Numerieke Integratie - Oefening 6

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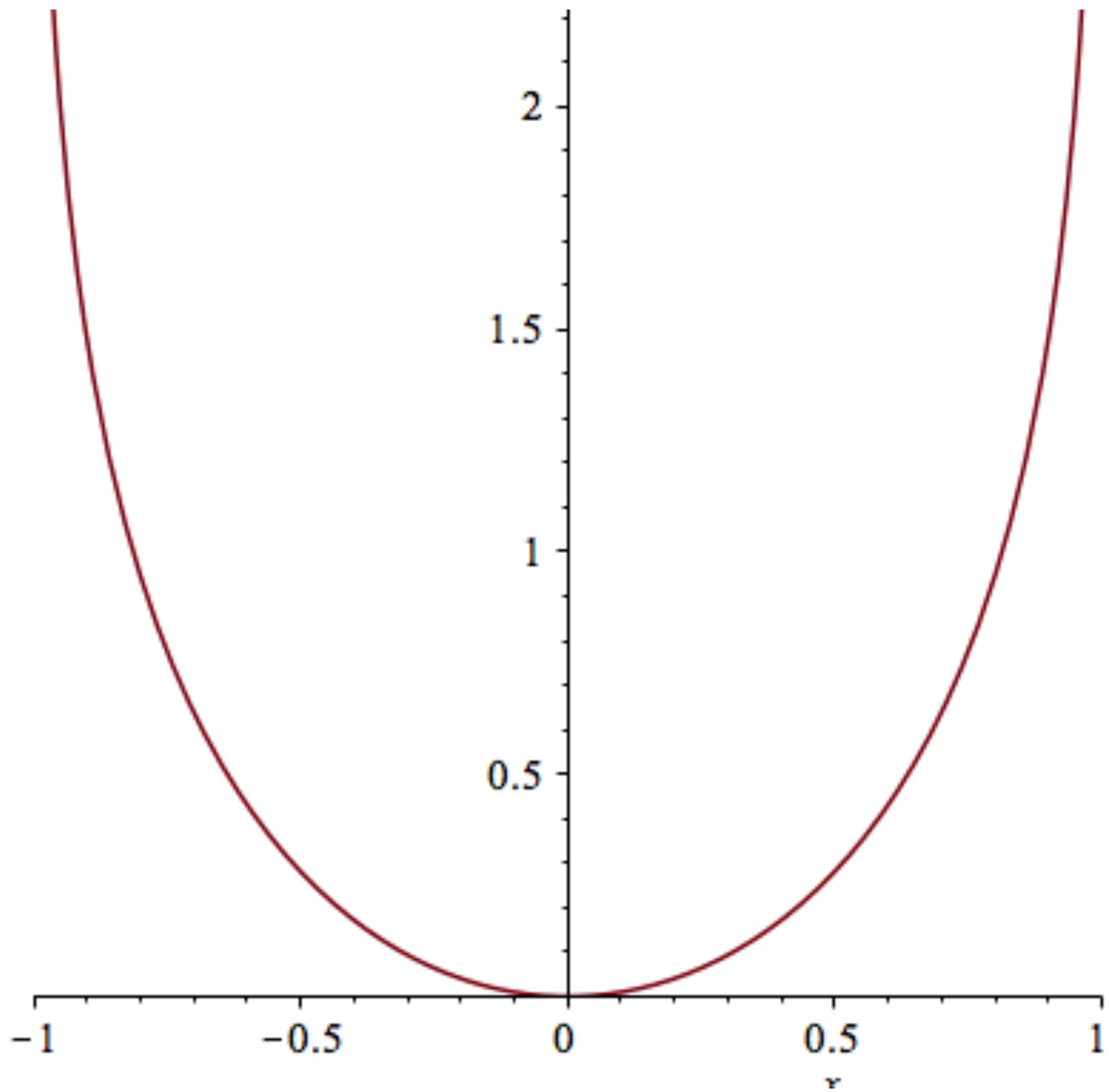
Oefening

1. Plot $f(x) = -\log(1 + x)\log(1 - x)$ op $[-1,1]$
2. Bereken $\int f(x)dx$ d.m.v. Maple
3. Bereken $\int f(x)dx$ d.m.v. de trapeziumregel en zorg dat de relatieve fout $< 2^{-23}$



