MPFR

The Multiple Precision Floating-Point Reliable Library Edition 2.2.0 September 2005

The MPFR team

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1 Introduction to MPFR

MPFR is a portable library written in C for arbitrary precision arithmetic on floating-point numbers. It is based on the GNU MP library. It aims to extend the class of floating-point numbers provided by the GNU MP library by a precise semantics. The main differences with the mpf class from GNU MP are:

- the mpfr code is portable, i.e. the result of any operation does not depend (or should not) on the machine word size mp_bits_per_limb (32 or 64 on most machines);
- the precision in bits can be set exactly to any valid value for each variable (including very small precision);
- mpfr provides the four rounding modes from the IEEE 754-1985 standard.

In particular, with a precision of 53 bits, mpfr should be able to exactly reproduce all computations with double-precision machine floating-point numbers (double type in C), except the default exponent range is much wider and subnormal numbers are not implemented but can be emulated.

This version of MPFR is released under the GNU Lesser General Public License. It is permitted to link MPFR to non-free programs, as long as when distributing them the MPFR source code and a means to re-link with a modified MPFR library is provided.

1.1 How to use this Manual

Everyone should read Chapter 4 [MPFR Basics], page 6. If you need to install the library yourself, you need to read Chapter 2 [Installing MPFR], page 3, too.

The rest of the manual can be used for later reference, although it is probably a good idea to glance through it.

2 Installing MPFR

2.1 How to install

Here are the steps needed to install the library on Unix systems (more details are provided in the 'INSTALL' file):

- 1. To build MPFR, you first have to install GNU MP (version 4.1 or higher) on your computer. You need a C compiler, preferably GCC, but any reasonable compiler should work. And you need a standard Unix 'make' program, plus some other standard Unix utility programs.
- 2. In the MPFR build directory, type './configure'

This will prepare the build and setup the options according to your system. If you get error messages, you might check that you use the same compiler and compile options as for GNU MP (see the 'INSTALL' file).

3. 'make'

This will compile MPFR, and create a library archive file 'libmpfr.a'. A dynamic library may be produced too (see configure).

4. 'make check'

This will make sure MPFR was built correctly. If you get error messages, please report this to 'mpfr@loria.fr'. (See Chapter 3 [Reporting Bugs], page 5, for information on what to include in useful bug reports.)

5. 'make install'

This will copy the files 'mpfr.h' and 'mpf2mpfr.h' to the directory '/usr/local/include', the file 'libmpfr.a' to the directory '/usr/local/lib', and the file 'mpfr.info' to the directory '/usr/local/info' (or if you passed the '--prefix' option to 'configure', using the prefix directory given as argument to '--prefix' instead of '/usr/local').

2.2 Other make targets

There are some other useful make targets:

• 'mpfr.info' or 'info'

Create an info version of the manual, in 'mpfr.info'.

• 'mpfr.dvi' or 'dvi'

Create a DVI version of the manual, in 'mpfr.dvi'.

'mpfr.ps'

Create a Postscript version of the manual, in 'mpfr.ps'.

'clean'

Delete all object files and archive files, but not the configuration files.

'distclean'

Delete all files not included in the distribution.

• 'uninstall'

Delete all files copied by 'make install'.

2.3 Known Build Problems

MPFR suffers from all bugs from the GNU MP library, plus many more.

Please report other problems to 'mpfr@loria.fr'. See Chapter 3 [Reporting Bugs], page 5. Some bug fixes are available on the MPFR web page http://www.mpfr.org/.

2.4 Getting the Latest Version of MPFR

The latest version of MPFR is available from http://www.mpfr.org/.

3 Reporting Bugs

If you think you have found a bug in the MPFR library, first have a look on the MPFR web page http://www.mpfr.org/: perhaps this bug is already known, in which case you may find there a workaround for it. Otherwise, please investigate and report it. We have made this library available to you, and it is not to ask too much from you, to ask you to report the bugs that you find.

There are a few things you should think about when you put your bug report together.

You have to send us a test case that makes it possible for us to reproduce the bug. Include instructions on how to run the test case.

You also have to explain what is wrong; if you get a crash, or if the results printed are incorrect and in that case, in what way.

Please include compiler version information in your bug report. This can be extracted using 'cc -V' on some machines, or, if you're using gcc, 'gcc -v'. Also, include the output from 'uname -a'.

If your bug report is good, we will do our best to help you to get a corrected version of the library; if the bug report is poor, we won't do anything about it (aside of chiding you to send better bug reports).

Send your bug report to: 'mpfr@loria.fr'.

If you think something in this manual is unclear, or downright incorrect, or if the language needs to be improved, please send a note to the same address.

4 MPFR Basics

All declarations needed to use MPFR are collected in the include file 'mpfr.h'. It is designed to work with both C and C++ compilers. You should include that file in any program using the MPFR library:

#include <mpfr.h>

4.1 Nomenclature and Types

A floating-point number or float for short, is an arbitrary precision mantissa with a limited precision exponent. The C data type for such objects is $mpfr_t$. A floating-point number can have three special values: Not-a-Number (NaN) or plus or minus Infinity. NaN represents an uninitialized object, the result of an invalid operation (like 0 divided by 0), or a value that cannot be determined (like +Infinity minus +Infinity). Moreover, like in the IEEE 754-1985 standard, zero is signed, i.e. there are both +0 and -0; the behavior is the same as in the IEEE 754-1985 standard and it is generalized to the other functions supported by MPFR.

The precision is the number of bits used to represent the mantissa of a floating-point number; the corresponding C data type is mp_prec_t. The precision can be any integer between MPFR_PREC_MIN and MPFR_PREC_MAX. In the current implementation, MPFR_PREC_MIN is equal to 2.

The rounding mode specifies the way to round the result of a floating-point operation, in case the exact result can not be represented exactly in the destination mantissa; the corresponding C data type is mp_rnd_t.

A *limb* means the part of a multi-precision number that fits in a single word. (We chose this word because a limb of the human body is analogous to a digit, only larger, and containing several digits.) Normally a limb contains 32 or 64 bits. The C data type for a limb is mp_limb_t.

4.2 Function Classes

There is only one class of functions in the MPFR library:

1. Functions for floating-point arithmetic, with names beginning with mpfr_. The associated type is mpfr_t.

4.3 MPFR Variable Conventions

As a general rule, all MPFR functions expect output arguments before input arguments. This notation is based on an analogy with the assignment operator.

MPFR allows you to use the same variable for both input and output in the same expression. For example, the main function for floating-point multiplication, $mpfr_mul$, can be used like this: $mpfr_mul$ (x, x, x, rnd_mode). This computes the square of x with rounding mode rnd_mode and puts the result back in x.

Before you can assign to an MPFR variable, you need to initialize it by calling one of the special initialization functions. When you're done with a variable, you need to clear it out, using one of the functions for that purpose.

A variable should only be initialized once, or at least cleared out between each initialization. After a variable has been initialized, it may be assigned to any number of times.

For efficiency reasons, avoid to initialize and clear out a variable in loops. Instead, initialize it before entering the loop, and clear it out after the loop has exited.

You don't need to be concerned about allocating additional space for MPFR variables, since any variable has a mantissa of fixed size. Hence unless you change its precision, or clear and reinitialize it, a floating-point variable will have the same allocated space during all its life.

4.4 Rounding modes.

The following four rounding modes are supported:

• GMP_RNDN: round to nearest

• GMP_RNDZ: round towards zero

• GMP_RNDU: round towards plus infinity

• GMP_RNDD: round towards minus infinity

The 'round to nearest' mode works as in the IEEE 754-1985 standard: in case the number to be rounded lies exactly in the middle of two representable numbers, it is rounded to the one with the least significant bit set to zero. For example, the number 5/2, which is represented by (10.1) in binary, is rounded to (10.0)=2 with a precision of two bits, and not to (11.0)=3. This rule avoids the *drift* phenomenon mentioned by Knuth in volume 2 of The Art of Computer Programming (Section 4.2.2).

Most MPFR functions take as first argument the destination variable, as second and following arguments the input variables, as last argument a rounding mode, and have a return value of type int, called the *ternary value*. The value stored in the destination variable is exactly rounded, i.e. MPFR behaves as if it computed the result with an infinite precision, then rounded it to the precision of this variable. The input variables are regarded as exact (in particular, their precision does not affect the result).

