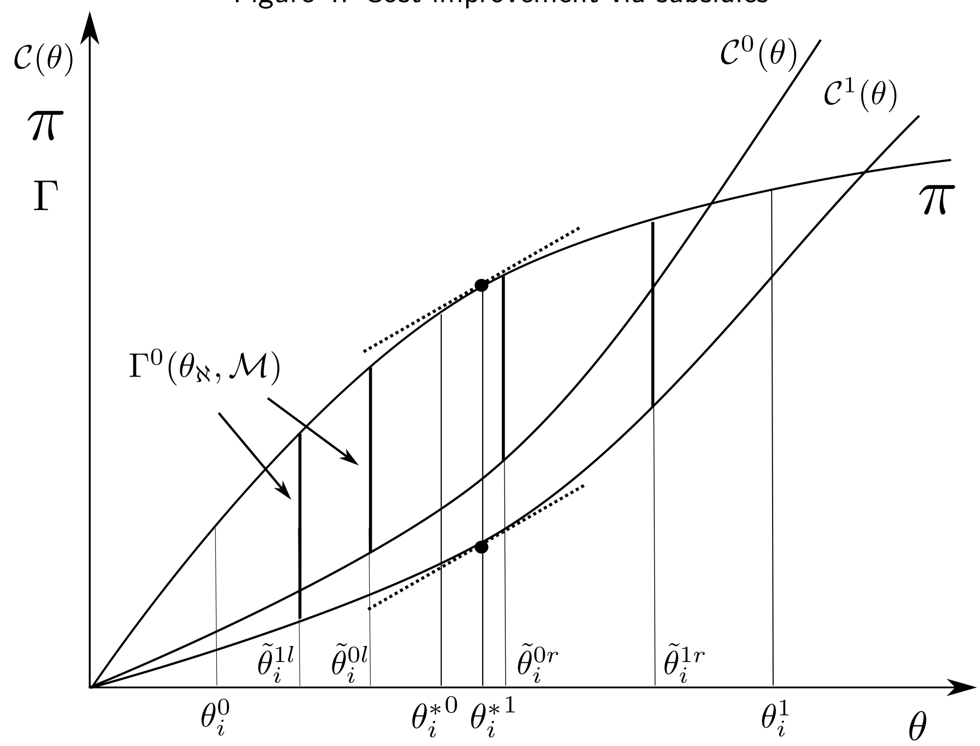
In the model this policy can be represented as an exogenous reduction in the costs of search, one that does not involve any changes in the arguments of the function, and that does not affect neither the payoff or the threshold. *Ceteris paribus*, a decrease in the cost increases the area of the locus formed between the payoff and the cost functions. At the new cost, with the threshold remaining at the same level, the satisficing criteria is met at a lower type (higher type) for those whose type search departure is located to the left (right) of θ_i^*, thus inducing individuals to adopt lower types (higher types) than those that would emerge at the original cost. Figure 3 presents this case with $C(\theta)^0$ being the initial cost, and $C(\theta)^1$ the cost after a cost-improving subsidy is introduced. If θ_i^0 is the search departure type, by Proposition 2 the individual will search types in an upward direction, searching for upper types. Given that the threshold has not been affected, it remains at $\Gamma(\theta)^0$, and since the locus $\int_{\underline{\theta}}^{\overline{\theta}'} \pi(\theta) - \mathcal{C}(\theta, \mathcal{M})\, d\theta$ has increased, the aspiration level (the threshold) is reached at a lower type. The individual adopts type $\tilde{\theta}_i^{1l}$, a type that is lower than that that would be adopted at the original cost $\mathcal{C}^0(\theta)$, that is $\tilde{\theta}_i^{0l}$. For those whose type search starts to the right of θ_i^* the effect is symmetric.

They end up adopting type $\tilde{\theta}_i^{1r}$ at the new cost instead of adopting $\tilde{\theta}_i^{0r}$, which leads them to adopt a type that is further from their true type. The next result formalises this observation.

Proposition 7. ***[Cost-improvement policy backfire]*** *Assume the benefits perceived by acquiring any type, and the threshold remain the same. A cost-improving policy that decreases the cost of acquiring any type, induces individuals to adopt a type further from the optimal type in comparison with the initial state.*

Proof of Proposition 7. Let a initial state with payoff function $\pi(\theta)$, costs $C(\theta)^0$, and threshold $\Gamma(\theta)$, produce an adopted type $\tilde{\theta}_i^0$. *Ceteris paribus*, assume a decrease in the cost of acquiring all types with a new cost function $C(\theta)^1$. Such a change will increase the area of the locus $\int_{\underline{\theta}}^{\overline{\theta}'} \pi(\theta) - \mathcal{C}(\theta, \mathcal{M})\, d\theta$. Let θ_i^0 be the search departure type, then from Proposition 2 the individual will search types in an upward direction if this type is located to the left of the optimal type, and in downward direction if the initial type is located to the right of the optimal type, with the individual searching for upper types in the former case, and for lower types in the latter. At the new cost, with $\Gamma(\cdot)$ and $\Pi(\cdot)$ remaining at the same level, the satisficing criteria is met at a lower type $\tilde{\theta}_i^1 < \tilde{\theta}_i^0$, for those whose type search departure is to the left of their true type, and at higher types for those starting to the right of it, $\tilde{\theta}_i^1 > \tilde{\theta}_i^0$. Thus inducing individuals to adopt lower or higher types in comparison with those that would emerge at the original cost levels. $\square$

