Numerieke Integratie
Oefening 1

Ruben Van Assche

Opgave

1. Bereken *ln(2)* met de samengestelde trapeziumregel

2. Efficiëntie?

Berekening trapeziumregel

$$\frac{b-a}{n}\left(\frac{1}{2}f(x_0)+f(x_1)+f(x_2)+\ldots+f(x_{n-1})+\frac{1}{2}f(x_n)\right).$$

- $n = 2^k$ met k = 1,2,3,…
- T(k) → benadering trapeziumregel met bepaalde k
- Stopconditie: $\left|\frac{T(k)-T(k+1))}{T(k+1)}\right| \leq 2^{-40}$

Berekening ln(2)

$$\int_1^2 \frac{1}{x} dx$$

```
Doub next() {
    Doub x,tnm,sum,del;
        Int it,j;
        n++;
        if (n == 1) {
                return (s=0.5*(b-a)*(func(a)+func(b)));
        } else {
                for (it=1,j=1;j<n-1;j++) it <<= 1;
                tnm=it;
                del=(b-a)/tnm;
                x=a+0.5*del;
                for (sum=0.0,j=0;j<it;j++,x+=del) sum += func(x);
                s=0.5*(s+(b-a)*sum/tnm);
                return s;
            }
    }
```

```cpp
double trapezium(std::function<double(double)> f, int k, double a,  double b){
  int n = pow(2, k);
  double h = (b-a)/n;

  double sum = 0.0;
  double x = a;

  for(int i = 1;i < n;i++){
    x += h;
    sum += h*f(x);
  }

  sum += (h/2)*f(a);
  sum += (h/2)*f(b);

  return sum;
}
```

Uitkomsten

- k = 20
- n = 1048576
- 0.693147180559991804 57
  (Maple: 0.693147180559945)
- Relatieve fout = 2.56594462955557767756e-13

Kunnen we dit optimaliseren?

Berekening f(x) waarden

$k = 3 \rightarrow n = 8$

$k = 4 \rightarrow n = 16$

n=2

$h(\frac{1}{2}f(1)+\frac{1}{2}f(2))+h(f(1.5))$

n=4

$\frac{h}{2}(\frac{1}{2}f(1)+\frac{1}{2}f(2))+\frac{h}{2}(f(1.5))+\frac{h}{2}(f(1.25)+f(1.75))$

n=8

$\frac{h}{4}(\frac{1}{2}f(1)+\frac{1}{2}f(2))+\frac{h}{4}(f(1.5))+\frac{h}{4}(f(1.25)+f(1.75))+\frac{h}{4}(f(1.125)+f(1.375)+f(1.65)+f(1.875))$

Hoe berekenen?

- T(k-1)/2
- + aantal termen($2^{log_2(n)-1}$)
- Voor i = 1, …, $\frac{2^{log_2(n)-1}-2}{2}$
  - f(a+h+2*ih)
  - f(b-h-2ih)

n	f()
2	/
4	1.25, 1.75
8	1.125, 1.375, 1.65, 1.875

```cpp
double calculateSum(std::function<double(double)> f, int k, double a, double b)
    if (k == 1) {
        double n = pow(2, k);
        double h = (b - a) / n;

        return h * f(a + h);
    }

    double nprev = pow(2, k - 1);
    double n = nprev * 2;
    double h = (b - a) / n;

    double incremental = h * 2;
    double toCalc = (nprev - 2) / 2;

    double sum;
    double posa = a + h;
    double posb = b - h;
    sum += h * f(posa);
    sum += h * f(posb);
    for (int i = 0; i < toCalc; i++) {
        posa += incremental;
        posb -= incremental;
        sum += h * f(posa);
        sum += h * f(posb);
    }

    return sum;
}
```

```cpp
// Calculates the trapeoid rule till the relative error is smaller then maxError
int trapeziumewitherror(std::function<double(double)> f, double a, double b, double maxError){
    int k = 1;
    double prevsum = 0.0;
    double sum = 0.0;

    while(true){
        // Set prevsum equal to the previous sum
        prevsum = sum;

        double n = pow(2, k);

        if (k == 1) {
            double h = (b - a) / n;

            sum += (h/2) * (f(a) + f(b));
            sum += calculateSum(f, k, a, b);
        } else {
            sum /= 2;
            sum += calculateSum(f, k, a, b);
        }

        // Calculate error
        double error = (prevsum-sum)/(sum);

        if(fabs(error) <= maxError){
            std::cout << "Found integration using " << n << " intervals(n = " << k;
            std::cout << ") with an error of " << error << " and solution: " << sum << std::endl;
            break;
        }

        // Raise intervals
        std::cout << "Using " << n << " intervals: " << sum << " error: " << error << std::endl;
        k++;
    }
```

```cpp
int main() {
    std::cout.precision(20);

    int a = 1;
    int b = 2;
    std::function<double(double)> f = [](double x) {
        return 1.0 / x;
    };

    int k = trapeziumewitherror(f, 1, 2, pow(2, -40));

    std::cout << "Calculate with n = " << k << " using basic trapezoid rule" << std::endl;
    std::cout << trapezium(f, k, 1, 2);
}
```

Uitkomsten

- k = 20
- n = 1048576
- 0.693147180559991804 57
  (Maple: 0.693147180559945)
- Relatieve fout = 2.56594462955557767756e-13