Unless documented otherwise, functions returning an int return a ternary value. If the ternary value is zero, it means that the value stored in the destination variable is the exact result of the corresponding mathematical function. If the ternary value is positive (resp. negative), it means the value stored in the destination variable is greater (resp. lower) than the exact result. For example with the GMP_RNDU rounding mode, the ternary value is usually positive, except when the result is exact, in which case it is zero. In the case of an infinite result, it is considered as inexact when it was obtained by overflow, and exact otherwise. A NaN result (Not-a-Number) always corresponds to an exact return value. The opposite of a returned ternary value is guaranteed to be representable in an int.

Unless documented otherwise, functions returning a 1 (or any other value specified in this manual) for special cases (like acos(0)) should return an overflow or an underflow if 1 is not representable in the current exponent range.

5 MPFR Interface

The floating-point functions expect arguments of type mpfr_t.

The MPFR floating-point functions have an interface that is similar to the GNU MP integer functions. The function prefix for floating-point operations is mpfr_.

There is one significant characteristic of floating-point numbers that has motivated a difference between this function class and other GNU MP function classes: the inherent inexactness of floating-point arithmetic. The user has to specify the precision for each variable. A computation that assigns a variable will take place with the precision of the assigned variable; the cost of that computation should not depend from the precision of variables used as input (on average).

The semantics of a calculation in MPFR is specified as follows: Compute the requested operation exactly (with "infinite accuracy"), and round the result to the precision of the destination variable, with the given rounding mode. The MPFR floating-point functions are intended to be a smooth extension of the IEEE 754-1985 arithmetic. The results obtained on one computer should not differ from the results obtained on a computer with a different word size.

MPFR does not keep track of the accuracy of a computation. This is left to the user or to a higher layer. As a consequence, if two variables are used to store only a few significant bits, and their product is stored in a variable with large precision, then MPFR will still compute the result with full precision.

5.1 Initialization Functions

An mpfr_t object must be initialized before storing the first value in it. The functions mpfr_init and mpfr_init2 are used for that purpose.

void mpfr_init2 (mpfr_t x, mp_prec_t prec)

[Function]

Initialize x, set its precision to be **exactly** prec bits and its value to NaN. (Warning: the corresponding mpf functions initialize to zero instead.)

Normally, a variable should be initialized once only or at least be cleared, using mpfr_clear, between initializations. To change the precision of a variable which has already been initialized, use mpfr_set_prec. The precision *prec* must be an integer between MPFR_PREC_MIN and MPFR_PREC_MAX (otherwise the behavior is undefined).

void mpfr_clear (mpfr_t x)

[Function]

Free the space occupied by x. Make sure to call this function for all mpfr_t variables when you are done with them.

void mpfr_init (mpfr_t x)

[Function]

Initialize x and set its value to NaN.

Normally, a variable should be initialized once only or at least be cleared, using $mpfr_clear$, between initializations. The precision of x is the default precision, which can be changed by a call to $mpfr_set_default_prec$.

void mpfr_set_default_prec (mp_prec_t prec)

[Function]

Set the default precision to be **exactly** prec bits. The precision of a variable means the number of bits used to store its mantissa. All subsequent calls to **mpfr_init** will use this precision, but previously initialized variables are unaffected. This default precision is set to 53 bits initially. The precision can be any integer between MPFR_PREC_MIN and MPFR_PREC_MAX.

```
mp_prec_t mpfr_get_default_prec (void)
Return the default MPFR precision in bits.
```

[Function]

Here is an example on how to initialize floating-point variables:

```
f
  mpfr_t x, y;
  mpfr_init (x); /* use default precision */
  mpfr_init2 (y, 256); /* precision exactly 256 bits */
    ...
  /* When the program is about to exit, do ... */
  mpfr_clear (x);
  mpfr_clear (y);
}
```

The following functions are useful for changing the precision during a calculation. A typical use would be for adjusting the precision gradually in iterative algorithms like Newton-Raphson, making the computation precision closely match the actual accurate part of the numbers.

```
void mpfr_set_prec (mpfr_t x, mp_prec_t prec)
```

[Function]

Reset the precision of x to be **exactly** prec bits, and set its value to NaN. The previous value stored in x is lost. It is equivalent to a call to $mpfr_clear(x)$ followed by a call to $mpfr_init2(x, prec)$, but more efficient as no allocation is done in case the current allocated space for the mantissa of x is enough. The precision prec can be any integer between MPFR_PREC_MIN and MPFR_PREC_MAX.

In case you want to keep the previous value stored in x, use mpfr_prec_round instead.

```
mp_prec_t mpfr_get_prec (mpfr_t x)
```

[Function]

Return the precision actually used for assignments of x, i.e. the number of bits used to store its mantissa.

5.2 Assignment Functions

These functions assign new values to already initialized floats (see Section 5.1 [Initialization Functions], page 8). When using any functions using intmax_t, you must include <stdint.h> or <inttypes.h> before 'mpfr.h', to allow 'mpfr.h' to define prototypes for these functions.

```
int mpfr_set (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
                                                                             [Function]
int mpfr_set_ui (mpfr_t rop, unsigned long int op, mp_rnd_t rnd)
                                                                             [Function]
int mpfr_set_si (mpfr_t rop, long int op, mp_rnd_t rnd)
                                                                             [Function]
int mpfr_set_uj (mpfr_t rop, uintmax_t op, mp_rnd_t rnd)
                                                                             [Function]
int mpfr_set_sj (mpfr_t rop, intmax_t op, mp_rnd_t rnd)
                                                                             [Function]
int mpfr_set_d (mpfr_t rop, double op, mp_rnd_t rnd)
                                                                             [Function]
int mpfr_set_ld (mpfr_t rop, long double op, mp_rnd_t rnd)
                                                                             [Function]
int mpfr_set_z (mpfr_t rop, mpz_t op, mp_rnd_t rnd)
                                                                             [Function]
int mpfr_set_q (mpfr_t rop, mpq_t op, mp_rnd_t rnd)
                                                                             [Function]
int mpfr_set_f (mpfr_t rop, mpf_t op, mp_rnd_t rnd)
                                                                             [Function]
```

Set the value of rop from op, rounded towards the given direction rnd. Note that the input 0 is converted to +0 by mpfr_set_ui, mpfr_set_si, mpfr_set_sj, mpfr_set_uj, mpfr_set_z, mpfr_set_q and mpfr_set_f, regardless of the rounding mode. If the system doesn't support the IEEE-754 standard, mpfr_set_d and mpfr_set_ld might not preserve the signed zeros. mpfr_set_q might not be able to work if the numerator (or the denominator) can not be representable as a mpfr_t.

Note: If you want to store a floating-point constant to a mpfr_t, you should use mpfr_set_str instead of mpfr_set_d or mpfr_set_ld. Otherwise the floating-point constant will be first converted into a reduced-precision (e.g., 53-bit) binary number before MPFR can work with it.

Set the value of rop from $op \times 2^e$, rounded towards the given direction rnd. Note that the input 0 is converted to +0.

int mpfr_set_str (mpfr_t rop, const char *s, int base, mp_rnd_t rnd) [Function] Set rop to the value of the whole string s in base base, rounded in the direction rnd. See the documentation of mpfr_strtofr for a detailed description of the valid string formats. This function returns 0 if the entire string up to the final null character is a valid number in base base; otherwise it returns -1, and rop may have changed.

int mpfr_strtofr (mpfr_t rop, const char *nptr, char **endptr, int base, mp_rnd_t rnd) [Function]

Read a floating-point number from a string nptr in base base, rounded in the direction rnd. If successful, the result is stored in rop and *endptr points to the character just after those parsed. If str doesn't start with a valid number then rop is set to zero and the value of nptr is stored in the location referenced by endptr.

Parsing follows the standard C strtod function. This means optional leading whitespace, an optional + or -, mantissa digits with an optional decimal point, and an optional exponent consisting of an e or E (if $base \leq 10$) or Q, an optional sign, and digits. The decimal point can be either the one defined by the current locale or the period (the first one is accepted for consistency with the C standard and the practice, the second one is accepted to allow the programmer to provide MPFR numbers from strings in a way that does not depend on the current locale). A hexadecimal mantissa can be given with a leading Q or Q, in which case Q or Q may introduce an optional binary exponent, indicating the power of Q by which the mantissa is to be scaled. A binary mantissa can be given with a leading Q or Q or Q, in which case Q, Q, Q may introduce the binary exponent. The exponent is always written in base Q.

In addition, infinity, inf (if $base \le 10$) or @inf@ with an optional sign, or nan, nan(n-char-sequence) (if $base \le 10$), @nan@ or @nan@(n-char-sequence) all case insensitive (as Latin letters), can be given. A n-char-sequence is a non-empty string containing only digits, Latin letters and the underscore (0, 1, 2, ..., 9, a, b, ..., z, A, B, ..., Z, ...).

