Global quantum thermometry

Drawing optimal estimates for temperature from arbitrary datasets

Jesús Rubio

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Key work

arXiv:2011.13018

Theory seminar
University of Lancaster

2nd Sep 2021

Global Quantum Thermometry

Oct
2020

Optimal Probes for Global Quantum Thermometry

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(Dated: October 28, 2020)

Global Quantum Thermometry

(This talk)

Global Quantum Thermometry

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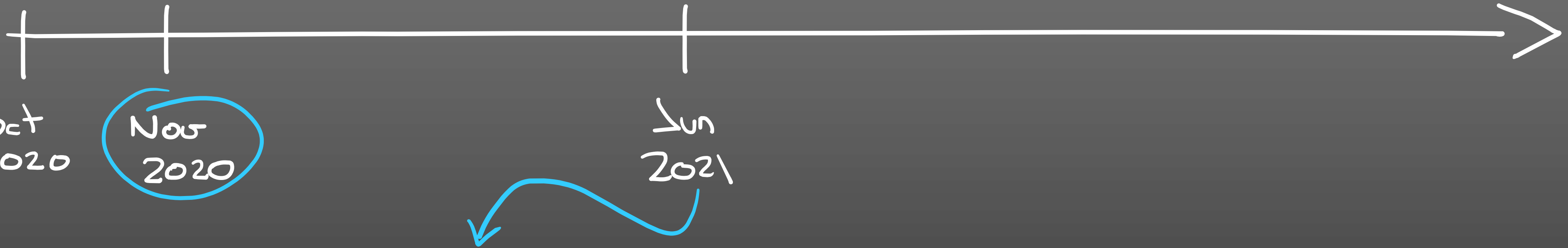
(Dated: 30th November 2020)

~~Global~~ Bayesian Quantum Thermometry



Bayesian estimation for collisional thermometry

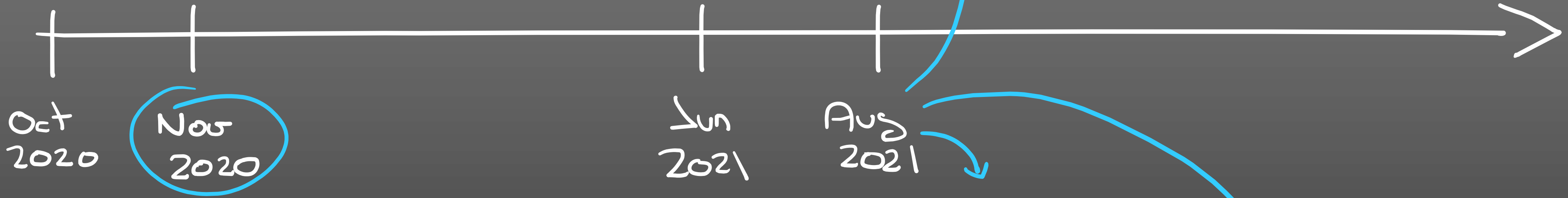
Gabriel O. Alves[1,*] and Gabriel T. Landi[1,†]

[1]*Instituto de Física da Universidade de São Paulo, 05314-970 São Paulo, Brazil.*

(Dated: June 24, 2021)

~~Global~~ Quantum Thermometry

Bayesian



Non-informative Bayesian Quantum Thermometry

Julia Boeyens,[1] Stella Seah,[2] and Stefan Nimmrichter[1]

[1] *Naturwissenschaftlich-Technische Fakultät, Universität Siegen, Siegen 57068, Germany*
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(Dated: August 18, 2021)

Fundamental limits in Bayesian thermometry and attainability via adaptive strategies

Mohammad Mehboudi,[1,*] Mathias R. Jørgensen,[2,†] Stella Seah,[1]
Jonatan B. Brask,[2] Jan Kołodyński,[3] and Martí Perarnau-Llobet[1,‡]

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Bayesian quantum thermometry based on thermodynamic length

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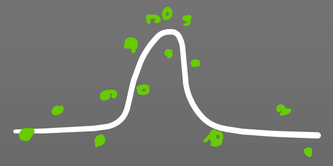
(Dated: August 19, 2021)

Our plan for today:

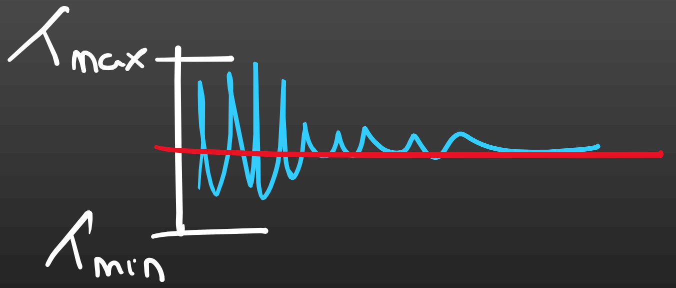
A tale of three thermometries:

