typevar, X, Y	type variables			
termvar, x, y	term variables			
typ, T, S	::=			types
	|	X		
	|	**Top**		top type
	|	$T_1 \to T_2$		function types
	|	**Forall** $X <: T_1 . T_2$	bind X in T_2	
	|	**True**		
	|	**False**		
	|	T_1 **Or** T_2		
	|	$T[X \leadsto T_1]$	M	
exp, e, v	::=			expressions
	|	x		variables
	|	$\lambda x.e$	bind x in e	abstractions
	|	$\lambda X <: T.e$	bind X in e	type abstraction
	|	$e[T]$		type applications
	|	$e_1\ e_2$		applications
	|	**true**		
	|	**false**		
	|	**if** e_1 **then** e_2 **else** e_3		
	|	$e_1 == e_2$		
	|	e_1 **and** e_2		
	|	$e_1[x \leadsto e_2]$	M	
	|	$e[X \leadsto T]$	M	
	|	(e)	S	
binding, b	::=			bindings
	|	**bind_typ** T		
	|	**bind_sub** T		
ctx, Γ	::=			typing context
	|	$\bullet$		empty context
	|	$\Gamma, x : T$		type binding
	|	$\Gamma, X <: T$		subtype binding
terminals	::=			
	|	λ		
	|	$\longrightarrow$		
	|	$\Longrightarrow$		
	|	$\to$		
	|	$\vdash$		
	|	$\in$		
	|	fv		
	|	$\leadsto$		
formula	::=			
	|	*judgement*		
	|	**is_value** v		

$$
\begin{array}{lll}
 & | & \mathsf{uniq}\,\Gamma \\
 & | & x : T \in \Gamma \\
 & | & X <: T \in \Gamma \\
 & | & x\ \mathbf{notin}\ \mathsf{fv}\, e \\
 & | & x\ \mathbf{notin}\ \mathbf{dom}\,\Gamma \\
 & | & X\ \mathbf{notin}\ \mathbf{dom}\,\Gamma
\end{array}
$$

$$
\begin{array}{llll}
\mathit{WFTyp} & ::= & & \\
 & | & \Gamma \vdash T & \text{Well-Formed Type rules} \\
 & & & \\
\mathit{WFEnv} & ::= & & \\
 & | & \vdash \Gamma & \text{Well-Formed Environment rules} \\
 & & & \\
\mathit{JSubTyping} & ::= & & \\
 & | & \Gamma \vdash T_1 <: T_2 & \text{Subtyping rules} \\
 & & & \\
\mathit{JTyping} & ::= & & \\
 & | & \Gamma \vdash e : T & \text{Typing rules} \\
 & & & \\
\mathit{JEval} & ::= & & \\
 & | & e \longrightarrow e' & \text{Small-step operational semantics} \\
 & & & \\
\mathit{judgement} & ::= & & \\
 & | & \mathit{WFTyp} & \\
 & | & \mathit{WFEnv} & \\
 & | & \mathit{JSubTyping} & \\
 & | & \mathit{JTyping} & \\
 & | & \mathit{JEval} & \\
 & & & \\
\mathit{user\_syntax} & ::= & & \\
 & | & \mathit{typevar} & \\
 & | & \mathit{termvar} & \\
 & | & \mathit{typ} & \\
 & | & \mathit{exp} & \\
 & | & \mathit{binding} & \\
 & | & \mathit{ctx} & \\
 & | & \mathit{terminals} & \\
 & | & \mathit{formula} &
\end{array}
$$

$\boxed{\Gamma \vdash T}$ Well-Formed Type rules

$$\frac{}{\Gamma \vdash \mathbf{Top}} \quad \text{WF\_TYP\_TOP}$$

$$\frac{}{\Gamma \vdash \mathbf{True}} \quad \text{WF\_TYP\_TRUE}$$

$$\frac{}{\Gamma \vdash \mathbf{False}} \quad \text{WF\_TYP\_FALSE}$$

$$\frac{\Gamma \vdash T_1 \qquad \Gamma \vdash T_2}{\Gamma \vdash T_1 \to T_2} \quad \text{WF\_TYP\_ARROW}$$

$$\frac{\Gamma \vdash T_1 \quad \Gamma, X <: T_1 \vdash T_2}{\Gamma \vdash \textbf{Forall}\, X <: T_1 . T_2} \quad \text{WF\_TYP\_FORALL}$$