There must be at least one digit in the mantissa for the number to be valid. If an exponent has no digits it's ignored and parsing stops after the mantissa. If an 0x, 0X, 0b or 0B is not followed by hexadecimal/binary digits, parsing stops after the first 0: the subject sequence is defined as the longest initial subsequence of the input string, starting with the first non-white-space character, that is of the expected form. The subject sequence contains no characters if the input string is not of the expected form.

Note that in the hex format the exponent P represents a power of 2, whereas @ represents a power of the base (i.e. 16).

If the argument base is different from 0, it must be in the range 2 to 36. Case is ignored; uppercase and lowercase letters have the same value.

If base is 0, then it tries to identify the used base: if the mantissa begins with the 0x prefix, it assumes that base is 16. If it begins with 0b, it assumes that base is 2. Otherwise, it assumes it is 10.

It returns a usual ternary value. If *endptr* is not a null pointer, a pointer to the character after the last character used in the conversion is stored in the location referenced by *endptr*.

```
void mpfr_set_inf (mpfr_t x, int sign) [Function]
void mpfr_set_nan (mpfr_t x) [Function]
Set the variable x to infinity or NaN (Not-a-Number) respectively. In mpfr_set_inf_x is
```

Set the variable x to infinity or NaN (Not-a-Number) respectively. In $mpfr_set_inf$, x is set to plus infinity iff sign is nonnegative.

```
void mpfr_swap (mpfr_t x, mpfr_t y)
```

[Function]

Swap the values x and y efficiently. Warning: the precisions are exchanged too; in case the precisions are different, mpfr_swap is thus not equivalent to three mpfr_set calls using a third auxiliary variable.

5.3 Combined Initialization and Assignment Functions

```
int mpfr_init_set (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
                                                                               [Macro]
int mpfr_init_set_ui (mpfr_t rop, unsigned long int op, mp_rnd_t rnd)
                                                                               [Macro]
int mpfr_init_set_si (mpfr_t rop, signed long int op, mp_rnd_t rnd)
                                                                               [Macro]
int mpfr_init_set_d (mpfr_t rop, double op, mp_rnd_t rnd)
                                                                               [Macro]
int mpfr_init_set_ld (mpfr_t rop, long double op, mp_rnd_t rnd)
                                                                               [Macro]
int mpfr_init_set_z (mpfr_t rop, mpz_t op, mp_rnd_t rnd)
                                                                               [Macro]
int mpfr_init_set_q (mpfr_t rop, mpq_t op, mp_rnd_t rnd)
                                                                               [Macro]
int mpfr_init_set_f (mpfr_t rop, mpf_t op, mp_rnd_t rnd)
                                                                               [Macro]
  Initialize rop and set its value from op, rounded in the direction rnd. The precision of rop
```

Initialize rop and set its value from op, rounded in the direction rnd. The precision of rop will be taken from the active default precision, as set by mpfr_set_default_prec.

int mpfr_init_set_str (mpfr_t x, const char *s, int base, mp_rnd_t rnd) [Function] Initialize x and set its value from the string s in base base, rounded in the direction rnd. See mpfr_set_str.

5.4 Conversion Functions

```
double mpfr_get_d (mpfr_t op, mp_rnd_t rnd) [Function] long double mpfr_get_ld (mpfr_t op, mp_rnd_t rnd) [Function] Convert op to a double (respectively long double), using the rounding mode rnd. If the system doesn't support IEEE 754 standard, this function might not preserve the signed zeros.
```

```
double mpfr_get_d_2exp (long *exp, mpfr_t op, mp_rnd_t rnd) [Function] Return d and set exp such that 0.5 \le |d| < 1 and d \times 2^{exp} equals op rounded to double precision, using the given rounding mode.
```

```
long mpfr_get_si (mpfr_t op, mp_rnd_t rnd) [Function]
unsigned long mpfr_get_ui (mpfr_t op, mp_rnd_t rnd) [Function]
intmax_t mpfr_get_sj (mpfr_t op, mp_rnd_t rnd) [Function]
```

uintmax_t mpfr_get_uj (mpfr_t op, mp_rnd_t rnd)

[Function]

Convert op to a long, an unsigned long, an intmax_t or an uintmax_t (respectively) after rounding it with respect to rnd. If op is NaN, the result is undefined. If op is too big for the return type, it returns the maximum or the minimum of the corresponding C type, depending on the direction of the overflow. The flag erange is set too. See also mpfr_fits_slong_p, mpfr_fits_ulong_p, mpfr_fits_intmax_p and mpfr_fits_uintmax_p.

mp_exp_t mpfr_get_z_exp (mpz_t rop, mpfr_t op)

[Function]

Put the scaled mantissa of op (regarded as an integer, with the precision of op) into rop, and return the exponent exp (which may be outside the current exponent range) such that op exactly equals $rop \times 2^{exp}$. If the exponent is not representable in the mp_exp_t type, the behavior is undefined.

void mpfr_get_z (mpz_t rop, mpfr_t op, mp_rnd_t rnd)

[Function]

Convert op to a mpz_t, after rounding it with respect to rnd. If op is NaN or Inf, the result is undefined.

int mpfr_get_f (mpf_t rop, mpfr_t op, mp_rnd_t rnd)

[Function]

Convert op to a mpf_t, after rounding it with respect to rnd. Return zero iff no error occurred, in particular a non-zero value is returned if op is NaN or Inf, which do not exist in mpf.

Convert op to a string of digits in base b, with rounding in the direction rnd. The base may vary from 2 to 36.

The generated string is a fraction, with an implicit radix point immediately to the left of the first digit. For example, the number -3.1416 would be returned as "-31416" in the string and 1 written at expptr. If rnd is to nearest, and op is exactly in the middle of two possible outputs, the one with an even last digit is chosen (for an odd base, this may not correspond to an even mantissa).

If n is zero, the number of digits of the mantissa is chosen large enough so that re-reading the printed value with the same precision, assuming both output and input use rounding to nearest, will recover the original value of op. More precisely, the chosen precision is $1+\lceil n\frac{\log 2}{\log b}\rceil$. This is the minimal precision depending on n and b only that satisfies the above property.

If str is a null pointer, space for the mantissa is allocated using the current allocation function, and a pointer to the string is returned. To free the returned string, you must use mpfr_free_str.

If str is not a null pointer, it should point to a block of storage large enough for the mantissa, i.e., at least max(n + 2, 7). The extra two bytes are for a possible minus sign, and for the terminating null character.

If n is 0, note that the space requirements for str in this case will be impossible for the user to predetermine. Therefore, one needs to pass a null pointer for the string argument whenever n is 0.

If the input number is an ordinary number, the exponent is written through the pointer expptr (the current minimal exponent for 0).

A pointer to the string is returned, unless there is an error, in which case a null pointer is returned.

```
void mpfr_free_str (char *str)
```

[Function]

[Function]

Free a string allocated by mpfr_get_str using the current unallocation function (preliminary interface). The block is assumed to be strlen(str)+1 bytes. For more information about how it is done: see section "Custom Allocation" in GNU MP.

```
int mpfr_fits_ulong_p (mpfr_t op, mp_rnd_t rnd)
                                                                           [Function]
int mpfr_fits_slong_p (mpfr_t op, mp_rnd_t rnd)
                                                                           [Function]
int mpfr_fits_uint_p (mpfr_t op, mp_rnd_t rnd)
                                                                           [Function]
int mpfr_fits_sint_p (mpfr_t op, mp_rnd_t rnd)
                                                                           [Function]
int mpfr_fits_ushort_p (mpfr_t op, mp_rnd_t rnd)
                                                                           [Function]
int mpfr_fits_sshort_p (mpfr_t op, mp_rnd_t rnd)
                                                                           [Function]
int mpfr_fits_intmax_p (mpfr_t op, mp_rnd_t rnd)
                                                                           [Function]
int mpfr_fits_uintmax_p (mpfr_t op, mp_rnd_t rnd)
                                                                          [Function]
```

Return non-zero if op would fit in the respective C data type, when rounded to an integer in the direction rnd.

