# GNU MPFR

The Multiple Precision Floating-Point Reliable Library Edition 2.4.2 November 2009

The MPFR team

mpfr@loria.fr

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## **MPFR** Copying Conditions

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The precise conditions of the license for the GNU MPFR library are found in the Lesser General Public License that accompanies the source code. See the file COPYING.LIB.

## 1 Introduction to MPFR

MPFR is a portable library written in C for arbitrary precision arithmetic on floating-point numbers. It is based on the GNU MP library. It aims to extend the class of floating-point numbers provided by the GNU MP library by a precise semantics. The main differences with the mpf class from GNU MP are:

- the MPFR code is portable, i.e. the result of any operation does not depend (or should not) on the machine word size mp\_bits\_per\_limb (32 or 64 on most machines);
- the precision in bits can be set exactly to any valid value for each variable (including very small precision);
- MPFR provides the four rounding modes from the IEEE 754-1985 standard.

In particular, with a precision of 53 bits, MPFR should be able to exactly reproduce all computations with double-precision machine floating-point numbers (e.g., double type in C, with a C implementation that rigorously follows Annex F of the ISO C99 standard and FP\_CONTRACT pragma set to OFF) on the four arithmetic operations and the square root, except the default exponent range is much wider and subnormal numbers are not implemented (but can be emulated).

This version of MPFR is released under the GNU Lesser General Public License, Version 2.1 or any later version. It is permitted to link MPFR to most non-free programs, as long as when distributing them the MPFR source code and a means to re-link with a modified MPFR library is provided.

#### 1.1 How to Use This Manual

Everyone should read Chapter 4 [MPFR Basics], page 6. If you need to install the library yourself, you need to read Chapter 2 [Installing MPFR], page 3, too.

The rest of the manual can be used for later reference, although it is probably a good idea to glance through it.

## 2 Installing MPFR

#### 2.1 How to Install

Here are the steps needed to install the library on Unix systems (more details are provided in the 'INSTALL' file):

1. To build MPFR, you first have to install GNU MP (version 4.1 or higher) on your computer. You need a C compiler, preferably GCC, but any reasonable compiler should work. And you need a standard Unix 'make' program, plus some other standard Unix utility programs. Then, in the MPFR build directory, type the following commands.

#### 2. './configure'

This will prepare the build and setup the options according to your system. You can give options to specify the install directories (instead of the default '/usr/local'), threading support, and so on. See the 'INSTALL' file and/or the output of './configure --help' for more information, in particular if you get error messages.

#### 3. 'make'

This will compile MPFR, and create a library archive file 'libmpfr.a'. On most platforms, a dynamic library will be produced too (see configure).

#### 4. 'make check'

This will make sure MPFR was built correctly. If you get error messages, please report this to 'mpfr@loria.fr'. (See Chapter 3 [Reporting Bugs], page 5, for information on what to include in useful bug reports.)

#### 5. 'make install'

This will copy the files 'mpfr.h' and 'mpf2mpfr.h' to the directory '/usr/local/include', the library files ('libmpfr.a' and possibly others) to the directory '/usr/local/lib', the file 'mpfr.info' to the directory '/usr/local/share/info', and some other documentation files to the directory '/usr/local/share/doc/mpfr' (or if you passed the '--prefix' option to 'configure', using the prefix directory given as argument to '--prefix' instead of '/usr/local').

## 2.2 Other 'make' Targets

There are some other useful make targets:

• 'mpfr.info' or 'info'

Create or update an info version of the manual, in 'mpfr.info'.

This file is already provided in the MPFR archives.

'mpfr.pdf' or 'pdf'

Create a PDF version of the manual, in 'mpfr.pdf'.

• 'mpfr.dvi' or 'dvi'

Create a DVI version of the manual, in 'mpfr.dvi'.

• 'mpfr.ps' or 'ps'

Create a Postscript version of the manual, in 'mpfr.ps'.

• 'mpfr.html' or 'html'

Create a HTML version of the manual, in several pages in the directory 'mpfr.html'; if you want only one output HTML file, then type 'makeinfo --html --no-split mpfr.texi' instead.

• 'clean'

Delete all object files and archive files, but not the configuration files.

• 'distclean'

Delete all generated files not included in the distribution.

• 'uninstall'

Delete all files copied by 'make install'.

#### 2.3 Build Problems

In case of problem, please read the 'INSTALL' file carefully before reporting a bug, in particular section "In case of problem". Some problems are due to bad configuration on the user side (not specific to MPFR). Problems are also mentioned in the FAQ http://www.mpfr.org/faq.html.

Please report problems to 'mpfr@loria.fr'. See Chapter 3 [Reporting Bugs], page 5. Some bug fixes are available on the MPFR 2.4.2 web page http://www.mpfr.org/mpfr-2.4.2/.

## 2.4 Getting the Latest Version of MPFR

The latest version of MPFR is available from ftp://ftp.gnu.org/gnu/mpfr/ or http://www.mpfr.org/.

## 3 Reporting Bugs

If you think you have found a bug in the MPFR library, first have a look on the MPFR 2.4.2 web page http://www.mpfr.org/mpfr-2.4.2/ and the FAQ http://www.mpfr.org/faq.html: perhaps this bug is already known, in which case you may find there a workaround for it. Otherwise, please investigate and report it. We have made this library available to you, and it is not to ask too much from you, to ask you to report the bugs that you find.

There are a few things you should think about when you put your bug report together.

You have to send us a test case that makes it possible for us to reproduce the bug. Include instructions on how to run the test case.

You also have to explain what is wrong; if you get a crash, or if the results printed are incorrect and in that case, in what way.

Please include compiler version information in your bug report. This can be extracted using 'cc -V' on some machines, or, if you're using gcc, 'gcc -v'. Also, include the output from 'uname -a' and the MPFR version (the GMP version may be useful too).

If your bug report is good, we will do our best to help you to get a corrected version of the library; if the bug report is poor, we will not do anything about it (aside of chiding you to send better bug reports).

Send your bug report to: 'mpfr@loria.fr'.

If you think something in this manual is unclear, or downright incorrect, or if the language needs to be improved, please send a note to the same address.

#### 4 MPFR Basics

#### 4.1 Headers and Libraries

All declarations needed to use MPFR are collected in the include file 'mpfr.h'. It is designed to work with both C and C++ compilers. You should include that file in any program using the MPFR library:

```
#include <mpfr.h>
```

Note however that prototypes for MPFR functions with FILE \* parameters are provided only if <stdio.h> is included too (before 'mpfr.h').

```
#include <stdio.h>
#include <mpfr.h>
```

Likewise <stdarg.h> (or <varargs.h>) is required for prototypes with va\_list parameters, such as mpfr\_vprintf.

And for any functions using intmax\_t, you must include <stdint.h> or <inttypes.h> before 'mpfr.h', to allow 'mpfr.h' to define prototypes for these functions. Moreover, users of C++ compilers under some platforms may need to define the \_\_STDC\_CONSTANT\_MACROS macro (before <stdint.h> or <inttypes.h> has been included) or MPFR\_USE\_INTMAX\_T (before 'mpfr.h' has been included), at least for portability; of course, it is possible to do that on the command line, e.g., with -DMPFR\_USE\_INTMAX\_T.

You can avoid the use of MPFR macros encapsulating functions by defining the 'MPFR\_USE\_NO\_MACRO' macro before 'mpfr.h' is included. In general this should not be necessary, but this can be useful when debugging user code: with some macros, the compiler may emit spurious warnings with some warning options, and macros can prevent some prototype checking.

All programs using MPFR must link against both 'libmpfr' and 'libgmp' libraries. On a typical Unix-like system this can be done with '-lmpfr -lgmp' (in that order), for example

```
gcc myprogram.c -lmpfr -lgmp
```

MPFR is built using Libtool and an application can use that to link if desired, see GNU Libtool.

If MPFR has been installed to a non-standard location, then it may be necessary to set up environment variables such as 'C\_INCLUDE\_PATH' and 'LIBRARY\_PATH', or use '-I' and '-L' compiler options, in order to point to the right directories. For a shared library, it may also be necessary to set up some sort of run-time library path (e.g., 'LD\_LIBRARY\_PATH') on some systems. Please read the 'INSTALL' file for additional information.

## 4.2 Nomenclature and Types

A floating-point number or float for short, is an arbitrary precision significand (also called mantissa) with a limited precision exponent. The C data type for such objects is  $mpfr_t$  (internally defined as a one-element array of a structure, and  $mpfr_ptr$  is the C data type representing a pointer to this structure). A floating-point number can have three special values: Not-a-Number (NaN) or plus or minus Infinity. NaN represents an uninitialized object, the result of an invalid operation (like 0 divided by 0), or a value that cannot be determined (like +Infinity minus +Infinity). Moreover, like in the IEEE 754 standard, zero is signed, i.e. there are both +0 and -0; the behavior is the same as in the IEEE 754 standard and it is generalized to the other functions supported by MPFR. Unless documented otherwise, the sign bit of a NaN is unspecified.

The precision is the number of bits used to represent the significand of a floating-point number; the corresponding C data type is mp\_prec\_t. The precision can be any integer between MPFR\_PREC\_MIN and MPFR\_PREC\_MAX. In the current implementation, MPFR\_PREC\_MIN is equal to 2.

Warning! MPFR needs to increase the precision internally, in order to provide accurate results (and in particular, correct rounding). Do not attempt to set the precision to any value near MPFR\_PREC\_MAX, otherwise MPFR will abort due to an assertion failure. Moreover, you may reach some memory limit on your platform, in which case the program may abort, crash or have undefined behavior (depending on your C implementation).

The rounding mode specifies the way to round the result of a floating-point operation, in case the exact result can not be represented exactly in the destination significand; the corresponding C data type is mp\_rnd\_t.

A *limb* means the part of a multi-precision number that fits in a single word. (We chose this word because a limb of the human body is analogous to a digit, only larger, and containing several digits.) Normally a limb contains 32 or 64 bits. The C data type for a limb is mp\_limb\_t.

#### 4.3 Function Classes

There is only one class of functions in the MPFR library:

1. Functions for floating-point arithmetic, with names beginning with mpfr\_. The associated type is mpfr\_t.

#### 4.4 MPFR Variable Conventions

As a general rule, all MPFR functions expect output arguments before input arguments. This notation is based on an analogy with the assignment operator.

MPFR allows you to use the same variable for both input and output in the same expression. For example, the main function for floating-point multiplication,  $mpfr_mul$ , can be used like this:  $mpfr_mul$  (x, x, x, rnd\_mode). This computes the square of x with rounding mode rnd\_mode and puts the result back in x.

Before you can assign to an MPFR variable, you need to initialize it by calling one of the special initialization functions. When you're done with a variable, you need to clear it out, using one of the functions for that purpose.

A variable should only be initialized once, or at least cleared out between each initialization. After a variable has been initialized, it may be assigned to any number of times.

For efficiency reasons, avoid to initialize and clear out a variable in loops. Instead, initialize it before entering the loop, and clear it out after the loop has exited.

You do not need to be concerned about allocating additional space for MPFR variables, since any variable has a significand of fixed size. Hence unless you change its precision, or clear and reinitialize it, a floating-point variable will have the same allocated space during all its life.

## 4.5 Rounding Modes

The following four rounding modes are supported:

- GMP\_RNDN: round to nearest (roundTiesToEven in IEEE 754-2008)
- GMP\_RNDZ: round toward zero (roundTowardZero in IEEE 754-2008)
- GMP\_RNDU: round toward plus infinity (roundTowardPositive in IEEE 754-2008)
- GMP\_RNDD: round toward minus infinity (roundTowardNegative in IEEE 754-2008)

The 'round to nearest' mode works as in the IEEE 754 standard: in case the number to be rounded lies exactly in the middle of two representable numbers, it is rounded to the one with the least significant bit set to zero. For example, the number 5/2, which is represented by (10.1) in binary, is rounded to (10.0)=2 with a precision of two bits, and not to (11.0)=3. This rule avoids the *drift* phenomenon mentioned by Knuth in volume 2 of The Art of Computer Programming (Section 4.2.2).

Most MPFR functions take as first argument the destination variable, as second and following arguments the input variables, as last argument a rounding mode, and have a return value of type int, called the *ternary value*. The value stored in the destination variable is correctly rounded, i.e. MPFR behaves as if it computed the result with an infinite precision, then rounded it to the precision of this variable. The input variables are regarded as exact (in particular, their precision does not affect the result).

