Logistic Map

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Introduction

The logistic map is a mathematical model that plays a significant role in understanding population dynamics, chaos theory, and various other scientific disciplines. In this report, we explore the behavior of the logistic map for different values of the control parameter "r" and its applications in diverse fields.

The Logistic Map Model

The logistic map is represented by the equation:

$$\boldsymbol{x_{n+1} = r \times x_n(1 - x_n)}$$

x_n: Represents the population size or any quantity of interest at iteration "n".

$\boldsymbol{x_{n+1}}$: Denotes the population size at the next iteration "n + 1".

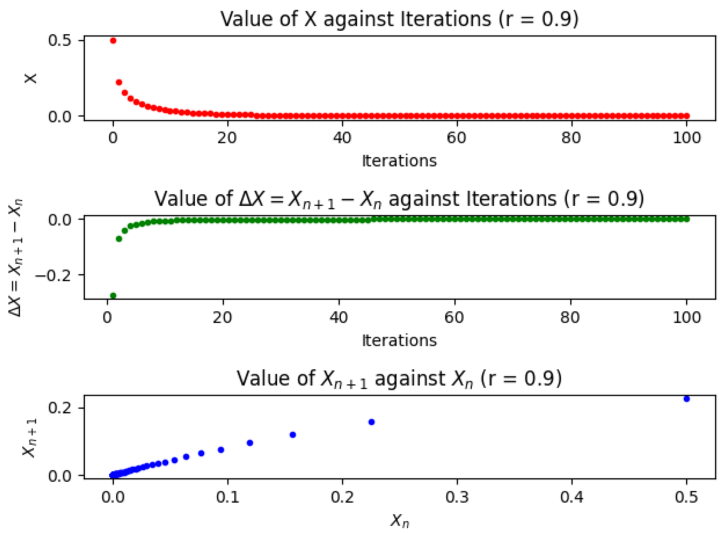
r : The control parameter that influences the system's behavior.

Behavior of the Logistic Map

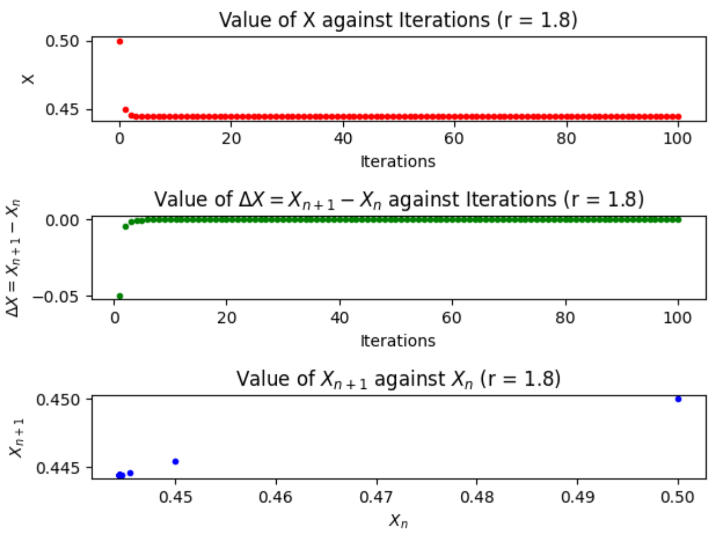
The logistic map exhibits various behaviors based on the value of "r." In the following graphs, the evolution of the logistic map is presented for the following values of $r = \{0.9,\ 1.8,\ 2.6,\ 3.2,\ 3.9,\ 3.99\}$, with an initial value of $x_0 = 0.5$. The simulations were run for 100 iterations. Three graphs were generated for each of the r values. The first graph represents the evolution of x_n values over past N iterations (in our case 100 iterations), the second graph shows the variation of x_n over past N iterations, and the third graph illustrates the relationship between future generations and present generations, with x_{n+1} values on the ordinate axis and x_n values on the abscissa axis. These are separated into three regimes:

- **Stable Regime**

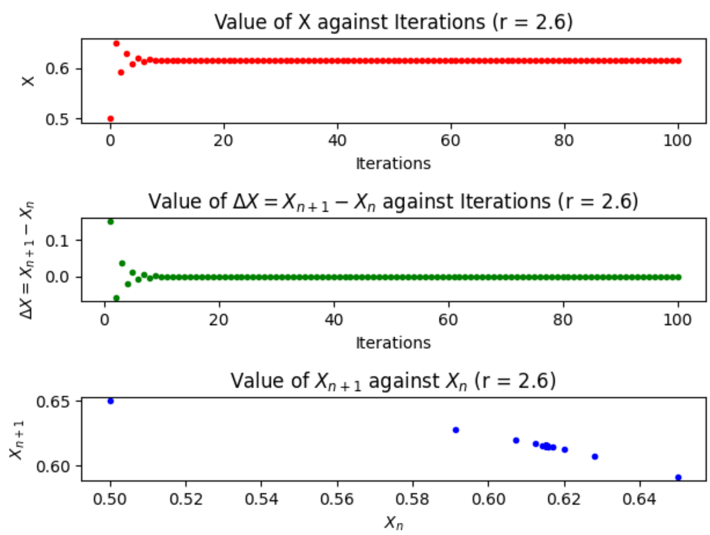
When "r" is equal to 0.9, we observe that x stabilizes at zero, indicating the presence of an attractor for $x = 0$, this tends to be the population's behavior when the growth rate “r” is less than 1. This behavior aligns with the characteristics of a stable regime. In such cases, the plot shows a single cluster of points resembling a parabolic curve, which signifies the system's convergence to a stable equilibrium. Additionally, we can see that the population is decreasing as it exhibits a negative delta until it stabilizes at values that tend towards zero. Lastly, by observing the third graph, we can conclude that the system enters a stable state by accumulating many points in the region, for example, in the proximity of (0, 0).



When "r" is equal to 1.8, we can observe in the first and second graphs that the function is decreasing and x stabilizes at values very close to 0.44… This behavior can be observed in the second graph, where the variation is negative until it cancels out, indicating the existence of an attractor for values very close to the asymptote $x_n = 0.44$. Through the third graph, we can see a significant concentration of points in the proximity of $x_n = 0.44(4)$, which supports the idea of the existence of an attractor.

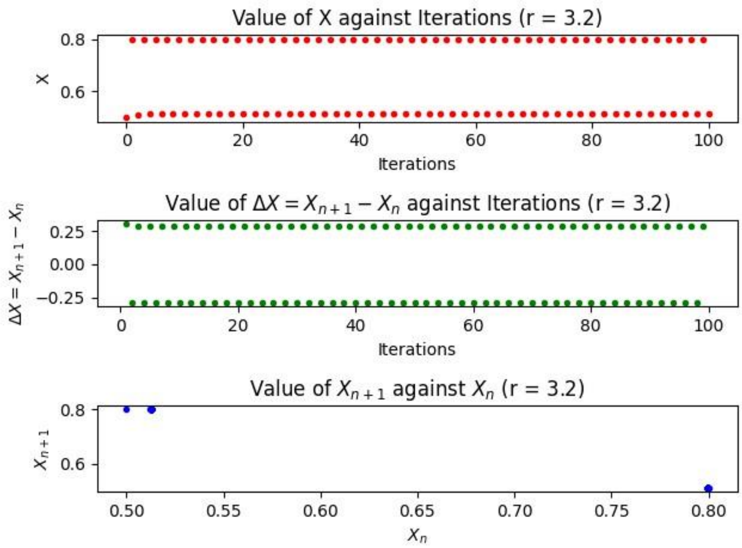


When "r" is equal to 2.6, we can observe in the first and second graphs that the function exhibits an initially irregular behavior for values of x, and as we progress through the iterations, we obtain values very close to 0.615… This behavior can be observed in the second graph, where the variation shows random growth and decline values, but after fifteen iterations, the variation appears to be zero, indicating the existence of an attractor for values very close to the asymptote $x = 0.615$. Through the third graph, we can see a significant concentration of points in the vicinity of the range $x_n \in [0.6{,}0.62]$, which supports the idea of the existence of an attractor.