5.5 Basic Arithmetic Functions

```
int mpfr_add (mpfr_t rop, mpfr_t op1, mpfr_t op2, mp_rnd_t rnd)
                                                                                [Function]
int mpfr_add_ui (mpfr_t rop, mpfr_t op1, unsigned long int op2, mp_rnd_t
                                                                                [Function]
         rnd)
int mpfr_add_si (mpfr_t rop, mpfr_t op1, long int op2, mp_rnd_t rnd)
                                                                                [Function]
int mpfr_add_z (mpfr_t rop, mpfr_t op1, mpz_t op2, mp_rnd_t rnd)
                                                                                [Function]
int mpfr_add_q (mpfr_t rop, mpfr_t op1, mpq_t op2, mp_rnd_t rnd)
                                                                                [Function]
  Set rop to op1 + op2 rounded in the direction rnd. For types having no signed zero, it is
  considered unsigned (i.e. (+0) + 0 = (+0) and (-0) + 0 = (-0)).
int mpfr_sub (mpfr_t rop, mpfr_t op1, mpfr_t op2, mp_rnd_t rnd)
                                                                                [Function]
int mpfr_ui_sub (mpfr_t rop, unsigned long int op1, mpfr_t op2, mp_rnd_t
                                                                                [Function]
int mpfr_sub_ui (mpfr_t rop, mpfr_t op1, unsigned long int op2, mp_rnd_t
                                                                               [Function]
         rnd)
int mpfr_si_sub (mpfr_t rop, long int op1, mpfr_t op2, mp_rnd_t rnd)
                                                                                [Function]
int mpfr_sub_si (mpfr_t rop, mpfr_t op1, long int op2, mp_rnd_t rnd)
                                                                                [Function]
int mpfr_sub_z (mpfr_t rop, mpfr_t op1, mpz_t op2, mp_rnd_t rnd)
                                                                                [Function]
int mpfr_sub_q (mpfr_t rop, mpfr_t op1, mpq_t op2, mp_rnd_t rnd)
                                                                                [Function]
  Set rop to op1 - op2 rounded in the direction rnd. For types having no signed zero, it is
  considered unsigned (i.e. (+0) - 0 = (+0), (-0) - 0 = (-0), 0 - (+0) = (-0) and 0 - (-0)
  = (+0).
int mpfr_mul (mpfr_t rop, mpfr_t op1, mpfr_t op2, mp_rnd_t rnd)
                                                                                [Function]
int mpfr_mul_ui (mpfr_t rop, mpfr_t op1, unsigned long int op2, mp_rnd_t
                                                                                [Function]
         rnd)
int mpfr_mul_si (mpfr_t rop, mpfr_t op1, long int op2, mp_rnd_t rnd)
                                                                                [Function]
int mpfr_mul_z (mpfr_t rop, mpfr_t op1, mpz_t op2, mp_rnd_t rnd)
                                                                                [Function]
int mpfr_mul_q (mpfr_t rop, mpfr_t op1, mpq_t op2, mp_rnd_t rnd)
                                                                                [Function]
  Set rop to op1 \times op2 rounded in the direction rnd. When a result is zero, its sign is the product
  of the signs of the operands (for types having no signed zero, it is considered positive).
```

Set rop to op^2 rounded in the direction rnd.

int mpfr_sqr (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)

int mpfr_div (mpfr_t rop, mpfr_t op1, mpfr_t op2, mp_rnd_t rnd) [Function]

```
int mpfr_ui_div (mpfr_t rop, unsigned long int op1, mpfr_t op2, mp_rnd_t
                                                                                [Function]
int mpfr_div_ui (mpfr_t rop, mpfr_t op1, unsigned long int op2, mp_rnd_t
                                                                                [Function]
int mpfr_si_div (mpfr_t rop, long int op1, mpfr_t op2, mp_rnd_t rnd)
                                                                                [Function]
int mpfr_div_si (mpfr_t rop, mpfr_t op1, long int op2, mp_rnd_t rnd)
                                                                                [Function]
int mpfr_div_z (mpfr_t rop, mpfr_t op1, mpz_t op2, mp_rnd_t rnd)
                                                                                [Function]
int mpfr_div_q (mpfr_t rop, mpfr_t op1, mpq_t op2, mp_rnd_t rnd)
                                                                                [Function]
  Set rop to op1/op2 rounded in the direction rnd. When a result is zero, its sign is the product
  of the signs of the operands (for types having no signed zero, it is considered positive).
                                                                                [Function]
int mpfr_sqrt (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
int mpfr_sqrt_ui (mpfr_t rop, unsigned long int op, mp_rnd_t rnd)
                                                                                [Function]
  Set rop to \sqrt{op} rounded in the direction rnd. Return -0 if op is -0 (to be consistent with
  the IEEE 754-1985 standard). Set rop to NaN if op is negative.
int mpfr_cbrt (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
                                                                                [Function]
int mpfr_root (mpfr_t rop, mpfr_t op, unsigned long int k, mp_rnd_t rnd)
                                                                                [Function]
  Set rop to the cubic root (resp. the kth root) of op rounded in the direction rnd. An odd
  (resp. even) root of a negative number (including -Inf) returns a negative number (resp.
  NaN). The kth root of -0 is defined to be -0, whatever the parity of k.
int mpfr_pow (mpfr_t rop, mpfr_t op1, mpfr_t op2, mp_rnd_t rnd)
                                                                                [Function]
int mpfr_pow_ui (mpfr_t rop, mpfr_t op1, unsigned long int op2, mp_rnd_t
                                                                                [Function]
         rnd)
int mpfr_pow_si (mpfr_t rop, mpfr_t op1, long int op2, mp_rnd_t rnd)
                                                                                [Function]
int mpfr_pow_z (mpfr_t rop, mpfr_t op1, mpz_t op2, mp_rnd_t rnd)
                                                                                [Function]
int mpfr_ui_pow_ui (mpfr_t rop, unsigned long int op1, unsigned long int
                                                                                [Function]
          op2, mp_rnd_t rnd)
int mpfr_ui_pow (mpfr_t rop, unsigned long int op1, mpfr_t op2, mp_rnd_t
                                                                                [Function]
  Set rop to op1^{op2}, rounded in the direction rnd. Special values are currently handled as
```

described in the ISO C99 standard for the pow function (note this may change in future versions):

- pow(± 0 , y) returns plus or minus infinity for y a negative odd integer.
- $pow(\pm 0, y)$ returns plus infinity for y negative and not an odd integer.
- $pow(\pm 0, y)$ returns plus or minus zero for y a positive odd integer.
- $pow(\pm 0, y)$ returns plus zero for y positive and not an odd integer.
- pow(-1, \pm inf) returns 1.
- pow(+1, y) returns 1 for any y, even a NaN.
- pow(x, y) returns NaN for finite negative x and finite non-integer y.
- pow(x, -inf) returns plus infinity for 0 < |x| < 1, and plus zero for |x| > 1.
- pow(x, +inf) returns plus zero for 0 < |x| < 1, and plus infinity for |x| > 1.
- pow(-inf, y) returns minus zero for y a negative odd integer.
- pow(-inf, y) returns plus zero for y negative and not an odd integer.
- pow(-inf, y) returns minus infinity for y a positive odd integer.
- pow(-inf, y) returns plus infinity for y positive and not an odd integer.
- pow(+inf, y) returns plus zero for y negative, and plus infinity for y positive.

int mpfr_neg (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function]
Set rop to -op rounded in the direction rnd. Just changes the sign if rop and op are the same variable.

int mpfr_abs (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function] Set rop to the absolute value of op, rounded in the direction rnd. Just changes the sign if rop and op are the same variable.

int mpfr_mul_2si (mpfr_t rop, mpfr_t op1, long int op2, mp_rnd_t rnd) [Function] Set rop to $op1 \times 2^{op2}$ rounded in the direction rnd. Just increases the exponent by op2 when rop and op1 are identical.

int mpfr_div_2si (mpfr_t rop, mpfr_t op1, long int op2, mp_rnd_t rnd) [Function] Set rop to $op1/2^{op2}$ rounded in the direction rnd. Just decreases the exponent by op2 when rop and op1 are identical.

5.6 Comparison Functions

```
int mpfr_cmp (mpfr_t op1, mpfr_t op2)
                                                                             [Function]
int mpfr_cmp_ui (mpfr_t op1, unsigned long int op2)
                                                                             [Function]
int mpfr_cmp_si (mpfr_t op1, signed long int op2)
                                                                             [Function]
int mpfr_cmp_d (mpfr_t op1, double op2)
                                                                             [Function]
int mpfr_cmp_ld (mpfr_t op1, long double op2)
                                                                             [Function]
int mpfr_cmp_z (mpfr_t op1, mpz_t op2)
                                                                             [Function]
int mpfr_cmp_q (mpfr_t op1, mpq_t op2)
                                                                             [Function]
int mpfr_cmp_f (mpfr_t op1, mpf_t op2)
                                                                             [Function]
```

Compare op1 and op2. Return a positive value if op1 > op2, zero if op1 = op2, and a negative value if op1 < op2. Both op1 and op2 are considered to their full own precision, which may differ. If one of the operands is NaN (Not-a-Number), return zero and set the erange flag.