As a consequence, in case of a non-zero real rounded result, the error on the result is less or equal to 1/2 ulp (unit in the last place) of the target in the rounding to nearest mode, and less than 1 ulp of the target in the directed rounding modes (a ulp is the weight of the least significant represented bit of the target after rounding).

Unless documented otherwise, functions returning an int return a ternary value. If the ternary value is zero, it means that the value stored in the destination variable is the exact result of the corresponding mathematical function. If the ternary value is positive (resp. negative), it means the value stored in the destination variable is greater (resp. lower) than the exact result. For example with the GMP\_RNDU rounding mode, the ternary value is usually positive, except when the result is exact, in which case it is zero. In the case of an infinite result, it is considered as inexact when it was obtained by overflow, and exact otherwise. A NaN result (Not-a-Number) always corresponds to an exact return value. The opposite of a returned ternary value is guaranteed to be representable in an int.

Unless documented otherwise, functions returning a 1 (or any other value specified in this manual) for special cases (like acos(0)) should return an overflow or an underflow if 1 is not representable in the current exponent range.

## 4.6 Floating-Point Values on Special Numbers

This section specifies the floating-point values (of type mpfr\_t) returned by MPFR functions. For functions returning several values (like mpfr\_sin\_cos), the rules apply to each result separately.

Functions can have one or several input arguments. An input point is a mapping from these input arguments to the set of the MPFR numbers. When none of its components are NaN, an input point can also be seen as a tuple in the extended real numbers (the set of the real numbers with both infinities).

When the input point is in the domain of the mathematical function, the result is rounded as described in Section "Rounding Modes" (but see below for the specification of the sign of an exact zero). Otherwise the general rules from this section apply unless stated otherwise in the description of the MPFR function (Chapter 5 [MPFR Interface], page 11).

When the input point is not in the domain of the mathematical function but is in its closure in the extended real numbers and the function can be extended by continuity, the result is the obtained limit. Examples:  $mpfr_hypot$  on (+Inf,0) gives +Inf. But  $mpfr_pow$  cannot be defined on (1,+Inf) using this rule, as one can find sequences  $(x_n,y_n)$  such that  $x_n$  goes to 1,  $y_n$  goes to +Inf and  $(x_n)^{y_n}$  goes to any positive value when n goes to the infinity.

When the input point is in the closure of the domain of the mathematical function and an input argument is +0 (resp. -0), one considers the limit when the corresponding argument approaches 0 from above (resp. below). If the limit is not defined (e.g.,  $mpfr_log$  on -0), the behavior must be specified in the description of the MPFR function.

When the result is equal to 0, its sign is determined by considering the limit as if the input point were not in the domain: If one approaches 0 from above (resp. below), the result is +0 (resp. -0). In the other cases, the sign must be specified in the description of the MPFR function. Example: mpfr\_sin on +0 gives +0.

When the input point is not in the closure of the domain of the function, the result is NaN. Example:  $mpfr\_sqrt$  on -17 gives NaN.

When an input argument is NaN, the result is NaN, possibly except when a partial function is constant on the finite floating-point numbers; such a case is always explicitly specified in Chapter 5 [MPFR Interface], page 11. Example:  $mpfr_hypot$  on (NaN,0) gives NaN, but  $mpfr_hypot$  on (NaN,+Inf) gives +Inf (as specified in Section 5.7 [Special Functions], page 21), since for any finite input x,  $mpfr_hypot$  on (x,+Inf) gives +Inf.

## 4.7 Exceptions

MPFR supports 5 exception types:

- Underflow: An underflow occurs when the exact result of a function is a non-zero real number and the result obtained after the rounding, assuming an unbounded exponent range (for the rounding), has an exponent smaller than the minimum exponent of the current range. In the round-to-nearest mode, the halfway case is rounded toward zero.
  - Note: This is not the single definition of the underflow. MPFR chooses to consider the underflow after rounding. The underflow before rounding can also be defined. For instance, consider a function that has the exact result  $7 \times 2^{e-4}$ , where e is the smallest exponent (for a significand between 1/2 and 1) in the current range, with a 2-bit target precision and rounding toward plus infinity. The exact result has the exponent e-1. With the underflow before rounding, such a function call would yield an underflow, as e-1 is outside the current exponent range. However, MPFR first considers the rounded result assuming an unbounded exponent range. The exact result cannot be represented exactly in precision 2, and here, it is rounded to  $0.5 \times 2^e$ , which is representable in the current exponent range. As a consequence, this will not yield an underflow in MPFR.
- Overflow: An overflow occurs when the exact result of a function is a non-zero real number and the result obtained after the rounding, assuming an unbounded exponent range (for the rounding), has an exponent larger than the maximum exponent of the current range. In the round-to-nearest mode, the result is infinite.
- NaN: A NaN exception occurs when the result of a function is a NaN.
- Inexact: An inexact exception occurs when the result of a function cannot be represented exactly and must be rounded.
- Range error: A range exception occurs when a function that does not return a MPFR number (such as comparisons and conversions to an integer) has an invalid result (e.g. an argument is NaN in mpfr\_cmp or in a conversion to an integer).

MPFR has a global flag for each exception, which can be cleared, set or tested by functions described in Section 5.13 [Exception Related Functions], page 32.

Differences with the ISO C99 standard:

• In C, only quiet NaNs are specified, and a NaN propagation does not raise an invalid exception. Unless explicitly stated otherwise, MPFR sets the NaN flag whenever a NaN

is generated, even when a NaN is propagated (e.g. in NaN + NaN), as if all NaNs were signaling.

• An invalid exception in C corresponds to either a NaN exception or a range error in MPFR.

## 4.8 Memory Handling

MPFR functions may create caches, e.g. when computing constants such as  $\pi$ , either because the user has called a function like mpfr\_const\_pi directly or because such a function was called internally by the MPFR library itself to compute some other function.

At any time, the user can free the various caches with mpfr\_free\_cache. It is strongly advised to do that before terminating a thread, or before exiting when using tools like 'valgrind' (to avoid memory leaks being reported).

MPFR internal data such as flags, the exponent range, the default precision and rounding mode, and caches (i.e., data that are not accessed via parameters) are either global (if MPFR has not been compiled as thread safe) or per-thread (thread local storage).

## 5 MPFR Interface

The floating-point functions expect arguments of type mpfr\_t.

The MPFR floating-point functions have an interface that is similar to the GNU MP integer functions. The function prefix for floating-point operations is mpfr\_.

There is one significant characteristic of floating-point numbers that has motivated a difference between this function class and other GNU MP function classes: the inherent inexactness of floating-point arithmetic. The user has to specify the precision for each variable. A computation that assigns a variable will take place with the precision of the assigned variable; the cost of that computation should not depend from the precision of variables used as input (on average).

The semantics of a calculation in MPFR is specified as follows: Compute the requested operation exactly (with "infinite accuracy"), and round the result to the precision of the destination variable, with the given rounding mode. The MPFR floating-point functions are intended to be a smooth extension of the IEEE 754 arithmetic. The results obtained on one computer should not differ from the results obtained on a computer with a different word size.

MPFR does not keep track of the accuracy of a computation. This is left to the user or to a higher layer. As a consequence, if two variables are used to store only a few significant bits, and their product is stored in a variable with large precision, then MPFR will still compute the result with full precision.

The value of the standard C macro errno may be set to non-zero by any MPFR function or macro, whether or not there is an error.

#### 5.1 Initialization Functions

An mpfr\_t object must be initialized before storing the first value in it. The functions mpfr\_init and mpfr\_init2 are used for that purpose.

#### void mpfr\_init2 (mpfr\_t x, mp\_prec\_t prec)

[Function]

Initialize x, set its precision to be **exactly** prec bits and its value to NaN. (Warning: the corresponding mpf functions initialize to zero instead.)

Normally, a variable should be initialized once only or at least be cleared, using mpfr\_clear, between initializations. To change the precision of a variable which has already been initialized, use mpfr\_set\_prec. The precision *prec* must be an integer between MPFR\_PREC\_MIN and MPFR\_PREC\_MAX (otherwise the behavior is undefined).

#### void mpfr\_inits2 (mp\_prec\_t prec, mpfr\_t x, ...)

[Function]

Initialize all the mpfr\_t variables of the given va\_list, set their precision to be exactly prec bits and their value to NaN. See mpfr\_init2 for more details. The va\_list is assumed to be composed only of type mpfr\_t (or equivalently mpfr\_ptr). It begins from x. It ends when it encounters a null pointer (whose type must also be mpfr\_ptr).

#### void mpfr\_clear (mpfr\_t x)

[Function]

Free the space occupied by x. Make sure to call this function for all  $mpfr_t$  variables when you are done with them.

#### void mpfr\_clears (mpfr\_t x, ...)

[Function]

Free the space occupied by all the mpfr\_t variables of the given va\_list. See mpfr\_clear for more details. The va\_list is assumed to be composed only of type mpfr\_t (or equivalently

 $mpfr_ptr$ ). It begins from x. It ends when it encounters a null pointer (whose type must also be  $mpfr_ptr$ ).

Here is an example of how to use multiple initialization functions:

```
{
    mpfr_t x, y, z, t;
    mpfr_inits2 (256, x, y, z, t, (mpfr_ptr) 0);
    ...
    mpfr_clears (x, y, z, t, (mpfr_ptr) 0);
}
```

```
void mpfr_init (mpfr_t x)
```

[Function]

Initialize x and set its value to NaN.

Normally, a variable should be initialized once only or at least be cleared, using  $mpfr\_clear$ , between initializations. The precision of x is the default precision, which can be changed by a call to  $mpfr\_set\_default\_prec$ .

Warning! In a given program, some other libraries might change the default precision and not restore it. Thus it is safer to use mpfr\_init2.

#### void mpfr\_inits (mpfr\_t x, ...)

[Function]

Initialize all the mpfr\_t variables of the given va\_list, set their precision to be the default precision and their value to NaN. See mpfr\_init for more details. The va\_list is assumed to be composed only of type mpfr\_t (or equivalently mpfr\_ptr). It begins from x. It ends when it encounters a null pointer (whose type must also be mpfr\_ptr).

Warning! In a given program, some other libraries might change the default precision and not restore it. Thus it is safer to use mpfr\_inits2.

#### MPFR\_DECL\_INIT (name, prec)

[Macro]

This macro declares name as an automatic variable of type mpfr\_t, initializes it and sets its precision to be exactly prec bits and its value to NaN. name must be a valid identifier. You must use this macro in the declaration section. This macro is much faster than using mpfr\_init2 but has some drawbacks:

- You must not call mpfr\_clear with variables created with this macro (the storage is allocated at the point of declaration and deallocated when the brace-level is exited).
- You cannot change their precision.
- You **should not** create variables with huge precision with this macro.
- Your compiler must support 'Non-Constant Initializers' (standard in C++ and ISO C99) and 'Token Pasting' (standard in ISO C89). If prec is not a constant expression, your compiler must support 'variable-length automatic arrays' (standard in ISO C99). 'GCC 2.95.3' and above supports all these features. If you compile your program with gcc in c89 mode and with '-pedantic', you may want to define the MPFR\_USE\_EXTENSION macro to avoid warnings due to the MPFR\_DECL\_INIT implementation.

#### void mpfr\_set\_default\_prec (mp\_prec\_t prec)

[Function]

Set the default precision to be **exactly** prec bits. The precision of a variable means the number of bits used to store its significand. All subsequent calls to mpfr\_init will use this precision, but previously initialized variables are unaffected. This default precision is set to 53 bits initially. The precision can be any integer between MPFR\_PREC\_MIN and MPFR\_PREC\_MAX.

```
mp_prec_t mpfr_get_default_prec (void)
Return the default MPFR precision in bits.
```

[Function]

Here is an example on how to initialize floating-point variables:

The following functions are useful for changing the precision during a calculation. A typical use would be for adjusting the precision gradually in iterative algorithms like Newton-Raphson, making the computation precision closely match the actual accurate part of the numbers.

```
void mpfr_set_prec (mpfr_t x, mp_prec_t prec)
```

[Function]

Reset the precision of x to be **exactly** prec bits, and set its value to NaN. The previous value stored in x is lost. It is equivalent to a call to  $mpfr_clear(x)$  followed by a call to  $mpfr_init2(x, prec)$ , but more efficient as no allocation is done in case the current allocated space for the significand of x is enough. The precision prec can be any integer between MPFR\_PREC\_MIN and MPFR\_PREC\_MAX.