$$\frac{\Gamma \vdash T_1 \quad \Gamma \vdash T_2}{\Gamma \vdash T_1 \,\textbf{Or}\, T_2} \quad \text{WF\_TYP\_UNION}$$

$$\frac{X <: T \in \Gamma}{\Gamma \vdash X} \quad \text{WF\_TYP\_VAR}$$

$\boxed{\vdash \Gamma}$ Well-Formed Environment rules

$$\frac{}{\vdash \bullet} \quad \text{WF\_ENV\_EMPTY}$$

$$\frac{\vdash \Gamma \quad \Gamma \vdash T \quad X \,\textbf{notin}\,\textbf{dom}\,\Gamma}{\vdash \Gamma, X <: T} \quad \text{WF\_ENV\_SUB}$$

$$\frac{\vdash \Gamma \quad \Gamma \vdash T \quad x \,\textbf{notin}\,\textbf{dom}\,\Gamma}{\vdash \Gamma, x : T} \quad \text{WF\_ENV\_TYP}$$

$\boxed{\Gamma \vdash T_1 <: T_2}$ Subtyping rules

$$\frac{\vdash \Gamma \quad \Gamma \vdash T}{\Gamma \vdash T <: \textbf{Top}} \quad \text{SUB\_TOP}$$

$$\frac{\vdash \Gamma}{\Gamma \vdash \textbf{True} <: \textbf{True}} \quad \text{SUB\_REFL\_TRUE}$$

$$\frac{\vdash \Gamma}{\Gamma \vdash \textbf{False} <: \textbf{False}} \quad \text{SUB\_REFL\_FALSE}$$

$$\frac{\vdash \Gamma \quad \Gamma \vdash X}{\Gamma \vdash X <: X} \quad \text{SUB\_REFL\_TVAR}$$

$$\frac{X <: T_1 \in \Gamma \quad \Gamma \vdash T_1 <: T_2}{\Gamma \vdash X <: T_2} \quad \text{SUB\_TRANS\_TVAR}$$

$$\frac{\Gamma \vdash T_1 <: S_1 \quad \Gamma \vdash T_2 <: S_2}{\Gamma \vdash S_1 \to S_2 <: T_1 \to T_2} \quad \text{SUB\_ARROW}$$

$$\frac{\Gamma \vdash T_1 <: S_1 \quad \Gamma, X <: T_1 \vdash S_2 <: T_2}{\Gamma \vdash \textbf{Forall}\, X <: S_1 . S_2 <: \textbf{Forall}\, X <: T_1 . T_2} \quad \text{SUB\_FORALL}$$

$$\frac{\vdash \Gamma \quad \Gamma \vdash T_2 \quad \Gamma \vdash S <: T_1}{\Gamma \vdash S \,\textbf{Or}\, T_2 <: T_1 \,\textbf{Or}\, T_2} \quad \text{SUB\_UNION\_L}$$

$$\frac{\vdash \Gamma \quad \Gamma \vdash T_1 \quad \Gamma \vdash S <: T_2}{\Gamma \vdash T_1 \,\textbf{Or}\, S <: T_1 \,\textbf{Or}\, T_2} \quad \text{SUB\_UNION\_R}$$

$$\frac{\Gamma \vdash S_1 <: T \quad \Gamma \vdash S_2 <: T}{\Gamma \vdash S_1 \,\textbf{Or}\, S_2 <: T} \quad \text{SUB\_UNION\_M}$$

$\boxed{\Gamma \vdash e : T}$ Typing rules

$$\frac{\vdash \Gamma \quad x : T \in \Gamma}{\Gamma \vdash x : T} \quad \text{TYPING\_VAR}$$

$$\frac{\Gamma, x : T_1 \vdash e : T_2}{\Gamma \vdash \lambda x.e : T_1 \to T_2} \quad \text{TYPING\_ABS}$$

$$\frac{\Gamma \vdash e_1 : T_1 \to T_2 \quad \Gamma \vdash e_2 : T_1}{\Gamma \vdash e_1\, e_2 : T_2} \quad \text{TYPING\_APP}$$

$$\frac{\Gamma, X <: T_1 \vdash e : T_2}{\Gamma \vdash \lambda X <: T_1.e : \textbf{Forall}\, X <: T_1.T_2} \quad \text{TYPING\_TABS}$$