Note: These functions may be useful to distinguish the three possible cases. If you need to distinguish two cases only, it is recommended to use the predicate functions (e.g., mpfr_equal_p for the equality) described below; they behave like the IEEE-754 comparisons, in particular when one or both arguments are NaN. But only floating-point numbers can be compared (you may need to do a conversion first).

```
int mpfr_cmp_ui_2exp (mpfr_t op1, unsigned long int op2, mp_exp_t e) [Function] int mpfr_cmp_si_2exp (mpfr_t op1, long int op2, mp_exp_t e) [Function] Compare op1 and op2 \times 2^e. Similar as above.
```

int mpfr_cmpabs (mpfr_t op1, mpfr_t op2) [Function] Compare |op1| and |op2|. Return a positive value if |op1| > |op2|, zero if |op1| = |op2|, and a negative value if |op1| < |op2|. If one of the operands is NaN (Not-a-Number), return zero and set the erange flag.

```
int mpfr_nan_p (mpfr_t op)
int mpfr_inf_p (mpfr_t op)
int mpfr_number_p (mpfr_t op)
[Function]
```

int mpfr_zero_p (mpfr_t op)

[Function]

Return non-zero if op is respectively Not-a-Number (NaN), an infinity, an ordinary number (i.e. neither Not-a-Number nor an infinity) or zero. Return zero otherwise.

int mpfr_sgn (mpfr_t op)

[Macro]

Return a positive value if op > 0, zero if op = 0, and a negative value if op < 0. Its result is undefined when op is NaN (Not-a-Number).

int mpfr_greater_p (mpfr_t op1, mpfr_t op2)

[Function]

Return non-zero if op1 > op2, zero otherwise.

int mpfr_greaterequal_p (mpfr_t op1, mpfr_t op2)

[Function]

Return non-zero if $op1 \ge op2$, zero otherwise.

int mpfr_less_p ($mpfr_t op1$, $mpfr_t op2$) Return non-zero if op1 < op2, zero otherwise. [Function]

int mpfr_lessequal_p (mpfr_t op1, mpfr_t op2)

[Function]

Return non-zero if $op1 \leq op2$, zero otherwise.

int mpfr_lessgreater_p (mpfr_t op1, mpfr_t op2)

[Function]

Return non-zero if op1 < op2 or op1 > op2 (i.e. neither op1, nor op2 is NaN, and $op1 \neq op2$), zero otherwise (i.e. op1 and/or op2 are NaN, or op1 = op2).

int mpfr_equal_p (mpfr_t op1, mpfr_t op2)

[Function]

Return non-zero if op1 = op2, zero otherwise (i.e. op1 and/or op2 are NaN, or $op1 \neq op2$).

int mpfr_unordered_p (mpfr_t op1, mpfr_t op2)

[Function]

Return non-zero if op1 or op2 is a NaN (i.e. they cannot be compared), zero otherwise.

5.7 Special Functions

All those functions, except explicitly stated, return zero for an exact return value, a positive value for a return value larger than the exact result, and a negative value otherwise.

```
int mpfr_log (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
```

[Function]

int mpfr_log2 (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)

[Function]

int mpfr_log10 (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)

[Function]

Set rop to the natural logarithm of op, $\log_2 op$ or $\log_{10} op$, respectively, rounded in the direction rnd.

```
int mpfr_exp (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
```

[Function]

int mpfr_exp2 (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)

[Function]

int mpfr_exp10 (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)

[Function]

Set rop to the exponential of op, to 2^{op} or to 10^{op} , respectively, rounded in the direction rnd.

```
int mpfr_cos (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
```

[Function]

int mpfr_sin (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)

[Function]

int mpfr_tan (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)

[Function]

Set rop to the cosine of op, sine of op, tangent of op, rounded in the direction rnd.

[Function]

int mpfr_csc (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)

[Function]

direction rnd.

```
int mpfr_cot (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
                                                                                  [Function]
  Set rop to the secant of op, cosecant of op, cotangent of op, rounded in the direction rnd.
int mpfr_sin_cos (mpfr_t sop, mpfr_t cop, mpfr_t op, mp_rnd_t rnd)
  Set simultaneously sop to the sine of op and cop to the cosine of op, rounded in the direction
  rnd with the corresponding precisions of sop and cop. Return 0 iff both results are exact.
int mpfr_acos (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
                                                                                  [Function]
int mpfr_asin (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
                                                                                  [Function]
int mpfr_atan (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
                                                                                  [Function]
  Set rop to the arc-cosine, arc-sine or arc-tangent of op, rounded in the direction rnd.
int mpfr_atan2 (mpfr_t rop, mpfr_t y, mpfr_t x, mp_rnd_t rnd)
  Set rop to the arc-tangent2 of y and x, rounded in the direction rnd: if x > 0, atan2(y, x)
  = atan (y/x); if x < 0, atan2(y, x) = sign(y)*(PI - atan <math>(|y/x|)).
  atan2(y, 0) does not raise any floating-point exception. Special values are currently handled
  as described in the ISO C99 standard for the atan2 function (note this may change in future
  versions):
    • atan2(+0, -0) returns +\pi.
    • atan2(-0, -0) returns -\pi.
    • atan2(+0, +0) returns +0.
    • atan2(-0, +0) returns -0.
    • atan2(+0, x) returns +\pi for x < 0.
    • atan2(-0, x) returns -\pi for x < 0.
    • atan2(+0, x) returns +0 for x > 0.
    • atan2(-0, x) returns -0 for x > 0.
    • atan2(y, 0) returns -\pi/2 for y < 0.
    • atan2(y, 0) returns +\pi/2 for y > 0.
    • atan2(+INF, -INF) returns +3 * \pi/4.
    • atan2(-INF, -INF) returns -3 * \pi/4.
    • atan2(+INF, +INF) returns +\pi/4.
    • atan2(-INF, +INF) returns -\pi/4.
    • atan2(+INF, x) returns +\pi/2 for finite x.
    • atan2(-INF, x) returns -\pi/2 for finite x.
    • atan2(y, -INF) returns +\pi for finite y > 0.
    • atan2(y, -INF) returns -\pi for finite y < 0.
    • atan2(y, +INF) returns +0 for finite y > 0.
    • atan2(y, +INF) returns -0 for finite y < 0.
int mpfr_cosh (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
                                                                                  [Function]
int mpfr_sinh (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
                                                                                  [Function]
int mpfr_tanh (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
                                                                                  [Function]
  Set rop to the hyperbolic cosine, sine or tangent of op, rounded in the direction rnd.
int mpfr_sech (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
                                                                                  [Function]
int mpfr_csch (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
                                                                                  [Function]
int mpfr_coth (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
                                                                                  [Function]
```

Set rop to the hyperbolic secant of op, cosecant of op, cotangent of op, rounded in the

```
int mpfr_acosh (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function]
int mpfr_asinh (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function]
int mpfr_atanh (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function]
Set rop to the inverse hyperbolic cosine, sine or tangent of op, rounded in the direction rnd.
```

int mpfr_fac_ui (mpfr_t rop, unsigned long int op, mp_rnd_t rnd) [Function] Set rop to the factorial of the unsigned long int op, rounded in the direction rnd.

int mpfr_log1p (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function] Set rop to the logarithm of one plus op, rounded in the direction rnd.

int mpfr_expm1 (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function] Set rop to the exponential of op minus one, rounded in the direction rnd.

int mpfr_eint (mpfr_t y, mpfr_t x, mp_rnd_t rnd) [Function] Set y to the exponential integral of x, rounded in the direction rnd. For positive x, the exponential integral is the sum of Euler's constant, of the logarithm of x, and of the sum for k from 1 to infinity of $x^k/k!$. For negative x, the returned value is NaN.

int mpfr_gamma (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function] int mpfr_lngamma (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function] Set rop to the value of the Gamma function on op, and its logarithm respectively, rounded in the direction rnd. When op is a negative integer, NaN is returned.

int mpfr_zeta (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function] Set rop to the value of the Riemann Zeta function on op, rounded in the direction rnd.

int mpfr_erf (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function] Set rop to the value of the error function on op, rounded in the direction rnd.

int mpfr_erfc (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function]
Set rop to the value of the complementary error function on op, rounded in the direction rnd.