In case you want to keep the previous value stored in x, use mpfr\_prec\_round instead.

```
mp_prec_t mpfr_get_prec (mpfr_t x)
```

[Function]

Return the precision actually used for assignments of x, i.e. the number of bits used to store its significand.

## 5.2 Assignment Functions

These functions assign new values to already initialized floats (see Section 5.1 [Initialization Functions], page 11).

```
int mpfr_set (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
                                                                            [Function]
int mpfr_set_ui (mpfr_t rop, unsigned long int op, mp_rnd_t rnd)
                                                                            [Function]
int mpfr_set_si (mpfr_t rop, long int op, mp_rnd_t rnd)
                                                                            [Function]
int mpfr_set_uj (mpfr_t rop, uintmax_t op, mp_rnd_t rnd)
                                                                            [Function]
int mpfr_set_sj (mpfr_t rop, intmax_t op, mp_rnd_t rnd)
                                                                            [Function]
int mpfr_set_d (mpfr_t rop, double op, mp_rnd_t rnd)
                                                                            [Function]
int mpfr_set_ld (mpfr_t rop, long double op, mp_rnd_t rnd)
                                                                            [Function]
int mpfr_set_decimal64 (mpfr_t rop, _Decimal64 op, mp_rnd_t rnd)
                                                                            [Function]
int mpfr_set_z (mpfr_t rop, mpz_t op, mp_rnd_t rnd)
                                                                            [Function]
int mpfr_set_q (mpfr_t rop, mpq_t op, mp_rnd_t rnd)
                                                                            [Function]
int mpfr_set_f (mpfr_t rop, mpf_t op, mp_rnd_t rnd)
                                                                            [Function]
```

Set the value of rop from op, rounded toward the given direction rnd. Note that the input 0 is converted to +0 by mpfr\_set\_ui, mpfr\_set\_si, mpfr\_set\_sj, mpfr\_set\_uj, mpfr\_set\_z, mpfr\_set\_q and mpfr\_set\_f, regardless of the rounding mode. If the system does not support the IEEE 754 standard, mpfr\_set\_d, mpfr\_set\_ld and mpfr\_set\_decimal64 might not preserve the signed zeros. The mpfr\_set\_decimal64 function is built only with the

configure option '--enable-decimal-float', which also requires '--with-gmp-build', and when the compiler or system provides the '\_Decimal64' data type (GCC version 4.2.0 is known to support this data type, but only when configured with '--enable-decimal-float' too). mpfr\_set\_q might not be able to work if the numerator (or the denominator) can not be representable as a mpfr\_t.

Note: If you want to store a floating-point constant to a  $mpfr_t$ , you should use  $mpfr_set_str$  (or one of the MPFR constant functions, such as  $mpfr_const_pi$  for  $\pi$ ) instead of  $mpfr_set_d$ ,  $mpfr_set_ld$  or  $mpfr_set_decimal64$ . Otherwise the floating-point constant will be first converted into a reduced-precision (e.g., 53-bit) binary number before MPFR can work with it.

Set the value of rop from  $op \times 2^e$ , rounded toward the given direction rnd. Note that the input 0 is converted to +0.

int mpfr\_set\_str (mpfr\_t rop, const char \*s, int base, mp\_rnd\_t rnd) [Function] Set rop to the value of the string s in base base, rounded in the direction rnd. See the documentation of mpfr\_strtofr for a detailed description of the valid string formats. Contrary to mpfr\_strtofr, mpfr\_set\_str requires the whole string to represent a valid floating-point number. This function returns 0 if the entire string up to the final null character is a valid number in base base; otherwise it returns -1, and rop may have changed.

Read a floating-point number from a string *nptr* in base *base*, rounded in the direction *rnd*; base must be either 0 (to detect the base, as described below) or a number from 2 to 36 (otherwise the behavior is undefined). If *nptr* starts with valid data, the result is stored in *rop* and \*endptr points to the character just after the valid data (if endptr is not a null pointer); otherwise *rop* is set to zero and the value of *nptr* is stored in the location referenced by endptr (if endptr is not a null pointer). The usual ternary value is returned.

Parsing follows the standard C strtod function with some extensions. Case is ignored. After optional leading whitespace, one has a subject sequence consisting of an optional sign (+ or -), and either numeric data or special data. The subject sequence is defined as the longest initial subsequence of the input string, starting with the first non-whitespace character, that is of the expected form.

The form of numeric data is a non-empty sequence of significand digits with an optional decimal point, and an optional exponent consisting of an exponent prefix followed by an optional sign and a non-empty sequence of decimal digits. A significand digit is either a decimal digit or a Latin letter (62 possible characters), with  $a=10,\,b=11,\,\ldots,\,z=35;$  its value must be strictly less than the base. The decimal point can be either the one defined by the current locale or the period (the first one is accepted for consistency with the C standard and the practice, the second one is accepted to allow the programmer to provide MPFR numbers from strings in a way that does not depend on the current locale). The exponent prefix can be e or E for bases up to 10, or O in any base; it indicates a multiplication by

a power of the base. In bases 2 and 16, the exponent prefix can also be p or P, in which case it introduces a binary exponent: it indicates a multiplication by a power of 2 (there is a difference only for base 16). The value of an exponent is always written in base 10. In base 2, the significand can start with 0b or OB, and in base 16, it can start with 0x or OX.

If the argument base is 0, then the base is automatically detected as follows. If the significand starts with 0b or 0B, base 2 is assumed. If the significand starts with 0x or 0X, base 16 is assumed. Otherwise base 10 is assumed.

Note: The exponent must contain at least a digit. Otherwise the possible exponent prefix and sign are not part of the number (which ends with the significand). Similarly, if 0b, 0B, 0x or 0X is not followed by a binary/hexadecimal digit, then the subject sequence stops at the character 0.

Special data (for infinities and NaN) can be @inf@ or @nan@(n-char-sequence), and if  $base \leq 16$ , it can also be infinity, inf, nan or nan(n-char-sequence-opt), all case insensitive. A n-char-sequence-opt is a possibly empty string containing only digits, Latin letters and the underscore  $(0, 1, 2, \ldots, 9, a, b, \ldots, z, A, B, \ldots, Z, \_)$ . Note: one has an optional sign for all data, even NaN.

Set the variable x to infinity or NaN (Not-a-Number) respectively. In  $mpfr_set_inf$ , x is set to plus infinity iff sign is nonnegative.

```
void mpfr_swap (mpfr_t x, mpfr_t y)
```

[Function]

Swap the values x and y efficiently. Warning: the precisions are exchanged too; in case the precisions are different,  $mpfr_swap$  is thus not equivalent to three  $mpfr_set$  calls using a third auxiliary variable.

## 5.3 Combined Initialization and Assignment Functions

```
int mpfr_init_set (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
                                                                               [Macro]
int mpfr_init_set_ui (mpfr_t rop, unsigned long int op, mp_rnd_t rnd)
                                                                               [Macro]
int mpfr_init_set_si (mpfr_t rop, signed long int op, mp_rnd_t rnd)
                                                                               [Macro]
int mpfr_init_set_d (mpfr_t rop, double op, mp_rnd_t rnd)
                                                                               [Macro]
int mpfr_init_set_ld (mpfr_t rop, long double op, mp_rnd_t rnd)
                                                                               [Macro]
int mpfr_init_set_z (mpfr_t rop, mpz_t op, mp_rnd_t rnd)
                                                                               [Macro]
int mpfr_init_set_q (mpfr_t rop, mpq_t op, mp_rnd_t rnd)
                                                                               [Macro]
int mpfr_init_set_f (mpfr_t rop, mpf_t op, mp_rnd_t rnd)
                                                                               [Macro]
  Initialize rop and set its value from op, rounded in the direction rnd. The precision of rop
  will be taken from the active default precision, as set by mpfr_set_default_prec.
```

int mpfr\_init\_set\_str (mpfr\_t x, const char \*s, int base, mp\_rnd\_t rnd) [Function] Initialize x and set its value from the string s in base base, rounded in the direction rnd. See mpfr\_set\_str.

#### 5.4 Conversion Functions

```
double mpfr_get_d (mpfr_t op, mp_rnd_t rnd) [Function]
long double mpfr_get_ld (mpfr_t op, mp_rnd_t rnd) [Function]
_Decimal64 mpfr_get_decimal64 (mpfr_t op, mp_rnd_t rnd) [Function]
Convert op to a double (respectively _Decimal64 or long double), using the rounding mode
```

rnd. If op is NaN, some fixed NaN (either quiet or signaling) or the result of 0.0/0.0 is

returned. If op is  $\pm Inf$ , an infinity of the same sign or the result of  $\pm 1.0/0.0$  is returned. If op is zero, these functions return a zero, trying to preserve its sign, if possible. The mpfr\_get\_decimal64 function is built only under some conditions: see the documentation of mpfr\_set\_decimal64.

double mpfr\_get\_d\_2exp ( $long *exp, mpfr_t op, mp_rnd_t rnd$ ) [Function] long double mpfr\_get\_ld\_2exp ( $long *exp, mpfr_t op, mp_rnd_t rnd$ ) [Function] Return d and set exp such that  $0.5 \le |d| < 1$  and  $d \times 2^{exp}$  equals exp or rounded to double (resp. long double) precision, using the given rounding mode. If exp is zero, then a zero of the same sign (or an unsigned zero, if the implementation does not have signed zeros) is returned, and exp is set to 0. If exp is NaN or an infinity, then the corresponding double precision (resp. long-double precision) value is returned, and exp is undefined.

```
long mpfr_get_si (mpfr_t op, mp_rnd_t rnd) [Function]
unsigned long mpfr_get_ui (mpfr_t op, mp_rnd_t rnd) [Function]
intmax_t mpfr_get_sj (mpfr_t op, mp_rnd_t rnd) [Function]
uintmax_t mpfr_get_uj (mpfr_t op, mp_rnd_t rnd) [Function]
```

Convert op to a long, an unsigned long, an intmax\_t or an uintmax\_t (respectively) after rounding it with respect to rnd. If op is NaN, the result is undefined. If op is too big for the return type, it returns the maximum or the minimum of the corresponding C type, depending on the direction of the overflow. The erange flag is set too. See also mpfr\_fits\_slong\_p, mpfr\_fits\_ulong\_p, mpfr\_fits\_intmax\_p and mpfr\_fits\_uintmax\_p.

## mp\_exp\_t mpfr\_get\_z\_exp (mpz\_t rop, mpfr\_t op) [Function]

Put the scaled significand of op (regarded as an integer, with the precision of op) into rop, and return the exponent exp (which may be outside the current exponent range) such that op exactly equals  $rop \times 2^{exp}$ . If op is zero, the minimal exponent emin is returned. If the exponent is not representable in the exponent type, the behavior is undefined.

```
void mpfr_get_z (mpz_t rop, mpfr_t op, mp_rnd_t rnd) [Function] Convert op to a mpz_t, after rounding it with respect to rnd. If op is NaN or Inf, the result is undefined.
```

```
int mpfr_get_f (mpf_t rop, mpfr_t op, mp_rnd_t rnd) [Function] Convert op to a mpf_t, after rounding it with respect to rnd. Return zero iff no error occurred, in particular a non-zero value is returned if op is NaN or Inf, which do not exist in mpf.
```

Convert op to a string of digits in base b, with rounding in the direction rnd, where n is either zero (see below) or the number of significant digits; in the latter case, n must be greater or equal to 2. The base may vary from 2 to 36.

The generated string is a fraction, with an implicit radix point immediately to the left of the first digit. For example, the number -3.1416 would be returned as "-31416" in the string and 1 written at expptr. If rnd is to nearest, and op is exactly in the middle of two possible outputs, the one with an even significand is chosen: that significand is considered with the exponent of op. Note that for an odd base, this may not correspond to an even last digit: for example with 2 digits in base 7, 16 and a half is rounded to 20 which is 14 in decimal, 36 and a half is rounded to 40 which is 28 in decimal, and 66 and a half is rounded to 66 which is 48 in decimal.

If n is zero, the number of digits of the significand is chosen large enough so that re-reading the printed value with the same precision, assuming both output and input use rounding

to nearest, will recover the original value of op. More precisely, in most cases, the chosen precision of str is the minimal precision depending on p = PREC(op) and b only that satisfies the above property, i.e.,  $m = 1 + \lceil p \frac{\log 2}{\log b} \rceil$ , with p replaced by p-1 if b is a power of 2, but in some very rare cases, it might be m+1 (the smallest case for bases up to 62 is when p equals 186564318007 for bases 7 and 49).