Plot f(x)

```
plot(−log(1 + x) * log(1 − x))
```

Exacte Berekening Maple

- `int(-log(1 + x) * log(1 - x), x = -1..1)`

  $= -4+4*\ln(2)-2*\ln(2)^2 + 1/3 *\pi^2$

- `evalf(-4+4*ln(2)-2*ln(2)`^{2}` + 1/3 *`π^2`)`

  $= 1.101550828$

Berekening trapeziumregel

$$\frac{b-a}{n}\left(\frac{1}{2}f(x_0)+f(x_1)+f(x_2)+\ldots+f(x_{n-1})+\frac{1}{2}f(x_n)\right).$$

- $n = 2^k$ met k = 1,2,3,…
- T(k) → benadering trapeziumregel met bepaalde k
- Stopconditie: $\left|\frac{T(k)-T(k+1)}{T(k+1)}\right| \leq 2^{-23}$

```cpp
double trapezium(std::function<double(double)> f, int k, double a,  double b){
  int n = pow(2, k);
  double h = (b-a)/n;

  double sum = 0.0;
  double x = a;

  for(int i = 1;i < n;i++){
    x += h;
    sum += h*f(x);
  }

  sum += (h/2)*f(a);
  sum += (h/2)*f(b);

  return sum;
}
```

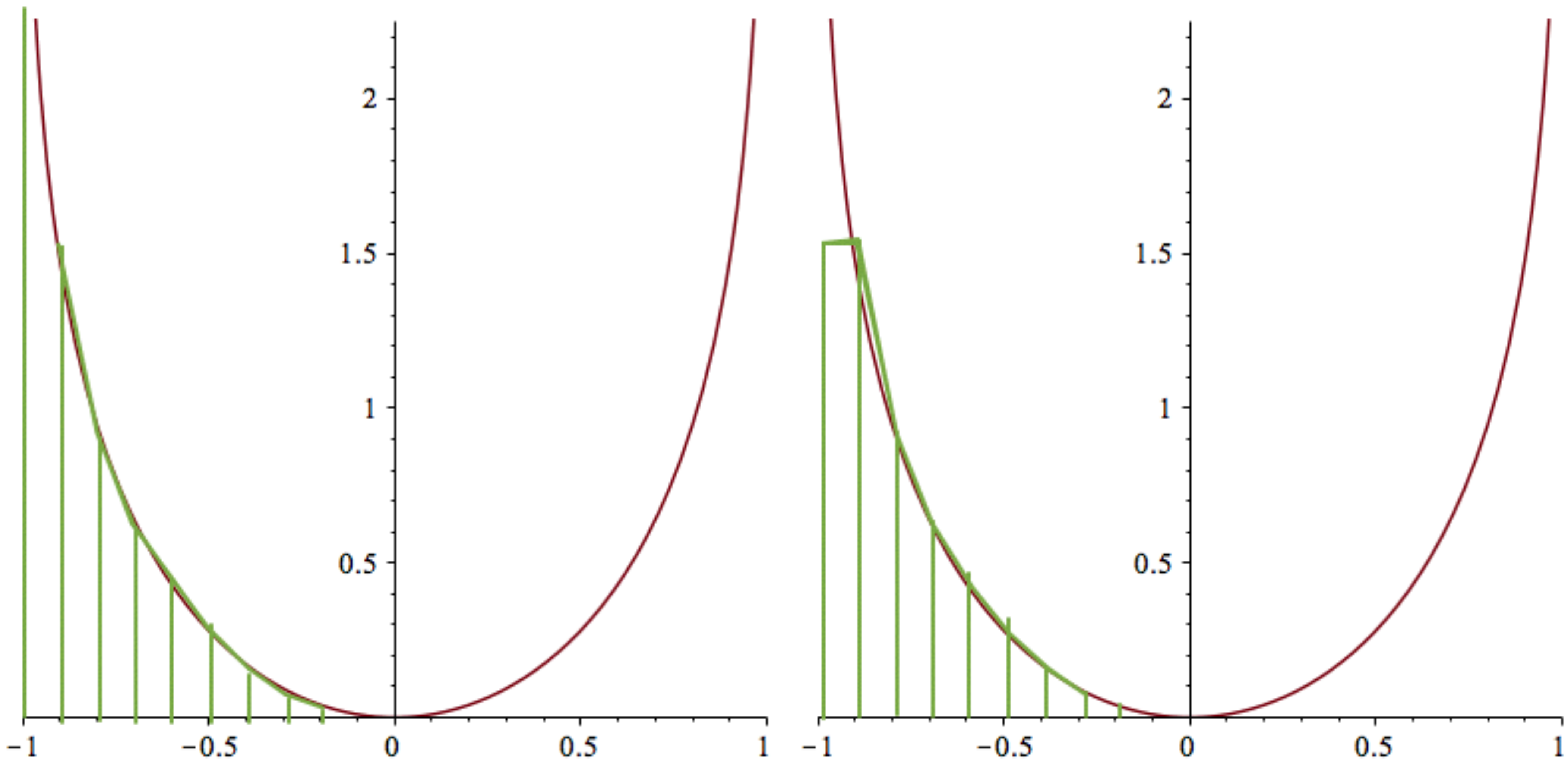
Probleem!

