

# Explanation of the Multiphoton Fluorescence Calculation

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## 1 Multiphoton Fluorescence Calculation

For an n-photon process, fluorescence flux per pulse is described as follows according to [1]

$$F \propto \sigma_n \times I_{peak}^n \times \tau \quad (1)$$

For an n-photon process, fluorescence flux per second is described as follows

$$F \propto \sigma_n \times I_{peak}^n \times \tau \times R \quad (2)$$

Where  $\sigma_n$  is the cross section for the n-photon process,  $I_{peak}$  is the peak intensity,  $\tau$  is the pulse width, and  $R$  is the repetition rates.

The peak power of a pulse is dependent on the temporal shape of a laser pulse. For a soliton pulse (*sech*<sup>2</sup> shape), the peak power can be determined by the following equation.

$$P_{peak} = 0.88 \times E_p / \tau \quad (3)$$

For a gaussian pulse shape, the constant factor is 0.94 instead of 0.88. We will assume the pulse is soliton shape in our following calculations, which is consistent with the laser pulses coming out of the Coherent Chameleon laser.

The conversion of the peak power to the peak intensity is a little bit tricky. For a laser beam with Gaussian profile, The peak intensity on the beam axis is

$$I_{peak} = \frac{P_{peak}}{\pi \times w^2/2} \quad (4)$$

The pulse energy  $E_p$  can be easily determined by

$$E_p = P_{avg}/R \quad (5)$$

So, we will have

$$P_{peak} = \frac{0.88 \times P_{avg}}{R \times \tau} \quad (6)$$

Combining all the above, we will have,

$$F = \sigma_n \times \left( \frac{1.76 \times P_{avg}}{\pi R \tau w^2} \right)^n \tau R \quad (7)$$

Please note that the  $\sigma_n$  is in unit of photons. As such, we need to covert the laser energy within one second to photons by dividing  $P_{avg}$  with  $hc/\lambda$ , where h is the planck constant, c is the speed of light and  $\lambda$  is the excitation wavelength.

The calculation then becomes

$$F = \sigma_n \times \left( \frac{1.76 \times P_{avg} \times \lambda}{\pi R \tau w^2 \times hc} \right)^n \tau R \quad (8)$$

## 2 Example Calculations I - Two Photon Fluorescence

## 3 Example Calculations II - Three Photon Fluorescence

## 4 Example Calculations III - Four Photon Fluorescence

## References

- [1] Xu, C., Zipfel, W., Shear, J. B., Williams, R. M., Webb, W. W. (1996). Proceedings of the National Academy of Sciences, 93(20), 10763-10768.