If str is a null pointer, space for the significand is allocated using the current allocation function, and a pointer to the string is returned. To free the returned string, you must use  $mpfr\_free\_str$ .

If str is not a null pointer, it should point to a block of storage large enough for the significand, i.e., at least max(n + 2, 7). The extra two bytes are for a possible minus sign, and for the terminating null character.

If the input number is an ordinary number, the exponent is written through the pointer expptr (the current minimal exponent for 0).

A pointer to the string is returned, unless there is an error, in which case a null pointer is returned.

```
void mpfr_free_str (char *str)
```

[Function]

Free a string allocated by mpfr\_get\_str using the current unallocation function (preliminary interface). The block is assumed to be strlen(str)+1 bytes. For more information about how it is done: see Section "Custom Allocation" in GNU MP.

```
int mpfr_fits_ulong_p (mpfr_t op, mp_rnd_t rnd)
                                                                           [Function]
int mpfr_fits_slong_p (mpfr_t op, mp_rnd_t rnd)
                                                                           [Function]
int mpfr_fits_uint_p (mpfr_t op, mp_rnd_t rnd)
                                                                           [Function]
int mpfr_fits_sint_p (mpfr_t op, mp_rnd_t rnd)
                                                                           [Function]
int mpfr_fits_ushort_p (mpfr_t op, mp_rnd_t rnd)
                                                                           [Function]
int mpfr_fits_sshort_p (mpfr_t op, mp_rnd_t rnd)
                                                                           [Function]
int mpfr_fits_intmax_p (mpfr_t op, mp_rnd_t rnd)
                                                                           [Function]
int mpfr_fits_uintmax_p (mpfr_t op, mp_rnd_t rnd)
                                                                           [Function]
```

Return non-zero if op would fit in the respective C data type, when rounded to an integer in the direction rnd.

#### 5.5 Basic Arithmetic Functions

```
int mpfr_add (mpfr_t rop, mpfr_t op1, mpfr_t op2, mp_rnd_t rnd)
                                                                               [Function]
int mpfr_add_ui (mpfr_t rop, mpfr_t op1, unsigned long int op2, mp_rnd_t
                                                                               [Function]
         rnd)
int mpfr_add_si (mpfr_t rop, mpfr_t op1, long int op2, mp_rnd_t rnd)
                                                                               [Function]
int mpfr_add_d (mpfr_t rop, mpfr_t op1, double op2, mp_rnd_t rnd)
                                                                               [Function]
int mpfr_add_z (mpfr_t rop, mpfr_t op1, mpz_t op2, mp_rnd_t rnd)
                                                                               [Function]
int mpfr_add_q (mpfr_t rop, mpfr_t op1, mpq_t op2, mp_rnd_t rnd)
                                                                               [Function]
  Set rop to op1 + op2 rounded in the direction rnd. For types having no signed zero, it is
  considered unsigned (i.e. (+0) + 0 = (+0) and (-0) + 0 = (-0)). The mpfr_add_d function
  assumes that the radix of the double type is a power of 2, with a precision at most that
  declared by the C implementation (macro IEEE_DBL_MANT_DIG, and if not defined 53 bits).
```

```
int mpfr_si_sub (mpfr_t rop, long int op1, mpfr_t op2, mp_rnd_t rnd)
                                                                                [Function]
int mpfr_sub_si (mpfr_t rop, mpfr_t op1, long int op2, mp_rnd_t rnd)
                                                                                [Function]
int mpfr_d_sub (mpfr_t rop, double op1, mpfr_t op2, mp_rnd_t rnd)
                                                                                [Function]
int mpfr_sub_d (mpfr_t rop, mpfr_t op1, double op2, mp_rnd_t rnd)
                                                                                [Function]
int mpfr_sub_z (mpfr_t rop, mpfr_t op1, mpz_t op2, mp_rnd_t rnd)
                                                                                [Function]
int mpfr_sub_q (mpfr_t rop, mpfr_t op1, mpq_t op2, mp_rnd_t rnd)
                                                                                [Function]
  Set rop to op1 - op2 rounded in the direction rnd. For types having no signed zero, it is
  considered unsigned (i.e. (+0) - 0 = (+0), (-0) - 0 = (-0), (-0) - (+0) = (-0) and (-0) - (-0)
  = (+0)). The same restrictions than for mpfr_add_d apply to mpfr_d_sub and mpfr_sub_d.
int mpfr_mul (mpfr_t rop, mpfr_t op1, mpfr_t op2, mp_rnd_t rnd)
                                                                                [Function]
int mpfr_mul_ui (mpfr_t rop, mpfr_t op1, unsigned long int op2, mp_rnd_t
                                                                                [Function]
         rnd)
int mpfr_mul_si (mpfr_t rop, mpfr_t op1, long int op2, mp_rnd_t rnd)
                                                                                [Function]
int mpfr_mul_d (mpfr_t rop, mpfr_t op1, double op2, mp_rnd_t rnd)
                                                                                [Function]
int mpfr_mul_z (mpfr_t rop, mpfr_t op1, mpz_t op2, mp_rnd_t rnd)
                                                                                [Function]
int mpfr_mul_q (mpfr_t rop, mpfr_t op1, mpq_t op2, mp_rnd_t rnd)
                                                                                [Function]
  Set rop to op1 \times op2 rounded in the direction rnd. When a result is zero, its sign is the
  product of the signs of the operands (for types having no signed zero, it is considered positive).
  The same restrictions than for mpfr_add_d apply to mpfr_mul_d.
int mpfr_sqr (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
                                                                                [Function]
  Set rop to op^2 rounded in the direction rnd.
int mpfr_div (mpfr_t rop, mpfr_t op1, mpfr_t op2, mp_rnd_t rnd)
                                                                                [Function]
int mpfr_ui_div (mpfr_t rop, unsigned long int op1, mpfr_t op2, mp_rnd_t
                                                                                [Function]
         rnd)
int mpfr_div_ui (mpfr_t rop, mpfr_t op1, unsigned long int op2, mp_rnd_t
                                                                               [Function]
         rnd)
int mpfr_si_div (mpfr_t rop, long int op1, mpfr_t op2, mp_rnd_t rnd)
                                                                                [Function]
int mpfr_div_si (mpfr_t rop, mpfr_t op1, long int op2, mp_rnd_t rnd)
                                                                                [Function]
int mpfr_d_div (mpfr_t rop, double op1, mpfr_t op2, mp_rnd_t rnd)
                                                                                [Function]
int mpfr_div_d (mpfr_t rop, mpfr_t op1, double op2, mp_rnd_t rnd)
                                                                                [Function]
int mpfr_div_z (mpfr_t rop, mpfr_t op1, mpz_t op2, mp_rnd_t rnd)
                                                                                [Function]
int mpfr_div_q (mpfr_t rop, mpfr_t op1, mpq_t op2, mp_rnd_t rnd)
                                                                                [Function]
  Set rop to op1/op2 rounded in the direction rnd. When a result is zero, its sign is the product
  of the signs of the operands (for types having no signed zero, it is considered positive). The
  same restrictions than for mpfr_add_d apply to mpfr_d_div and mpfr_div_d.
int mpfr_sqrt (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
                                                                                [Function]
int mpfr_sqrt_ui (mpfr_t rop, unsigned long int op, mp_rnd_t rnd)
                                                                                [Function]
  Set rop to \sqrt{op} rounded in the direction rnd. Return -0 if op is -0 (to be consistent with
  the IEEE 754 standard). Set rop to NaN if op is negative.
int mpfr_rec_sqrt (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
                                                                                [Function]
  Set rop to 1/\sqrt{op} rounded in the direction rnd. Return +Inf if op is \pm 0, and +0 if op is +Inf.
  Set rop to NaN if op is negative.
int mpfr_cbrt (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
                                                                                [Function]
int mpfr_root (mpfr_t rop, mpfr_t op, unsigned long int k, mp_rnd_t rnd)
                                                                               [Function]
  Set rop to the cubic root (resp. the kth root) of op rounded in the direction rnd. An odd
  (resp. even) root of a negative number (including -Inf) returns a negative number (resp.
```

NaN). The kth root of -0 is defined to be -0, whatever the parity of k.

Set rop to  $op1^{op2}$ , rounded in the direction rnd. Special values are currently handled as described in the ISO C99 standard for the pow function (note this may change in future versions):

- $pow(\pm 0, y)$  returns plus or minus infinity for y a negative odd integer.
- pow( $\pm 0$ , y) returns plus infinity for y negative and not an odd integer.
- $pow(\pm 0, y)$  returns plus or minus zero for y a positive odd integer.
- $pow(\pm 0, y)$  returns plus zero for y positive and not an odd integer.
- pow(-1,  $\pm$ Inf) returns 1.
- pow(+1, y) returns 1 for any y, even a NaN.
- pow(x,  $\pm$ 0) returns 1 for any x, even a NaN.
- pow(x, y) returns NaN for finite negative x and finite non-integer y.
- pow(x, -Inf) returns plus infinity for 0 < |x| < 1, and plus zero for |x| > 1.
- pow(x, +Inf) returns plus zero for 0 < |x| < 1, and plus infinity for |x| > 1.
- pow(-Inf, y) returns minus zero for y a negative odd integer.
- pow(-Inf, y) returns plus zero for y negative and not an odd integer.
- pow(-Inf, y) returns minus infinity for y a positive odd integer.
- pow(-Inf, y) returns plus infinity for y positive and not an odd integer.
- pow(+Inf, y) returns plus zero for y negative, and plus infinity for y positive.

# int mpfr\_neg (mpfr\_t rop, mpfr\_t op, mp\_rnd\_t rnd) [Function] Set rop to -op rounded in the direction rnd. Just changes the sign if rop and op are the same variable.

int mpfr\_abs (mpfr\_t rop, mpfr\_t op, mp\_rnd\_t rnd) [Function] Set rop to the absolute value of op, rounded in the direction rnd. Just changes the sign if rop and op are the same variable.

int mpfr\_dim ( $mpfr_t rop$ ,  $mpfr_t op1$ ,  $mpfr_t op2$ ,  $mp_rnd_t rnd$ ) [Function] Set rop to the positive difference of op1 and op2, i.e., op1 - op2 rounded in the direction rnd if op1 > op2, and +0 otherwise. Returns NaN when op1 or op2 is NaN.

int mpfr\_mul\_2si (mpfr\_t rop, mpfr\_t op1, long int op2, mp\_rnd\_t rnd) [Function] Set rop to  $op1 \times 2^{op2}$  rounded in the direction rnd. Just increases the exponent by op2 when rop and op1 are identical.

```
int mpfr_div_2ui (mpfr_t rop, mpfr_t op1, unsigned long int op2,
                                                                               [Function]
         mp_rnd_t rnd)
int mpfr_div_2si (mpfr_t rop, mpfr_t op1, long int op2, mp_rnd_t rnd)
                                                                               [Function]
  Set rop to op1/2^{op2} rounded in the direction rnd. Just decreases the exponent by op2 when
```

rop and op1 are identical.

## 5.6 Comparison Functions

```
int mpfr_cmp (mpfr_t op1, mpfr_t op2)
                                                                             [Function]
int mpfr_cmp_ui (mpfr_t op1, unsigned long int op2)
                                                                             [Function]
int mpfr_cmp_si (mpfr_t op1, signed long int op2)
                                                                             [Function]
int mpfr_cmp_d (mpfr_t op1, double op2)
                                                                             [Function]
int mpfr_cmp_ld (mpfr_t op1, long double op2)
                                                                             [Function]
int mpfr_cmp_z (mpfr_t op1, mpz_t op2)
                                                                             [Function]
int mpfr_cmp_q (mpfr_t op1, mpq_t op2)
                                                                             [Function]
int mpfr_cmp_f (mpfr_t op1, mpf_t op2)
                                                                             [Function]
  Compare op1 and op2. Return a positive value if op1 > op2, zero if op1 = op2, and a
```

negative value if op1 < op2. Both op1 and op2 are considered to their full own precision, which may differ. If one of the operands is NaN, set the erange flag and return zero.