① The experimental →



② The local →

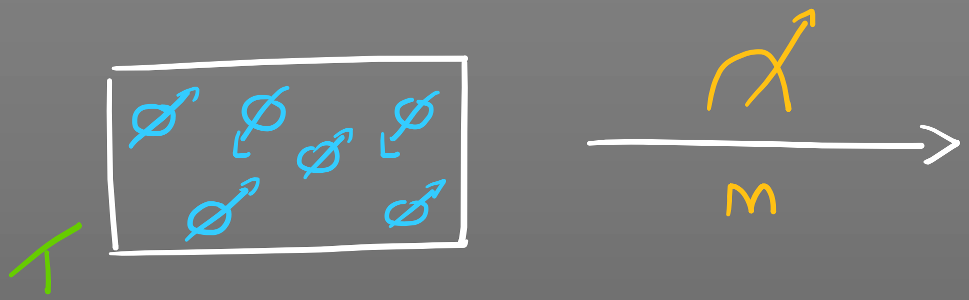
③ The global (a.k.a. the Bayesian) →



Data analysis in <u>experimental</u> <u>thermometry</u>:

essential aspects

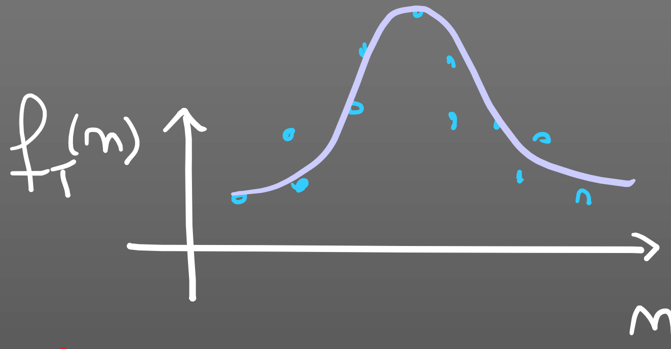
Formulation of the problem



$\vec{m} = (m_1, m_2, \dots, m_M)$

- Thermal equilibrium
- Any measurement:

  $E, x, p, \dots$



T-dependent model for distribution of measurements

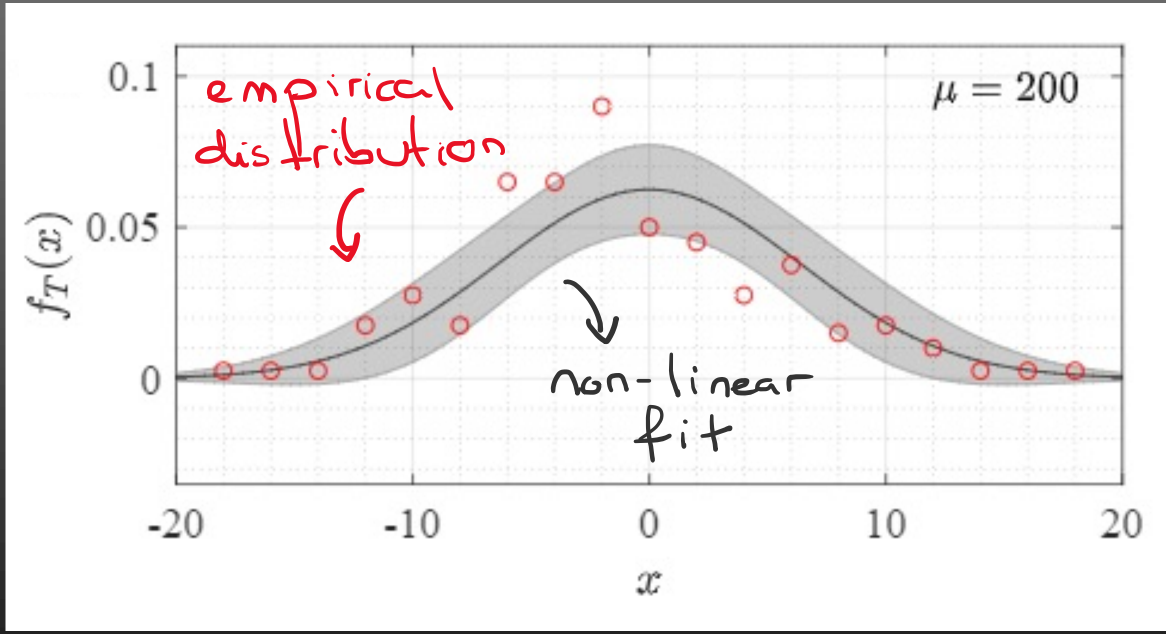
$f_T(m)$

$\longrightarrow$ Non-linear fit to empirical distribution to retrieve T

An example: position measurements [SIMULATION]

