Embedded Systems - Notes Week 7

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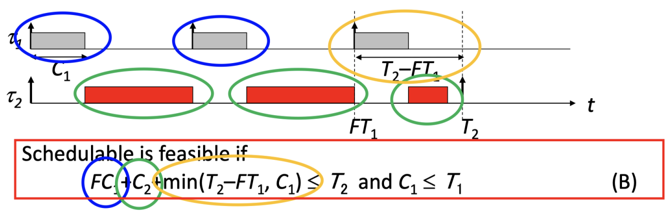
January 16, 2022



We need to show that $(A) \implies (B)$:

$C_1 + C_2 \leq T_1 \implies C_1 \leq T_1 C_1 + C_2 \leq T_1 \implies FC_1 + C_2 \leq FC_1 + F_c 2 \leq FT_1 \implies FC_1 + C_2 + \min(T_2 - FT_1, C_1) \leq FT_1 + \min(T_2 - F$

> Given tasks τ_1 and τ_2 with $T_1 < T_2$, then if the schedule is feasible by an arbitrary fixed priority assignment, it is also feasible by RM.

Schedulability analysis for RM: A set of periodic tasks is schedulable with RM if:

$$\sum_{i=1}^{n} \frac{C_i}{T_i} \leq n(2^{1/n} - 1)$$

This condition is sufficient but not necessary. The term $U = \sum_{i=1}^{n} \frac{C_i}{T_i}$ denotes the **processor utilization factor** U which is the fraction of processor time spent in the execution of the task set.

6.3.3 Deadline Monotonic Scheduling (DM)

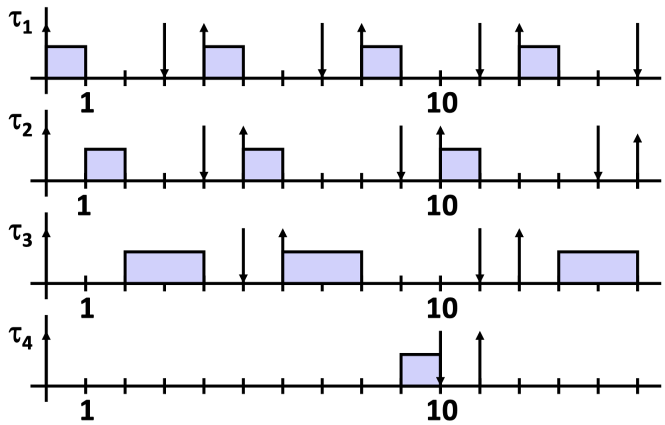
Assumptions for **deadline monotonic scheduling** are as in rate monotonic scheduling, but *deadlines may be smaller than the period,* i.e. $C_i \leq D_i \leq T_i$.

> Each task is assigned a priority. Tasks with smaller relative deadlines will have higher priorities. Jobs with higher priorities interrupt jobs with lower priorities.

Schedulability analysis for DM: A set of periodic tasks is schedulable with DM if:

$$\sum_{i=1}^{n} \frac{C_i}{D_i} \leq n(2^{1/n} - 1).$$

The condition is sufficient but not necessary. *Example* with $U = 0.874$, $\sum_{i=1}^{n} \frac{C_i}{D_i} = 1.08 > n(s^{1/n} - 1) = 0 - 757$:



There is also a *necessary and sufficient* schedulability test which is computationally more involved. It is based on the following observations:

- The worst-case processor demand occurs when all tasks are released simultaneously, that is, at their critical instances
- For each task i, the sum of its processing time and the interference imposed by higher priority task must be less than or equal to D_i
- A measure of the *worst-case inference* for task i can be computed as the sum of the processing times of all higher priority tasks released before some time t where tasks are ordered according to $m < n \iff D_m < D_n$:

$$I_i = \sum_{j=1}^{i-1} \lceil \frac{t}{T_j} \rceil C_j$$

- The *longest response time* R_i of a job of a periodic task i is computed, at the critical instant, as the sum of its computation time and the interference due to preemption by higher priority tasks: $R_i = C_i + I_i$
- Hence, the schedulability test needs to compute the smallest R_i that satisfies:

$$R_i = C_i + \sum_{j=1}^{i-1} \lceil \frac{R_i}{T_j} C_j,$$

for all tasks i. Then, $R_i \leq D_i$ must hold for all i. It can be shown that this condition is *necessary and sufficient.*