Note: These functions may be useful to distinguish the three possible cases. If you need to distinguish two cases only, it is recommended to use the predicate functions (e.g., mpfr\_ equal\_p for the equality) described below; they behave like the IEEE 754 comparisons, in particular when one or both arguments are NaN. But only floating-point numbers can be compared (you may need to do a conversion first).

```
int mpfr_cmp_ui_2exp (mpfr_t op1, unsigned long int op2, mp_exp_t e)
                                                                                   [Function]
int mpfr_cmp_si_2exp (mpfr_t op1, long int op2, mp_exp_t e)
                                                                                   [Function]
  Compare op1 and op2 \times 2<sup>e</sup>. Similar as above.
```

```
int mpfr_cmpabs (mpfr_t op1, mpfr_t op2)
```

[Function]

Compare |op1| and |op2|. Return a positive value if |op1| > |op2|, zero if |op1| = |op2|, and a negative value if |op1| < |op2|. If one of the operands is NaN, set the erange flag and return zero.

```
int mpfr_nan_p (mpfr_t op)
                                                                            [Function]
int mpfr_inf_p (mpfr_t op)
                                                                            [Function]
int mpfr_number_p (mpfr_t op)
                                                                            [Function]
int mpfr_zero_p (mpfr_t op)
                                                                            [Function]
```

Return non-zero if op is respectively NaN, an infinity, an ordinary number (i.e. neither NaN nor an infinity) or zero. Return zero otherwise.

```
int mpfr_sgn (mpfr_t op)
```

[Macro]

Return a positive value if op > 0, zero if op = 0, and a negative value if op < 0. If the operand is NaN, set the erange flag and return zero.

```
int mpfr_greater_p (mpfr_t op1, mpfr_t op2)
```

[Function]

Return non-zero if op1 > op2, zero otherwise.

```
int mpfr_greaterequal_p (mpfr_t op1, mpfr_t op2)
  Return non-zero if op1 > op2, zero otherwise.
```

[Function]

```
int mpfr_less_p (mpfr_t op1, mpfr_t op2)
                                                                                 [Function]
  Return non-zero if op1 < op2, zero otherwise.
int mpfr_lessequal_p (mpfr_t op1, mpfr_t op2)
                                                                                 [Function]
  Return non-zero if op1 \leq op2, zero otherwise.
int mpfr_lessgreater_p (mpfr_t op1, mpfr_t op2)
                                                                                 [Function]
  Return non-zero if op1 < op2 or op1 > op2 (i.e. neither op1, nor op2 is NaN, and op1 \neq op2),
  zero otherwise (i.e. op1 and/or op2 are NaN, or op1 = op2).
int mpfr_equal_p (mpfr_t op1, mpfr_t op2)
                                                                                 [Function]
  Return non-zero if op1 = op2, zero otherwise (i.e. op1 and/or op2 are NaN, or op1 \neq op2).
int mpfr_unordered_p (mpfr_t op1, mpfr_t op2)
                                                                                 [Function]
  Return non-zero if op1 or op2 is a NaN (i.e. they cannot be compared), zero otherwise.
5.7 Special Functions
All those functions, except explicitly stated, return zero for an exact return value, a positive
value for a return value larger than the exact result, and a negative value otherwise.
Important note: in some domains, computing special functions (either with correct or incor-
rect rounding) is expensive, even for small precision, for example the trigonometric and Bessel
functions for large argument.
int mpfr_log (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
                                                                                 [Function]
int mpfr_log2 (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
                                                                                 [Function]
int mpfr_log10 (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
                                                                                 [Function]
  Set rop to the natural logarithm of op, \log_2 op or \log_{10} op, respectively, rounded in the
  direction rnd. Return -Inf if op is -0 (i.e. the sign of the zero has no influence on the
  result).
int mpfr_exp (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
                                                                                 [Function]
int mpfr_exp2 (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
                                                                                 [Function]
int mpfr_exp10 (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
                                                                                 [Function]
  Set rop to the exponential of op, to 2^{op} or to 10^{op}, respectively, rounded in the direction rnd.
int mpfr_cos (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
                                                                                 [Function]
int mpfr_sin (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
                                                                                 [Function]
int mpfr_tan (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
                                                                                 [Function]
  Set rop to the cosine of op, sine of op, tangent of op, rounded in the direction rnd.
int mpfr_sec (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
                                                                                 [Function]
int mpfr_csc (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
                                                                                 [Function]
int mpfr_cot (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
                                                                                 [Function]
```

int mpfr\_sin\_cos (mpfr\_t sop, mpfr\_t cop, mpfr\_t op, mp\_rnd\_t rnd) [Function] Set simultaneously sop to the sine of op and cop to the cosine of op, rounded in the direction rnd with the corresponding precisions of sop and cop, which must be different variables. Return 0 iff both results are exact.

Set rop to the secant of op, cosecant of op, cotangent of op, rounded in the direction rnd.

```
int mpfr_acos (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function]
int mpfr_asin (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function]
```

```
int mpfr_atan (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
```

[Function]

Set rop to the arc-cosine, arc-sine or arc-tangent of op, rounded in the direction rnd. Note that since acos(-1) returns the floating-point number closest to  $\pi$  according to the given rounding mode, this number might not be in the output range  $0 \le rop < \pi$  of the arc-cosine function; still, the result lies in the image of the output range by the rounding function. The same holds for asin(-1), asin(1), atan(-Inf), atan(+Inf).

int mpfr\_atan2 (mpfr\_t rop, mpfr\_t y, mpfr\_t x, mp\_rnd\_t rnd) [Function] Set rop to the arc-tangent2 of y and x, rounded in the direction rnd: if x > 0, atan2(y, x) = atan (y/x); if x < 0, atan2(y, x) = sign(y)\*(Pi - atan (|y/x|)). As for atan, in case the exact mathematical result is  $+\pi$  or  $-\pi$ , its rounded result might be outside the function output range.

atan2(y, 0) does not raise any floating-point exception. Special values are currently handled as described in the ISO C99 standard for the atan2 function (note this may change in future versions):

- atan2(+0, -0) returns  $+\pi$ .
- atan2(-0, -0) returns  $-\pi$ .
- atan2(+0, +0) returns +0.
- atan2(-0, +0) returns -0.
- atan2(+0, x) returns  $+\pi$  for x < 0.
- atan2(-0, x) returns  $-\pi$  for x < 0.
- atan2(+0, x) returns +0 for x > 0.
- atan2(-0, x) returns -0 for x > 0.
- atan2(y, 0) returns  $-\pi/2$  for y < 0.
- atan2(y, 0) returns  $+\pi/2$  for y > 0.
- atan2(+Inf, -Inf) returns  $+3 * \pi/4$ .
- atan2(-Inf, -Inf) returns  $-3 * \pi/4$ .
- atan2(+Inf, +Inf) returns  $+\pi/4$ .
- atan2(-Inf, +Inf) returns  $-\pi/4$ .
- atan2(+Inf, x) returns  $+\pi/2$  for finite x.
- atan2(-Inf, x) returns  $-\pi/2$  for finite x.
- atan2(y, -Inf) returns  $+\pi$  for finite y > 0.
- atan2(y, -Inf) returns  $-\pi$  for finite y < 0.
- atan2(y, +Inf) returns +0 for finite y > 0.
- atan2(y, +Inf) returns -0 for finite y < 0.

```
int mpfr_cosh (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
int mpfr_sinh (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
int mpfr_tanh (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
[Function]
[Function]
```

Set rop to the hyperbolic cosine, sine or tangent of op, rounded in the direction rnd.

int mpfr\_sinh\_cosh (mpfr\_t sop, mpfr\_t cop, mpfr\_t op, mp\_rnd\_t rnd) [Function] Set simultaneously sop to the hyperbolic sine of op and cop to the hyperbolic cosine of op, rounded in the direction rnd with the corresponding precision of sop and cop which must be different variables. Return 0 iff both results are exact.

direction rnd.

int mpfr\_acosh (mpfr\_t rop, mpfr\_t op, mp\_rnd\_t rnd) [Function]
int mpfr\_asinh (mpfr\_t rop, mpfr\_t op, mp\_rnd\_t rnd) [Function]
int mpfr\_atanh (mpfr\_t rop, mpfr\_t op, mp\_rnd\_t rnd) [Function]

Set rop to the inverse hyperbolic cosine, sine or tangent of op, rounded in the direction rnd.

int mpfr\_fac\_ui (mpfr\_t rop, unsigned long int op, mp\_rnd\_t rnd) [Function] Set rop to the factorial of the unsigned long int op, rounded in the direction rnd.

int mpfr\_log1p (mpfr\_t rop, mpfr\_t op, mp\_rnd\_t rnd) [Function] Set rop to the logarithm of one plus op, rounded in the direction rnd.

int mpfr\_expm1 (mpfr\_t rop, mpfr\_t op, mp\_rnd\_t rnd) [Function] Set rop to the exponential of op minus one, rounded in the direction rnd.

int mpfr\_eint ( $mpfr_t y$ ,  $mpfr_t x$ ,  $mp_rnd_t rnd$ ) [Function] Set y to the exponential integral of x, rounded in the direction rnd. For positive x, the exponential integral is the sum of Euler's constant, of the logarithm of x, and of the sum for x from 1 to infinity of  $x^k/k/k!$ . For negative x, the returned value is NaN.

int mpfr\_li2 (mpfr\_t rop, mpfr\_t op, mp\_rnd\_t rnd\_mode) [Function] Set rop to real part of the dilogarithm of op, rounded in the direction rnd\_mode. The dilogarithm function is defined here as  $-\int_{t=0}^{x} log(1-t)/t dt$ .

int mpfr\_gamma (mpfr\_t rop, mpfr\_t op, mp\_rnd\_t rnd) [Function] Set rop to the value of the Gamma function on op, rounded in the direction rnd. When op is a negative integer, NaN is returned.

int mpfr\_lngamma ( $mpfr_t rop$ ,  $mpfr_t op$ ,  $mp_rnd_t rnd$ ) [Function] Set rop to the value of the logarithm of the Gamma function on op, rounded in the direction rnd. When  $-2k-1 \le x \le -2k$ , k being a non-negative integer, NaN is returned. See also mpfr\_lgamma.

int mpfr\_lgamma ( $mpfr_t rop$ , int \*signp,  $mpfr_t op$ ,  $mp_rnd_t rnd$ ) [Function] Set rop to the value of the logarithm of the absolute value of the Gamma function on op, rounded in the direction rnd. The sign (1 or -1) of Gamma(op) is returned in the object pointed to by signp. When op is an infinity or a non-positive integer, +Inf is returned. When op is NaN, -Inf or a negative integer, \*signp is undefined, and when op is  $\pm 0$ , \*signp is the sign of the zero.

int mpfr\_erf (mpfr\_t rop, mpfr\_t op, mp\_rnd\_t rnd) [Function] Set rop to the value of the error function on op, rounded in the direction rnd.

int mpfr\_erfc (mpfr\_t rop, mpfr\_t op, mp\_rnd\_t rnd)

[Function]

Set rop to the value of the complementary error function on op, rounded in the direction rnd.

```
int mpfr_j0 (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
```

[Function]

int mpfr\_j1 (mpfr\_t rop, mpfr\_t op, mp\_rnd\_t rnd)

[Function]

int mpfr\_jn (mpfr\_t rop, long n, mpfr\_t op, mp\_rnd\_t rnd)

[Function]

Set rop to the value of the first kind Bessel function of order 0, 1 and n on op, rounded in the direction rnd. When op is NaN, rop is always set to NaN. When op is plus or minus Infinity, rop is set to +0. When op is zero, and n is not zero, rop is +0 or -0 depending on the parity and sign of n, and the sign of op.

```
int mpfr_y0 (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
```

[Function]

int mpfr\_y1 (mpfr\_t rop, mpfr\_t op, mp\_rnd\_t rnd)

[Function]

int mpfr\_yn (mpfr\_t rop, long n, mpfr\_t op, mp\_rnd\_t rnd)

[Function]

Set rop to the value of the second kind Bessel function of order 0, 1 and n on op, rounded in the direction rnd. When op is NaN or negative, rop is always set to NaN. When op is +Inf, rop is +0. When op is zero, rop is +Inf or -Inf depending on the parity and sign of n.

int mpfr\_fma (mpfr\_t rop, mpfr\_t op1, mpfr\_t op2, mpfr\_t op3, mp\_rnd\_t rnd)

[Function]

Set rop to  $(op1 \times op2) + op3$ , rounded in the direction rnd.

int mpfr\_fms (mpfr\_t rop, mpfr\_t op1, mpfr\_t op2, mpfr\_t op3, mp\_rnd\_t

[Function]

Set rop to  $(op1 \times op2) - op3$ , rounded in the direction rnd.

int mpfr\_agm (mpfr\_t rop, mpfr\_t op1, mpfr\_t op2, mp\_rnd\_t rnd)

[Function]

Set rop to the arithmetic-geometric mean of op1 and op2, rounded in the direction rnd. The arithmetic-geometric mean is the common limit of the sequences u[n] and v[n], where u[0] = op1, v[0] = op2, u[n+1] is the arithmetic mean of u[n] and v[n], and v[n+1] is the geometric mean of u[n] and v[n]. If any operand is negative, the return value is NaN.

int mpfr\_hypot (mpfr\_t rop, mpfr\_t x, mpfr\_t y, mp\_rnd\_t rnd)

[Function]

Set rop to the Euclidean norm of x and y, i.e.  $\sqrt{x^2 + y^2}$ , rounded in the direction rnd. Special values are currently handled as described in Section F.9.4.3 of the ISO C99 standard, for the hypot function (note this may change in future versions): If x or y is an infinity, then plus infinity is returned in rop, even if the other number is NaN.

```
int mpfr_const_log2 (mpfr_t rop, mp_rnd_t rnd)
```

[Function]

int mpfr\_const\_pi (mpfr\_t rop, mp\_rnd\_t rnd)

[Function]

int mpfr\_const\_euler (mpfr\_t rop, mp\_rnd\_t rnd)

[Function]

int mpfr\_const\_catalan (mpfr\_t rop, mp\_rnd\_t rnd)

[Function]

Set rop to the logarithm of 2, the value of  $\pi$ , of Euler's constant 0.577..., of Catalan's constant 0.915..., respectively, rounded in the direction rnd. These functions cache the computed values to avoid other calculations if a lower or equal precision is requested. To free these caches, use mpfr\_free\_cache.