$$\frac{\Gamma \vdash e : \textbf{Forall}\, X <: T_1.T_2 \quad \Gamma \vdash S <: T_1}{\Gamma \vdash e[S] : T_2[X \rightsquigarrow S]} \quad \text{TYPING\_TAPP}$$

$$\frac{\Gamma \vdash e : S \quad \Gamma \vdash S <: T}{\Gamma \vdash e : T} \quad \text{TYPING\_SUB}$$

$$\frac{\vdash \Gamma}{\Gamma \vdash \textbf{true} : \textbf{True}} \quad \text{TYPING\_TRUE}$$

$$\frac{\vdash \Gamma}{\Gamma \vdash \textbf{false} : \textbf{False}} \quad \text{TYPING\_FALSE}$$

$$\frac{\Gamma \vdash T_1 \quad \Gamma \vdash T_2 \quad \Gamma \vdash e_1 : T_1 \quad \Gamma \vdash e_2 : T_2}{\Gamma \vdash e_1 == e_2 : \textbf{True Or False}} \quad \text{TYPING\_EQ}$$

$$\frac{\Gamma \vdash e_1 : \textbf{True Or False} \quad \Gamma \vdash e_2 : \textbf{True Or False}}{\Gamma \vdash e_1 \,\textbf{and}\, e_2 : \textbf{True Or False}} \quad \text{TYPING\_AND}$$

$\boxed{e \longrightarrow e'}$ Small-step operational semantics

$$\frac{e_1 \longrightarrow e_1'}{e_1\, e_2 \longrightarrow e_1'\, e_2} \quad \text{RED\_APP1}$$

$$\frac{e_2 \longrightarrow e_2'}{v_1\, e_2 \longrightarrow v_1\, e_2'} \quad \text{RED\_APP2}$$

$$\frac{}{(\lambda x.e_1)\, v_2 \longrightarrow e_1[x \rightsquigarrow v_2]} \quad \text{RED\_APP\_ABS}$$

$$\frac{e \longrightarrow e'}{e[T] \longrightarrow e'[T]} \quad \text{RED\_TAPP}$$

$$\frac{}{(\lambda X <: S.e)[T] \longrightarrow e_1[X \leadsto T]} \quad \text{RED\_TAPP\_TABS}$$

$$\frac{}{\textbf{if true then } e_1 \textbf{ else } e_2 \longrightarrow e_1} \quad \text{RED\_IF\_TRUE}$$

$$\frac{}{\textbf{if false then } e_1 \textbf{ else } e_2 \longrightarrow e_2} \quad \text{RED\_IF\_FALSE}$$

$$\frac{e \longrightarrow e'}{\textbf{if } e \textbf{ then } e_1 \textbf{ else } e_2 \longrightarrow \textbf{if } e' \textbf{ then } e_1 \textbf{ else } e_2} \quad \text{RED\_IF}$$

$$\frac{e_1 \longrightarrow e_1'}{e_1 == e_2 \longrightarrow e_1' == e_2} \quad \text{RED\_EQL}$$

$$\frac{e_2 \longrightarrow e_2'}{v_1 == e_2 \longrightarrow v_1 == e_2'} \quad \text{RED\_EQR}$$

$$\frac{}{v_1 == v_2 \longrightarrow \textbf{true}} \quad \text{RED\_EQ}$$

$$\frac{e_1 \longrightarrow e_1'}{e_1 \textbf{ and } e_2 \longrightarrow e_1' \textbf{ and } e_2} \quad \text{RED\_ANDL}$$

$$\frac{e_2 \longrightarrow e_2'}{v_1 \textbf{ and } e_2 \longrightarrow v_1 \textbf{ and } e_2'} \quad \text{RED\_ANDR}$$

$$\frac{}{\textbf{false and } v_2 \longrightarrow \textbf{false}} \quad \text{RED\_AND\_FALSE\_L}$$

$$\frac{}{v_1 \textbf{ and false} \longrightarrow \textbf{false}} \quad \text{RED\_AND\_FALSE\_R}$$

$$\frac{}{\textbf{true and true} \longrightarrow \textbf{true}} \quad \text{RED\_AND\_TRUE}$$

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Definition rules:        46 good    0 bad
Definition rule clauses: 105 good    0 bad
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