Figure 4: Cost improvement via subsidies



The intuition behind this result is the following. If the benefit of acquiring any type remains the same, and the aspirations of the individual (as measured by the threshold) do not change, a policy directed at decreasing the costs of accessing education aiming at more individuals attaining secondary education (type θ_i^0 in Figure 3) via generalised subsidies will reduce the cost of attaining all levels of education (all types). This policy will widen the net valuation not only for that type (secondary education) but for all types, this allows for the threshold to be reached closer to the lowest type, or closer to the origin in Figure 3. This result emerges as individuals can now reach their aspirations adopting lower types such as primary education (type θ_i^1 in Figure 3), as they can reach their aspirations at that lower type, which leads them to do so.

5 Final comments

In this paper a model of type unawareness and type search and adoption was presented. The paper adds to existing research offering a theoretical model that helps understand why and how suboptimal behaviour emerges among individuals, and the role that their social environment plays in shaping their type (identity), via aspirations dictating what is attainable for a given benefit and cost structure. The model also illustrates how rationally bounded behaviour arises from framework effects emerging from adopted and true type mismatches . The paper also presents policy implications, with results that show how inequality of opportunities hinders suboptimal choices, and how policies that subsidise costs can backfire if they are not accompanied by complementary policies aiming at improving their satisficing criteria, or aspirations.

In this setting, individuals start a type search process with the purpose of finding a type to adopt, and the search process ends when a satisficing criterion is met guided by a search stopping rule. The rule is characterised by three elements: the payoffs proceeding from adopting a given type, the type search and adoption cost, and a satisficing threshold. It was shown under which circumstances this process can lead to suboptimal type choices, resulting in adopted types not corresponding to individual's true types. The results show that agents can adopt a type that matches or mismatches their true type. In the former case, no problem arises, the type they adopt is the one that corresponds to the individual. In the latter case suboptimal type selection emerges. For individuals starting their type search process at low types, relative to their true type, the adopted type falls short of their true type, thus selection is sub-optimal in the sense that they do not realise their full capacity. For those starting their type search process at a level above their true type, type adoption happens at types higher that the true type that corresponds to them, sub-optimality in these cases arises from under-performance at higher types than that that their capacity indicates.

Additionally, the model of type search and adoption was linked to a two phase choice process. In a first phase type selection happens, then in a second phase the agent faces a choice problem in which the adopted type is used as guide in choice. It was shown under which circumstances type mismatch produces frames in extended choice problems. Also, results show why an adopted type working as a frame can limit choice alternatives, forming consideration sets from which choice is made, and that may not contain preference maximising choices that would be available in the absence of frames, affecting individual's well-being.

Two applications of the model show how inequality of opportunities can make the output of the type adoption process more salient, and how policies aiming at improving type adoption can backfire. When comparing results for individuals with higher perceived distance between types, what is interpreted as a measure of inequality, those in more disadvantage choose types that are lower than those that are in a more privileged position,thus worsening their mismatch. On the other hand, those individuals at the top of opportunities, although they keep choosing higher types with respect to their true type, end up adopting types that are closer to their true type. Thus, inequality of opportunities affects those at the bottom of the disadvantaged worsening their situation, but leads those at the top of the disadvantaged to adopt types closer to their true type in comparison with those at the top of the privileged.

In the second application, the effects of cost improving subsidies are analysed. The results show these type of policies can generate opposite effects to those that are been sought. If subsidies that decrease the cost of type search and adoption are applied in a generalised fashion, and not directed to the costs of adopting certain types, then individuals end up adopting types that are even further from their true types. This result emerges when thresholds and payoffs are not affected by the policy. If thresholds are interpreted as aspirations, under this cost improving policy, an increase in aspirations can help reach the objectives of the policy. This result highlights the importance of aspirations shaping the effectiveness of these kind of initiatives.

The model presented in this paper could be improved in a number of ways. For instance, the static nature of the model does not allow to study intertemporal effects that may emerge from individuals adjusting their types through time. Also, adding feedback effects from individuals to environments could make the model more realistic in certain ways, however, the complications added could make the model less tractable. The link between type search and adoption and choice in extended choice problems merits further investigation. For once, it could be interesting to build a model in which frames are formed on their own in the type search and adoption framework presented, *mutatis mutandis*, and then analyse its effects for choice in extended choice problems. Finally, a more abstract setting could allow for applications of the model to artificial intelligence and robotics, or to virtual environments.

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