#### void mpfr\_free\_cache (void)

[Function]

Free various caches used by MPFR internally, in particular the caches used by the functions computing constants (currently mpfr\_const\_log2, mpfr\_const\_pi, mpfr\_const\_euler and mpfr\_const\_catalan). You should call this function before terminating a thread, even if you did not call these functions directly (they could have been called internally).

Set rop to the sum of all elements of tab, whose size is n, rounded in the direction rnd. Warning, tab is a table of pointers to mpfr\_t, not a table of mpfr\_t (preliminary interface). If the returned int value is zero, rop is guaranteed to be the exact sum; otherwise rop might be smaller than, equal to, or larger than the exact sum (in accordance to the rounding mode). However, mpfr\_sum does guarantee the result is correctly rounded.

## 5.8 Input and Output Functions

This section describes functions that perform input from an input/output stream, and functions that output to an input/output stream. Passing a null pointer for a stream to any of these functions will make them read from stdin and write to stdout, respectively.

When using any of these functions, you must include the <stdio.h> standard header before 'mpfr.h', to allow 'mpfr.h' to define prototypes for these functions.

Output op on stream stream, as a string of digits in base base, rounded in the direction rnd. The base may vary from 2 to 36. Print n significant digits exactly, or if n is 0, enough digits so that op can be read back exactly (see mpfr\_get\_str).

In addition to the significant digits, a decimal point (defined by the current locale) at the right of the first digit and a trailing exponent in base 10, in the form 'eNNN', are printed. If base is greater than 10, '@' will be used instead of 'e' as exponent delimiter.

Return the number of bytes written, or if an error occurred, return 0.

Input a string in base base from stream stream, rounded in the direction rnd, and put the read float in rop.

This function reads a word (defined as a sequence of characters between whitespace) and parses it using mpfr\_set\_str (it may change). See the documentation of mpfr\_strtofr for a detailed description of the valid string formats.

Return the number of bytes read, or if an error occurred, return 0.

## 5.9 Formatted Output Functions

#### 5.9.1 Requirements

The class of mpfr\_printf functions provides formatted output in a similar manner as the standard C printf. These functions are defined only if your system supports ISO C variadic functions and the corresponding argument access macros.

When using any of these functions, you must include the <stdio.h> standard header before 'mpfr.h', to allow 'mpfr.h' to define prototypes for these functions.

#### 5.9.2 Format String

The format specification accepted by mpfr\_printf is an extension of the printf one. The conversion specification is of the form:

% [flags] [width] [.[precision]] [type] [rounding] conv

'flags', 'width', and 'precision' have the same meaning as for the standard printf (in particular, notice that the 'precision' is related to the number of digits displayed in the base chosen by 'conv' and not related to the internal precision of the mpfr\_t variable). mpfr\_printf accepts the same 'type' specifiers as GMP (except the non-standard and deprecated 'q', use 'll' instead), plus 'R' and 'P':

```
h'
          short
'hh'
          char
ʻj'
          intmax_t or uintmax_t
'1'
          long or wchar_t
11'
          long long
L'
          long double
't'
          ptrdiff_t
ʻz'
          size_t
'F'
          mpf_t, float conversions
          mpq_t, integer conversions
٠Q'
          mp_limb_t, integer conversions
·Μ'
          mp_limb_t array, integer conversions
'N'
'Z'
          mpz_t, integer conversions
٠р٬
          mp_prec_t, integer conversions
R'
          mpfr_t, float conversions
```

The 'type' specifiers have the same restrictions as those mentioned in the GMP documentation: see Section "Formatted Output Strings" in *GNU MP*. In particular, the 'type' specifiers (except 'R' and 'P' defined by MPFR) are supported only if they are supported by gmp\_printf in your GMP build; this implies that the standard specifiers, such as 't', must *also* be supported by your C library if you want to use them.

The 'rounding' field is specific to mpfr\_t arguments and should not be used with other types.

With conversion specification not involving 'P' and 'R' types, mpfr\_printf behaves exactly as gmp\_printf.

The 'P' type specifies that a following 'o', 'u', 'x', or 'X' conversion specifier applies to a mp\_prec\_t argument. It is needed because the mp\_prec\_t type does not necessarily correspond to an unsigned int or any fixed standard type. The 'precision' field specifies the minimum number of digits to appear. The default 'precision' is 1. For example:

```
mpfr_t x;
mp_prec_t p;
mpfr_init (x);
...
p = mpfr_get_prec (x);
mpfr_printf ("variable x with %Pu bits", p);
```

The 'R' type specifies that a following 'a', 'A', 'b', 'e', 'E', 'f', 'F', 'g', 'G', or 'n' conversion specifier applies to a mpfr\_t argument. The 'R' type can be followed by a 'rounding' specifier denoted by one of the following characters:

```
'U' round toward plus infinity
'D' round toward minus infinity
```

'Z' round toward zero

```
'N' round to nearest
```

'\*' rounding mode indicated by the mp\_rnd\_t argument just before the corresponding mpfr\_t variable.

The default rounding mode is rounding to nearest. The following three examples are equivalent:

```
mpfr_t x;
mpfr_init (x);
...
mpfr_printf ("%.128Rf", x);
mpfr_printf ("%.128RNf", x);
mpfr_printf ("%.128R*f", GMP_RNDN, x);
```

The output 'conv' specifiers allowed with mpfr\_t parameter are:

```
'a' 'A' hex float, C99 style
'b' binary output
'e' 'E' scientific format float
'f' 'F' fixed point float
'g' 'G' fixed or scientific float
```

The conversion specifier 'b' which displays the argument in binary is specific to mpfr\_t arguments and should not be used with other types. Other conversion specifiers have the same meaning as for a double argument.

In case of non-decimal output, only the significand is written in the specified base, the exponent is always displayed in decimal. Special values are always displayed as nan, -inf, and inf for 'a', 'b', 'e', 'f', and 'g' specifiers and NAN, -INF, and INF for 'A', 'E', 'F', and 'G' specifiers.

If the 'precision' field is not empty, the mpfr\_t number is rounded to the given precision in the direction specified by the rounding mode. If the precision is zero with rounding to nearest mode and one of the following 'conv' specifier: 'a', 'A', 'b', 'e', 'E', tie case is rounded to even when it lies between two values at the wanted precision which have the same exponent, otherwise, it is rounded away from zero. For instance, 85 is displayed as "8e+1" and 95 is displayed as "1e+2" with the format specification "%.ORNe". This also applies when the 'g' (resp. 'G') conversion specifier uses the 'e' (resp. 'E') style. If the precision is set to a value greater than the maximum value for an int, it will be silently reduced down to INT\_MAX.

If the 'precision' field is empty (as in %Re or %.Re) with 'conv' specifier 'e' and 'E', the number is displayed with enough digits so that it can be read back exactly, assuming that the input and output variables have the same precision and that the input and output rounding modes are both rounding to nearest (as for mpfr\_get\_str). The default precision for an empty 'precision' field with 'conv' specifiers 'f', 'F', 'g', and 'G' is 6.

#### 5.9.3 Functions

```
int mpfr_fprintf (FILE *stream, const char *template, ...) [Function] int mpfr_vfprintf (FILE *stream, const char *template, va_list ap) [Function] Print to the stream stream the optional arguments under the control of the template string template.
```

Return the number of characters written or a negative value if an error occurred. If the number of characters which ought to be written appears to exceed the maximum limit for an int, nothing is written in the stream, the function returns -1, sets the *erange* flag, and (in POSIX system only) error is set to EOVERFLOW.

```
int mpfr_printf (const char *template, ...) [Function] int mpfr_vprintf (const char *template, va_list ap) [Function] Print to stdout the optional arguments under the control of the template string template.
```

Return the number of characters written or a negative value if an error occurred. If the number of characters which ought to be written appears to exceed the maximum limit for an int, nothing is written in stdout, the function returns -1, sets the *erange* flag, and (in POSIX system only) error is set to EOVERFLOW.

```
int mpfr_sprintf (char *buf, const char *template, ...) [Function] int mpfr_vsprintf (char *buf, const char *template, va_list ap) [Function] Form a null-terminated string in buf. No overlap is permitted between buf and the other arguments.
```

Return the number of characters written in the array buf not counting the terminating null character or a negative value if an error occurred. If the number of characters which ought to be written appears to exceed the maximum limit for an int, nothing is written in buf, the function returns -1, sets the erange flag, and (in POSIX system only) errno is set to EOVERFLOW.

Form a null-terminated string in buf. If n is zero, nothing is written and buf may be a null pointer, otherwise, the n-1 first characters are written in buf and the n-th is a null character.

Return the number of characters that would have been written had n be sufficiently large, not counting the terminating null character or a negative value if an error occurred. If the number of characters produced by the optional arguments under the control of the template string template appears to exceed the maximum limit for an int, nothing is written in buf, the function returns -1, sets the erange flag, and (in POSIX system only) errno is set to EOVERFLOW.

```
int mpfr_asprintf (char **str, const char *template, ...) [Function] int mpfr_vasprintf (char **str, const char *template, va_list ap) [Function] Write their output as a null terminated string in a block of memory allocated using the
```

Write their output as a null terminated string in a block of memory allocated using the current allocation function. A pointer to the block is stored in str. The block of memory must be freed using mpfr\_free\_str.

The return value is the number of characters written in the string, excluding the null-terminator or a negative value if an error occurred. If the number of characters produced by the optional arguments under the control of the template string template appears to exceed the maximum limit for an int, str is a null pointer, the function returns -1, sets the erange flag, and (in POSIX system only) errno is set to EOVERFLOW.

## 5.10 Integer and Remainder Related Functions

```
int mpfr_rint (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
int mpfr_ceil (mpfr_t rop, mpfr_t op)
int mpfr_floor (mpfr_t rop, mpfr_t op)
int mpfr_round (mpfr_t rop, mpfr_t op)
int mpfr_trunc (mpfr_t rop, mpfr_t op)
int mpfr_trunc (mpfr_t rop, mpfr_t op)
[Function]
```

Set rop to op rounded to an integer. mpfr\_rint rounds to the nearest representable integer in the given rounding mode, mpfr\_ceil rounds to the next higher or equal representable integer,

mpfr\_floor to the next lower or equal representable integer, mpfr\_round to the nearest representable integer, rounding halfway cases away from zero (as in the roundTiesToAway mode of IEEE 754-2008), and mpfr\_trunc to the next representable integer toward zero.

The returned value is zero when the result is exact, positive when it is greater than the original value of op, and negative when it is smaller. More precisely, the returned value is 0 when op is an integer representable in rop, 1 or -1 when op is an integer that is not representable in rop, 2 or -2 when op is not an integer.