The longest response times R_i of the periodic tasks i can be computed iteratively by the following algorithm:

```
Algorithm: DM_guarantee(Gamma) {
    for (each tau_i in Gamma) {
        I = 0;
        do {
            R = I + C_i;
            if (R > D_i) return (UNSCHEDULABLE);
            I = Sum_{j = 1}^{i - 1} lceil R/T_j rceil C_j;
        } while (I + C_i > R);
    }
    return (SCHEDULABLE);
}
```

6.3.4 EDF Scheduling

Assumptions:

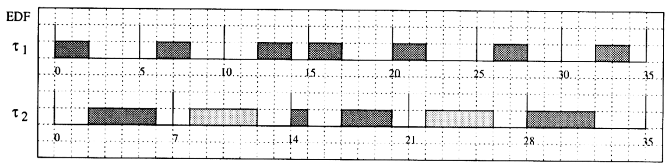
- Dynamic priority assignment
- Intrinsically preemptive

Algorithm: The currently executing task is preempted whenever another periodic instance with earlier deadline becomes active:

$$d_{i,\,j} = \Phi_i + (j-1)T_i + D_i$$

Optimality: No other algorithm can schedule a set of periodic tasks if the set cannot be scheduled by EDF.

Example:



A necessary and sufficient schedulability test for $D_i = T_i$:

A set of periodic tasks is schedulable with EDF if and only if $\sum_{i=1}^{n} \frac{C_i}{T_i} = U \leq 1$.

Here, U denotes the *average processor utilization.*

Remarks: If the deadline was missed at t_2, then define t_1 as a time before t_2 such that (a) the processor is continuously busy in $[t_1,\, t_2]$ and (b) the processor only executes tasks that have their arrival time and their deadline in $[t_1,\, t_2]$. Why does such a time t_1 exist? We find such a t_1 by starting at t_2 and going backwards in time always ensuring that the processor only executes tasks that have their deadline before or at t_2. Because of EDF, the processor will be busy shortly before t_2, and it executes on the task that has deadline after t_2. Suppose that we reach a time such that shortly before the processor works on a task with deadline after t_2 or the processor is idle, then we found t_1: we know that there is no execution on a task with deadline after t_2.

Within the interval $[t_1,\, t_2]$ the total *computation time demanded* by the periodic tasks is bounded by:

$$C_p(t_1,\, t_2) = \sum_{i=1}^{n} \lfloor \frac{t_2 - t_1}{T_i} C_i \leq \sum_{i=1}^{n} \frac{t_2 - t_1}{T_i} C_i = (t_2 - t_1)U$$

Since the deadline at time t_2 is missed, we must have $t_2 - t_1 < C_p(t_1,\, t_2) \leq (t_2 - t_1)U \implies U > 1$, which is *not possible.* This shows that if the utilization satisfies $U > 1$, then there is no valid schedule.

6.4 Real-Time Scheduling of Mixed Task Sets

6.4.1 Introduction

In many applications, there are aperiodic as well as periodic tasks:

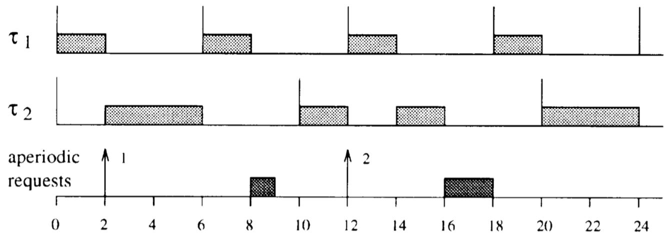
- *Periodic tasks:* time-driven, execute critical control activities with hard timing constraints aimed at guaranteeing regular activation rates.
- *Aperiodic tasks:* event-driven, may have hard, soft, or non-real-time requirements depending on the specific application
- *Sporadic tasks:* Offline guarantee of event-driven aperiodic tasks with critical timing constraints can be done only by making proper assumptions on the environment, that is, by assuming a *maximum arrival rate* for each critical event. Aperiodic tasks characterized by a minimum interarrival time are called *sporadic.*

6.4.2 Background Scheduling

Background scheduling is a simple solution for RM and EDF:

- Processing of aperiodic tasks in the background, i.e. execute if there aren't any pending periodic requests
- periodic tasks are not affected
- Response of aperiodic tasks may be prohibitively long and there is no possibility to assign a higher priority to them

Example with rate monotonic periodic scheduling:

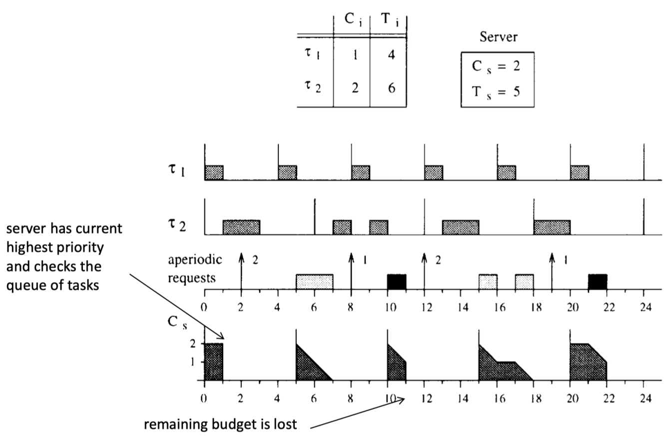


6.4.3 Rate-Monotonic Polling Server

The idea of **rate-monotonic polling servers** is to introduce an artificial periodic task whose purpose it is to service aperiodic requests as soon as possible, therefore we also call it "server". The function of the **polling server (PS)** is as follows:

- At regular intervals equal to T_s, a PS task is instantiated. When it has the highest current priority, it serves any pending aperiodic request within the limit of its capacity C_s.
- If no aperiodic requests are pending, PS suspends itself until the beginning of the next period.
- Its priority (period) can be chosen to match the response time requirement of the aperiodic tasks.

Example of rate-monotonic polling server scheduling:



The *schedulability analysis* of periodic tasks is:

- The interference by a server task is the same as the one introduced by an equivalent periodic task in rate-monotonic fixed-priority scheduling.
- A set of periodic tasks and a server task can be executed within their deadlines if (sufficient but not necessary):

$$\frac{C_s}{T_s} + \sum_{i=1}^{n} \frac{C_i}{T_i} \leq (n+1)(2^{1/(n+1)} - 1)$$

If we want to *guarantee the response time of aperiodic requests* with the assumption, that an aperiodic task is finished before a new aperiodic request arrives (with computation time C_a and deadline D_a), a sufficient schedulability test is:

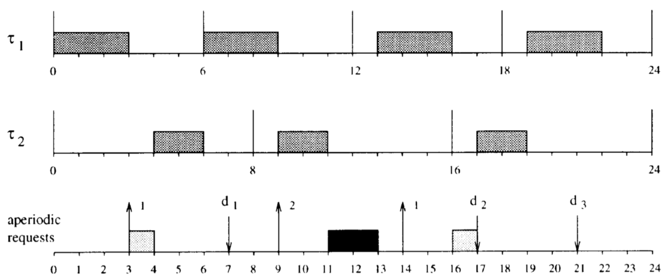
$$(1 + \lceil \frac{C_a}{C_s} \rceil) T_s \leq D_a$$

6.4.4 EDF - Total Bandwidth Server

The idea of an _EDF Total Bandwidth Server** is as follows:

- When the k-th aperiodic request arrives at time $t = r_k$, it receives a deadline $d_k = \max(r_k,\, d_{k-1}) + \frac{C_k}{U_s}$, where C_k is the execution time of the request and U_s is the server utilization factor (that is, its bandwidth). By definition, $d_0 = 0$.
- Once a deadline is assigned, the request is inserted into the ready queue of the system as any other periodic instance.

Example with $U_p = 0.75$, $U_s = 0.25$ and $U_p + U_s = 1$:



The *schedulability analysis* for an EDF total bandwidth server is as follows:

> Given a set of n periodic tasks with processor utilization U_p and a total bandwidth server with utilization U_2, the whole set is schedulable by EDF if and only if $U_p + U_s \leq 1$.

Proof: In each interval of time $[t_1,\, t_2]$, if C_{ape} is the total execution time demanded by an aperiodic request arrived at t_1 or later and served with deadlines less or equal to t_2, then

$$C_{ape} \leq (t_2 - t_1)U_s$$

If this has been proven, the proof of the schedulability test follows closely that of the periodic case.

Chapter 7: Shared Resources

7.1 Resource Sharing

7.1.1 Introduction

Examples of **shared resources** are data structures, variables, main memory area, files, set of registers, etc. Many shared resources do *not allow simultaneous access* but require *mutual exclusion.* These resources are called **exclusive resources.** In this case, no two threads are allowed to operate on the resource at the same time.

There are several methods available to protect exclusive resources, for example:

- Disabling interrupts and preemption, or
- Using concepts like semaphores and mutex that put threads into the blocked state if necessary.