Set rop to $op1 \times op2 + op3$, rounded in the direction rnd.

int mpfr_agm ($mpfr_t$ rop, $mpfr_t$ op1, $mpfr_t$ op2, mp_t rod trnd) [Function] Set rop to the arithmetic-geometric mean of op1 and op2, rounded in the direction rnd. The arithmetic-geometric mean is the common limit of the sequences u[n] and v[n], where u[0]=op1, v[0]=op2, u[n+1] is the arithmetic mean of u[n] and v[n], and v[n+1] is the geometric mean of u[n] and v[n]. If any operand is negative, the return value is NaN.

int mpfr_hypot (mpfr_t rop, mpfr_t x, mpfr_t y, mp_rnd_t rnd) [Function] Set rop to the Euclidean norm of x and y, i.e. $\sqrt{x^2 + y^2}$, rounded in the direction rnd. Special values are currently handled as described in Section F.9.4.3 of the ISO C99 standard, for the hypot function (note this may change in future versions): If x or y is an infinity, then plus infinity is returned in rop, even if the other number is NaN.

```
int mpfr_const_log2 (mpfr_t rop, mp_rnd_t rnd) [Function]
int mpfr_const_pi (mpfr_t rop, mp_rnd_t rnd) [Function]
int mpfr_const_euler (mpfr_t rop, mp_rnd_t rnd) [Function]
```

int mpfr_const_catalan (mpfr_t rop, mp_rnd_t rnd)

[Function]

Set rop to the logarithm of 2, the value of π , of Euler's constant 0.577..., of Catalan's constant 0.915..., respectively, rounded in the direction rnd. These functions cache the computed values to avoid other calculations if a lower or equal precision is requested. To free these caches, use mpfr_free_cache.

void mpfr_free_cache (void)

[Function]

Free the cache used by the functions computing constants if needed (currently mpfr_const_log2, mpfr_const_pi, mpfr_const_euler and mpfr_const_catalan).

Set ret to the sum of all elements of tab whose size is n, rounded in the direction rnd. Warning, tab is a table of pointers to mpfr_t, not a table of mpfr_t (preliminary interface). The returned int value is zero when the computed value is the exact value, and non-zero when this cannot be guaranteed, without giving the direction of the error as the other functions do.

5.8 Input and Output Functions

This section describes functions that perform input from an input/output stream, and functions that output to an input/output stream. Passing a null pointer for a *stream* argument to any of these functions will make them read from **stdin** and write to **stdout**, respectively.

When using any of these functions, you must include the <stdio.h> standard header before 'mpfr.h', to allow 'mpfr.h' to define prototypes for these functions.

Output op on stream stream, as a string of digits in base base, rounded in the direction rnd. The base may vary from 2 to 36. Print n significant digits exactly, or if n is 0, enough digits so that op can be read back exactly (see mpfr_get_str).

In addition to the significant digits, a decimal point (defined by the current locale) at the right of the first digit and a trailing exponent in base 10, in the form 'eNNN', are printed. If base is greater than 10, '@' will be used instead of 'e' as exponent delimiter.

Return the number of bytes written, or if an error occurred, return 0.

Input a string in base base from stream stream, rounded in the direction rnd, and put the read float in rop.

This function reads a word (defined as a sequence of characters between whitespace) and parses it using mpfr_set_str (it may change). See the documentation of mpfr_strtofr for a detailed description of the valid string formats.

Return the number of bytes read, or if an error occurred, return 0.

5.9 Integer Related Functions

```
int mpfr_rint (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)
int mpfr_ceil (mpfr_t rop, mpfr_t op)
int mpfr_floor (mpfr_t rop, mpfr_t op)
[Function]
```

```
int mpfr_round (mpfr_t rop, mpfr_t op) [Function]
int mpfr_trunc (mpfr_t rop, mpfr_t op) [Function]
```

Set rop to op rounded to an integer. mpfr_rint rounds to the nearest representable integer in the given rounding mode, mpfr_ceil rounds to the next higher or equal representable integer, mpfr_floor to the next lower or equal representable integer, mpfr_round to the nearest representable integer, rounding halfway cases away from zero, and mpfr_trunc to the next representable integer towards zero.

The returned value is zero when the result is exact, positive when it is greater than the original value of op, and negative when it is smaller. More precisely, the returned value is 0 when op is an integer representable in rop, 1 or -1 when op is an integer that is not representable in rop, 2 or -2 when op is not an integer.

Note that mpfr_round is different from mpfr_rint called with the rounding to the nearest mode (where halfway cases are rounded to an even integer or mantissa). Note also that no double rounding is performed; for instance, 4.5 (100.1 in binary) is rounded by mpfr_round to 4 (100 in binary) in 2-bit precision, though round(4.5) is equal to 5 and 5 (101 in binary) is rounded to 6 (110 in binary) in 2-bit precision.

```
int mpfr_rint_ceil (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function]
int mpfr_rint_floor (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function]
int mpfr_rint_round (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function]
int mpfr_rint_trunc (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function]
```

Set rop to op rounded to an integer. mpfr_rint_ceil rounds to the next higher or equal integer, mpfr_rint_floor to the next lower or equal integer, mpfr_rint_round to the nearest integer, rounding halfway cases away from zero, and mpfr_rint_trunc to the next integer towards zero. If the result is not representable, it is rounded in the direction rnd. The returned value is the ternary value associated with the considered round-to-integer function (regarded in the same way as any other mathematical function).

```
int mpfr_frac (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function]
Set rop to the fractional part of op, having the same sign as op, rounded in the direction rnd
```

Set rop to the fractional part of op, having the same sign as op, rounded in the direction rnd (unlike in mpfr_rint, rnd affects only how the exact fractional part is rounded, not how the fractional part is generated).

```
int mpfr_integer_p (mpfr_t op) [Function]
Return non-zero iff op is an integer.
```

5.10 Miscellaneous Functions

void mpfr_nexttoward (mpfr_t x, mpfr_t y) [Function]

If x or y is NaN, set x to NaN. Otherwise, if x is different from y, replace x by the next floating-point number (with the precision of x and the current exponent range) in the direction of y, if there is one (the infinite values are seen as the smallest and largest floating-point numbers). If the result is zero, it keeps the same sign. No underflow or overflow is generated.

```
void mpfr_nextbelow (mpfr_t x) [Function]
Equivalent to mpfr_nexttoward where y is minus infinity.
```

int mpfr_min ($mpfr_t rop$, $mpfr_t op1$, $mpfr_t op2$, $mp_t rod_t rod$) [Function] Set rop to the minimum of op1 and op2. If op1 and op2 are both NaN, then rop is set to NaN. If op1 or op2 is NaN, then rop is set to the numeric value. If op1 and op2 are zeros of different signs, then rop is set to -0.

int mpfr_max (mpfr_t rop, mpfr_t op1, mpfr_t op2, mp_rnd_t rnd) [Function] Set rop to the maximum of op1 and op2. If op1 and op2 are both NaN, then rop is set to NaN. If op1 or op2 is NaN, then rop is set to the numeric value. If op1 and op2 are zeros of different signs, then rop is set to +0.

int mpfr_urandomb (mpfr_t rop, gmp_randstate_t state)

[Function]

Generate a uniformly distributed random float in the interval $0 \le rop < 1$. Return 0, unless the exponent is not in the current exponent range, in which case rop is set to NaN and a non-zero value is returned.

void mpfr_random (mpfr_t rop)

[Function]

Generate a uniformly distributed random float in the interval $0 \le rop < 1$. This function is deprecated; mpfr_urandomb should be used instead.

void mpfr_random2 (mpfr_t rop, mp_size_t size, mp_exp_t exp)

[Function]

Generate a random float of at most size limbs, with long strings of zeros and ones in the binary representation. The exponent of the number is in the interval -exp to exp. This function is useful for testing functions and algorithms, since this kind of random numbers have proven to be more likely to trigger corner-case bugs. Negative random numbers are generated when size is negative. Put +0 in rop when size if zero.

mp_exp_t mpfr_get_exp (mpfr_t x)

[Function]

Get the exponent of x, assuming that x is a non-zero ordinary number. The behavior for NaN, Infinity or Zero is undefined.

int mpfr_set_exp (mpfr_t x, mp_exp_t e)

[Function]

Set the exponent of x if e is in the current exponent range, and return 0 (even if x is not a non-zero ordinary number); otherwise, return a non-zero value.

const char * mpfr_get_version (void)

[Function]

Return the MPFR version, as a null-terminated string.