Note that mpfr\_round is different from mpfr\_rint called with the rounding to nearest mode (where halfway cases are rounded to an even integer or significand). Note also that no double rounding is performed; for instance, 4.5 (100.1 in binary) is rounded by mpfr\_round to 4 (100 in binary) in 2-bit precision, though round(4.5) is equal to 5 and 5 (101 in binary) is rounded to 6 (110 in binary) in 2-bit precision.

Set rop to op rounded to an integer. mpfr\_rint\_ceil rounds to the next higher or equal integer, mpfr\_rint\_floor to the next lower or equal integer, mpfr\_rint\_round to the nearest integer, rounding halfway cases away from zero, and mpfr\_rint\_trunc to the next integer toward zero. If the result is not representable, it is rounded in the direction rnd. The returned value is the ternary value associated with the considered round-to-integer function (regarded in the same way as any other mathematical function).

```
int mpfr_frac (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function]
Set rop to the fractional part of op, having the same sign as op, rounded in the direction rnd (unlike in mpfr_rint, rnd affects only how the exact fractional part is rounded, not how the fractional part is generated).
```

int mpfr\_modf (mpfr\_t iop, mpfr\_t fop, mpfr\_t op, mp\_rnd\_t rnd) [Function] Set simultaneously iop to the integral part of op and fop to the fractional part of op, rounded in the direction rnd with the corresponding precision of iop and fop (equivalent to mpfr\_trunc(iop, op, rnd)) and mpfr\_frac(fop, op, rnd)). The variables iop and fop must be different. Return 0 iff both results are exact.

```
int mpfr_fmod (mpfr_t r, mpfr_t x, mpfr_t y, mp_rnd_t rnd) [Function] int mpfr_remainder (mpfr_t r, mpfr_t x, mpfr_t y, mp_rnd_t rnd) [Function] int mpfr_remquo (mpfr_t r, long^* q, mpfr_t x, mpfr_t y, mp_rnd_t rnd) [Function] Set r to the value of r and r and r are quotient of r divided by r, defined as follows: r is rounded toward zero for mpfr_fmod, and to the nearest integer (ties rounded to even) for mpfr_remainder and mpfr_remquo.
```

Special values are handled as described in Section F.9.7.1 of the ISO C99 standard: If x is infinite or y is zero, r is NaN. If y is infinite and x is finite, r is x rounded to the precision of r. If r is zero, it has the sign of x. The return value is the ternary value corresponding to r.

Additionally,  $mpfr_remquo$  stores the low significant bits from the quotient in \*q (more precisely the number of bits in a long minus one), with the sign of x divided by y (except if those low bits are all zero, in which case zero is returned). Note that x may be so large in magnitude relative to y that an exact representation of the quotient is not practical.  $mpfr_remainder$  and  $mpfr_remquo$  functions are useful for additive argument reduction.

```
int mpfr_integer_p (mpfr_t op)
Return non-zero iff op is an integer.
```

[Function]

## 5.11 Rounding Related Functions

void mpfr\_set\_default\_rounding\_mode (mp\_rnd\_t rnd)

[Function]

Set the default rounding mode to rnd. The default rounding mode is to nearest initially.

mp\_rnd\_t mpfr\_get\_default\_rounding\_mode (void)
Get the default rounding mode.

[Function]

int mpfr\_prec\_round (mpfr\_t x, mp\_prec\_t prec, mp\_rnd\_t rnd)

[Function]

Round x according to rnd with precision prec, which must be an integer between MPFR\_PREC\_MIN and MPFR\_PREC\_MAX (otherwise the behavior is undefined). If prec is greater or equal to the precision of x, then new space is allocated for the significand, and it is filled with zeros. Otherwise, the significand is rounded to precision prec with the given direction. In both cases, the precision of x is changed to prec.

int mpfr\_round\_prec (mpfr\_t x, mp\_rnd\_t rnd, mp\_prec\_t prec)
[This function is obsolete. Please use mpfr\_prec\_round instead.]

[Function]

Assuming b is an approximation of an unknown number x in the direction rnd1 with error at most two to the power E(b)-err where E(b) is the exponent of b, return a non-zero value if one is able to round correctly x to precision prec with the direction rnd2, and 0 otherwise (including for NaN and Inf). This function **does not modify** its arguments.

Note: if one wants to also determine the correct ternary value when rounding b to precision prec, a useful trick is the following:

```
if (mpfr_can_round (b, err, rnd1, GMP_RNDZ, prec + (rnd2 == GMP_RNDN)))
...
```

Indeed, if rnd2 is GMP\_RNDN, this will check if one can round to prec+1 bits with a directed rounding: if so, one can surely round to nearest to prec bits, and in addition one can determine the correct ternary value, which would not be the case when b is near from a value exactly representable on prec bits.

const char \* mpfr\_print\_rnd\_mode (mp\_rnd\_t rnd)

[Function]

Return the input string (GMP\_RNDD, GMP\_RNDU, GMP\_RNDN, GMP\_RNDZ) corresponding to the rounding mode *rnd* or a null pointer if *rnd* is an invalid rounding mode.

#### 5.12 Miscellaneous Functions

#### void mpfr\_nexttoward (mpfr\_t x, mpfr\_t y)

[Function]

If x or y is NaN, set x to NaN. Otherwise, if x is different from y, replace x by the next floating-point number (with the precision of x and the current exponent range) in the direction of y, if there is one (the infinite values are seen as the smallest and largest floating-point numbers). If the result is zero, it keeps the same sign. No underflow or overflow is generated.

void mpfr\_nextabove (mpfr\_t x)

[Function]

Equivalent to mpfr\_nexttoward where y is plus infinity.

#### void mpfr\_nextbelow (mpfr\_t x)

[Function]

Equivalent to mpfr\_nexttoward where y is minus infinity.

int mpfr\_min ( $mpfr_t rop$ ,  $mpfr_t op1$ ,  $mpfr_t op2$ ,  $mp_rnd_t rnd$ ) [Function] Set rop to the minimum of op1 and op2. If op1 and op2 are both NaN, then rop is set to NaN. If op1 or op2 is NaN, then rop is set to the numeric value. If op1 and op2 are zeros of different signs, then rop is set to -0.

int mpfr\_max (mpfr\_t rop, mpfr\_t op1, mpfr\_t op2, mp\_rnd\_t rnd) [Function] Set rop to the maximum of op1 and op2. If op1 and op2 are both NaN, then rop is set to NaN. If op1 or op2 is NaN, then rop is set to the numeric value. If op1 and op2 are zeros of different signs, then rop is set to +0.

#### int mpfr\_urandomb (mpfr\_t rop, gmp\_randstate\_t state)

[Function]

Generate a uniformly distributed random float in the interval  $0 \le rop < 1$ . More precisely, the number can be seen as a float with a random non-normalized significand and exponent 0, which is then normalized (thus if e denotes the exponent after normalization, then the least -e significant bits of the significand are always 0). Return 0, unless the exponent is not in the current exponent range, in which case rop is set to NaN and a non-zero value is returned (this should never happen in practice, except in very specific cases). The second argument is a gmp\_randstate\_t structure which should be created using the GMP gmp\_randinit function, see the GMP manual.

#### void mpfr\_random (mpfr\_t rop)

[Function]

Generate a uniformly distributed random float in the interval  $0 \le rop < 1$ .

This function is deprecated and will be suppressed in the next release; mpfr\_urandomb should be used instead.

#### void mpfr\_random2 (mpfr\_t rop, mp\_size\_t size, mp\_exp\_t exp)

[Function]

Generate a random float of at most *size* limbs, with long strings of zeros and ones in the binary representation. The exponent of the number is in the interval -exp to exp. This function is useful for testing functions and algorithms, since this kind of random numbers have proven to be more likely to trigger corner-case bugs. Negative random numbers are generated when *size* is negative. Put +0 in *rop* when size if zero. The internal state of the default pseudorandom number generator is modified by a call to this function (the same one as GMP if MPFR was built using '--with-gmp-build').

This function is deprecated and will be suppressed in the next release.

#### mp\_exp\_t mpfr\_get\_exp (mpfr\_t x)

[Function]

Get the exponent of x, assuming that x is a non-zero ordinary number and the significand is chosen in [1/2,1). The behavior for NaN, infinity or zero is undefined.

#### int mpfr\_set\_exp (mpfr\_t x, mp\_exp\_t e)

[Function]

Set the exponent of x if e is in the current exponent range, and return 0 (even if x is not a non-zero ordinary number); otherwise, return a non-zero value. The significand is assumed to be in [1/2,1).

#### int mpfr\_signbit (mpfr\_t op)

[Function]

Return a non-zero value iff op has its sign bit set (i.e. if it is negative, -0, or a NaN whose representation has its sign bit set).

int mpfr\_setsign (mpfr\_t rop, mpfr\_t op, int s, mp\_rnd\_t rnd) [Function] Set the value of rop from op, rounded toward the given direction rnd, then set (resp. clear) its sign bit if s is non-zero (resp. zero), even when op is a NaN.

int mpfr\_copysign (mpfr\_t rop, mpfr\_t op1, mpfr\_t op2, mp\_rnd\_t rnd) [Function] Set the value of rop from op1, rounded toward the given direction rnd, then set its sign bit to that of op2 (even when op1 or op2 is a NaN). This function is equivalent to mpfr\_setsign (rop, op1, mpfr\_signbit (op2), rnd).

#### const char \* mpfr\_get\_version (void)

[Function]

Return the MPFR version, as a null-terminated string.

```
MPFR_VERSION[Macro]MPFR_VERSION_MAJOR[Macro]MPFR_VERSION_MINOR[Macro]MPFR_VERSION_PATCHLEVEL[Macro]MPFR_VERSION_STRING[Macro]
```

MPFR\_VERSION is the version of MPFR as a preprocessing constant. MPFR\_VERSION\_MAJOR, MPFR\_VERSION\_MINOR and MPFR\_VERSION\_PATCHLEVEL are respectively the major, minor and patch level of MPFR version, as preprocessing constants. MPFR\_VERSION\_STRING is the version (with an optional suffix, used in development and pre-release versions) as a string constant, which can be compared to the result of mpfr\_get\_version to check at run time the header file and library used match:

```
if (strcmp (mpfr_get_version (), MPFR_VERSION_STRING))
  fprintf (stderr, "Warning: header and library do not match\n");
```

Note: Obtaining different strings is not necessarily an error, as in general, a program compiled with some old MPFR version can be dynamically linked with a newer MPFR library version (if allowed by the library versioning system).

#### long MPFR\_VERSION\_NUM (major, minor, patchlevel)

[Macro]

Create an integer in the same format as used by MPFR\_VERSION from the given major, minor and patchlevel. Here is an example of how to check the MPFR version at compile time:

```
#if (!defined(MPFR_VERSION) || (MPFR_VERSION<MPFR_VERSION_NUM(2,1,0)))
# error "Wrong MPFR version."
#endif</pre>
```

```
const char * mpfr_get_patches (void)
```

[Function]

Return a null-terminated string containing the ids of the patches applied to the MPFR library (contents of the 'PATCHES' file), separated by spaces. Note: If the program has been compiled with an older MPFR version and is dynamically linked with a new MPFR library version, the ids of the patches applied to the old (compile-time) MPFR version are not available (however this information should not have much interest in general).

## 5.13 Exception Related Functions

```
mp_exp_t mpfr_get_emin (void)
mp_exp_t mpfr_get_emax (void)
[Function]
```

Return the (current) smallest and largest exponents allowed for a floating-point variable. The smallest positive value of a floating-point variable is  $1/2 \times 2^{\text{emin}}$  and the largest value has the form  $(1 - \varepsilon) \times 2^{\text{emax}}$ .

```
\begin{array}{ll} \text{int mpfr\_set\_emin } (mp\_exp\_t \ exp) & [\text{Function}] \\ \text{int mpfr\_set\_emax } (mp\_exp\_t \ exp) & [\text{Function}] \end{array}
```

Set the smallest and largest exponents allowed for a floating-point variable. Return a non-zero value when  $\exp$  is not in the range accepted by the implementation (in that case the smallest or largest exponent is not changed), and zero otherwise. If the user changes the exponent range, it is her/his responsibility to check that all current floating-point variables are in the new allowed range (for example using mpfr\_check\_range), otherwise the subsequent behavior will be undefined, in the sense of the ISO C standard.

```
mp_exp_t mpfr_get_emin_min (void)
mp_exp_t mpfr_get_emin_max (void)
mp_exp_t mpfr_get_emax_min (void)
mp_exp_t mpfr_get_emax_max (void)
[Function]
[Function]
```

Return the minimum and maximum of the smallest and largest exponents allowed for mpfr\_set\_emin and mpfr\_set\_emax. These values are implementation dependent; it is possible to create a non portable program by writing mpfr\_set\_emax(mpfr\_get\_emax\_max()) and mpfr\_set\_emin(mpfr\_get\_emin\_min()) since the values of the smallest and largest exponents become implementation dependent.

```
int mpfr\_check\_range (mpfr\_t x, int t, mp\_rnd\_t rnd) [Function]
```

This function forces x to be in the current range of acceptable values, t being the current ternary value: negative if x is smaller than the exact value, positive if x is larger than the exact value and zero if x is exact (before the call). It generates an underflow or an overflow if the exponent of x is outside the current allowed range; the value of t may be used to avoid a double rounding. This function returns zero if the rounded result is equal to the exact one, a positive value if the rounded result is larger than the exact one, a negative value if the rounded result is smaller than the exact one. Note that unlike most functions, the result is compared to the exact one, not the input value x, i.e. the ternary value is propagated.

