Rigorous Software Engineering- Week 4 (Lectures)

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6. Static Anaylsis

We will learn a style called `abstract interpolation`, which is a general theory of how to do approximation systematically.

6.1 Abstract Interpolation

We can define abstract interpolation with the following steps:

1. Select/define an abstract domain: selected based on the type of properties you want to prove
2. Define abstract semantics for the language w.r.t. to the domain: prove sound w.r.t. concrete semantic
3. Iterate abstract transformers over the abstract domain: until we reach a fixed point

The `fixed point` is the *over-approximation* of the program.

6.2 Example Application of Abstract Interpolation

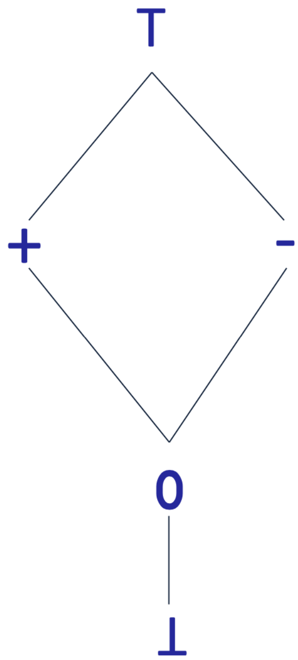
Lets prove an assertion to see how abstract interpolation works. Consider the following code:

```
    foo(int i) {
1:      int x = 5;
2:      int y = 7;

3:      if(i >= 0) {
4:          y = y + 1;
5:          i = i - 1;
6:          goto 3;
        }
7:      assert 0 <= x + y;
    }
```

Step 1: Select abstraction

Lets pick the `sign` abstraction, given as follows:



- $\top$: stands for all possible values
- $-$: stands for the negative values (≤ 0)
- $+$: stands for the positive values (≥ 0)
- 0: stands for zero
- $\perp$: unreachable numbers (for now)

An `abstract` program state is a map from variables to elements in the domain. Example:

pc	x	y	i
2	+	$\perp$	$\top$

We see that at pc (program counter) 2, i.e. right before the execution of the second line, x is positive, y is bottom, i.e. not yet defined, and i is either positive or negative.

Step 2: Define Transformers

An `abstract transformer` describes the effect of statement and expression evaluation on an abstract state.

It is important to remember that abstract transformers are defined per `programming language` once and for all, and not per-program!

This means thath any program in the programming language can use the same transformers.

A `correct abstract transformer` should always produce results that are superset of what a concrete transformer would produce.

Example of a sound transformer:

pc	x	y	i
4	$\top$	-	$\top$

$\Rightarrow$ `y := y + 1;` $\Rightarrow$

pc	x	y	i
5	$\top$	$\top$	$\top$

It is easy to be `sound` and `imprecise`, simply output $\top$. It is desirable though to be both `sound` and `precise`.

Step 3: Iterate to a fixed point

To start the analysis we start to iterate with the least abstract element. This means that initially, all 7 states in our code example from above look like:

pc	x	y	i
1-7	$\perp$	$\perp$	$\perp$

After executing some iterations we will see that we get a state which doesn't change anymore, a so called `fixed point`:

pc	x	y	i
1	$\perp$	$\perp$	$\top$

pc	x	y	i
2	+	$\perp$	$\top$

pc	x	y	i
3	+	+	$\top$

pc	x	y	i
4	+	+	+

pc	x	y	i
5	+	+	+

pc	x	y	i
6	+	+	$\top$

pc	x	y	i
7	+	+	-

Step 4: Check property

As we can see, our property we're trying to prove is $(0 \leq x + y)$. As both x and y are positive in pc 7, we have shown via abstract interpolation, that our assertion holds.

6.3 More on Abstract Interpolation

Joins

When we have two abstract elements A and B, we can `join` them to produce their least upper bound, denoted by $A \sqcup B$.

We then have that $A \sqsubseteq A \sqcup B$ and $B \sqsubseteq A \sqcup B$, where $D \sqsubseteq E$ means that E is `more abstract` than D.

Widening

With the interval abstraction we might not reach a fixed point. We therefore introduce a special operator called `widening operator`. It ensures termination at the expense of precision.

Whenever we want to join two states and we see that one variable is increasing, we directly go to infinity.

Example: Instead of joining $[7,\ 7]$ with $[8,\ 8]$ to $[7,\ 8]$, we widen it to $[7,\ \infty]$.