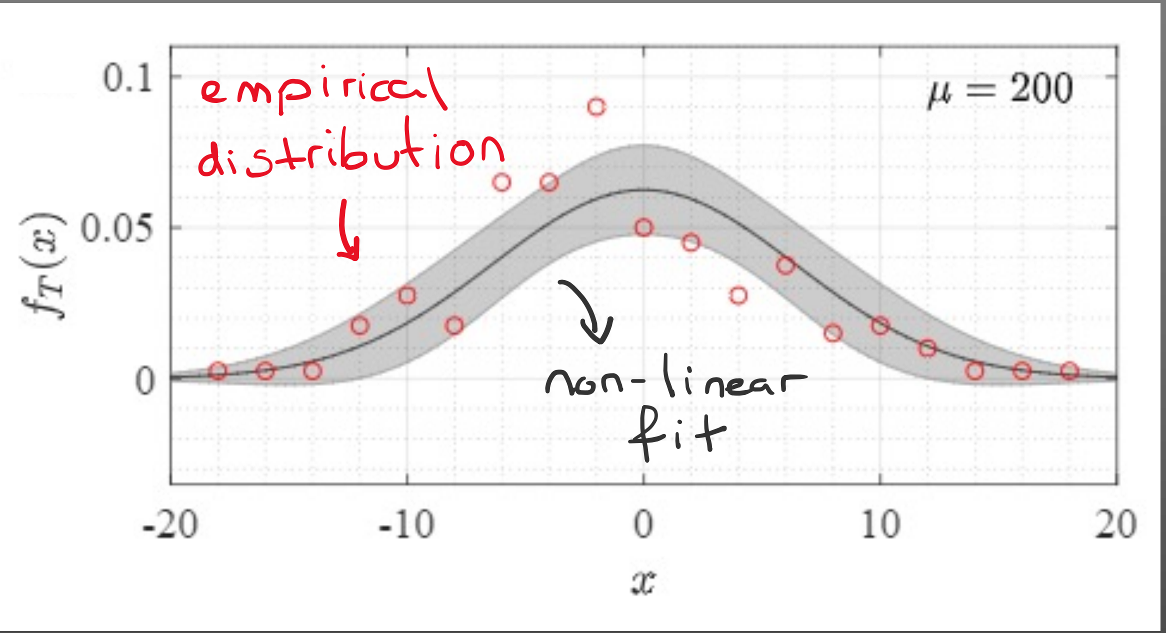
- **Model** ($\omega_0 = 1$ n.u., n.u. = natural units $\rightarrow k_B = 1$, etc)

$$f_T(x) = A\, e^{-\frac{x^2}{2\sigma^2}}, \quad \text{with } \sigma^2 = \frac{1}{2}\coth\left(\frac{1}{2T}\right)$$



$\rightarrow$ $\boxed{T = 6.4 \pm 1.7 \text{ n.u.}}$

($T_{true} = 6$ n.u.)



However, note that the histogram needs be reliable:

* Choice of bins?
* Sample size μ?

These can be bypassed

In search of fundamental limits to the precion in thermometry: experimental design within the local framework

* Precision-enhancement in quantum thermometry

- $f_T(m) \longleftrightarrow p(m|T)$ likelihood function [links m and T]

- In quantum systems:

$$p(m|T) = \mathrm{Tr}[\Pi(m)\,\rho(T)]$$

quantum state; in eq. thermometry

$$\rho \propto \sum_{\epsilon} \exp\left(-\frac{\epsilon}{k_B T}\right)|\epsilon\rangle\langle\epsilon|$$

measurement scheme $\begin{cases} \sum_m \Pi(m) = \mathbb{I} \\ \Pi(m) \geq 0, \ \forall m \end{cases}$

M. Mehboudi, A. Sanpera, and L. A. Correa, Thermometry in the quantum regime: recent theoretical progress, Journal of Physics A: Mathematical and Theoretical **52**, 303001 (2019).

- Using $p(m|T)$, we seek an estimator : $\vec{m} \longmapsto \tilde{T}(\vec{m})$
- We want $\tilde{T}(\vec{m})$ to be "good":

$$\Delta \tilde{T}^2 \geq \frac{1}{\mu F(T)} \geq \frac{1}{\mu F_q(T)}$$

M. Mehboudi, A. Sanpera, and L. A. Correa, Thermometry in the quantum regime: recent theoretical progress, Journal of Physics A: Mathematical and Theoretical **52**, 303001 (2019).