Note: If x is an infinity and t is different from zero (i.e., if the rounded result is an inexact infinity), then the overflow flag is set. This is useful because  $mpfr\_check\_range$  is typically called (at least in MPFR functions) after restoring the flags that could have been set due to internal computations.

```
int mpfr_subnormalize (mpfr_t x, int t, mp_rnd_t rnd) [Function]
```

This function rounds x emulating subnormal number arithmetic: if x is outside the subnormal exponent range, it just propagates the ternary value t; otherwise, it rounds x to precision EXP(x)-emin+1 according to rounding mode rnd and previous ternary value t, avoiding double rounding problems. More precisely in the subnormal domain, denoting by e the value of emin, x is rounded in fixed-point arithmetic to an integer multiple of  $2^{e-1}$ ; as a consequence,  $1.5 \times 2^{e-1}$  when t is zero is rounded to  $2^e$  with rounding to nearest.

PREC(x) is not modified by this function. rnd and t must be the used rounding mode for computing x and the returned ternary value when computing x. The subnormal exponent range is from emin to emin+PREC(x)-1. If the result cannot be represented in the current exponent range (due to a too small emax), the behavior is undefined. Note that unlike most functions, the result is compared to the exact one, not the input value x, i.e. the ternary value is propagated. This is a preliminary interface.

This is an example of how to emulate binary double IEEE 754 arithmetic (binary64 in IEEE 754-2008) using MPFR:

```
{
  mpfr_t xa, xb;
```

```
int i;
volatile double a, b;

mpfr_set_default_prec (53);
mpfr_set_emin (-1073);
mpfr_set_emax (1024);

mpfr_init (xa); mpfr_init (xb);

b = 34.3; mpfr_set_d (xb, b, GMP_RNDN);
a = 0x1.1235P-1021; mpfr_set_d (xa, a, GMP_RNDN);
i = mpfr_div (xa, xa, xb, GMP_RNDN);
i = mpfr_subnormalize (xa, i, GMP_RNDN);
mpfr_clear (xa); mpfr_clear (xb);
}
```

Warning: this emulates a double IEEE 754 arithmetic with correct rounding in the subnormal range, which may not be the case for your hardware.

```
void mpfr_clear_underflow (void)
                                                                              [Function]
void mpfr_clear_overflow (void)
                                                                               [Function]
void mpfr_clear_nanflag (void)
                                                                              [Function]
void mpfr_clear_inexflag (void)
                                                                              [Function]
void mpfr_clear_erangeflag (void)
                                                                              [Function]
  Clear the underflow, overflow, invalid, inexact and erange flags.
void mpfr_set_underflow (void)
                                                                              [Function]
void mpfr_set_overflow (void)
                                                                              [Function]
void mpfr_set_nanflag (void)
                                                                              [Function]
void mpfr_set_inexflag (void)
                                                                              [Function]
void mpfr_set_erangeflag (void)
                                                                              [Function]
  Set the underflow, overflow, invalid, inexact and erange flags.
void mpfr_clear_flags (void)
                                                                              [Function]
  Clear all global flags (underflow, overflow, inexact, invalid, erange).
int mpfr_underflow_p (void)
                                                                              [Function]
int mpfr_overflow_p (void)
                                                                              [Function]
int mpfr_nanflag_p (void)
                                                                              [Function]
int mpfr_inexflag_p (void)
                                                                               [Function]
int mpfr_erangeflag_p (void)
                                                                              [Function]
  Return the corresponding (underflow, overflow, invalid, inexact, erange) flag, which is non-
```

# 5.14 Compatibility With MPF

A header file 'mpf2mpfr.h' is included in the distribution of MPFR for compatibility with the GNU MP class MPF. After inserting the following two lines after the #include <gmp.h> line,

```
#include <mpfr.h>
#include <mpf2mpfr.h>
```

zero iff the flag is set.

any program written for MPF can be compiled directly with MPFR without any changes. All operations are then performed with the default MPFR rounding mode, which can be reset with mpfr\_set\_default\_rounding\_mode.

Warning: the mpf\_init and mpf\_init2 functions initialize to zero, whereas the corresponding MPFR functions initialize to NaN: this is useful to detect uninitialized values, but is slightly incompatible with mpf.

void mpfr\_set\_prec\_raw (mpfr\_t x, mp\_prec\_t prec) [Function] Reset the precision of x to be exactly prec bits. The only difference with mpfr\_set\_prec is that prec is assumed to be small enough so that the significand fits into the current allocated memory space for x. Otherwise the behavior is undefined.

int mpfr\_eq (mpfr\_t op1, mpfr\_t op2, unsigned long int op3) [Function] Return non-zero if op1 and op2 are both non-zero ordinary numbers with the same exponent and the same first op3 bits, both zero, or both infinities of the same sign. Return zero otherwise. This function is defined for compatibility with mpf. Do not use it if you want to know whether two numbers are close to each other; for instance, 1.011111 and 1.100000 are currently regarded as different for any value of op3 larger than 1 (but this may change in the next release).

void mpfr\_reldiff (mpfr\_t rop, mpfr\_t op1, mpfr\_t op2, mp\_rnd\_t rnd) [Function] Compute the relative difference between op1 and op2 and store the result in rop. This function does not guarantee the correct rounding on the relative difference; it just computes |op1 - op2|/op1, using the rounding mode rnd for all operations and the precision of rop.

int mpfr\_mul\_2exp (mpfr\_t rop, mpfr\_t op1, unsigned long int op2, mp\_rnd\_t rnd) [Function]

See mpfr\_mul\_2ui and mpfr\_div\_2ui. These functions are only kept for compatibility with MPF.

## 5.15 Custom Interface

Some applications use a stack to handle the memory and their objects. However, the MPFR memory design is not well suited for such a thing. So that such applications are able to use MPFR, an auxiliary memory interface has been created: the Custom Interface.

The following interface allows them to use MPFR in two ways:

- Either they directly store the MPFR FP number as a mpfr\_t on the stack.
- Either they store their own representation of a FP number on the stack and construct a new temporary mpfr\_t each time it is needed.

Nothing has to be done to destroy the FP numbers except garbaging the used memory: all the memory stuff (allocating, destroying, garbaging) is kept to the application.

Each function in this interface is also implemented as a macro for efficiency reasons: for example mpfr\_custom\_init (s, p) uses the macro, while (mpfr\_custom\_init) (s, p) uses the function.

Note 1: MPFR functions may still initialize temporary FP numbers using standard mpfr\_init. See Custom Allocation (GNU MP).

Note 2: MPFR functions may use the cached functions (mpfr\_const\_pi for example), even if they are not explicitly called. You have to call mpfr\_free\_cache each time you garbage the memory iff mpfr\_init, through GMP Custom Allocation, allocates its memory on the application stack.

Note 3: This interface is preliminary.

# size\_t mpfr\_custom\_get\_size (mp\_prec\_t prec)

[Function]

Return the needed size in bytes to store the significand of a FP number of precision prec.

# void mpfr\_custom\_init (void \*significand, mp\_prec\_t prec)

[Function]

Initialize a significand of precision *prec*. *significand* must be an area of mpfr\_custom\_get\_size (prec) bytes at least and be suitably aligned for an array of mp\_limb\_t.

void mpfr\_custom\_init\_set (mpfr\_t x, int kind, mp\_exp\_t exp,

[Function]

mp\_prec\_t prec, void \*significand)

Perform a dummy initialization of a mpfr\_t and set it to:

- if ABS(kind) == MPFR\_NAN\_KIND, x is set to NaN;
- if ABS(kind) == MPFR\_INF\_KIND, x is set to the infinity of sign sign(kind);
- if ABS(kind) == MPFR\_ZERO\_KIND, x is set to the zero of sign sign(kind);
- if ABS(kind) == MPFR\_REGULAR\_KIND, x is set to a regular number:  $x = sign(kind)*significand*2^exp$

In all cases, it uses *significand* directly for further computing involving x. It will not allocate anything. A FP number initialized with this function cannot be resized using mpfr\_set\_prec, or cleared using mpfr\_clear! *significand* must have been initialized with mpfr\_custom\_init using the same precision *prec*.

## int mpfr\_custom\_get\_kind (mpfr\_t x)

[Function]

Return the current kind of a mpfr\_t as used by mpfr\_custom\_init\_set. The behavior of this function for any mpfr\_t not initialized with mpfr\_custom\_init\_set is undefined.

## void \* mpfr\_custom\_get\_mantissa (mpfr\_t x)

[Function]

Return a pointer to the significand used by a mpfr\_t initialized with mpfr\_custom\_init\_set. The behavior of this function for any mpfr\_t not initialized with mpfr\_custom\_init\_set is undefined.

# mp\_exp\_t mpfr\_custom\_get\_exp (mpfr\_t x)

[Function]

Return the exponent of x, assuming that x is a non-zero ordinary number. The return value for NaN, Infinity or Zero is unspecified but does not produce any trap. The behavior of this function for any  $mpfr_t$  not initialized with  $mpfr_custom_init_set$  is undefined.

## void mpfr\_custom\_move (mpfr\_t x, void \*new\_position)

[Function]

Inform MPFR that the significand has moved due to a garbage collect and update its new position to new\_position. However the application has to move the significand and the mpfr\_t itself. The behavior of this function for any mpfr\_t not initialized with mpfr\_custom\_init\_set is undefined.

See the test suite for examples.

# 5.16 Internals

The following types and functions were mainly designed for the implementation of MPFR, but may be useful for users too. However no upward compatibility is guaranteed. You may need to include 'mpfr-impl.h' to use them.

The mpfr\_t type consists of four fields.

- The \_mpfr\_prec field is used to store the precision of the variable (in bits); this is not less than MPFR\_PREC\_MIN.
- The \_mpfr\_sign field is used to store the sign of the variable.
- The  $_{\rm mpfr\_exp}$  field stores the exponent. An exponent of 0 means a radix point just above the most significant limb. Non-zero values n are a multiplier  $2^n$  relative to that point. A NaN, an infinity and a zero are indicated by a special value of the exponent.
- Finally, the \_mpfr\_d is a pointer to the limbs, least significant limbs stored first. The number of limbs in use is controlled by \_mpfr\_prec, namely ceil(\_mpfr\_prec/mp\_bits\_ per\_limb). Non-singular values always have the most significant bit of the most significant limb set to 1. When the precision does not correspond to a whole number of limbs, the excess bits at the low end of the data are zero.

# Contributors

The main developers of MPFR are Guillaume Hanrot, Vincent Lefèvre, Patrick Pélissier, Philippe Théveny and Paul Zimmermann.

Sylvie Boldo from ENS-Lyon, France, contributed the functions mpfr\_agm and mpfr\_log. Emmanuel Jeandel, from ENS-Lyon too, contributed the generic hypergeometric code, as well as the mpfr\_exp3, a first implementation of the sine and cosine, and improved versions of mpfr\_const\_log2 and mpfr\_const\_pi. Mathieu Dutour contributed the functions mpfr\_atan and mpfr\_asin, and a previous version of mpfr\_gamma; David Daney contributed the hyperbolic and inverse hyperbolic functions, the base-2 exponential, and the factorial function. Fabrice Rouillier contributed the original version of 'mul\_ui.c', the 'gmp\_op.c' file, and helped to the Microsoft Windows porting. Jean-Luc Rémy contributed the mpfr\_zeta code. Ludovic Meunier helped in the design of the mpfr\_erf code. Damien Stehlé contributed the mpfr\_get\_ld\_2exp function.

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