

# EnpRisk - Lecture Notes Week 7

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## 0.0.1 More on Chaos

It can be shown that the logistic map is equivalent to the **tent map**:

$$y(n+1) = 2y(n) \mod 1,$$

with  $x_n = \sin^2(2\pi y_n)$ .

This explains the origin of the chaotic behavior as fundamentally embedded the mathematical properties of the digits of irrational numbers, which are of measure 1 among the real numbers.

Chaos is not random, but due to a *deterministic* map  $x : A \rightarrow B$ , satisfying the following properties:

- $x$  is low dimensional, i.e.  $x$  is only dependent on a "small" number of variables
- $x$  is deterministic, i.e. the next value can always be predicted exactly
- $x$  is sensitive to initial values, i.e. slight changes in the initial value can drastically change the output
- trajectories of  $x$  are reinjected, i.e. although slight differences in initial values can lead to trajectories arbitrarily far apart from each other, there will be a point at which the two trajectories are again arbitrarily close to each other.

## 0.1 The Diffusion of Innovation

### 0.1.1 Introduction

The *diffusion of innovation* describes a game of personal preference versus social trends. It answers questions such as "How does a new technology spread?" and "How are new products adopted by people in society?".

Five different categories of adopters can be defined:

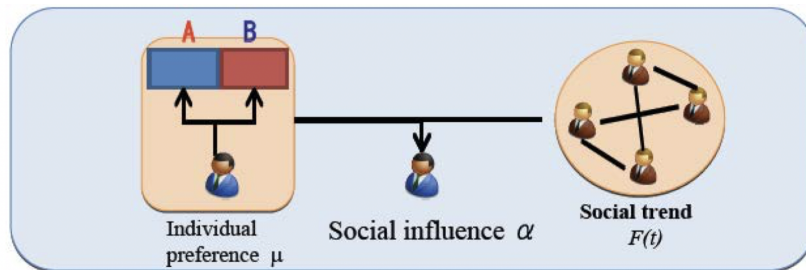
1. Innovators (2.5%) are risk-takers
2. Early adopters (13.5%) are selective
3. Early majority (34%) take their time
4. Late majority (34%) adopt in reaction to peer pressure
5. Laggards (16%) are traditional

The **penetration rate** is like the population size as discussed in Part 1, it follows an S-shaped curve and saturates at 100% when the full population has adopted a new technology. The **penetration speed** is comparable to the production rate, it follows a Gaussian shaped curve.

### 0.1.2 The Agent Based Model

Let us introduce some definitions:

- $F(t)$  is the **penetration rate**, it is the fraction of population that has adopted new technology, it follows an S-shaped curve and is cumulative
- $F(t+1) - F(t)$  is the **penetration rate**, it is the fraction of the population that adopts the new technology in the following time-step, it follows a Gaussian-shaped curve
- $p(t+1)$  gives the probability that an agent will adopt the new technology in the next time-step



**Probability of choosing A at time t**

$$p(t+1) = (1-\alpha)\underset{\text{preference}}{\mu} + \alpha\underset{\text{social trend}}{F(t)} \quad F(t) = A(t) / \{A(t) + B(t)\}$$

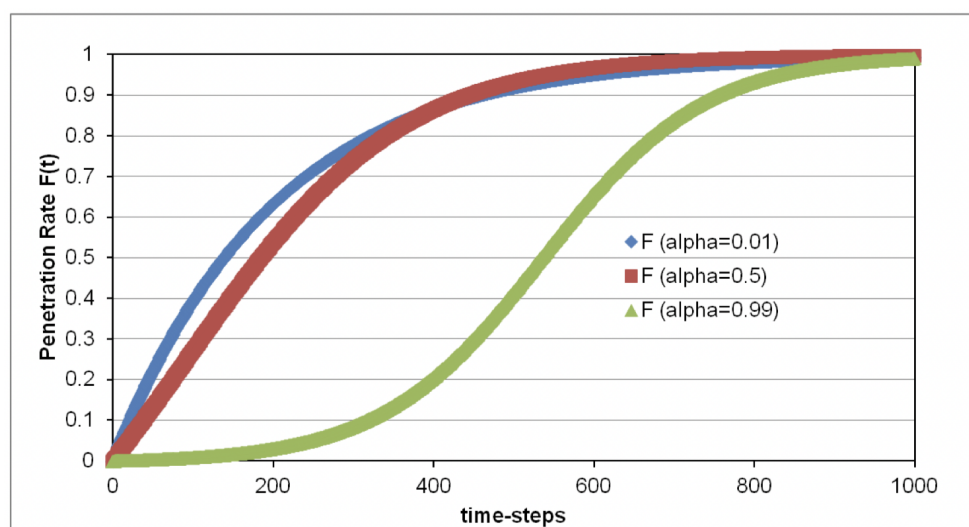
**F(t): the proportion of the agents to choose A**

$\alpha \in [0, 1]$  : social influence factor

More intuitively:

- No personal preference and only social interaction:  $\mu = 0$ ,  $F(t+1) = F(t) + \alpha \cdot F(t) \cdot (1 - F(t))$
- Only personal preference and no social interaction:  $\alpha = 0$ ,  $F(t+1) = F(t) + \mu \cdot (1 - F(t))$

The following figure shows an example with  $\mu = 0.5$ :



## 0.2 Case Study: Valuation of Facebook

The methodology works as follows:

- We model the Monthly Active Users:  $MAU(t)$
- We model the revenues per user:  $r(t)$
- We define a profit margin:  $p$
- *Profits at time:*  $r(t) \cdot MAU(t) \cdot p$
- We define the value of the company as the sum of all the discounted future profits using a discount rate  $d$

In the end, we have:

$$valuation = \sum_{t=1}^{end} \frac{r(t)MAU(t)p}{(1+d)^t} = \sum_{t=1}^{end} \frac{profits(t)}{(1+d)^t}$$

In conclusion, we have developed a new methodology to compute the fundamental value of social networking companies based on the dynamics of their users and revenues per user:

	user's dynamics	rev/user dynamics
Facebook	logistic function	logistic function
Zynga	more complex	more complex

Based on that, we can compute the intrinsic value of social networking companies.

The three *important laws of valuation and investment* are:

- Prediction is no extrapolation, but understanding the underlying process.
- Understand the technicalities of the market and of investment banking.
- Always **RTFM - Read The Fucking Manual!**