Fisher info. (FI):

$$F(T) = \int \frac{dm}{p(m|T)} \left[\frac{\partial p(m|T)}{\partial T} \right]^2$$

Quantum FI:

$$F_q(T) = \mathrm{Tr}\left[\rho(T) L(T)\right]$$

depends on $\rho(T)$ + plus informs optimal measurements

- Using $\Delta \tilde{T}^2 \geq \frac{1}{\mu F_q(T)}$, we can optimise:
  - → Energy spectrum (state)

$$\rho \propto \sum_{\epsilon} \exp\left(-\frac{\epsilon}{k_B T}\right)|\epsilon\rangle\langle\epsilon|$$

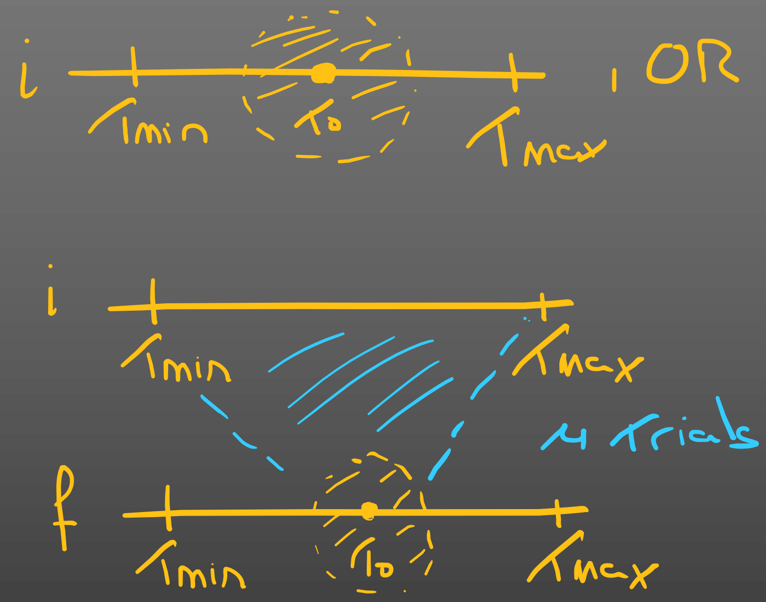
  - → Measured quantities (via $F_q(T) = \mathrm{Tr}[\rho(T) L(T)]$)

- With the optimal $p(\vec{m}|T)$, find the optimal $\tilde{T}(\vec{m})$

⚠ However:

→ Exponential family + unbiasedness, OR

→



M. Mehboudi, A. Sanpera, and L. A. Correa, Thermometry in the quantum regime: recent theoretical progress, Journal of Physics A: Mathematical and Theoretical **52**, 303001 (2019).

R. Demkowicz-Dobrzański, M. Jarzyna, and J. Kołodyński, Quantum Limits in Optical Interferometry, Progress in Optics **60**, 345 (2015).

☒ Using $\Delta \tilde{T}^2 \geq \frac{1}{\mu F_q(T)}$, we can optimise:

→ Energy spectrum (state)

$$\rho \propto \sum_{\epsilon} \exp\left(-\frac{\epsilon}{k_B T}\right)|\epsilon\rangle\langle\epsilon|$$

→ Measured quanties

(via $F_q(T) = \mathrm{Tr}\left[\rho(T) L(T)\right]$)

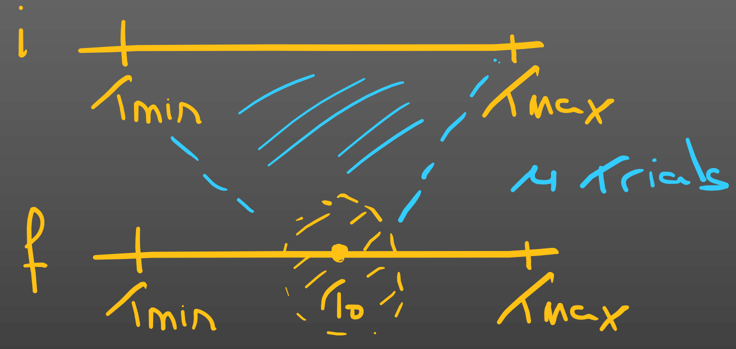
☒ With the optimal $p(\vec{m}|T)$, find the optimal $\tilde{T}(\vec{m})$

⚠ However:

→ Exponential family + unbiasedness, OR

→ 

, OR

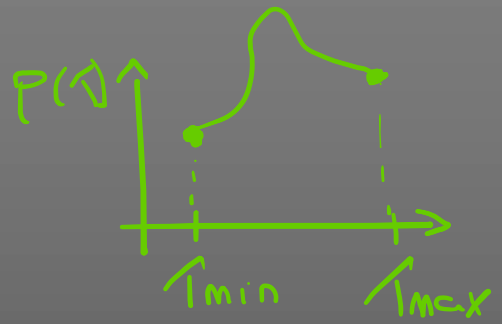
→ 

⇒ LOCAL thermometry

Probability theory à la Bayes
meets thermometry

Basic notions

- Link between $\vec{m}$ and τ $\longrightarrow$ $p(\vec{m}|\tau)$
- prior information $\longrightarrow$ $p(\tau)$



- Physically-motivated criterion of performance:

$$\bar{\epsilon} := \int d\vec{m}\, d\theta\, p(\theta)\, p(\vec{m}|\theta)\, D[\tilde{\theta}(\vec{m}), \theta]$$

hypothesis about τ

- Our task:

$\tilde{\theta}(m)$ that minimises $\bar{\epsilon}$

→ How do we choose the prior and the deviation function?

$\boxed{/\!/\!/}$ We only assume: scale invariance

$$\begin{cases} \epsilon \longrightarrow \epsilon' = \gamma \epsilon \\ (k_B T) \longrightarrow (k_B T') = \gamma (k_B T) \end{cases} \Rightarrow$$

Ratios $\frac{\epsilon}{k_B T}$ unchanged!

① $\Rightarrow$ $p(\lambda) \propto \frac{1}{T}$

R. E. Kass and L. Wasserman, The Selection of Prior Distributions by Formal Rules, Journal of the American Statistical Association **91**, 1343 (1996).
U. von Toussaint, Bayesian inference in physics, Reviews of Modern Physics **83**, 943 (2011).

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② $\Rightarrow$ $\bar{\epsilon}_{mle} = \int d\vec{m} \, \frac{d\theta}{\theta} \, p(\vec{m}|\theta) \log^2\left[\frac{\tilde{\theta}(\vec{m})}{\theta}\right]$

General minimisation (not just bounds or fits!)

① Physically-sound figure of merit

$$\bar{\epsilon}_{\mathrm{mle}} = \int dE\, d\theta\, p(E,\theta) \log^2\left[\frac{\tilde{\theta}(E)}{\theta}\right]$$

② Calculus of variations

$$\delta\epsilon[\tilde{\theta}(E)] = \delta \int dE\, \mathcal{L}[\tilde{\theta}(E), E] = 0$$

③ The solution:

$$\frac{k_B\tilde{\vartheta}(E)}{\varepsilon_0} = \exp\left[\int d\theta\, p(\theta|E) \log\left(\frac{k_B\theta}{\varepsilon_0}\right)\right]$$

④.a Fundamental limits:

$$\bar{\epsilon}_{\mathrm{opt}} = \bar{\epsilon}_{\mathrm{p}} - \mathcal{K}$$

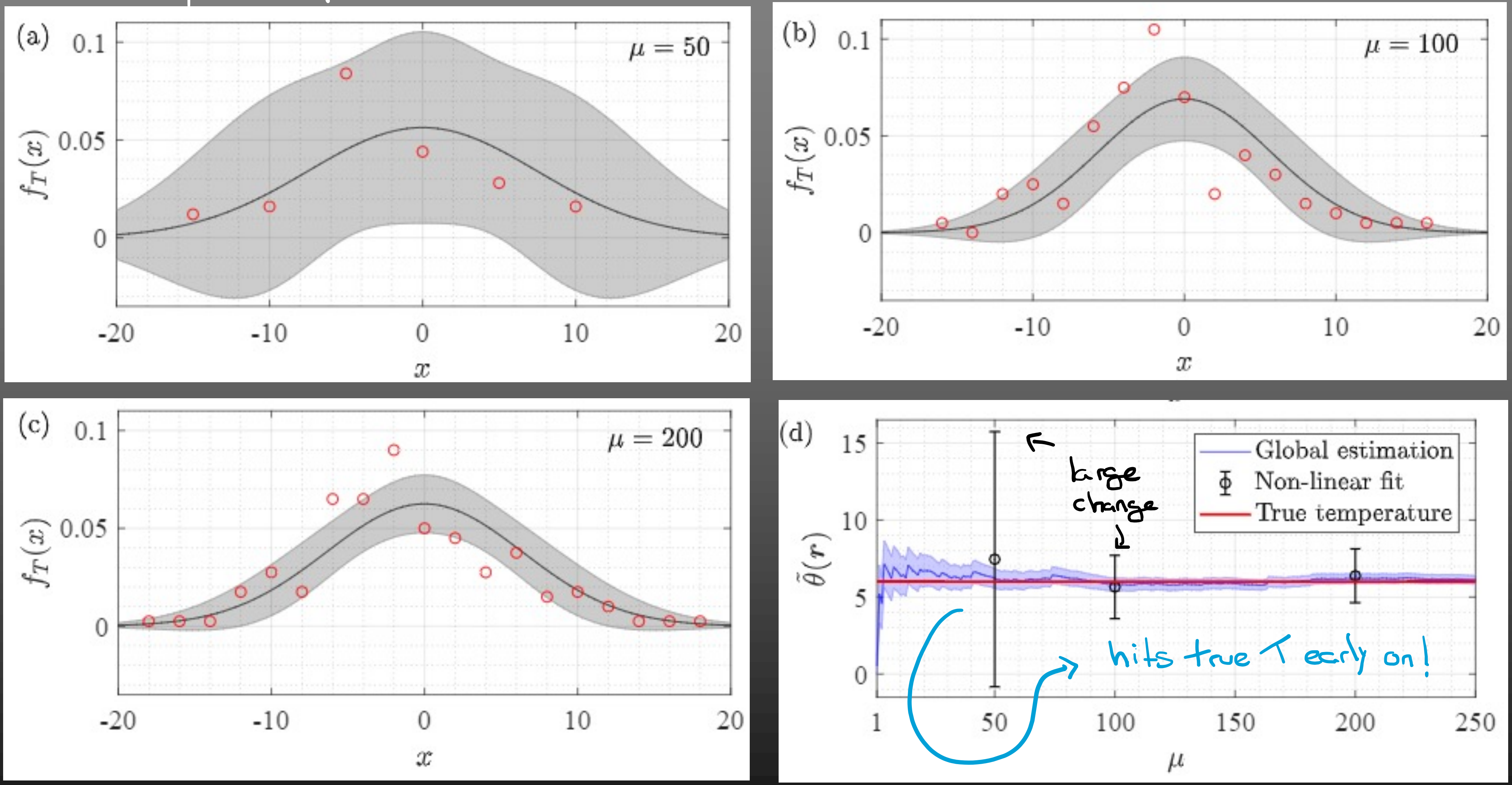
$$\bar{\epsilon}_{\mathrm{p}} := \int d\theta\, p(\theta) \log^2(\tilde{\vartheta}_{\mathrm{p}}/\theta)$$

$$\mathcal{K} := \int dE\, p(E) \log^2\left[\frac{\tilde{\vartheta}(E)}{\tilde{\vartheta}_p}\right]$$

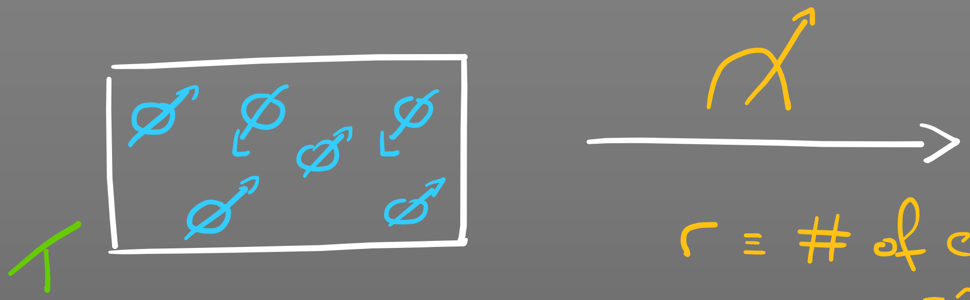
④.b In experiments:

$$\Delta\tilde{\Theta}(\vec{m})^2 = \tilde{\Theta}(m)^2 \int d\Theta\, p(\Theta|\vec{m}) \log^2\left[\frac{\tilde{\Theta}(\vec{m})}{\Theta}\right]$$