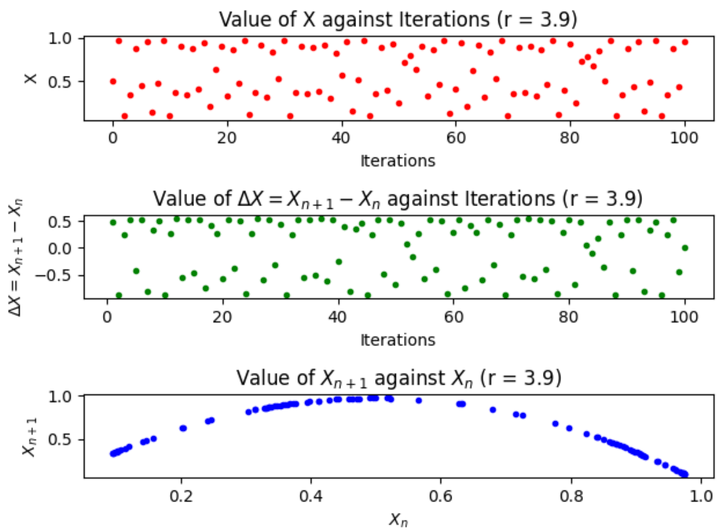
- **Periodic Regime**

  When "r" is equal to 3.2, we can observe through the first and second graphs that the function exhibits a periodic behavior, oscillating between $x = 0.51$ and $x = 0.8$. These can be considered as the attractors of the function, showing a periodicity of two, meaning a cycle that repeats every two points. In the third graph, it is possible to observe that there are two clusters that group most values obtained in the first hundred iterations. This indicates that the system follows a periodic cycle, varying between $x = 0.51$ and $x = 0.8$.

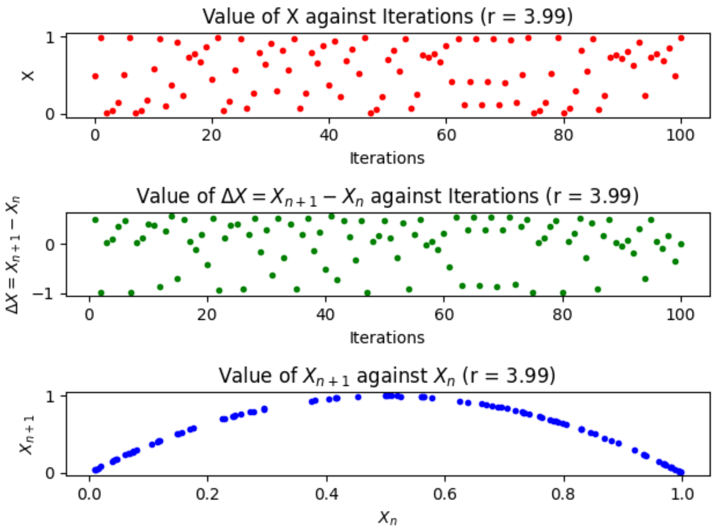


- **Chaotic Regime**

When "r" is equal to 3.9, it is possible to observe through the first and second graphs that the plot appears scattered, reflecting chaotic behavior where small differences in the initial conditions lead to unpredictable results, making it no longer possible to identify a set of attractors or any periodicity. In the third graph, it is no longer possible to observe well-defined clusters of points as seen in the previously analyzed regimes. On the other hand, we observe that the values of x_n appear to be scattered creating a parabola.



When "r" is equal to 3.99, it is possible to observe behavior similar to what was previously seen for "r" values of 3.9. We remain in a chaotic system, but it is noticeable that the degree of chaos is much higher, making the results even more unpredictable and resembling random behavior without periodicity or attraction.



In this exercise, we investigate the chaotic behavior of the logistic map for high "r" values. Starting with two nearly identical initial points like 0.5 and 0.499, we track how quickly their trajectories diverge until they differ by 0.5. We note the number of iterations required increase as the value of r decreases. Then, we slightly decrease "r" and repeat the process, comparing the divergence rate. This exercise helps us understand how minor parameter variations within the chaotic regime affect trajectory divergence. It illustrates the sensitivity of the logistic map to changes in "r" and sheds light on the intricate dynamics of chaotic systems.