Alle $T(k) = \infty$

Grenzen zijn ∞

$$\frac{b-a}{n} * (\frac{1}{2} f_0 + f_1 + f_2 + ... + f_{n-1} + \frac{1}{2} f_n)$$

- Wanneer f_0 en $f_n = \infty$ ➞ gehele expressie = ∞
- Oplossing:
    - $f_0 = f_1$
    - $f_n = f_{n-1}$



2
1.5
1
0.5
−1
−0.5
0
0.5
1
2
1.5
1
0.5
−1
−0.5
0
0.5
1

```cpp
double trapezium(std::function<double(double)> f, int k, double a,  double b){
  int n = pow(2, k);
  double h = (b-a)/n;

  double sum = 0.0;
  double x = a;

  for(int i = 1;i < n;i++){
    x += h;
    sum += h*f(x);
  }

  sum += (h/2)*f(a+h);
  sum += (h/2)*f(b-h);

  return sum;
}
```

Uitkomsten

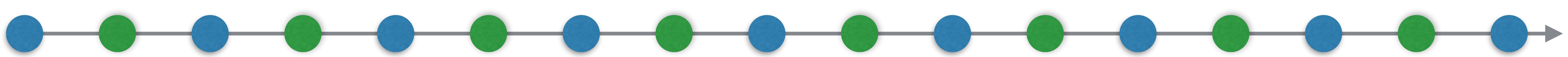
- k = 27
- n = 134217728
- 1.1015507219573130726075760321 7
  (Maple: 1.101550828)
- Relatieve fout =
  8.985730539048047128070160604 47e-08

Kunnen we dit optimaliseren?

Berekening f(x) waarden

$k = 3 \rightarrow n = 8$

$k = 4 \rightarrow n = 16$



n=2

$h(\frac{1}{2}f(-1) + \frac{1}{2}f(1)) + h(f(0))$

n=4

$\frac{h}{2}(\frac{1}{2}f(-1) + \frac{1}{2}f(1)) + \frac{h}{2}(f(0)) + \frac{h}{2}(f(-0.5) + f(0.5))$

n=8

$\frac{h}{4}(\frac{1}{2}f(-1) + \frac{1}{2}f(1)) + \frac{h}{4}(f(0)) + \frac{h}{4}(f(-0.5) + f(0.5)) + \frac{h}{4}(f(-0.75) + f(-0.25) + f(0.25) + f(0.75))$

Hoe berekenen?

- T(k-1)/2
- + aantal termen($2^{log_2(n)-1}$)
- Voor i = 1, …, $\frac{2^{log_2(n)-1}-2}{2}$
    - f(a+h+2*ih)
    - f(b-h-2ih)

n	f()
2	/
4	-0.5, 0.5
8	-0.75, -0.25, 0.75, 0.25
16	-0.875, -0.625, -0.375, -0.125, 0.875, 0.625, 0.375, 0.125

```cpp
double calculateSum(std::function<double(double)> f, int k, double a, double b){
  if(k == 1){
    double n = pow(2, k);
    double h = (b-a)/n;

    return h*f(0);
  }

   double nprev = pow(2, k-1);
   double n = nprev*2;
   double h = (b-a)/n;

   double incremental = h*2;
   double toCalc = (nprev-2)/2;

   double sum;
   double pos = a+h;
   sum += h*f(pos);
   sum += h*f(-pos);
   for(int i = 0;i < toCalc;i++){
     pos += incremental;
     sum += h*f(pos);
     sum += h*f(-pos);
   }

   return sum;
}
```

```
double trapeziume(std::function<double(double)> f, int k, double a,  double b){
  double sum = 0.0;
  for(int i = 1;i <= k;i++){
    double n = pow(2, i);

    if(i == 1){
      double h = (b-a)/n;

      sum += (h/2)*(f(a+h)+f(b-h));
      sum += calculateSum(f, i, a, b);
    }else{
      sum /= 2;
      sum += calculateSum(f, i, a, b);
    }
  }

  return sum;
}

};
```

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