MPFR_VERSION
MPFR_VERSION_MAJOR
MPFR_VERSION_MINOR
MPFR_VERSION_PATCHLEVEL
MPFR_VERSION_STRING

[Macro]

[Macro] [Macro]

[Macro]

MPFR_VERSION is the version of MPFR as a preprocessing constant. MPFR_VERSION_MAJOR, MPFR_VERSION_MINOR and MPFR_VERSION_PATCHLEVEL are respectively the major, minor and patch level of MPFR version, as preprocessing constants. MPFR_VERSION_STRING is the version as a string constant, which can be compared to the result of mpfr_get_version to check at run time the header file and library used match:

```
if (strcmp (mpfr_get_version (), MPFR_VERSION_STRING))
  fprintf (stderr, "Error, header and library files do not match\n");
```

long MPFR_VERSION_NUM (major, minor, patchlevel)

[Macro]

Create an integer in the same format as used by MPFR_VERSION from the given major, minor and patchlevel. Here is an example of how to check the MPFR version at compile time:

```
#if (!defined(MPFR_VERSION) || (MPFR_VERSION<MPFR_VERSION_NUM(2,1,0)))
# error "Wrong MPFR version."
#endif</pre>
```

5.11 Rounding Modes

```
void mpfr_set_default_rounding_mode (mp_rnd_t rnd)
```

[Function]

Set the default rounding mode to rnd. The default rounding mode is to nearest initially.

```
mp_rnd_t mpfr_get_default_rounding_mode (void)
```

[Function]

Get the default rounding mode.

```
int mpfr_prec_round (mpfr_t x, mp_prec_t prec, mp_rnd_t rnd)
```

[Function]

Round x according to rnd with precision prec, which must be an integer between MPFR_PREC_MIN and MPFR_PREC_MAX (otherwise the behavior is undefined). If prec is greater or equal to the precision of x, then new space is allocated for the mantissa, and it is filled with zeros. Otherwise, the mantissa is rounded to precision prec with the given direction. In both cases, the precision of x is changed to prec.

```
int mpfr_round_prec (mpfr_t x, mp_rnd_t rnd, mp_prec_t prec) [This function is obsolete. Please use mpfr_prec_round instead.]
```

[Function]

```
const char * mpfr_print_rnd_mode (mp_rnd_t rnd)
```

[Function]

Return the input string (GMP_RNDD, GMP_RNDU, GMP_RNDN, GMP_RNDZ) corresponding to the rounding mode rnd or a null pointer if rnd is an invalid rounding mode.

5.12 Exceptions

```
mp_exp_t mpfr_get_emin (void)
mp_exp_t mpfr_get_emax (void)
```

[Function]

[Function]

Return the (current) smallest and largest exponents allowed for a floating-point variable. The smallest positive value of a floating-point variable is $1/2 \times 2^{\text{emin}}$ and the largest value has the form $(1 - \varepsilon) \times 2^{\text{emax}}$.

```
int mpfr_set_emin (mp_exp_t exp)
```

[Function]

[Function]

int mpfr_set_emax (mp_exp_t exp)

Set the smallest and largest exponents allowed for a floating-point variable. Return a non-zero value when \exp is not in the range accepted by the implementation (in that case the smallest or largest exponent is not changed), and zero otherwise. If the user changes the exponent range, it is her/his responsibility to check that all current floating-point variables are in the new allowed range (for example using mpfr_check_range), otherwise the subsequent behavior will be undefined, in the sense of the ISO C standard.

```
mp_exp_t mpfr_get_emin_min (void) [Function]
mp_exp_t mpfr_get_emin_max (void) [Function]
mp_exp_t mpfr_get_emax_min (void) [Function]
mp_exp_t mpfr_get_emax_max (void) [Function]
```

Return the minimum and maximum of the smallest and largest exponents allowed for mpfr_set_emin and mpfr_set_emax. These values are implementation dependent; it is possible

to create a non portable program by writing mpfr_set_emax(mpfr_get_emax_max()) and mpfr_set_emin(mpfr_get_emin_min()) since the values of the smallest and largest exponents become implementation dependent.

int mpfr_check_range (mpfr_t x, int t, mp_rnd_t rnd)

[Function]

This function forces x to be in the current range of acceptable values, t being the current ternary value: negative if x is smaller than the exact value, positive if x is larger than the exact value and zero if x is exact (before the call). It generates an underflow or an overflow if the exponent of x is outside the current allowed range; the value of t may be used to avoid a double rounding. This function returns zero if the rounded result is equal to the exact one, a positive value if the rounded result is larger than the exact one, a negative value if the rounded result is smaller than the exact one. Note that unlike most functions, the result is compared to the exact one, not the input value x, i.e. the ternary value is propagated.

int mpfr_subnormalize (mpfr_t x, int t, mp_rnd_t rnd)

[Function]

This function rounds x emulating subnormal number arithmetic: if x is outside the subnormal exponent range, it just propagates the ternary value t; otherwise, it rounds x to precision $\mathsf{EXP}(x) - \mathsf{emin+1}$ according to rounding mode rnd and previous ternary value t, avoiding double rounding problems. $\mathsf{PREC}(x)$ is not modified by this function. rnd and t must be the used rounding mode for computing x and the returned ternary value when computing x. The subnormal exponent range is from $\mathsf{emin+PREC}(x)-1$. This functions assumes that $\mathsf{emax-emin} >= \mathsf{PREC}(x)$. Note that unlike most functions, the result is compared to the exact one, not the input value x, i.e. the ternary value is propagated. This is a preliminary interface.

This is an example of how to emulate double IEEE-754 arithmetic using MPFR:

```
f{
    mpfr_t xa, xb;
    int i;
    volatile double a, b;

mpfr_set_default_prec (53);
    mpfr_set_emin (-1073);
    mpfr_set_emax (1021);

mpfr_init (xa); mpfr_init (xb);

b = 34.3; mpfr_set_d (xb, b, GMP_RNDN);
    a = 0x1.1235P-1021; mpfr_set_d (xa, a, GMP_RNDN);

a /= b;
    i = mpfr_div (xa, xa, xb, GMP_RNDN);
    i = mpfr_subnormalize (xa, i, GMP_RNDN);
    i = mpfr_subnormalize (xa, i, GMP_RNDN);
```

Warning: this emulates a double IEEE-754 arithmetic with exact rounding in the subnormal range, which may not be the case for your hardware.

```
void mpfr_clear_underflow (void)[Function]void mpfr_clear_overflow (void)[Function]void mpfr_clear_nanflag (void)[Function]void mpfr_clear_inexflag (void)[Function]
```

void mpfr_clear_erangeflag (void)

[Function]

Clear the underflow, overflow, invalid, inexact and erange flags.

```
void mpfr_set_underflow (void)[Function]void mpfr_set_overflow (void)[Function]void mpfr_set_nanflag (void)[Function]void mpfr_set_inexflag (void)[Function]void mpfr_set_erangeflag (void)[Function]
```

Set the underflow, overflow, invalid, inexact and erange flags.

void mpfr_clear_flags (void)

[Function]

Clear all global flags (underflow, overflow, inexact, invalid, erange).

```
int mpfr_underflow_p (void) [Function]
int mpfr_overflow_p (void) [Function]
int mpfr_nanflag_p (void) [Function]
int mpfr_inexflag_p (void) [Function]
int mpfr_erangeflag_p (void) [Function]
```

Return the corresponding (underflow, overflow, invalid, inexact, erange) flag, which is non-zero iff the flag is set.

5.13 Advanced Functions

All the given interfaces are preliminary. They might change incompatibly in future revisions.

MPFR_DECL_INIT (name, prec)

[Macro]

This macro declares name as an automatic variable of type mpfr_t, initializes it and sets its precision to be exactly prec bits and its value to NaN. name must be a valid identifier. You must use this macro in the declaration section. This macro is much faster than using mpfr_init2 but has some drawbacks:

- You **must not** call mpfr_clear with variables created with this macro (The storage is allocated at the point of declaration and deallocated when the brace-level is exited.).
- You can not change their precision.
- You should not create variables with huge precision with this macro.
- Your compiler must support 'Non-Constant Initializers' (standard in C++ and ISO C99) and 'Token Pasting' (standard in ISO C89). If prec is not a compiler constant, your compiler must support 'Variable-length automatic arrays' (standard in ISO C99). 'GCC 2.95.3' supports all these features.

void mpfr_inits (mpfr_t x, ...)

[Function]

Initialize all the mpfr_t variables of the given va_list, set their precision to be the default precision and their value to NaN. See mpfr_init for more details. The va_list is assumed to be composed only of type mpfr_t. It begins from x. It ends when it encounters a null pointer.

void mpfr_inits2 (mp_prec_t prec, mpfr_t x, ...)