* Example: position thermometry



* Example: gas of n $1/2$-spin particles in thermal equilibrium



$r \equiv$ # of excited spins

$\vec{r} = (r_1, r_2, \dots, r_\mu)$

$\mu \equiv$ number of trials

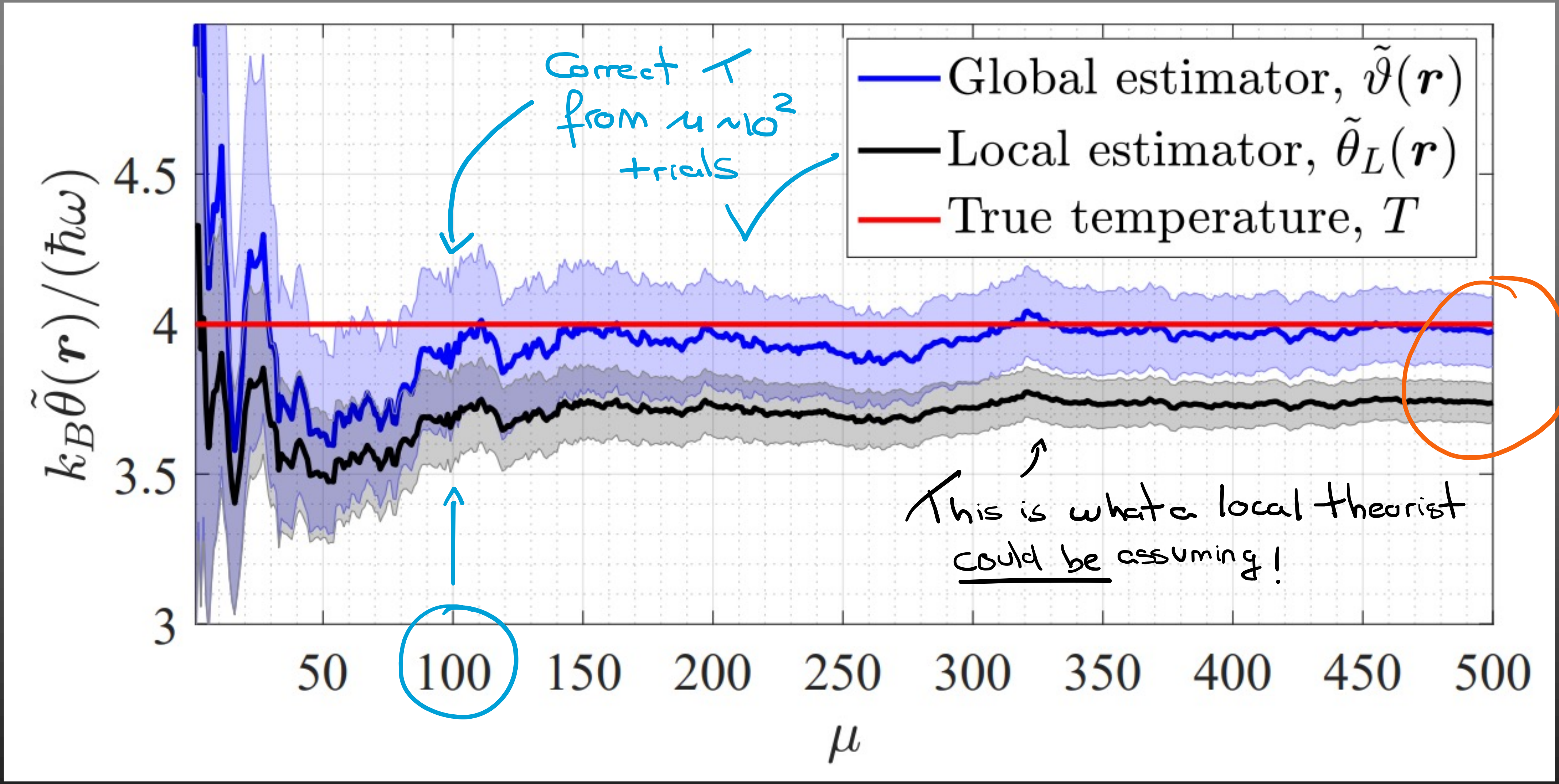
- Prior info: $\frac{k_B T}{\hbar\omega}$ lies on: ?

$\frac{k_B T_{min}}{\hbar\omega} = 0.1$, $\frac{k_B T_{max}}{\hbar\omega} = 10$, $p(T) \propto \frac{1}{T}$ in such range

- Physics:

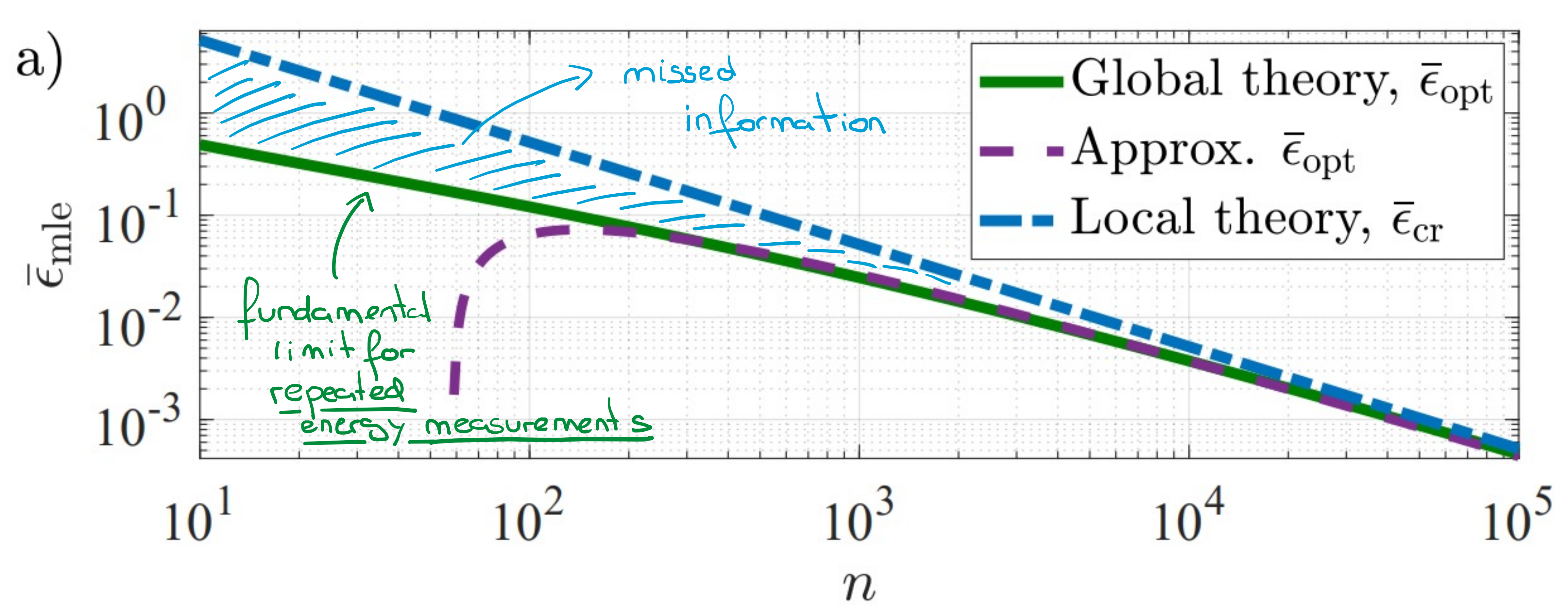
$$p(r|\theta) = \binom{n}{r} \frac{\exp[-r\hbar\omega/(k_B\theta)]}{Z[\hbar\omega/(k_B\theta)]}$$

→ [SIMULATION OF] Experimental estimation



The local approach is biased after all

→ Fundamental limits to precision in thermometry

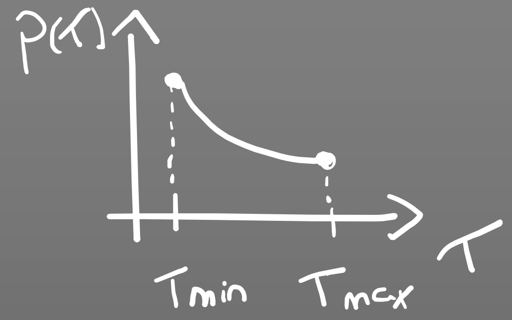


other examples →

Quantum metrology in the presence of limited data

To cite this article: Jesús Rubio and Jacob Dunningham 2019 *New J. Phys.* **21** 043037

* Example: The importance of acknowledging scale invariance



- Prior info.:

1

$T_{min} = 0.01\,K$ $\qquad$ $T_{max} = 100\,K$

2 orders of magnitude $\qquad$ 2 orders of magnitude

- Naive approach:

$$p(T) = \frac{1}{T_{max} - T_{min}} \longrightarrow \langle T \rangle = \int dT\, p(T)\, T \simeq 50\,K$$

- Global thermometry:

$$p(T) = \frac{1}{T \log\left(\frac{T_{max}}{T_{min}}\right)} \longrightarrow \tilde{T} = \exp\left[\int dT\, p(T) \log(T)\right] = 1\,K \quad \checkmark$$

Why should we care?

→ Reduce # of measurements needed to hit a given precision (smarter assignment of resources).

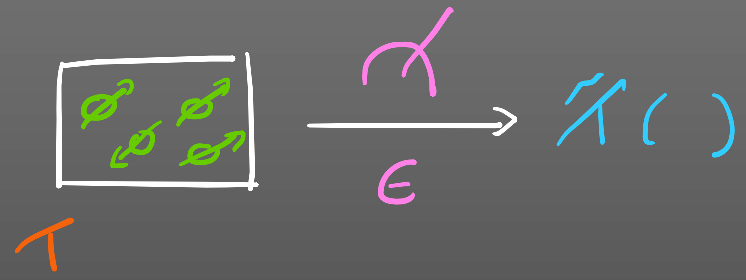
→ Universality and consistency; in general, a much more transparent estimation framework with minimal assumptions

→ A more natural and satisfactory dialogue between theory and experiments.

Take-home message:

→ Global (quantum) thermometry can be expected to be either as good as traditional methods or better, particularly in scenarios with limited data.

Thank you for your attention!



To learn more:

→ arXiv: 2011.13018

→ J.Rubio-Jimenez@exeter.ac.uk