[Function]

Initialize all the mpfr_t variables of the given va_list, set their precision to be exactly prec bits and their value to NaN. See mpfr_init2 for more details. The va_list is assumed to be composed only of type mpfr_t. It begins from x. It ends when it encounters a null pointer.

```
void mpfr_clears (mpfr_t x, ...)
```

[Function]

Free the space occupied by all the mpfr_t variables of the given va_list. See mpfr_clear for more details. The va_list is assumed to be composed only of type mpfr_t. It begins from x. It ends when it encounters a null pointer.

Here is an example of how to use multiple initialization functions:

```
{
  mpfr_t x, y, z, t;
  mpfr_inits2 (256, x, y, z, t, (void *) 0);
  ...
  mpfr_clears (x, y, z, t, (void *) 0);
}
```

5.14 Compatibility with MPF

A header file 'mpf2mpfr.h' is included in the distribution of MPFR for compatibility with the GNU MP class MPF. After inserting the following two lines after the #include <gmp.h> line,

```
#include <mpfr.h>
#include <mpf2mpfr.h>
```

any program written for MPF can be compiled directly with MPFR without any changes. All operations are then performed with the default MPFR rounding mode, which can be reset with mpfr_set_default_rounding_mode.

Warning: the mpf_init and mpf_init2 functions initialize to zero, whereas the corresponding mpfr functions initialize to NaN: this is useful to detect uninitialized values, but is slightly incompatible with mpf.

```
void mpfr_set_prec_raw (mpfr_t x, mp_prec_t prec)
```

[Function]

Reset the precision of x to be **exactly** prec bits. The only difference with mpfr_set_prec is that prec is assumed to be small enough so that the mantissa fits into the current allocated memory space for x. Otherwise the behavior is undefined.

```
int mpfr_eq (mpfr_t op1, mpfr_t op2, unsigned long int op3)
```

[Function]

Return non-zero if op1 and op2 are both non-zero ordinary numbers with the same exponent and the same first op3 bits, both zero, or both infinities of the same sign. Return zero otherwise. This function is defined for compatibility with mpf, but does not make much sense.

void mpfr_reldiff (mpfr_t rop, mpfr_t op1, mpfr_t op2, mp_rnd_t rnd) [Function] Compute the relative difference between op1 and op2 and store the result in rop. This function does not guarantee the exact rounding on the relative difference; it just computes |op1 - op2|/op1, using the rounding mode rnd for all operations and the precision of rop.

See mpfr_mul_2ui and mpfr_div_2ui. These functions are only kept for compatibility with MPF.

5.15 Custom interface

Some applications use a stack to handle the memory and their objects. However, the MPFR memory design is not well suited for such a thing. So that such applications are able to use MPFR, an auxiliary memory interface has been created: the Custom Interface.

The following interface allows them to use MPFR in two ways:

- Either they directly store the MPFR FP number as a mpfr_t on the stack.
- Either they store their own representation of a FP number on the stack and construct a new temporary mpfr_t each time it is needed.

Nothing has to be done to destroy the FP numbers except garbaging the used memory: all the memory stuff (allocating, destroying, garbaging) is kept to the application.

Each function is this interface is also implemented as a macro for efficiency reasons.

Note 1: MPFR functions may still initialize temporary FP numbers using standard mpfr_init. See Custom Allocation (GNU MP).

Note 2: MPFR functions may use the cached functions (mpfr_const_pi for example), even if they are not explicitly called. You have to call mpfr_free_cache each time you garbage the memory iff mpfr_init, through GMP Custom Allocation, allocates its memory on the application stack.

Note 3: This interface is preliminary.

```
size_t mpfr_custom_get_size (mp_prec_t prec) [Function]
Return the needed size in bytes to store the mantissa of a FP number of precision prec.
```

```
void mpfr_custom_init (void *mantissa, mp_prec_t prec) [Function]
Initialize a mantissa of precision prec. mantissa must be an area of mpfr_custom_get_size
(prec) bytes at least and be suitably aligned for an array of mp_limb_t.
```

Perform a dummy initialization of a mpfr_t and set it to:

- if ABS(kind) == MPFR_NAN_LIND, x is set to NaN;
- if ABS(kind) == MPFR_INF_KIND, x is set to the infinity of sign sign(kind);
- if ABS(kind) == MPFR_ZERO_KIND, x is set to the zero of sign sign(kind);
- if ABS(kind) == MPFR_REGULAR_KIND, x is set to a regular number: x = sign(kind)*mantissa*2^exp

In all cases, it uses mantissa directly for further computing involving x. It will not allocate anything. A FP number initialized with this function cannot be resized using mpfr_set_prec, or cleared using mpfr_clear! mantissa must have been initialized with mpfr_custom_init using the same precision prec.

this function for any mpfr_t not initialized with mpfr_custom_init_set is undefined.

```
void *mpfr_custom_get_mantissa (mpfr_t x) [Function]
Return a pointer to the mantissa used by a mpfr_t initialized with mpfr_custom_init_set.
The behavior of this function for any mpfr_t not initialized with mpfr_custom_init_set is undefined.
```

mp_exp_t mpfr_custom_get_exp (mpfr_t x)

[Function]

Return the exponent of x, assuming that x is a non-zero ordinary number. The return value for NaN, Infinity or Zero is unspecified but doesn't produced any trap. The behavior of this function for any mpfr_t not initialized with mpfr_custom_init_set is undefined.

void mpfr_custom_move (mpfr_t x, void *new_position)

[Function]

Inform MPFR that the mantissa has moved due to a garbage collect and update its new position to new_position. However the application has to move the mantissa and the mpfr_t itself. The behavior of this function for any mpfr_t not initialized with mpfr_custom_init_set is undefined.

See the test suite for examples.

5.16 Internals

The following types and functions were mainly designed for the implementation of mpfr, but may be useful for users too. However no upward compatibility is guaranteed. You may need to include 'mpfr-impl.h' to use them.

The mpfr_t type consists of four fields.

- The _mpfr_prec field is used to store the precision of the variable (in bits); this is not less than MPFR_PREC_MIN.
- The _mpfr_sign field is used to store the sign of the variable.
- The $_{\tt mpfr_exp}$ field stores the exponent. An exponent of 0 means a radix point just above the most significant limb. Non-zero values n are a multiplier 2^n relative to that point. A NaN, an Infinity and a Zero are indicated by a special value of the exponent.
- Finally, the _mpfr_d is a pointer to the limbs, least significant limbs stored first. The number of limbs in use is controlled by _mpfr_prec, namely ceil(_mpfr_prec/mp_bits_ per_limb). Non-singular values always have the most significant bit of the most significant limb set to 1. When the precision does not correspond to a whole number of limbs, the excess bits at the low end of the data are zero.

Assuming b is an approximation of an unknown number x in the direction rnd1 with error at most two to the power E(b)-err where E(b) is the exponent of b, return a non-zero value if one is able to round exactly x to precision prec with the direction rnd2, and 0 otherwise (including for NaN and Inf). This function **does not modify** its arguments.

double mpfr_get_d1 (mpfr_t op)

[Function]

Convert op to a double, using the default MPFR rounding mode (see function mpfr_set_default_rounding_mode). This function is obsolete.

Contributors

The main developers consist of Guillaume Hanrot, Vincent Lefèvre, Patrick Pélissier and Paul Zimmermann.

We would like to thank Jean-Michel Muller and Joris van der Hoeven for very fruitful discussions at the beginning of that project, Torbjörn Granlund and Kevin Ryde for their help about design issues, and Nathalie Revol for her careful reading of a previous version of this documentation. Kevin Ryde did a tremendous job for the portability of MPFR, and integrating it into GMP 4.x; alas the GMP developers decided in January 2004 not to include MPFR any more.

Sylvie Boldo from ENS-Lyon, France, contributed the functions mpfr_agm and mpfr_log. Emmanuel Jeandel, from ENS-Lyon too, contributed the generic hypergeometric code in generic.c, as well as the mpfr_exp3, a first implementation of the sine and cosine, and improved versions of mpfr_const_log2 and mpfr_const_pi. Mathieu Dutour contributed the functions mpfr_atan and mpfr_asin, and a previous version of mpfr_gamma; David Daney contributed the hyperbolic and inverse hyperbolic functions, the base-2 exponential, and the factorial function. Fabrice Rouillier contributed the original version of 'mul_ui.c', the 'gmp_op.c' file, and helped to the Windows porting. Jean-Luc Rémy contributed the mpfr_zeta code. Ludovic Meunier helped in the design of the mpfr_erf code.

The development of the MPFR library would not have been possible without the continuous support of LORIA, INRIA and INRIA Lorraine. The development of MPFR was also supported by a grant (202F0659 00 MPN 121) from the Conseil Régional de Lorraine in 2002.

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Version 1.1, March 2000

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